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Expectations Formation in Macroeconomic Agent-based Models

DOCTORAL THESIS

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Chapter 1

Introduction & Summary

In almost any economic theory or paradigm, the expectations of agents regarding the future are crucial in determining what happens in the present. This is particularly obvious in modern mainstream macroeconomics; in the New Keynesian model, for instance, the current values of the output gap and inflation are chiefly determined by their respective expected values (Gali, 2015, Ch. 3). Expectations are typically held to be ‘rational’, but a great deal of work has focussed on the consequences of relaxing this assumption (Evans and Honkapohja, 2001; Massaro, 2012; Hommes, 2013). Turning to less conventional approaches, (post-) Keynesian macroeconomics builds directly on Keynes (1936) whose work in many ways centres on expectations formation, be it the importance of ‘animal spirits’ and ‘the state of long term expectation’ in determining investment under conditions of fundamental uncertainty, or the view of stock markets as a ‘beauty contest’. The work of Hyman Minsky, which is the theme of chapter 3, draws extensively on this Keynesian view of the role of expectations and as demonstrated by Lavoie (2014, Ch. 2), this is also an important feature of post-Keynesian economics more generally. At the other end of the spectrum, the importance of expectations is also emphasised in Austrian economics (Holcombe, 2014), particularly its Hayekian strand (Butos and Koppl, 1993) which is regarded as one of the foundational paradigms underlying complexity economics (Wible, 2000) and is centrally concerned with the role of markets as ‘information processing systems’ (Bowles et al., 2017). Institutional and Evolutionary economics both draw on the work of Herbert Simon (Hodgson, 2007) which lays the foundation for depictions of bounded rational behaviour (Simon, 1982), including in the formation of expectations. Institutional economics emphasises
reflexive, mutually sustaining relationships between institutions and expectations (Hodgson, 1998), while Evolutionary economics is concerned with how expectations may adapt and evolve over time, and why a system may or may not settle on some particular expectation eventually (Arthur, 2006). And finally, despite its strong materialist orientation and its focus on class dynamics and systemic contradictions of capitalism, a role for expectations can also be discerned in Marxian economics, particularly when considering business cycle theory and the key role of (expected) profitability (Sherman, 1979; Reuten, 2002; Shaikh, 2016, Ch. 13).

As the title would suggest, expectations formation, particularly under bounded rationality, is also the unifying theme of the papers comprising this thesis. Moreover, the chief focus of the thesis is on the analysis of short-run, business cycle dynamics, and particular attention is paid to the influence of various policy tools thereupon. Methodologically, all three papers are similar in that all three are based on the analysis of computational simulation models, all of which make use of an agent-based approach. The models presented in chapters 2 and 3 are purpose-built to address the respective research questions while chapter 4 takes a widely used existing framework as its point of departure. Chapters 2 and 3 in particular also emphasise the concept of stock-flow consistency. As detailed in the literature review contained in chapter 2, there has in recent years been an increasing trend of merging the agent-based and stock-flow consistent approaches to computational macroeconomics, including through the construction of ‘hybrid’ models such as those presented in chapters 2 and 3. These chapters provide good examples of the complementarities between the agent-based and stock-flow consistent approaches, with the former being able to provide a microeconomic dimension for otherwise highly aggregative models, while the latter represents an important disciplining device and consistency check in the construction of macroeconomic models. Given the paradigmatic origin of these two approaches to macroeconomic modelling, and in view of many of the behavioural assumptions underlying the presented models, the thesis is most closely related to the post-Keynesian and evolutionary approaches to economics.

As discussed in the literature review contained in chapter 2, research on the broadly defined concept of bounded rationality is a long-standing component of the macroeconomic literature and indeed, bounded rational behaviour may be regarded as one of the key characteristics of
agent-based models as these typically depict complex systems which inherently preclude the incorporation of rational behaviour as commonly defined. Nevertheless, detailed research specifically concerned with bounded rational expectations formation in macroeconomic agent-based models is a relatively recent addition to the literature. The thesis makes contributions on this front through applying canonical approaches to the modelling of bounded rational expectations - namely heterogeneous expectations and strategy switching behaviour (Hommes, 2013; Franke and Westerhoff, 2017) as well as least squares learning (Evans and Honkapohja, 2001) - to the analysed models. Each individual chapter places a particular focus on the analysis of heterogeneity and expectations formation in one particular economic sector; banks in the case of chapter 2, firms in chapter 3 and households in chapter 4. In so doing, the three chapters jointly offer a rich analysis of the diverse roles played by the expectations formation of individual agents in giving rise to macroeconomic dynamics. In all cases, the goal is to contribute to a better understanding of the importance of expectations in a class of macroeconomic models which has thus far chiefly relied on a limited range of fairly simplistic expectations formation heuristics.

Chapter 2 presents a hybrid agent-based stock-flow-consistent model featuring heterogeneous banks, purposely built to examine the effects of variations in banks’ individual expectations formation and forecasting behaviour on macroeconomic dynamics and to conduct policy experiments with a focus on monetary and prudential policy. The model is initialised to a deterministic stationary state and a subset of its free parameters are calibrated empirically in order to reproduce characteristics of UK macro-time-series data. Experiments carried out on the baseline focus on the expectations formation and forecasting behaviour of banks through allowing banks to switch between forecasting strategies and having them engage in least squares learning. Overall, simple heuristics turn out to be remarkably robust in the model. In the baseline, which represents a relatively stable environment, the use of arguably more sophisticated expectations formation mechanisms makes little difference to simulation results in terms of aggregate outcomes. In a modified version of the baseline representing a less stable environment alternative heuristics may in fact be destabilising. To conclude the paper, a range of policy experiments is conducted, showing that an appropriate mix of monetary and prudential policy can consid-
ably attenuate the macroeconomic volatility produced by the model and also tame the instability which alternative expectations formation mechanisms can give rise to.

**Chapter 3** presents a fully formalised version of Hyman Minsky’s two-price model of capital investment embedded in a macroeconomic model consisting of an agent-based sector of consumption goods firms as well as three strongly simplified aggregated sectors. In an innovation to the literature on Minsky models, the model is calibrated empirically using moments drawn from US data, demonstrating that it is capable of producing plausible time series. Simulations show that the individual investment and financing choices made by the agent-based firms lead to the emergence of business cycles at the aggregate level. Key aspects of the model can be closely related to central concepts from the financial accelerator literature and moreover, it is demonstrated through simulation experiments that expectations and sentiment dynamics play an important role, being able to amplify or dampen the cycles. It is also shown that the introduction of fiscal policy, monetary policy, or a restriction on firm dividend payouts can contribute to a stabilisation of the model.

**Chapter 4** uses a macroeconomic agent-based model building on Delli Gatti et al. (2011) to investigate the influence of agents’ expectations and consumption choices on government expenditure multipliers. Following a thorough investigation of the size of the multiplier in the pre-existing baseline model, a modification is introduced, allowing agents to engage in intertemporal optimisation of consumption subject to a budget constraint which is based on estimates of future income. It is found that this alternative consumption behaviour leads both to an increase in welfare derived from consumption and to a considerable reduction in macroeconomic fluctuations. Compared to the baseline, the fiscal multiplier is strongly affected by this alternative consumption behaviour, becoming significantly smaller. Nevertheless, expansionary government expenditure shocks are welfare-improving in both cases. In a further step, agents’ beliefs about the effects of government expenditure shocks on future income are explicitly introduced. In the case of exogenously imposed beliefs coupled either with adaptation of individual beliefs or switching behaviour between different types of beliefs, it is shown that both optimistic and pessimistic expectations can be temporarily self-fulfilling and either increase or decrease the value of the multiplier. Both forms of belief dynamics also allow for the incorporation of an-
ouncement effects of fiscal policy. In a final experiment, agents are allowed to engage in least- squares learning in order to gain an estimate of the effect of government expenditure shocks on future income. It is shown that under least squares learning, beliefs are ‘rational’ insofar as they lead to broadly correct predictions on average. The paper hence contributes to addressing aspects of the Lucas critique as applied to macro-ABMs, since agents react systematically (and reasonably) to announcements of changes in fiscal policy.

While there is no strong overarching conclusion uniting all three papers, all of them emphasise the potentially strong influence of agents’ expectations on macroeconomic volatility and show that depending on the specification, expectations can be both a stabilising and destabilising factor. This means that moving forward, it is worth paying increased attention to the depiction of expectations formation in agent-based models. Moreover, through the wide range of policy experiments conducted, all three papers serve to emphasise the important role of stabilisation policies in systems exhibiting endogenous fluctuations and chapter 4 in particular highlights the potential dependence of policy effectiveness on expectations. At the same time, some of the obtained results serve as a caution that in complex systems, policy interventions must be carefully calibrated lest they themselves become a source of instability.

Each individual chapter of this thesis is self-contained with its own introduction, literature review and conclusion and can hence be read independently. All referenced works appear in a common bibliography at the end of the thesis.
Chapter 2

Heterogeneous expectations, forecasting behaviour and policy experiments in a hybrid agent-based stock-flow-consistent model

2.1 Introduction

This paper presents a hybrid agent-based stock-flow consistent (AB-SFC) macro-model with an agent-based banking sector. Its purpose is to investigate the effects of various assumptions concerning banks’ expectations formation and forecasting behaviour and, along the policy dimension, the impacts of monetary policy and prudential regulation. The hybrid model is constructed by fusing a macroeconomic stock-flow consistent model featuring households, firms, a government and a central bank with an agent-based banking sector which interacts with the aggregate portions of the model.

The model is initialised to a deterministic stationary state using UK data as a rough guide to give rise to realistic initial values, whereby the aggregate stock-flow consistent structure is utilised to reduce the degrees of freedom. A subset of the remaining free parameters is then calibrated empirically using the method of simulated moments, utilising a set of statistics calculated from UK macro time-series data, with the result that the model can reproduce these quite closely. Following validation exercises and a presentation of the dynamics produced by the baseline simulation

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1This chapter was originally published as Reissl, S. (2020), “Heterogeneous expectations, forecasting behaviour and policy experiments in a hybrid agent-based stock-flow-consistent model”, Journal of Evolutionary Economics, advance access.
thus obtained, I carry out two sets of experiments. Firstly I experiment with allowing banks to use a variety of forecasting heuristics in their expectations formation and decision-making, including heterogeneous expectations with heuristic switching and OLS-learning. The result is that these changes produce little to no difference in the overall dynamics when implemented in the baseline model, which provides a relatively stable environment. More sophisticated heuristics do not appear able to significantly outperform simpler ones, giving rise to very similar simulation results. When implemented in a modified version of the baseline model representing a less stable environment, however, it turns out that varying the expectations formation mechanism of banks can have a strongly destabilising impact. In addition, it is shown that when banks use an alternative, arguably more sophisticated heuristic for setting their interest rates, this can produce inferior outcomes for them. These results are in line with existing research and the concept of ‘ecological rationality’ which emphasises that the fitness of behavioural rules is highly context-dependent. The second set of experiments concerns the implementation of various stabilisation policies. It is shown that an appropriate mix of monetary and prudential policy can considerably reduce the macroeconomic fluctuations present in the baseline simulation, a result which is strengthened when fiscal policy is added to the policy mix. A different policy mix is necessary, however, to contain the instability triggered by alternative expectations formation heuristics in the modified version of the model. Along the policy dimension, the paper hence makes a case for concerted action incorporating a range of different tools and highlights the possible dependence of policy effectiveness on expectations formation mechanisms used by agents.

The paper contributes to research on hybrid AB-SFC models in that the particular focus on bank heterogeneity is novel to the literature. Moreover, the literature examining expectations formation in macroeconomic agent-based models in detail is at present still underdeveloped. Finally, the paper contributes to the increasing empirical orientation of the AB-SFC literature through the application of an empirical calibration algorithm to the presented model.

The paper is structured as follows: Section 2.2 gives a brief motivation for this research and reviews some relevant literature. Section 2.3 outlines the structure of the model and the behavioural assumptions. Section 2.4 discusses the initialisation and calibration strategy and
presents the baseline simulation. Section 2.5 contains the results of the experiments carried out on the baseline. Section 2.6 concludes the paper. Appendix 2.A presents the traditional balance sheet and transactions-flow matrices summarising the stock-flow consistent structure of the model. Initial and parameter values for the baseline simulation can be found in appendix 2.B. Appendix [2.C] contains a sensitivity analysis on several parameters which are not included in the empirical calibration procedure.

2.2 Motivation and literature review

The purpose of this paper is to combine insights from various strands of the literature to advance research on agent-based stock-flow consistent (AB-SFC) models. Over the past 10 to 15 years there have been substantial advances in the use of agent-based models (ABMs) in macroeconomics, leading to the emergence of a number of frameworks which have been applied to a variety of topics in macroeconomic research. Among others, these include the family of ‘Complex Adaptive Trivial Systems’ (CATS) models (Delli Gatti et al., 2011; Assenza et al., 2015), the various incarnations of the Eurace model (Cincotti et al., 2010; Dawid et al., 2012), and the Keynes+Schumpeter (K+S) model (Dosi et al., 2010). The basic goal of all these frameworks is to provide an alternative way to ‘micro-found’ macroeconomic models rooted in the complex adaptive systems paradigm, emphasising both micro-micro and micro-macro interactions, adaptation, as well as emergent properties. Agent-based approaches, including to macroeconomics, have also attracted increasing interest among policy-makers (see e.g. Turrell, 2016; Haldane and Turrell, 2018, 2019). Dawid and Delli Gatti (2018) provide a comprehensive review of agent-based macroeconomics and compare the major different frameworks in detail.

A by now fairly closely related strand of the literature which emerged out of the post-Keynesian tradition in macroeconomic research is that of stock-flow consistent (SFC) models (see God-ley and Lavoie (2007), who develop the approach, as well as Caverzasi and Godin (2015) and Nikiforos and Zezza (2017) for surveys). Stock-flow consistent models are aggregative (i.e. not ‘micro-founded’), depicting dynamics at the sectoral level, and aim in particular at jointly modelling national accounts variables and flow-of-funds variables within a fully consistent ac-
counting framework. This approach provides an important disciplining device and consistency check in writing large-scale computational models and is essential in comprehensive depictions of real-financial interactions. While by now there exist a range of large-scale pure SFC-models, including one developed in a central bank (Burgess et al., 2016), there is also a growing literature which combines stock-flow consistent frameworks with agent-based modelling in various ways (Dawid et al., 2012; Michell, 2014; Seppecher, 2016). Among these, Caiani et al. (2016) stand out in their emphasis upon the SFC structure of their model and the creative use thereof in initialisation and calibration.

The present paper follows the trend of combining agent-based and SFC modelling techniques and in particular represents a contribution to the development of hybrid agent-based models in which certain parts or sectors of the economy are modelled in an aggregate/structural way or using representative agents whilst others (typically one sector) are disaggregated and modelled using AB methods. Examples of this include Assenza et al. (2007) and Assenza and Delli Gatti (2013) who apply this approach, using heterogeneous firms, to a financial accelerator model in which firms differ in terms of their financial robustness which in turn affects investment, following (Greenwald and Stiglitz, 1993). Michell (2014) uses an agent-based firm sector within an otherwise aggregate SFC framework to model the ideas of Steindl (1952) regarding monopolisation and stagnation along with Minsky’s (1986) trichotomy of hedge, speculative and Ponzi finance. Pedrosa and Lang (2018) construct a more complex model than that of Michell (2014) to investigate similar issues. Botta et al. (2019) focus on heterogeneity among households to investigate inequality dynamics in a financialised economy. The advantage of such a hybrid approach is that important insights arising from agent heterogeneity can be gained from a hybrid model without the necessity of constructing a fully agent-based framework, instead focusing only on a subset of sectors. While, as indicated above, there exist several canonical macroeconomic agent-based modelling frameworks, this is not the case for pure SFC models and the class of hybrid models described in this paragraph. Rather, these models are typically purpose-built for a given research question.\footnote{Although they are often broadly comparable in that SFC models frequently incorporate behavioural assumptions based on the post-Keynesian paradigm (Lavoie, 2014).} The present paper follows this approach, presenting a model purpose-built to discuss banks’ expectations formation under bounded rationality.
and to conduct policy experiments with a particular focus on monetary policy and prudential regulation. The emphasis is hence on introducing heterogeneity only within the banking sector; an approach which to my knowledge is novel to the literature. The hybrid approach allows me to pay particularly close attention to the banking sector which is modelled in great detail in order to investigate the effect of banks’ behaviour and particularly their expectations formation on macroeconomic dynamics. This stands in contrast to many existing non-mainstream macroeconomic models, particularly those with a post-Keynesian flavour, in which banks are frequently modelled as relatively passive entities.

The issues of bounded rationality, learning and expectations formation are relatively longstanding components of the macroeconomic literature. Bounded rationality is a broad concept, with contributions ranging from works such as that of Sargent (1993) which arguably involves only minimal departures from full rationality, via the heuristics and biases approach of (new) Behavioural Economics (Kahneman and Tversky, 2000) to the ‘procedural/ecological’ rationality concepts of Simon (1982) and Gigerenzer (2008) which aim to replace the traditional concept of perfect rationality altogether.

Any departure from full rationality in the traditional sense raises several thorny issues, especially how economic agents are envisioned to form expectations in the absence of full rationality. Several ways of tackling this problem have been proposed. Evans and Honkapohja (2001) develop the so-called e-learning approach whereby one can derive conditions under which agents, through attempting to estimate model parameters, may be able to ‘learn’ the rational expectations equilibrium of a model even in the absence of full rationality and perfect information. Hommes (2013) is a book-length treatment of the idea, stemming from the seminal contribution of Brock and Hommes (1997), that agents may switch between a number of different forecasting strategies based on their relative performance, and possibly the cost of acquiring the necessary information. Arifovic (2000) discusses the use of evolutionary learning algorithms in various macroeconomic settings.

While expectations formation, including under bounded rationality, is thus widely discussed in the literature, such considerations have had relatively little impact in AB and SFC models

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3Indeed, even in many otherwise fully-fledged macro-ABMs (e.g. in Assenza et al. (2015) and Seppecher (2012)), it is assumed for simplicity that there exists a unique/representative bank.

4This latter paradigm is arguably closest to the concept of rationality espoused in macroeconomic ABMs.
in which simple adaptive or naive expectations are often assumed without much discussion. Thus for instance, Dosi et al. (2017a) appears to so far be the only paper explicitly applying structural heterogeneity of expectations in a macroeconomic ABM. In a recent contribution, Catullo et al. (2020) allow firms to use machine learning in order to form expectations about sales in a macroeconomic ABM setting. The literature explicitly analysing expectations formation in macroeconomic ABMs is hence still relatively small. In most ABMs agents are by necessity boundedly rational and endowed with imperfect information, but specific modelling choices with regard to how agents form expectations are seldom discussed in detail, meaning that there is much room to contribute to the existing literature. In this context, it should however be noted that the modelling of learning more generally, which is of course closely related to expectations formation and adaptation of behavioural rules, has played a much more prominent role in the macro-ABM literature (see e.g. Dawid et al., 2012; Salle et al., 2012; Landini et al., 2014; Seppecher et al., 2019), with various forms of genetic algorithms (Dawid, 1999) being a popular choice to depict the adaptation of agents to a changing economic environment. The present paper contributes to the literature along similar lines as Dosi et al. (2017a) and Catullo et al. (2020), focussing on the consequences of varying assumptions about agents’ expectations formation. Despite not being based on any pre-existing framework, the model presented here incorporates a range of behavioural assumptions which are common in the AB and/or SFC literature and hence the simulation results discussed below should be of some general interest to researchers in the area.

Recent years have seen major advances in the development of AB(-SFC) modelling as a viable alternative paradigm in macroeconomic analysis. Chief among these has been the work carried out on empirical estimation/calibration and validation of macro-ABMs, moving away from the rather informal validation protocols which had been standard in the earlier literature (Windrum et al., 2007). Grazzini and Richiardi (2015) discuss the use of simulated minimum distance estimators as developed e.g. by Gilli and Winker (2003), and Grazzini et al. (2017) suggest the application of Bayesian methods for the estimation of macroeconomic ABMs. Lamperti et al. (2018) show how machine learning surrogates can be used to empirically calibrate macro-ABMs in a computationally economical manner. Barde and van der Hoog (2017) present a de-
tailed validation protocol for large-scale ABMs using the Markov Information Criterion (Barde, 2017) and stochastic kriging to interpolate the response of the model to parameter changes. Guerini and Moneta (2017) suggest a validation method based on comparing structural vector autoregressive models estimated on both empirical and simulated data. These contributions represent a major step in increasing the credibility and empirical orientation of macro-ABMs as one particular weakness of this approach has always been the large number of parameters contained in any reasonably detailed model. While a range of different empirical estimation/calibration methods for macroeconomic ABMs have hence recently become available, their application to relatively complex models, especially newly developed ones, is still not standard in the literature. In applying a simulated minimum distance approach - in particular the method of simulated moments - to the model developed here, this paper hence contributes to the increasing empirical orientation of the macro-ABM literature. In addition, the paper represents, to my knowledge, the first attempt to apply an empirical calibration algorithm to a hybrid AB-SFC model containing only one agent-based sector, meaning that it should also be of some interest to researchers working on pure SFC models, the empirical grounding of which is also somewhat underdeveloped.

The chosen focus for this paper, as already mentioned, lies on the expectations formation of the banking sector. On the policy front, the detailed modelling of the banking sector also provides opportunity to contribute to recent debates surrounding the appropriate conduct of prudential regulation policy and its possible interactions with monetary policy (Galati and Moessner, 2012; Barwell, 2013; Claessens et al., 2013; Freixas et al., 2015). Prudential policy has begun to gain importance in the ABM literature, with several of the major frameworks being used to conduct policy experiments in financial regulation (e.g. Popoyan et al., 2017; Salle and Seppecher, 2018; Krug, 2018; van der Hoog, 2018; van der Hoog and Dawid, 2019). By contrast, there have been relatively few treatments of this topic in pure aggregative SFC models (exceptions include Nikolaidi (2015), Detzer (2016) and Burgess et al. (2016)). The model presented here is purposely constructed so as to incorporate a rich structure of prudential policy levers and potential feedback effects of monetary and prudential policy which are detailed in the model description below.
2.3 Model outline

The current section provides an overview of the model and its behavioural assumptions, beginning with its general sectoral structure.

2.3.1 General structure

The macroeconomic sectoral structure of the model is summarised in figure 2.1. The more traditional balance sheet and transactions flow matrices representing the aggregate structure of the model (i.e. excluding transactions occurring within the banking sector) are shown in tables 2.12 and 2.13 in appendix 2.A.

As can be seen, the model consists of 5 sectors, namely households, firms, the government, the central bank and the banks. The first four sectors are modelled as aggregates without explicit micro-foundations whilst the banks are disaggregated. In particular, the model contains an oligopolistic banking sector consisting of 12 individual banks which are structurally identical.
(i.e. they all hold the same types of assets and liabilities) but may differ w.r.t. their decision-making and the precise composition and size of their balance sheets. The following sub-sections provide a sector-by-sector overview of the behavioural assumptions. The basic tick-length in simulations of this model is one week (with one year being composed of 48 weeks), and it is assumed that while all endogenous variables are computed on a weekly basis, main decision variables adjust to their target or desired values at differing speeds in an adaptive fashion, as will be detailed below.

2.3.2 Households

Every week, households compute a plan for desired consumption according to the consumption function

\[ c_d = \alpha_1 \cdot yd^e + \frac{\alpha_2}{48} \cdot v_{h,-1}, \]

where \( \alpha_1 \) is the propensity to consume out of weekly disposable income (which is not a fixed parameter but a endogenous variable specified below), \( yd^e \) is expected real household disposable income for the week, \( \alpha_2 \) is the annual propensity to consume out of accumulated wealth (which similarly to \( \alpha_1 \) is endogenous) and \( v_h \) is real household wealth. The expectations formation mechanism for all expected values in the model is discussed in section 2.3.7. The motivation of this consumption rule, which is standard in the SFC literature, bears similarity to many of the canonical ABM rules described by Dawid and Delli Gatti (2018) and is also used in the benchmark AB-SFC of Caiani et al. (2016), is that if disposable income and household wealth entering the function are defined in a manner consistent with the Haig-Simons definition of income, then this rule implicitly defines a target steady/stationary state household wealth to disposable income ratio to which households adjust over time.\(^5\) One variation on the usual assumption of constant consumption propensities (\( \alpha_1 \) and \( \alpha_2 \)) is that here these are assumed to depend on the (expected) real rate of return on households’ assets (government bonds, deposits and houses),\(^6\) \( \text{rr}_{h}^e \), according to a logistic function, meaning that the target ratio of wealth to

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\(^5\)In a stationary state it must be the case that \( v_h = v_{h,-1} \), which implies \( c_d = yd = yd^e = c \) and hence, if all changes in net worth are correctly accounted for, \( \frac{v_h}{yd^e} = \frac{1-\alpha_1}{\alpha_2} \).

\(^6\)In addition to these assets, households are also assumed to privately own the firms and banks in the model. However, as firm and bank equity are assumed to be non-tradeable their rates of return do not enter into the com-
disposable income (and hence households’ saving) also becomes a function of this return rate. \( \alpha_1 \) is determined by

\[
\alpha_1 = \alpha_1^L + \frac{\alpha_1^U - \alpha_1^L}{1 + e^{(\sigma_{MPC} \cdot r - \sigma_{MPC}^2)}},
\]

(2.2)

where \( \alpha_1^L \) and \( \alpha_1^U \) are the lower and upper bounds of the consumption propensity, \( \sigma_{MPC}^1 \) determines the slope of the function and \( \sigma_{MPC}^2 \) is a parameter shifting the function horizontally. The same functional form also determines \( \alpha_2 \). Household consumption demand hence becomes a decreasing function of interest rates in the model (in particular government bond and deposit rates). This introduces a feedback effect of monetary policy (as well as banks’ interest rate setting behaviour) on economic activity which is a basic building block of the New Keynesian framework (Galí, 2015) but which is largely absent from the AB and SFC literatures. While the strong link between interest rates and consumption implied by the standard New Keynesian model may be viewed as unrealistic, it does appear reasonable to suppose that the return rate households can expect on their savings should have some (if small) effect on their consumption demand. Empirically, the link between interest rates and consumption is still contentious. Some studies, such as Jahm (1998) do find a significant and relatively strong negative effect, but most often the effect is found to be weak at best or sometimes even to have the ‘wrong’ sign (e.g. Weber, 1970; Campbell and Mankiw, 1998; Taylor, 1999). Broadly in line with this, the calibration procedure described below does indeed arrive at a value for the relevant parameter \( \sigma_{MPC}^1 \) which implies a fairly low sensitivity of the consumption propensities to the return rate. Both desired consumption and the ‘desired’ consumption propensities are computed every period, but it is assumed, using an adaptive mechanism, that consumption adjusts more quickly towards the desired level than do the consumption propensities. The idea is the following: equation (2.1) is interpreted as giving an aggregate level of desired consumption of all households represented by the modelled aggregate household sector. At the same time, I assume that households on average update their consumption every quarter, i.e. every 12 periods. Accordingly, I assume that every period (week), 1/12 of the gap between actual and desired consumption (which may of course itself change from period to period) is closed. The same mechanism is applied to the
consumption propensities, but here I assume an updating frequency of 24 periods. This mechanism enables me to mimic asynchronous adaptive decision-making at differing frequencies even in the case of sectors modelled as aggregates, whilst sticking to the basic model-time unit of one week. Importantly, this imparts a degree of real stickiness to the model which is central in enabling the model to generate realistic macroeconomic dynamics. The adjustment mechanism is applied throughout the model to decision variables pertaining to the aggregate sectors, which adapt according to a process of the form

\[ x_{\text{actual}}^{\text{new}} = x_{\text{actual}}^{\text{old}} + \frac{x_{\text{desired}} - x_{\text{actual}}^{\text{old}}}{\text{horizon}}, \]  

(2.3)

where \( x \) is the decision variable in question and \( \text{horizon} \) is the adjustment speed which might be 4, 12, 24 or 48 as the case may be.\(^7\) Beyond the determination of consumption and the consumption propensities, the mechanism is also applied to the determination of demand on the housing market, households’ portfolio decisions, wage-setting, firms’ pricing and investment decisions as well as their leverage adjustment and dividend payouts described below. Where appropriate, adaptive expectations formation as described in section 2.3.7 is also specified so as to match the stickiness of the variable to be predicted.

Households are assumed to privately own the firms and banks in the model, i.e. there are no traded firm or bank shares. Consequently, in allocating their financial savings to different financial assets, households have a choice between bank deposits and government bonds. Households’ demand for government bonds is given by a Brainard-Tobin portfolio equation (Brainard and Tobin, 1968; Kemp-Benedict and Godin, 2017), which is standard in the SFC literature (though not in ABMs). In this case, deposits act as the buffer stock absorbing shocks and errors in expectations, hence the share of financial wealth held as deposits by households is a residual. It is assumed that households revise their portfolio decisions at a frequency of 12 periods (quarterly).

The final two important behavioural assumptions regarding households are their demand for

\(^7\)One problem with this is that with the exception of prices and wages, there does not appear to be empirical research from which to gain an idea of the appropriate adjustment speeds, meaning that in many cases the adjustment speeds are calibrated in accordance with what appears broadly reasonable. In the cases of consumption and investment, the adjustment speeds are set in line with the stylised fact that consumption is less volatile than investment.
housing, and the dynamics of the wage rate. Households form a ‘notional’ demand for houses according to

$$H_{nd}^n = \rho_0 + \rho_1 (V_{h,-1} - V^*_h) + \rho_2 \cdot LTV - \rho_3 (\bar{r}_{M,r}^e - \bar{r}_{M,r}^n),$$  \hspace{1cm} (2.4)

where LTV is the maximum loan-to-value ratio (which is constant in the baseline but represents a possible prudential policy lever due to its effect on notional housing demand), \(V^*_h\) is the current target level of nominal household wealth derived from equation (2.1), and \(\bar{r}_{M,r}^e\) is the expected average real interest rate charged on mortgages, with \(\bar{r}_{M,r}^n\) being a ‘normal’ or conventional value, set equal to the value in the initial stationary state which serves to anchor mortgage demand.\(^8\) There is hence no ‘direct’ speculative element in housing demand (in the sense that, for instance, (expected) house prices do not enter directly into the function), although appreciation of the housing stock obviously has a positive impact on \(V_h\). With notional housing demand, the updating time horizon using equation (2.3) is assumed to be one quarter. Based on the notional housing demand, households formulate a demand for mortgages based on the LTV (it is assumed that households always attempt to use the maximum permissible LTV ratio). This demand for mortgages, which may or may not be fully satisfied based on banks’ credit rationing behaviour, in turn gives rise to an ‘effective’ demand for houses equal to

$$H_{d}^{ef} = \min(H_{nd}^n, M_s + (1 - LTV) \cdot H_{d}^n),$$  \hspace{1cm} (2.5)

where \(M_s\) is the total amount of mortgages supplied by banks in the current period and \((1 - LTV) \cdot H_{d}^n\) is the part of housing demand not financed by mortgages (\(LTV \cdot H_{d}^n\) being the demand for mortgages). Banks’ behaviour (as well as prudential policy, which, as described below, has an impact on rationing behaviour) hence affects the housing market both through mortgage rates and the extent to which mortgage demand is rationed. The supply of houses is determined by the assumption that in each period, a constant fraction \(\eta\) of a constant total stock of houses in the model are up for sale. The price of houses is then determined by market

\(^8\)This modelling choice is also expedient for the subsequent calibration of the model to a deterministic stationary state as in the stationary state, the deviation of the mortgage rate from its ‘normal’ value is assumed to be zero and hence the term drops out of the equation.
Regarding wages, it is assumed that the (desired) nominal wage rate is determined by a Phillips-curve-type equation of the form

\[ W = (W_n + \beta \cdot (u_h^e - u_n))(1 + \pi_h^e) \]  \hspace{1cm} (2.6)

which is supposed to mimic the aggregate outcome of a wage-bargaining process. \( u_h^e \) is the households’ expected rate of industrial capacity utilisation, \( \pi_h^e \) is their (semi-annualised) expected rate of inflation whilst \( u_n \) is an exogenous ‘normal’ rate of capacity utilisation. The wage rate is anchored around \( W_n \), its level in the stationary state, which appears reasonable since there is no long-run growth in the model and factor productivities are fixed. The actual wage rate adjusts to the desired one with a time-horizon of 24 periods which appears roughly consistent with, perhaps somewhat shorter than, available evidence regarding the duration of wage-spells (e.g. Barattieri et al., 2014).

2.3.3 Firms

Firms produce a homogeneous good used both for consumption and investment according to a Leontief production function the coefficients of which are fixed throughout. The good is demanded by households, the government and firms themselves (for capital investment) and it is assumed that demand is in general satisfied instantaneously. However, the Leontief production function in principle implies a maximum level of output which can be produced given the existing capital stock if capacity utilisation = 1, such that firms may be incapable of satisfying all demand. Accordingly it is assumed that if total demand exceeds capacity, consumption demand is rationed. Unless aggregate demand exceeds firms’ productive capacity \( \overline{y} \), output is hence demand-determined, i.e.

\[ y = \min(c_d + i_d + g_d, \overline{y}). \]  \hspace{1cm} (2.7)

The production function together with actual production implies a demand for labour which is assumed to always be fully satisfied by households at the going wage rate. Firms set the price for their output according to a fixed mark-up over the sum of unit labour cost and ‘unit interest
cost’, defined here as a one-year moving average of firms’ net interest payments over output.\footnote{This implies that monetary policy prima facie has an ambiguous effect on inflation; increases in the central bank rate will tend to decrease aggregate demand and economic activity, which will tend to lead to lower wages and hence prices, but will also increase unit interest cost, which tends to lead to higher prices. The actual effect on the price level will depend on the relative strength of these effects. Such contradictory feedback channels can also be found in some DSGE models (e.g. Christiano et al., 2005).}

The price level adjusts with a horizon of 24 periods which is broadly consistent with empirical evidence Álvarez et al. (2006).

To determine the demand for investment goods, firms compute a desired growth rate of the capital stock according to

\[
g^d_k = \gamma_1 \cdot \frac{u^e_f - u_n}{u_n} - \gamma_2 \cdot \frac{\overline{r_{L,r}}^e - \overline{r_{L,r}}^n}{\overline{r_L}^n},
\]

where \(\overline{r_{L,r}}^e\) is the expected average real interest rate charged on bank loans (with \(\overline{r_{L,r}}^n\) being once more a ‘normal’ or conventional level given by the value in the initial stationary state which serves as an anchor to the desired growth rate of capital) and \(u^e_f\) is firms’ expectation of the future rate of capacity utilisation. This formulation for the investment function is similar to the one adopted by Caiani et al. (2016) with the exception that instead of a profit rate, the interest rate on loans enters the investment function. This is done to provide a direct channel from banks’ interest rate setting to firms’ investment behaviour although as in the case of consumption demand, the inclusion of such a link may be viewed as somewhat controversial.

As with consumption, the effect of interest rates on investment is empirically contentious, with some studies finding rather strong effects (e.g Bernanke, 1983), while in many others the link is found to be weak (Hall et al., 1977; Stockhammer and Grafl, 2010; Arestis et al., 2012). Once again, the result of the calibration procedure described below appears to bear this out as the calibrated value of \(\gamma_2\) turns out to be small. The formulation of investment demand as a function of capacity utilisation, implying that firms invest more in periods of high utilisation, targeting a ‘normal’ level of utilisation, is standard in the SFC literature and also quite common in ABMs (Dawid and Delli Gatti, 2018). The incorporation of the interest rate on loans introduces an additional feedback effect of monetary policy (as well as the behaviour of banks) on aggregate demand. The desired growth rate of capital is assumed to adapt at a frequency of one quarter. Together with the depreciation of capital (which takes place at a fixed rate) the desired growth
rate of capital implies a demand for investment goods. Despite the presence of a capital stock and investment, the model does not feature long-run growth but rather aims to depict business cycle fluctuations around a stationary state.

It is assumed that firms possess a fixed target for their leverage ratio (defined as loans over capital stock). Based on firms’ existing stocks of loans and capital, current loan repayments and depreciation as well as their current investment plans, one can derive a gap between current and target leverage and firms attempt to slowly close this gap by using appropriate combinations of loans and flows of current net revenue in financing their investment (indeed, if leverage is below target, firms may also take out new loans exceeding current investment in order to increase leverage). The actual combination of internal and external finance may differ from the planned one due to possible rationing of loan demand which is described below. If firms are unable to obtain the amount of loans for which they apply due to credit rationing, investment expenditure is curtailed accordingly. Deposits with the banking sector act as the buffer stock of the firm sector, just as they do for households, absorbing unexpected fluctuations in revenues and expenditures. Firms’ dividend payouts are determined by firms’ profit after deducting net revenues used to internally finance investment, adapting with a frequency of 24 periods.

2.3.4 Government

The government collects taxes on a one-year moving average of household income (wages, interest and profits/dividends accruing to households) at a fixed rate $\tau$. In addition it may levy a tax on firms’ retained earnings if it suffers persistent deficits. In the baseline model, the real value of government spending is assumed to be fixed. Deficits are covered as they occur by the issuance of government bonds of an amount corresponding to the deficit in the current period, while in the case of a surplus, repayments are made to households and the central bank proportionally to their respective holdings of bonds. Government bonds hence in principle have an infinite maturity (i.e. the government does not have to repay or roll over particular bonds at a specific time), but can be repaid when a surplus allows the government to do so. These assumptions are made so as to keep the government bond market relatively simple. A more elaborate maturity and issuance structure of bonds would greatly complicate the model without,
in my view, adding much insight on the main objectives of the paper, which are to investigate the role of expectations formation among the heterogeneous banks and the impacts of monetary and prudential policy. For similar reasons, banks do not participate in the government bond market as the focus lies on modelling banks’ lending behaviour to the private sector, the associated competition and balance sheet dynamics, as well as the expectations banks have to form in this context. The balance sheet structure assumed for banks in the model is sufficient to allow for quite rich asset and liability management behaviour, as outlined in the description of bank behaviour in section 2.3.6.

New government bonds are offered in the first instance to households and the government varies the interest rate on bonds in an attempt to clear the market. It does so by equating the updated stock of government bonds (following any issuance in the current period) with the previous period’s demand for bonds from households (emanating, as outlined above, from a portfolio equation) by adjusting the interest rate.

2.3.5 Central Bank

The central bank sets a nominal deposit rate according to a Taylor-type pure inflation-targeting rule:

\[ r_{cb,d} = r_0 + \pi^e + \phi_\pi \cdot (\pi^e - \pi^t), \tag{2.9} \]

where \( \pi^e \) is the central bank’s expected inflation rate and \( \pi^t \) is its target, set equal to 0. This rate is adjusted once every month and then remains constant for the following 4 periods. The central bank’s lending rate is given by a constant mark-up over its deposit rate, giving rise to a corridor system. In addition, I suppose that the central bank has in mind a target interbank rate in the middle of this corridor and continuously carries out open-market operations in order to steer the level of central bank reserves to a level consistent with this target. It does so by purchasing and selling government bonds from/to the households, transferring/withdrawal reserves to/from the banking sector which in turn increases/decreases households’ deposits by the corresponding amount (for simplicity I assume that the households are always willing to enter into such transactions). Given the sequence of events within a period, the central bank is in fact
always able to perfectly target the correct level of reserves, meaning that in practice banks are never ‘in the Bank’ to acquire advances and similarly there is never an aggregate excess level of reserves since these are drained by the central bank.

If necessary, the central bank also acts as a lender of last resort to the government by purchasing residuals of newly issued bonds. All central bank profits are transferred to the government (and all losses are reimbursed by the government). In addition, the central bank is the prudential policy-maker in the model. At present, the model includes three prudential policy ratios, namely the capital adequacy ratio, the liquidity coverage ratio and a maximum loan-to-value ratio on mortgages, all applying to banks. In the baseline, the targets for all these regulatory ratios are assumed constant.

The capital adequacy ratio of a bank $i$ is given by

$$CAR^i = \frac{v_{bi}}{\omega_1 \cdot M^i + \omega_2 \cdot L^i},$$

(2.10)

where $M$ are mortgages, $L$ are loans to firms and $v_{bi}$ is the banks’ capital buffer, equal to $v_b + e_b$, the bank’s net worth plus the fixed amount of bank equity held by households. The $\omega$’s are risk-weights whereby the asset assumed to be the riskier one, loans to firms, is given a weight of 1 in the calculation of the capital adequacy ratio and mortgages receive a weight $< 1$. These risk weights are used below to determine the extent to which different sources of credit demand are rationed, and the default probabilities on loans and mortgages described below are set so as to be in line with these risk weights. The liquidity coverage ratio is in essence a minimum reserve requirement applying to deposits defined in line with the Basel III framework (Basel Committee on Banking Supervision, 2010, 2013).

2.3.6 Banks

The agent-based banks possess the richest behavioural structure of all the sectors in the model. Each bank must set three interest rates, namely the rate of interest on deposits, on loans to firms, and on mortgages. It is assumed that each period, a random sample of banks is drawn (such that on average, each bank is drawn once every 4 periods) and these are allowed to adjust their
interest rate in a given period (meaning that on average, each bank can adjust its interest rate once a month). The deposit rate offered by a bank $i$ is given by

$$r^i_d = \bar{r}_{cb} + \varepsilon_{d1} \cdot cl_i. \tag{2.11}$$

$\bar{r}_{cb}$ is a one-quarter moving average of the midpoint of the central bank’s interest rate corridor and $\varepsilon_{d1}$ is a parameter $< 0$. $cl_i$ is an indicator for bank $i$’s clearing position (i.e. the difference between all transactions during a period representing an inflow of reserves and those representing an outflow of reserves from bank $i$’s balance sheet). This indicator is calculated as

$$cl_i = 1 + \tanh(\varepsilon_{d2} \cdot clear_i) \tag{2.12}$$

whereby $clear_i$ is a one-quarter moving average of bank $i$’s clearing position. The intuition is that a bank which persistently finds itself with a negative clearing position (i.e. experiencing a constant drain of reserves) will increase its deposit rate in order to attract more deposits, and vice-versa. Inflows of deposits represent the cheapest source of funding for banks in the model; in particular they are by construction cheaper than to borrow reserves on the interbank market or from the central bank. The hyperbolic tangent is chosen as a functional form so as to place an upper and lower bound on the value of $cl_i$. The lending rates of bank $i$ are given by

$$r^i_M = \theta_{M,i} \cdot (\bar{r}_{cb} + default_{M,i})$$
$$r^i_L = \theta_{L,i} \cdot (\bar{r}_{cb} + default_{L,i}), \tag{2.13}$$

where the $\theta$’s are gross mark-ups (the mark-ups are $> 1$ and evolve endogenously as described below). $default$ signifies the current default rate on mortgages and loans of bank $i$, which are added to $\bar{r}_{cb}$ in an attempt by the bank to cover for expected losses based on its current assessment of default rates. In addition to setting its interest rates, a bank can also decide to engage in direct rationing of credit. Each bank calculates the gap between its current risk-weighted assets and the maximum allowed given the target capital adequacy ratio and its expected capital buffer. If this gap is greater than zero, meaning that risk-weighted assets are too high, banks ration credit directly. In particular, in each period they attempt to close $\frac{1}{48}$ of the gap in risk-
weighted assets by rationing both mortgage and loan demand according to their relative risk weights.\textsuperscript{10} This way of modelling credit rationing is somewhat similar to the one used in the family of models building on Delli Gatti et al. (2011) such as Assenza et al. (2015) and Assenza et al. (2018a). Through potentially curtailing both investment and housing demand (which in turn will feed back into household wealth and hence consumption), banks’ rationing behaviour which is partly determined by their expectations about their own capital buffer hence has an impact on aggregate demand.

A central element in the modelling of the banking sector is that of the distribution of loan demand and deposits between the different banks. It is assumed that the aggregate amount of deposits of households and firms is distributed between the banks in each period according to

\[
\text{share}_D^i = \left( \frac{D_{d,1}^i - 1}{D_{d,1}} \cdot \tilde{r}_d^i \cdot \epsilon_d, i \right),
\]

(2.14)

where \( D \) are deposits, \( \tilde{r}_d^i \) is bank \( i \)'s relative deposit rate and \( \epsilon_d \) is an autocorrelated, normally distributed random shock centered on 1 with standard deviation \( \sigma_{dis} \). The shares thus calculated are then normalised and multiplied by the total amount of deposits in order to determine the amount held by each individual bank. The share of mortgage demand received by each bank is calculated as follows:

\[
\text{share}_M^i = \left( \frac{M_{d,1}^i - 1}{M_{d,1}} \cdot \tilde{r}_M^i \cdot \text{ration}_{M,i}^i \cdot \epsilon_{M,i} \right).
\]

(2.15)

\( M_d \) is mortgage demand, \( \tilde{r}_M^i \) is the inverse of bank \( i \)'s relative rate on mortgages and \( \text{ration}_{M,i}^i \) is an indicator of the relative intensity of the rationing of mortgages by bank \( i \). This is first calculated as

\[
\text{ration}_{M,i}^i = \frac{\xi_1}{1 + e^{\xi_2 \left( \frac{M_{d,1}^i - 1}{M_{d,1}^i} + 1 \right)}},
\]

(2.16)

and then normalised so that \( \sum \text{ration}_{M,i}^i = 1 \). The equation implies that banks which rationed mortgage demand in the previous period will tend to lose market shares in the current period.

\textsuperscript{10}It is assumed that banks always grant loans to firms which are purely aimed at financing replacement investment and only ration loan demand exceeding that needed for replacement investment.
with the logistic functional form bounding the value of $\hat{\text{ration}}_{M,i}$. The distribution of loan demand between the banks takes place in identical fashion to that of mortgages. One potential problem with this formulation of deposit and loan distribution is that if a bank loses its entire market share, there is no way for it to re-enter the market. For this reason, a small lower bound is imposed on the market share of each bank to give it the possibility to re-capture market shares it previously lost. In each period, a given fraction of firm loans and mortgages held by each bank are repaid. Defaults evolve according to

$$def_{M,i} = \zeta_M \cdot lev_h \cdot \epsilon_{M,i}^{def}$$

$$M_{np,i} = def_{M,i} \cdot M_{i-1},$$

and symmetrically for firm loans. $\zeta_M$ is a fixed parameter, $lev_h$ is a one-year moving average of the ratio of mortgages to the housing stock\(^{11}\) meaning that defaults tend to increase as households become more highly leveraged in the housing market, and $\epsilon_{M,i}^{def}$ is a random variable drawn from a logistic distribution which is not only autocorrelated but also cross-correlated across banks. This implies that default shocks are not completely idiosyncratic across banks but instead contain a ‘systemic’ element hitting all banks at the same time. To construct these random default shocks, I first generate a matrix of cross-correlated normal random variables and then transform them into draws from a logistic distribution the location parameter of which is the relative interest rate of each bank and the scale parameter, $s_{def}$, of which is empirically calibrated below. I assume that defaults on firm loans are more frequent than those on mortgages on average and this is reflected in the risk weights of the two assets.

In addition to its decisions on credit rationing (and indirectly through the effects of defaults on both equity and the interest rates), the capital adequacy ratio also feeds into banks’ dividend policy. In particular, banks form an expectation about their future capital buffer and compare it to a target value (based on the target capital adequacy ratio). Every quarter, they calculate a mean of the deviations over the previous 12 periods and then, for the following 12 periods, pay a dividend equal to current profits plus $\frac{1}{12}$ of the deviation (while the deviation may be negative, the total dividend must be positive or zero). Banks’ dividend payouts (as well as banks’ expec-

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\(^{11}\)In the case of firms, $lev_f$ is a one-year moving average of the ratio of loans to capital.
tations which feed into their determination), making up part of households’ disposable income, have a feedback effect on aggregate demand through consumption expenditure.

A further element of the banking sectors’ behaviour concerns the interbank market. Banks’ final demand for reserves is determined by their deposits and the target liquidity coverage ratio set by the regulator. In order to calculate their demand for/supply of funds on the interbank market, each bank calculates a clearing position netting all its in- and outflows of reserves over the present period. After adding this clearing position to their previous stock of reserves, banks end up with a ‘prior’ stock of reserves which is compared to their target stock, thereby determining whether they will demand or supply funds on the interbank market.

Demand and supply on the interbank market are aggregated and matched, and whichever side of the market is short is rationed proportionately. For instance, if total demand on the interbank market is higher than total supply, each bank on the demand side receives funds equal to

\[
\text{individual demand} = \frac{\text{total demand}}{\text{total supply}}.
\]

The interbank market is hence modelled in an extremely simple fashion and in and of itself has only a very slight impact on simulation results through its determination of the interbank rate (see below). By construction there are no defaults or possibilities of the interbank market freezing up. It merely represents a straightforward way to redistribute reserves among banks in accordance with their reserve targets. As outlined above, however, inflows and outflows of reserves during the period (i.e. before interaction on the interbank market) do play an important role in that they determine banks’ clearing position and hence their behaviour in competition for deposits.

If banks are unable to obtain all the reserves they need on the interbank market, they request advances from the central bank. These advances are always granted on demand at the central bank lending rate, which is however higher than the interbank rate, since the latter by construction falls within the corridor and is given by

\[
r_{IB} = r_{cb,d} + \frac{r_{cb,l} - r_{cb,d}}{1 + e^{-\sigma_{IB}(R_{gap})}}, \quad \text{(2.18)}
\]

where \( R_{gap} \) is the aggregate gap between reserves prior to the central bank’s intervention and target reserves. This functional form ensures that the interbank rate is bounded within the
interest rate corridor set by the central bank, which is reasonable as all banks have access to the central bank’s lending and deposit facilities.

Recall that when setting interest rates on loans and mortgages, banks add the default rate to a moving average of the central bank rate and then apply a mark-up on this sum. In the baseline model, each bank adaptively changes its mark-ups every time it is allowed to alter its interest rates. In particular, a given bank \( i \) will follow the following rule:

- If revenues of bank \( i \) on the asset (mortgages or loans) were higher during the past month than in the month before that, and bank \( i \)’s rate is lower than the sector average, increase the mark-up.
- In the opposite case, decrease the mark-up
- Otherwise, leave the mark-up unchanged.

To revise the mark-up, banks draw a normally distributed random number centered on the parameter \( \text{step} \) (which is however constrained to always be non-negative) and then increase or decrease the mark-up by this amount. Overall this mechanism is intended to depict a type of heuristic search for the most profitable mark-up rate, and it involves banks implicitly making predictions about the relationships between (relative) interest rates and revenues. This heuristic for setting the price of loans is similar to the one used by Assenza et al. (2015) to model the pricing decisions of firms. When setting interest rates, banks’ information set includes the current-period default rates on their portfolios of mortgages and loans and they use these current default rates as a prediction of future ones in setting their interest rates. Both of these mechanisms will be altered as part of the experiments presented in section 2.5.1.

Finally, the model does not contain a bankruptcy mechanism for banks which does not represent a problem for the present work as no bank has so far gone bankrupt in any simulation. For future applications which may include more extreme scenarios which could trigger the failure of one or more banks, a bankruptcy mechanism should be added.
2.3.7 Expectations

In the baseline all expectations are modelled following an adaptive mechanism:

\[ x^e = x^e_{-1} + \psi_{ad} \cdot (x_{-1} - x^e_{-1}) \] \hspace{1cm} (2.19)

To take into account that decision variables in the model adjust at different speeds, \( x_{-1} \) in the equation above may be a moving average of some length corresponding to the adjustment speed of the forecasted variable. While the focus of the model lies on the expectations of banks, expectations also enter into the behaviour of other sectors for both theoretical and computational reasons. Households forecast their disposable income, their wealth, mortgage interest rates, the rate of inflation, and the composite rate of return on their assets. Firms forecast the average interest rate on loans, as well as their capacity utilisation. The central bank forms expectations about inflation as well as capacity utilisation. Banks must form expectations about their capital buffer, \( v_{bb} \). In addition, as outlined above, they engage in a type of forecasting or expectations formation when setting mark-ups and (possibly) in forming perceptions about default rates, which however is different from the adaptive expectations mechanism. The first experiment reported in section 2.5.1 consists in replacing the adaptive expectations mechanism in the banking sector with the model of heterogeneous expectations formation and heuristic switching proposed by Brock and Hommes (1997). Specifically I use the version presented in Anufriev and Hommes (2012) where it is assumed that agents can switch between four different specifications for expected variables given by

\[ x^{e1} = x^{e1}_{-1} + \psi_{ad} \cdot (x_{-1} - x^{e1}_{-1}) \]
\[ x^{e2} = x_{-1} + \psi_{tf1} \cdot (x_{-1} - x_{-2}) \]
\[ x^{e3} = x_{-1} + \psi_{tf2} \cdot (x_{-1} - x_{-2}) \]
\[ x^{e4} = \psi_{aa} \cdot \overline{x_{-1}} + (1 - \psi_{aa}) \cdot x_{-1} + (x_{-1} - x_{-2}) \]. \hspace{1cm} (2.20)

The first rule is the same adaptive one used in the baseline, which is now augmented by two trend-following rules (one weak and one strong, i.e. \( \psi_{tf1} < 0 \) and \( \psi_{tf2} > 1 \)) and an ‘anchoring and adjustment’ mechanism in which the anchor is the moving average of \( x \). Agents switch
between these four mechanisms based on a fitness function calculated using the error between expected values and realisations of the forecasted variables as detailed in Anufriev and Hommes (2012). It is important to note that these rules stem from specific experimental settings (which differ from the environment provided by the model presented here; for instance, Anufriev and Hommes (2012) conduct an asset pricing experiment), meaning that their general applicability remains an open question. Nevertheless it is interesting to investigate their effect on the model presented here. In addition, section 2.5.1 discusses experiments in which the banks can use simple econometric techniques to conduct forecasts and form expectations.

2.4 Calibration & baseline simulation

Before simulation, the model is calibrated to a deterministic stationary state. This is done using a script in which initial values as well as a range of parameters can be sequentially calculated based on the imposition of successive restrictions on some characteristics of the stationary state such as the capital stock, the stock of housing, the investment and government spending to income ratios, and so on. Where possible, these values are chosen so as to correspond roughly to those of the economy of the United Kingdom.\(^\text{12}\) To give an example of how this calibration procedure works, once I impose a stationary state level of the capital stock, the capacity utilisation rate, and a parameter value for the capital depreciation rate, then investment demand, the capital to full capacity output ratio and real GDP are implied by these imposed values jointly with the assumed Leontieff production function and the assumption that the simulation begins in a stationary state. In a stationary state it must be the case that

\[
i = i_d = \delta_k \cdot k,
\]

i.e. capital investment must equal depreciation for the capital stock to be constant. Next, note that the production function implies

\[
\kappa \cdot y = u \cdot k
\]

where \(\kappa\) is the capital to full capacity output ratio. Next, I can substitute for \(k\) from equation

\(^\text{12}\)In the case of stock variables, the chosen initial period is 1995 Q1.
Having previously imposed a stationary state value for \( \frac{i}{y} \), this gives me the value for \( \kappa \) which in turn I can use to get a value for \( y \) from equation (2.22). The rest of the initialisation protocol proceeds similarly. For instance, by imposing a stationary state value of the government expenditure to output ratio, I get a stationary state level of government expenditure and furthermore a value for consumption since
\[
c = y - i - g.  
\] (2.24)

In many cases, the stock-flow consistent accounting structure of the model is useful in this initialisation exercise as accounting conventions dictate the values of certain variables once a sufficient number of others are determined. Despite the imposition of successive restrictions, this procedure leaves a range of parameter values unidentified (in particular those appearing in behavioural equations written in terms of deviations from ‘normal’ or stationary state values). A subset of these are empirically calibrated below, while the rest are set to values which give rise to reasonable results and are subjected to a sensitivity analysis in appendix 2.C. Further details on the initialisation and parametrisation of the baseline simulation can be found in appendix 2.B.

For the empirical calibration of free parameters I make use of the simulated minimum distance approach described by Grazzini and Richiardi (2015) by applying the method of simulated moments (see Gilli and Winker, 2003; Franke and Westerhoff, 2012; Schmitt, 2020) in order to empirically calibrate 8 of the model’s free parameters. This is done through maximising an objective function involving a set of 8 moments/statistics calculated from empirical data along with their equivalents generated by model simulations. In particular, the function to be maximised is
\[
O(\theta) = -\left(m_d(\theta_0) - m(\theta)\right)' \cdot W \cdot \left(m_d(\theta_0) - m(\theta)\right)  
\] (2.25)
where \( \theta \) is a vector of model parameters (with \( \theta_0 \) being the vector of their ‘true’ values), \( m_d \) is a vector of empirical moments and \( m(\theta) \) is a vector of simulated moments. \( W \) is a weighting matrix. Following Franke and Westerhoff (2012), the weighting matrix used here is the inverse
of the variance-covariance matrix of the empirical moments/statistics, which is obtained through the use of bootstrapping. This ensures that the variance of the empirical moments is taken into account in the calibration procedure.

As outlined by Grazzini and Richiardi (2015), building on Grazzini (2012), the use of simulated minimum distance estimators in agent-based models raises the issues of stationarity and ergodicity, in that a simulated minimum distance estimator will only be consistent if the simulated moments/statistics used are stationary and ergodic. Note that equation (2.25) is somewhat misleading in that in an agent-based model, $m$ may be a function not only of $\theta$ but also of the random seed $s$ and the vector of initial conditions $y_0$ and may in particular be non-ergodic w.r.t. the random seed and/or the initial conditions. Having observed the behaviour of the model across a large number of simulations, it appears reasonable to assume that the stationarity assumption is fulfilled for the simulated moments I use, in particular since I apply the HP-filter to the simulated data before calculation of the objective function. The ergodicity assumption w.r.t. the random seed and initial conditions is somewhat more problematic but I can at least partly overcome this issue on the one hand by choosing initial conditions based on empirical information as far as possible and subsequently keeping them fixed across simulations, and on the other hand by defining $m$ as the Monte-Carlo average of moments from a set of simulations with different random seeds, for which in turn the ergodicity assumption appears less heroic.

More broadly, the empirical calibration procedure is used here primarily to arrive at a reasonable baseline simulation without having to fully parametrise the model by hand, rather than to consistently estimate the ‘true’ values of the parameters (all the more so since, as outlined below, I am not able to cover the entire parameter space in my simulations and instead rely on sampling). The time-series I choose for the empirical calibration procedure are quarterly real GDP, real consumption, real investment and the CPI for the UK from 1994 Q2 until 2019 Q1, such that the length of the empirical time series is equal to that of the simulated ones (all simulations shown below, as well as those used for the empirical calibration have a post-transient duration of 25 years). I apply the HP-filter to each empirical time series and then calculate the standard deviation and first order autocorrelation of each series’ percentage-deviation from its trend component. The same procedure is applied to the simulated quarterly time series which are
constructed from the weekly model output. The vector of parameters I am aiming to calibrate consists of the parameters shown in table 2.1.

Table 2.1: Empirically calibrated parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>Investment sensitivity to capacity utilisation</td>
<td>-3 : -0.5</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>Investment sensitivity to loan rate</td>
<td>0.5 : 1.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Phillips curve slope</td>
<td>0.5 : 0.95</td>
</tr>
<tr>
<td>$\sigma_{MPC}^1$</td>
<td>Consumption propensity adjustment</td>
<td>0.25 : 0.95</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Housing demand sensitivity to wealth</td>
<td>1 : 10</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>Housing demand sensitivity to mortgage rate</td>
<td>0.1 : 3</td>
</tr>
<tr>
<td>$AR_{def}$</td>
<td>Default shock autocorrelation</td>
<td>0.3 : 0.5</td>
</tr>
<tr>
<td>$s_{def}$</td>
<td>Default shock scale parameter</td>
<td>-5 : -1</td>
</tr>
</tbody>
</table>

The empirical calibration proceeds by sampling the parameter space made up of the eight parameters within the ranges shown in table 2.1 above using latin hypercube sampling, simulating each parameter configuration 100 times with different (reproducible) seeds and calculating the values of the objective function. Sampling is then repeated around points which appear promising in terms of the value of the objective function until eventually a satisfactory configuration is reached in the sense that further sampling and simulation generates no notable improvements in the value of the objective function.\(^{13}\)

Table 2.2: Comparison of empirical and simulated standard deviations and first order autocorrelations

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
<th>Investment</th>
<th>CPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD Empirical</td>
<td>0.01085</td>
<td>0.00949</td>
<td>0.03581</td>
<td>0.00495</td>
</tr>
<tr>
<td>SD Simulated</td>
<td>0.01093</td>
<td>0.00942</td>
<td>0.03545</td>
<td>0.00497</td>
</tr>
<tr>
<td>AC Empirical</td>
<td>0.89237</td>
<td>0.81905</td>
<td>0.70851</td>
<td>0.87244</td>
</tr>
<tr>
<td>AC Simulated</td>
<td>0.87366</td>
<td>0.89859</td>
<td>0.82969</td>
<td>0.82332</td>
</tr>
</tbody>
</table>

\(^{13}\)There is clearly a danger for this algorithm to get ‘stuck’ at a local maximum of the objective function, but there is little I can do regarding this issue given limited time and computational resources, and the obtained results seem reasonably good.
Table 2.2 provides a comparison of the empirical statistics and those generated by the model following calibration. It can be seen that the model is able to reproduce the standard deviations of the empirical time series very closely. It performs somewhat less well on the first order autocorrelations, in particular those of consumption and investment for which the simulated series show a higher autocorrelation than their empirical counterparts. This higher autocorrelation of consumption and investment however appears necessary in order for the model to be able to reproduce the autocorrelation of GDP due to the absence of other components of GDP in the model and government spending being constant in the baseline.

A final remark regarding initialisation and parametrisation concerns the agent-based banks. Rather than start with completely identical agents, as is sometimes done in the literature, I instead impose a heterogeneous initialisation by giving banks balance sheets of differing sizes. The reason for this is that I am primarily interested in the behaviour of heterogeneous banks rather than the endogenous emergence of heterogeneity. The initial distribution is detailed in appendix 2.B. Following calibration of the model, I move on to the baseline simulation. This is conducted using 100 Monte Carlo repetitions. Stochastic elements in the simulation emanate from the default process, the distribution of aggregate flows of loan and mortgage demand as well as deposits between banks, the re-setting of bank interest rates at random intervals and the random amount by which banks’ mark-ups change in case they are revised. After discarding a transient of 480 periods, the baseline simulates the behaviour of the model for 1200 periods, corresponding to 25 years (it is assumed that one year is made up of 48 weeks).14

Starting from the deterministic stationary state, the stochastic elements characterising the behaviour of the agent-based banks are sufficient to make the economy diverge from the stationary state and converge to a pattern of irregular cyclical movements. Figures 2.2 to 2.6 give an idea of the dynamics of the model by showing aggregate time-series from one individual, representative run. Figure 2.6 reports the sectoral financial balances at a quarterly frequency since the weekly data is too noisy to allow for a legible graphical representation.

Investment demand which reacts to both utilisation gaps and financing conditions (in the form of interest rates and credit rationing) is clearly the main driver of fluctuations in aggregate

14 The scripts necessary to reproduce the simulations can be downloaded from https://github.com/SReissl/JEEC.
income. Consumption is more persistent, being driven by a combination of relatively slow-moving factors including changes in the wage rate, fluctuations in the consumption propensities, and the impact of cycles in the price of housing on household wealth.

Figure 2.2: Real GDP and components (excluding constant g) for a single run

Figure 2.3: Annualised rate of inflation

Figure 2.4 shows that even average lending rates are quite volatile, reacting to developments in the cross-correlated default shocks and revisions in the mark-ups. In addition, since the model produces fairly volatile inflation rates, the pure inflation-targeting monetary policy leads to strong fluctuations in the central bank rates which in turn feed through into the rates offered by banks. Credit rationing is not particularly prevalent but increases strongly at the peaks of economic booms, contributing to the consequent downturn.
Turning to the dynamics at the level of individual banks, it can be seen in figure 2.7 that banks are quite successful at remaining close to the fixed target for the capital adequacy ratio. Fluctuations in the capital adequacy ratio are correlated across banks to some degree, but at times individual banks diverge from the common trend. Figure 2.8 shows that the size-distribution of banks in terms of the length of their balance sheets is relatively constant and in particular that there appears to be no tendency towards monopolisation of all loan markets by any single bank. Instead, the oligopolistic structure appears stable.
A look at the distribution of loans and mortgages, as in figure 2.9, reveals an interesting pattern. It can be seen that in general, banks never lose significant market shares in both loan and mortgage markets at the same time and even that instead, a bank which loses shares in one market tends to increase its share in the other one. This is a general property of the simulation results which can be found across the individual Monte Carlo runs of the model. In some simulations, one individual bank eventually gains a considerable share in one market (loans or mortgages) but at the same time, its share in the other market declines strongly. This phenomenon is not a feature built into the model but rather can be considered an emergent property of the system.
While the model dynamics are qualitatively quite similar across different Monte Carlo repetitions, the cyclical movements almost disappear when taking the mean of simulated time-series as the peaks and troughs of cyclical movements do not necessarily coincide across different runs. To get an idea of the dynamics generated by the model, I must hence focus on analysing individual runs as was done above, together with second moments of simulated data and various other summary statistics. In addition to calibrating the model so that the volatility and first-order autocorrelation of the main macro time-series corresponds to that of their empirical counterparts, I also follow Assenza et al. (2015) in taking a look at the cross-correlations and higher-order autocorrelations of these series. Figures 2.10 and 2.11 provide an overview of these, using quarterly and filtered simulated data. Figure 2.10 (where the solid lines represent...
the simulated quarterly data, the bars are Monte-Carlo standard deviations and the dashed lines represent the empirical data) shows that the model does a reasonably good job at reproducing empirical autocorrelations although the simulated time series of investment and the price level are somewhat less persistent than the empirical ones.

Figure 2.10: Autocorrelations

Figure 2.11: Cross-correlations

Figure 2.11 shows the cross-correlations of real output at time $t$ with output, investment, consumption and the price level at time $t - lag$. Again the fit appears reasonably good, with the
exception of that of the price-level. In the case of the latter the model produces a much stronger
cyclicality than that which is observed in the empirical data so that overall the model does not
appear to reproduce price and inflation dynamics especially well (this is also underlined by
figure 2.12 which shows that the model appears to produce regular cyclical dynamics in the
inflation rate which are not present to the same extent in the data).

![Figure 2.12: Autocorrelation of inflation](image)

Given the focus of the present paper on the agent-based banking sector, it appears appropriate to
give a closer examination to the role of the banking sector and particularly bank heterogeneity
in producing the observed model dynamics. The banking sector clearly is an important driver
of macroeconomic fluctuations in the model, with fluctuations in interest rates and credit avail-
ability impacting both investment and consumption expenditure. The interbank market, being
modelled in a strongly simplified form, plays a mostly passive role aimed at redistributing re-
serves among banks; the model by assumption does not allow for the possibility of defaults
and a freeze-up of the interbank market. The specific effect of including multiple banks with
heterogeneous balance sheet compositions in the model can be gauged rather simply through
a counterfactual experiment, by running a version of the model in which there is only a single
bank. Table 2.3 (where the numbers in parentheses represent the 95% confidence interval from

\[\text{Given the absence of competition, the loan and mortgage interest rate setting mechanism used by the single}
\text{bank is changed such that it no longer compares its rate to the average rate (since these will obviously always be equal) but rather increases the mark-up on loans (mortgages) by a stochastic amount if its revenues on loans (mortgages) have been growing in the recent past and decreases it if the latter have been falling.}\]
a Wilcoxon signed-rank test) shows that in the absence of bank heterogeneity, macroeconomic volatility increases strongly.\textsuperscript{16} Competition between banks, which leads to a redistribution of loan demand to banks which offer lower rates and are less likely to ration credit hence appears to have a stabilising influence on the system as it is able to stabilise the flow of credit to the private sector to a certain degree. With only a single bank, default shocks are no longer partly idiosyncratic as in the baseline but rather become systemic, meaning that overall they have a greater influence on interest rates and credit rationing.

Table 2.3: Standard deviations; baseline & no heterogeneity

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.01083 (0.01041; 0.01126)</td>
<td>0.00930 (0.00889; 0.00975)</td>
</tr>
<tr>
<td>No het.</td>
<td>0.02110 (0.02027; 0.02195)</td>
<td>0.01876 (0.01785; 0.01974)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Investment</th>
<th>CPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.03517 (0.03396; 0.03637)</td>
<td>0.00483 (0.00466; 0.00504)</td>
</tr>
<tr>
<td>No het.</td>
<td>0.08668 (0.08243; 0.09117)</td>
<td>0.01028 (0.00966; 0.01098)</td>
</tr>
</tbody>
</table>

2.5 Simulation Experiments

Having presented the baseline simulation, the current section reports a range of experiments which were carried out on the model. The first set focuses on the expectations formation and forecasting behaviour of banks while the second concerns the implementation of various policy tools.

2.5.1 Expectations & Forecasting

Due to the importance of banks’ balance sheet management for the observed model dynamics, a straightforward experiment to carry out is to replace banks’ adaptive expectations formation

\textsuperscript{16}The numbers in table 2.3 as well as all other tables below show the point-estimates and 95\% confidence intervals from a Wilcoxon signed-rank test on the simulated statistics across the 100 MC repetitions of the respective simulations. The numbers reported in table 2.2 on the other hand represent the unconditional means of the statistics which were used in the empirical calibration, explaining the slight difference between the baseline numbers reported in table 2.2 and those shown below.
process with the heterogeneous expectations and heuristic switching mechanism outlined in section 2.3.7. While banks’ expectations in the baseline model are not homogeneous insofar as banks may hold different expectations at any given point in time, all banks make use of the same expectations formation rule, namely the one given by equation (2.16). In this experiment, by contrast, they may switch between different expectations formation mechanisms. As was explained in the model description, banks must make forecasts about the composition of their own balance sheet, in particular their future capital buffer, to use as an input in their decision-making about credit rationing and dividend payments. Interestingly, however, an implementation of the mechanism described in Anufriev and Hommes (2012) for banks in the present version of the model appears to have little effect on simulation results. Banks do indeed switch between different heuristics and tend to prefer a mix between the strong and weak trend following rules with occasional use of the adaptive and the anchoring and adjustment rules. Notably, no individual dominant forecasting strategy appears to emerge.

Table 2.4 reports the means and standard deviations of banks’ errors in forecasting their own capital buffer in the baseline and under heterogeneous expectations with heuristic switching. While the heuristic switching case provides a clear improvement over simple adaptive ones in terms of the standard deviation of forecast errors, in the baseline model simple adaptive expectations by themselves turn out to be a fairly decent forecasting heuristic upon which the alternative heuristic cannot improve sufficiently to significantly alter the dynamics of the model. The decrease in the standard deviation of forecast errors shows that under heuristic switching, banks tend to make smaller mistakes. An examination of the simulation data shows that this leads to a small increase in the average level of bank profits and a decrease in their volatility, as well as a decrease in the incidence of credit rationing as banks make smaller mistakes in forecasting their own capital buffer. However, none of these differences are large enough to be statistically

---

17I follow both Dosi et al. (2017a) and Catullo et al. (2020) in varying the expectations formation mechanism of only one type of agent. While this allows me to more clearly isolate the effects of changes in the way in which banks form expectations, it may somewhat bias the results since other agents continue to make use of simple adaptive expectations.

18The parametrisation of the mechanism is as follows: $\psi_{ad} = 0.5, \psi_{aa} = 0.5, \psi_{df1} = 0.75, \psi_{df2} = 1.3$, intensity of choice = 5, memory parameter = 0.7. The functional forms exactly follow those suggested by Anufriev and Hommes (2012).

19Note in particular that average forecast errors of both adaptive expectations and heterogeneous expectations with heuristic switching are not significantly different from zero.
significant as, despite the large difference between the standard deviations shown in table 2.4, banks’ forecast errors are already fairly small even under simple adaptive expectations. Table 2.5 consequently shows that the introduction of heterogeneous expectations and heuristic switching for banks makes no significant difference for the simulated moments of the macroeconomic time-series. As discussed further below, the mixture of forecasting heuristics selected by banks based on the fitness criterion turns out to be so similar to adaptive expectations in the baseline model that simulation results change only very little.

Table 2.4: Forecast errors; baseline & heuristic switching

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-0.02693</td>
<td>2.78979</td>
</tr>
<tr>
<td></td>
<td>(-0.19648; 0.13969)</td>
<td>(2.61177; 2.98783)</td>
</tr>
<tr>
<td>Switching</td>
<td>-0.00486</td>
<td>0.37219</td>
</tr>
<tr>
<td></td>
<td>(-0.02211; 0.01260)</td>
<td>(0.35034; 0.39679)</td>
</tr>
</tbody>
</table>

Table 2.5: MC-average standard deviations; baseline & heuristic switching

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.01083</td>
<td>0.00930</td>
</tr>
<tr>
<td></td>
<td>(0.01041; 0.01126)</td>
<td>(0.00889; 0.00975)</td>
</tr>
<tr>
<td>Switching</td>
<td>0.01071</td>
<td>0.00910</td>
</tr>
<tr>
<td></td>
<td>(0.01030; 0.01110)</td>
<td>(0.00869; 0.00952)</td>
</tr>
<tr>
<td></td>
<td><strong>Investment</strong></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td>0.03517</td>
<td>0.00483</td>
</tr>
<tr>
<td></td>
<td>(0.03396; 0.03637)</td>
<td>(0.00466; 0.00504)</td>
</tr>
<tr>
<td>Switching</td>
<td>0.03501</td>
<td>0.00474</td>
</tr>
<tr>
<td></td>
<td>(0.03378; 0.03609)</td>
<td>(0.00461; 0.00489)</td>
</tr>
</tbody>
</table>

Instead of allowing banks to switch between the forecasting rules suggested by Hommes, I can instead also allow them to attempt to make forecasts using econometric methods, as is sometimes done in conventional models (see Evans and Honkapohja, 2001). Here I allow banks to estimate an AR(1) model of their own capital buffer using OLS regression on all past observations of their own capital buffer and make the forecast based on the estimated parameters of this model. Due to the use of OLS, this learning algorithm falls into the decreasing gain category (Evans and Honkapohja, 2001, Ch. 1), meaning that the period-to-period change in the
estimated parameters tends to decrease as more data is accumulated.

Table 2.6: Forecast errors; baseline & OLS-leaning

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-0.02693 (-0.19648; 0.13969)</td>
<td>2.78979 (2.61177; 2.98783)</td>
</tr>
<tr>
<td>OLS</td>
<td>0.01704 (-0.08766; 0.11795)</td>
<td>1.37412 (1.28921; 1.46873)</td>
</tr>
</tbody>
</table>

Table 2.7: MC-average standard deviations; baseline & OLS-learning

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.01083 (0.01041; 0.01126)</td>
<td>0.00930 (0.00889; 0.00975)</td>
</tr>
<tr>
<td>OLS</td>
<td>0.01088 (0.01049; 0.01131)</td>
<td>0.00926 (0.00879; 0.00969)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Investment</th>
<th>CPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.03517 (0.03396; 0.03637)</td>
<td>0.00483 (0.00466; 0.00504)</td>
</tr>
<tr>
<td>OLS</td>
<td>0.03571 (0.03446; 0.03699)</td>
<td>0.00492 (0.00473; 0.00512)</td>
</tr>
</tbody>
</table>

The result of this exercise is summarised in tables 2.6 and 2.7. As in the case of heuristic switching, the standard deviation of the forecast error decreases, though not to the same degree.

In terms of macroeconomic outcomes this experiment is little different from the previous one; OLS-learning is able to improve upon simple adaptive expectations in terms of the standard deviation of forecast errors but for the same reasons given above, the improvement makes little difference to model outcomes. As in Catullo et al. (2020), the use of alternative expectations formation mechanisms hence leads to more accurate outcomes at the individual level, but in the case of the present model the improvement does not appear sufficient to significantly affect macroeconomic outcomes.

The model contains two more points at which the behaviour of banks may be modified to allow for some more sophisticated prediction behaviour. Recall from the model description that in setting their lending rates, banks apply a mark-up over the sum of the central bank rate and the current default rate of loans/mortgages. Suppose that instead, banks attempt to estimate future
default probabilities and also use estimation techniques in order to make a decision on mark-up revisions. I implement this notion in the following way: For default probabilities, I once more suppose that the banks use AR(1) models estimated using OLS on all past observations of the default rates on loans and mortgages to make one-period ahead forecasts. With regard to the mark-ups, recall that banks use two criteria for revision, namely whether their revenues from each asset have been growing and whether their interest rates are high relative to those of other banks. I now allow them to make a forecast of their future revenues using an econometric model to make a judgement as to whether their revenues are going to increase over the next month with the proposed mark-up revision. In particular, each bank which is allowed to re-set its interest rates in a given period estimates the model:

\[ i_{L,t}^i = c + \varphi_1 \cdot i_{L,t-1}^i + \varphi_2 \cdot \bar{r}_{L,t-1} + \varphi_3 \cdot r_{L,t-1}^L + \varphi_4 \cdot (r_{L,t-1}^L)^2 + \varepsilon_t \]  

(2.26)

using all past observations, where \( i_L \) are revenues on loans and \( r_L \) is the interest rate on loans, with \( \bar{r}_L \) being its average prevailing across the banking sector. Banks hence attempt to estimate the effect of their own lending rate on their revenues, controlling for the average rate charged in the economy. The equivalent model is also estimated for mortgages. Based on the estimated coefficients \( \varphi_3 \) and \( \varphi_4 \) of equation (2.26) the bank then estimates potential gains in revenue from increasing or decreasing its mark-up. It does so by randomly drawing a change in the mark-up using the same distribution as in the baseline, and then, also taking into account its forecast of default probabilities, calculating the interest rate implied by increasing as well as decreasing the mark-up on loans (or mortgages) by that amount. The bank then uses the estimated parameters \( \varphi_3 \) and \( \varphi_4 \) to calculate whether an increased, decreased or unchanged mark-up is expected to produce the highest revenue and sets its mark-up accordingly.

In contrast to the previous experiments, this modification of the model does have a statistically significant impact on simulation outcomes. In particular, allowing the banks to use the new, arguably more sophisticated behavioural rules outlined above leads to a considerable reduction in the volatility of investment at the macroeconomic level as under the alternative behavioural rules, banks’ lending rates become less volatile. The standard deviation of consumption increases somewhat due in particular to a slight increase in the volatility of dividend payments.
from firms and especially banks whose profits appear to become less stable using the alternative interest rate setting mechanism. At the microeconomic level, as shown in table 2.9, banks’ profits on average decrease when they use the alternative method of forecasting revenues and defaults in setting their interest rates, suggesting that the simple heuristics used in the baseline model are superior to the more sophisticated ones implemented in this experiment. At the microeconomic level, this result is in line with those obtained by Dosi et al. (2017a), who find that in their model more sophisticated heuristics are generally less successful than simpler ones. Catullo et al. (2020), on the other hand, find that at the microeconomic level, the use of more sophisticated expectations formation mechanisms lead to increased profits for firms, which however leads to inferior outcomes at the macroeconomic level.

Table 2.8: MC-average standard deviations; baseline & alternative behavioural rules

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.01083 (0.01041; 0.01126)</td>
<td>0.00930 (0.00889; 0.00975)</td>
</tr>
<tr>
<td>Alternative</td>
<td>0.01076 (0.01043; 0.01115)</td>
<td>0.01024 (0.00984; 0.01070)</td>
</tr>
<tr>
<td>Investment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td>0.03517 (0.03396; 0.03637)</td>
<td>0.00483 (0.00466; 0.00504)</td>
</tr>
<tr>
<td>Alternative</td>
<td>0.03034 (0.02934; 0.03121)</td>
<td>0.00529 (0.00507; 0.00549)</td>
</tr>
</tbody>
</table>

Table 2.9: Bank profits (weekly); baseline & alternative behavioural rules

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>37.85227 (36.95857; 38.80818)</td>
<td>9.86132 (9.41118; 10.33429)</td>
</tr>
<tr>
<td>Alternative</td>
<td>29.88634 (28.45792; 31.42544)</td>
<td>12.83906 (11.91618; 13.84539)</td>
</tr>
</tbody>
</table>

The results of the above experiments raise several issues worth discussing. It may appear curious that replacing banks’ adaptive expectations formation mechanism with heterogeneous expectations and heuristic switching or OLS learning as in the first two experiments would have next to no effect on simulation results. One might suspect based on this result that banks’ expectations in fact do not play an important role in the model. This assertion can be tested by
a simulation in which banks, rather than form expectations about their capital buffer, simply assume that it will at all times be equal to its target. This leads banks to frequently make large forecast errors which in turn leads to increased volatility at the aggregate level. The robustness of the model to an implementation of heterogeneous expectations with heuristic switching or OLS learning for banks can be explained by the fact that in the present model, which is stationary and incorporates numerous rigidities imparting stickiness to endogenous variables, adaptive expectations, heuristic switching and OLS learning in fact yield very similar expectations. Figure 2.13 shows a comparison of banks’ average expectation of their capital buffer under the three different expectations formation rules (using a snapshot of 250 periods from one individual simulation for better legibility), demonstrating that the differences between the predictions of the three heuristics are minimal. Given the negligible differences between predictions, banks’ behaviour will be next to identical under all three heuristics, explaining that overall simulation results are also close to identical. By contrast, the modifications made to the model in the third experiment, particularly the way banks forecast their revenues, represents a major change in the behavioural rules underlying the model, making it unsurprising that simulation results are more strongly affected.

![Figure 2.13](image.png)

Figure 2.13: Comparing adaptive expectations, heuristic switching and OLS learning in the baseline

However, it also turns out that some minor modifications to the model can make it much more
sensitive to the expectations formation mechanism used by banks. In particular, one can construct an environment that is somewhat more unstable than the baseline by reducing the adjustment horizons of aggregate consumption, housing demand and investment from 12 periods to 4 and increasing the speed at which banks adjust their credit rationing and dividend payout behaviour such that they now attempt to hit their target for the capital adequacy ratio period-to-period rather than over time. The effect of this is to make the environment in which banks must form their expectations more unstable and unpredictable. This leads to an increase in macroeconomic volatility under any expectations formation mechanism.

Figure 2.14: Modified model GDP under adaptive expectations, heuristic switching and OLS learning

Figure 2.14 plots the MC averages of GDP for the three types of expectations formation together with 95% confidence intervals. As can be seen, OLS learning still delivers similar outcomes as adaptive expectations, but the use of heuristic switching puts the model on an explosive path. Across MC repetitions, it causes a collapse in output during the transient phase. This is due to excessive credit rationing by banks as a consequence of large forecast errors they make at the beginning of the simulation. The deep downturn leads to a large increase in government debt. The recovery from this collapse in turn appears to invariably lead the economy onto an explosive trajectory fuelled by a positive feedback loop between a growing stock of government debt and interest payments on government debt (which feed back on the real economy as they
form a part of household disposable income but also lead to ever-growing public deficits). This type of feedback mechanism between interest payments on government debt and the stock of government bonds is not uncommon in the SFC literature (see e.g. Lavoie and Reissl, 2019), but in the present context it is particularly strong.

The results of the experiments reported above build upon those obtained by Dosi et al. (2017a) and Catullo et al. (2020). The former find that alternative expectations formation mechanisms may yield inferior individual results and can act as a source of instability, while the alternative expectations formation mechanisms used by the latter are more accurate but still lead to inferior macroeconomic outcomes. The present paper underlines the context-dependence of such results. For instance, the K+S model used by Dosi et al. (2017a) represents a highly complex and fast-moving environment which also incorporates technological change and long-term growth. By contrast, the baseline model presented in this paper constructs a stationary and relatively stable environment in which alternative expectations formation mechanisms perform fairly well (although interestingly they do not outperform the simpler one by a great margin). If the environment is made more unstable, or the modification to the behavioural rules is more fundamental, however, the results move closer to those obtained by Dosi et al. (2017a). The framework used by Catullo et al. (2020) can also be regarded as somewhat more complex than the one presented in this paper, but they take great care to adapt and calibrate the alternative forecasting methods to the context in which they are used, leading them to be more accurate than naive expectations. These general results are very much in line with the concept of ecological rationality advanced by Gigerenzer (2008), which emphasises that the ‘rationality’ or suitability of a particular behavioural rule is always dependent on the context in which it is applied.

2.5.2 Policy

To conclude my investigation of the model, I conduct several policy experiments, starting from the unmodified baseline model. To begin with, I undertake a parameter sweep of the Taylor rule to analyse the effects of monetary policy. Recall that in the baseline, the central bank follows a
pure inflation-targeting policy rule. For the experiment, I generalise the policy rule to

$$r_{cb,d} = r_0 + \pi^e + \phi_{\pi} \cdot (\pi^e - \pi^t) + \phi_u \cdot (u_{cb}^e - u_n)$$  \hspace{1cm} (2.27)$$

meaning that the central bank can also react to gaps between expected capacity utilisation and its normal or conventional value. In the baseline, $\phi_{\pi} = 0.25$ so that the Taylor principle holds (recall that $\pi_t = 0$). I then simulate the model for a range of values for both parameters, the range being $-1$ to $1$ for $\phi_{\pi}$ and $0$ to $1.5$ for $\phi_u$ with step-size $0.25$ in both cases. All parameter combinations are simulated for $100$ MC-repetitions as in the baseline. Note that if $\phi_{\pi} < 0$, the Taylor principle does not hold and when $\phi_{\pi} = -1$ monetary policy does not react to inflation dynamics at all. Figures 2.15 and 2.16 show the response of the standard deviations of (filtered) real output and the (filtered) price-level to variations in $\phi_{\pi}$ (axis label $\pi$) and $\phi_u$ (axis label $u$) using heatmaps.

It can be seen that simulation results are fairly sensitive to changes in the parametrisation of the monetary policy rule. A look at the results concerning $\phi_{\pi}$ suggests that price level volatility is minimised around the value of $\phi_{\pi}$ in the baseline ($0.25$), with $\phi_u$ being close to $0$. Output volatility, on the other hand, is minimised then $\phi_u$ is close to zero while $\phi_u$ reacts moderately to utilisation gaps, suggesting a weak trade-off between price and output stabilisation. Overly strong reactions of monetary policy to output gaps, on the other hand, tend to lead to greater volatility in both output and inflation (indeed for high values of $\phi_u$ the model gives rise to extreme volatility or breaks down completely, which explains the missing observations in the plots). Similarly, very strong (but also very weak) reactions of monetary policy to inflation appear disadvantageous for macroeconomic stability.

Overall this means that, within this model, adherence to the Taylor principle is indeed helpful for price stabilisation, but a too strong response of monetary policy to inflation may also be disadvantageous. Overall, the generalised Taylor rule incorporating capacity utilisation as a proxy for output does not appear able to strongly improve on the outcomes of the baseline simulation in terms of limiting macroeconomic volatility.
Following the parameter sweep of the monetary policy rule, I experiment with several different policy tools aiming to increase macroeconomic stability relative to the baseline simulation, namely an activist fiscal policy, an alternative monetary policy rule, an endogenous maximum loan-to-value ratio on mortgages, and an endogenous target capital adequacy ratio. Real government expenditure, which is constant in the baseline, is endogenised as

\[ g = g_0 \cdot (1 - \hat{c})^{2.5}, \]  

(2.28)
where $\hat{c}$ is the annualised growth rate of private consumption over the preceding quarter. The government sets a target level of expenditure according to the equation above on a quarterly basis and then gradually adjusts its spending to the desired level over the next quarter. The monetary policy rule is amended with the specific aim of stabilising investment expenditure. The central bank continues to re-set its interest rate on a monthly basis, but does so according to

$$r_{cb,d} = r_0 + \pi^e + \phi_\pi \cdot (\pi^e - \pi^t) + 0.15 \cdot \hat{i}_d,$$

(2.29)

where $\hat{i}_d$ is the annualised growth rate of private investment demand over the previous month. The endogenous maximum loan-to-value ratio on mortgages is implemented as

$$LTV = LTV^* \cdot \frac{2}{1 + e^{10(\bar{p}_h - p_h^*)}},$$

(2.30)

where $LTV^*$ is the fixed maximum loan-to-value ratio from the baseline, $\bar{p}_h$ is the average price of housing over the past month and $p_h^*$ is a target level for the house price, set equal to the value of the price of housing prevailing in the initial stationary state for convenience. It is assumed that the central bank resets a ‘target’ maximum LTV according to the equation above once every month and then adjusts the maximum LTV to this target over the following 4 periods. Finally, the endogenous target capital adequacy ratio for banks is given by

$$CAR^t = CAR^* \cdot \frac{2}{1 + e^{8(-lev_f - lev_f^*)}},$$

(2.31)

where $CAR^*$ is the fixed target capital adequacy ratio from the baseline, $\bar{lev}_f$ is the average leverage ratio of firms over the previous month and $lev_f^*$ is the fixed target leverage ratio used by firms in the baseline, which is here assumed to also be taken as a target by the central bank. Just as the endogenous maximum LTV ratio, the endogenous target capital adequacy ratio is re-set once every month by the central bank and then adjusted gradually to that level over the following 4 periods.
Table 2.10: MC-average standard deviations; baseline & policy tools

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
<th>Investment</th>
<th>CPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.01083</td>
<td>0.00930</td>
<td>0.03517</td>
<td>0.00483</td>
</tr>
<tr>
<td>Fiscal</td>
<td>0.00739*</td>
<td>0.00554*</td>
<td>0.03259*</td>
<td>0.00471</td>
</tr>
<tr>
<td>Monetary</td>
<td>0.00943*</td>
<td>0.00951</td>
<td>0.03030*</td>
<td>0.00865*</td>
</tr>
<tr>
<td>LTV</td>
<td>0.01022</td>
<td>0.00794*</td>
<td>0.03503</td>
<td>0.00482</td>
</tr>
<tr>
<td>CAR</td>
<td>0.00705*</td>
<td>0.00804*</td>
<td>0.02883*</td>
<td>0.00484</td>
</tr>
</tbody>
</table>

Table 2.10 summarises the effects of the four policy tools outlined above, implemented individually, in terms of their impact on macroeconomic volatility. Stars indicate standard deviations which become significantly lower (or higher) under the respective policy relative to the baseline, based on the 5% confidence intervals of a Wilcoxon signed rank test. As is common in the AB(-SFC) literature, fiscal policy turns out to be highly effective at reducing output volatility. As the fiscal policy rule is tied to the growth of private consumption, the reduction in output volatility is primarily achieved through a reduction in the volatility of consumption. The endogenous capital adequacy ratio also appears very effective as a tool to stabilise GDP, with the greatest impact being a reduction in the volatility of investment. The alternative monetary policy rule also significantly reduces the volatility of investment and GDP but does not significantly affect consumption and has the drawback of considerably increasing fluctuations in the price level. Finally, the endogenous maximum LTV ratio is able to decrease consumption volatility, but its impact is not quite strong enough for this to also translate into a significant reduction of output volatility.

As a final experiment I investigate the joint impacts of the policy tools considered above. For joint implementation, I reduce the strength of each individual tool as it turns out that when implemented jointly, the policy tools may themselves be a source of additional volatility if they are calibrated to react too strongly. Accordingly, I reduce the parameter in the fiscal policy rule from 2.5 to 1, the one in the alternative monetary policy rule from 0.15 to 0.1, the one for the endogenous maximum LTV from 10 to 5 and the one for the endogenous target capital adequacy
ratio from 8 to 4. Table 2.11 summarises the effects of jointly implementing the alternative monetary policy rule and the endogenous maximum LTV and target capital adequacy ratios at first without and then with the addition of an activist fiscal policy.

Table 2.11: MC-average standard deviations; baseline & policy mixes

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.01083</td>
<td>0.00930</td>
</tr>
<tr>
<td></td>
<td>(0.01041; 0.01126)</td>
<td>(0.00889; 0.00975)</td>
</tr>
<tr>
<td>MP, LTV &amp; CAR</td>
<td>0.00574</td>
<td>0.00651</td>
</tr>
<tr>
<td></td>
<td>(0.00557; 0.00591)</td>
<td>(0.00627; 0.00672)</td>
</tr>
<tr>
<td>MP, FP, LTV &amp; CAR</td>
<td>0.00496</td>
<td>0.00543</td>
</tr>
<tr>
<td></td>
<td>(0.00478; 0.00510)</td>
<td>(0.00526; 0.00563)</td>
</tr>
<tr>
<td>Investment</td>
<td>0.03517</td>
<td>0.00483</td>
</tr>
<tr>
<td></td>
<td>(0.03396; 0.03637)</td>
<td>(0.00466; 0.00504)</td>
</tr>
<tr>
<td>MP, LTV &amp; CAR</td>
<td>0.02233</td>
<td>0.00643</td>
</tr>
<tr>
<td></td>
<td>(0.02160; 0.02309)</td>
<td>(0.00623; 0.00662)</td>
</tr>
<tr>
<td>MP, FP, LTV &amp; CAR</td>
<td>0.02226</td>
<td>0.00651</td>
</tr>
<tr>
<td></td>
<td>(0.02154; 0.02302)</td>
<td>(0.00629; 0.00672)</td>
</tr>
</tbody>
</table>

It can be seen that the alternative monetary policy rule jointly with the two prudential policy tools can produce a considerable reduction in the volatility of output and its components, a result which becomes even stronger when fiscal policy is added to the mix. However, both policy packages lead to a significant increase in the volatility of the price level. Depending on the policy tool(s) used, there hence appears to be a trade-off between output and price-level/inflation-stabilisation in the model. Despite reducing overall macroeconomic volatility, both policy mixes somewhat increase short-term fluctuations in both wage and interest costs which translates into a more volatile price level. The overall conclusion of these experiments is that monetary, prudential and fiscal policies can, both individually and interacting in a mutually reinforcing manner, promote the attenuation of business cycles in real GDP and its components. However, such effects may come at the price of at least partly sacrificing other policy objectives, such as inflation control in the case of the present analysis. In addition, both the calibration and the timing of policy interventions, especially the frequency at which policies are altered and the speed at which they adjust play an important role in determining the success of a policy.
rule in this model. Too strong/weak or too frequent/infrequent intervention may well reduce the effectiveness of a particular policy or even turn it into a source of additional macroeconomic volatility. This suggests that prudential policies such as the ones tested here can be a valuable addition to policy-makers’ toolboxes and that they can interact favourably with monetary policy, but also that they must be very carefully calibrated and closely coordinated with other policy measures.

All policy experiments presented above were re-run under both heterogeneous expectations with heuristic switching and OLS learning for banks in the unmodified model. Just as in the baseline, the use of these two alternative heuristics does not have a significant impact on simulation outcomes, meaning that the presented policy interventions are able to produce equivalent reductions in macroeconomic volatility also under alternative expectations formation regimes. A somewhat different picture emerges when looking at the modified model, i.e. the one featuring a greater degree of volatility, which was discussed at the end of section 2.5.1. In section 2.5.1 it was shown that in the modified model, OLS learning leads to very similar results to those obtained under simple adaptive expectations. Re-running the policy experiments presented above under both OLS learning and adaptive expectations within the modified model reveals that in both cases, they deliver a similar degree of reduction in volatility as when they are applied in the unmodified model. None of the policy measures discussed above appear able, however, to contain the instability produced by the modified model under heterogeneous expectations which was shown in figure 2.14. In order to prevent the model from settling onto an explosive path, a different mix of policy interventions is necessary. For this experiment, the macro-prudential policy rules for the maximum LTV and target capital adequacy ratio presented above are combined with a generalised Taylor rule as in equation (2.27), with $\phi_u = 0.5$, and a fairly strong activist fiscal policy of the form

$$g = g_0 \cdot \left(\frac{\nu_t}{\bar{\nu}}\right)^4, \quad (2.32)$$

where $\bar{\nu}$ is a one-quarter average of industrial capacity utilisation. It is found that the use of this policy mix can indeed prevent the model from settling on the explosive path, as is demonstrated by figure 2.17, which shows the MC-averages of GDP with and without the application of the policy mix. By having fiscal and monetary policy target capacity utilisation while macro-
prudential policy stabilises financial variables, the policy mix is able to attenuate the initial deep downturns characterising all simulations of the modified model under heuristic switching, and hence to prevent the subsequent explosive dynamics. This last experiment hence indicates that the effectiveness of particular policy measures may be sensitive to the expectations formation heuristic used by agents and that if expectations trigger extreme fluctuations and instability, policy measures may have to be adapted relative to their use under ‘normal’ conditions.

Figure 2.17: Using policy to stabilise the modified model under heuristic switching

2.6 Conclusion

This paper presented a hybrid agent-based stock-flow-consistent macroeconomic model with a multi-agent banking sector and used it to investigate the effects of alternative assumptions regarding the expectations formation and forecasting behaviour of banks. In the baseline model, the use of heterogeneous expectations with heuristic switching and OLS learning to replace banks’ simple adaptive expectations about their future capital buffer produced little discernible difference in simulation results as the predictions of all three expectations formation mechanisms are virtually identical. A more fundamental alteration of banks’ behaviour, concerning the way in which they forecast their revenues under different interest rates, by contrast, turned out to produce inferior outcomes for banks in terms of their average profit. Moreover, it was shown that when the baseline model is altered to produce a somewhat more unstable and unpre-
dictable environment, alternative expectations formation heuristics can be a source of instability. These results build upon and contextualise those reached by Dosi et al. (2017a) and Catullo et al. (2020). It was shown here that simple heuristics can be remarkably robust and indeed produce superior macroeconomic outcomes in some circumstances, but the paper also underlined the context-dependence of such conclusions. This latter point is also strongly emphasised by Gigerenzer (2008) in his discussion of the concept of ‘ecological rationality’.

In addition, the paper presented a range of policy experiments involving prudential, fiscal and alternative monetary policy rules to analyse their effects on macroeconomic stability. It was shown that an endogenous maximum loan-to-value ratio on mortgages, an endogenous target capital adequacy ratio, an activist fiscal policy and an alternative monetary policy rule aiming to stabilise investment can all, to varying degrees, have a stabilising effect when implemented individually. Moreover, it was found that when implemented jointly, these policies can have a mutually reinforcing effect leading to a considerable reduction in macroeconomic volatility. These results, however, strongly depend on the timing and strength of policy interventions, and may also be sensitive to the expectations formation mechanism used by the banking sector. While a joint use of multiple policy tools can hence lead to superior outcomes in terms of macroeconomic stabilisation, policy interventions must be very carefully coordinated and designed in order for them to be successful.
Appendices

Appendix 2.A: Additional tables

Tables 2.11 and 2.12 below show the traditional balance sheet and transactions flow matrices which provide an overview of the aggregate SFC structure of the model.

Table 2.12: Balance Sheet Matrix

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Gov.</th>
<th>Central Bank</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank Deposits</td>
<td>+$D_h$</td>
<td>+$D_f$</td>
<td>−$D$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Reserves</td>
<td>+$R$</td>
<td></td>
<td>−$R$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>CB Advances</td>
<td>−$A$</td>
<td></td>
<td>+$A$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Gov. Bonds</td>
<td>+$GB_h$</td>
<td>−$GB$</td>
<td>+$GB_{cb}$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Mortgages</td>
<td>−$M$</td>
<td>+$M$</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Loans</td>
<td>−$L$</td>
<td></td>
<td>+$L$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Firms’ Equity</td>
<td>+$E_f$</td>
<td>−$E_f$</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Banks’ Equity</td>
<td>+$E_b$</td>
<td>−$E_b$</td>
<td></td>
<td></td>
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<td>0</td>
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<tr>
<td>Fixed Capital</td>
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<td>+$p \cdot k$</td>
<td></td>
<td></td>
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<td>$p \cdot k$</td>
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<tr>
<td>Houses</td>
<td>+$p_h \cdot h$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$p_h \cdot h$</td>
</tr>
<tr>
<td>Σ</td>
<td>$V_h$</td>
<td>$V_f$</td>
<td>$V_b$</td>
<td>$V_g$</td>
<td>$V_{cb}$</td>
<td>$p \cdot k + p_h \cdot h$</td>
</tr>
</tbody>
</table>
Table 2.13: Transactions Flow Matrix

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Government</th>
<th>Central Bank</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>$-C$</td>
<td>$+C$</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Investment</td>
<td>$+I$</td>
<td>$-I$</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Gov. Spending</td>
<td>$+G$</td>
<td></td>
<td></td>
<td></td>
<td>$-G$</td>
<td>0</td>
</tr>
<tr>
<td>Taxes</td>
<td>$-T ax_h$</td>
<td>$-T ax_f$</td>
<td></td>
<td></td>
<td>$+T ax$</td>
<td>0</td>
</tr>
<tr>
<td>Wages</td>
<td>$+W$</td>
<td></td>
<td></td>
<td></td>
<td>$-W$</td>
<td>0</td>
</tr>
<tr>
<td>Firm Dividends</td>
<td>$+Div_f$</td>
<td></td>
<td>$-Div_f$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Bank Dividends</td>
<td>$+Div_b$</td>
<td></td>
<td></td>
<td></td>
<td>$-Div_b$</td>
<td>0</td>
</tr>
<tr>
<td>CB profits</td>
<td></td>
<td></td>
<td>$+PCB$</td>
<td></td>
<td>$-PCB$</td>
<td>0</td>
</tr>
<tr>
<td>Interest Mortgage</td>
<td>$-iM$</td>
<td></td>
<td>$+iM$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Interest Loans</td>
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<td></td>
<td>$-iL$</td>
<td></td>
<td>$+iL$</td>
<td>0</td>
</tr>
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<td>Interest Deposits</td>
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<td>$+iD_f$</td>
<td>$-iD$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Int. Gov. Bonds</td>
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<td></td>
<td></td>
<td></td>
<td>$-iGB$ $+iGB_{cb}$</td>
<td>0</td>
</tr>
<tr>
<td>Int. CB advances</td>
<td></td>
<td></td>
<td>$-iA$</td>
<td></td>
<td>$+iA$</td>
<td>0</td>
</tr>
<tr>
<td>Int. on reserves</td>
<td></td>
<td></td>
<td>$+iR$</td>
<td></td>
<td>$-iR$</td>
<td>0</td>
</tr>
<tr>
<td>Saving</td>
<td>$(Sav_h)$</td>
<td>$-Sav_f$</td>
<td>$+Sav_f$</td>
<td>$(Sav_b)$</td>
<td>$(Sav_g)$ $+DA$</td>
<td>0</td>
</tr>
<tr>
<td>Δ Deposits</td>
<td>$-DA_h$</td>
<td>$-DA_f$</td>
<td>$+DA$</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Δ Gov. Bonds</td>
<td>$-ΔGB_h$</td>
<td></td>
<td></td>
<td></td>
<td>$+ΔGB$ $-ΔGB_{cb}$</td>
<td>0</td>
</tr>
<tr>
<td>Δ CB advances</td>
<td></td>
<td></td>
<td>$+ΔA$</td>
<td></td>
<td>$-ΔA$</td>
<td>0</td>
</tr>
<tr>
<td>Δ Reserves</td>
<td></td>
<td></td>
<td>$-ΔR$</td>
<td></td>
<td>$+ΔR$</td>
<td>0</td>
</tr>
<tr>
<td>Δ Mortgages</td>
<td>$+ΔM-M_{np}$</td>
<td></td>
<td></td>
<td>$-ΔM-M_{np}$</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Δ Loans</td>
<td></td>
<td>$+ΔL-L_{np}$</td>
<td></td>
<td>$-ΔL-L_{np}$</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Σ</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Appendix 2.B: Initialisation, parameter values and data sources

The sources of the data used to empirically calibrate the model are as follows:

- Real GDP (quarterly): Office for national statistics; Source dataset: QNA; CDID: ABMI
- Real consumption (quarterly): Office for national statistics; Source dataset: PN2; CDID: ABJR
- Investment (quarterly): OECD; Subject P51
- Price level/CPI (quarterly): Office for national statistics; Source dataset: MM23; CDID: D7BT

Table 2.14 below shows the values of all parameters and exogenous variables used in the baseline simulation. In addition it shows whether a given value is empirically calibrated (“emp”), imposed to produce the initial stationary state (“pre-SS”), implied by the stationary state (“SS-given”), or free. Where applicable, the range of values used for the sensitivity analysis is also shown. For parameters and initial values which need to be set “pre-SS” (i.e. they are needed to identify the initial stationary state rather than being implied by the latter or being calibrated empirically), I try where possible to use rough empirical values. Thus for instance, the fixed housing stock and the initial capital stock are set so as to roughly correspond to their empirical counterparts in the UK in 1995 Q1 according to the national balance sheet. Similarly, conditions such as the ratios of government consumption and capital investment to GDP, the labour share in GDP, depreciation and labour productivity are set to values close to their empirical counterparts. The conditions thus imposed are kept fixed across all simulations. Once a sufficient number of such conditions have been imposed, a large part of the remaining free parameters and initial values is implied by those already set together with the SFC structure and the assumption of a stationary state. Of the rest (category “free”), a subset is calibrated empirically while most others are subjected to a sensitivity analysis which is discussed in appendix 2.C.
| Symbol | Remark | Description | Value | Sensitivity range  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_U$</td>
<td>free</td>
<td>upper bound of MPC out of income</td>
<td>0.85</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>free</td>
<td>lower bound of MPC out of income</td>
<td>0.75</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\alpha_U$</td>
<td>free</td>
<td>upper bound of MPC out of wealth</td>
<td>0.055</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>free</td>
<td>lower bound of MPC out of wealth</td>
<td>0.045</td>
<td>N.A.</td>
</tr>
<tr>
<td>LTV</td>
<td>pre-SS</td>
<td>Maximum loan-to-value ratio</td>
<td>0.75</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\psi_{ad}$</td>
<td>free</td>
<td>Adaptation parameter in adaptive expectations</td>
<td>0.5</td>
<td>0.35 - 0.65 (0.025)</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>pre-SS</td>
<td>Inflation target</td>
<td>0</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>pre-SS</td>
<td>Capital depreciation rate (weekly)</td>
<td>0.002385</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>pre-SS</td>
<td>Labour productivity</td>
<td>24</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\theta$</td>
<td>pre-SS</td>
<td>Firms’ mark-up</td>
<td>0.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>$lev_f^l$</td>
<td>pre-SS</td>
<td>Firms’ target leverage</td>
<td>0.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>$r_0$</td>
<td>pre-SS</td>
<td>Taylor rule intercept</td>
<td>0.04</td>
<td>N.A.</td>
</tr>
<tr>
<td>$r_1$</td>
<td>pre-SS</td>
<td>interest corridor width</td>
<td>0.01</td>
<td>N.A.</td>
</tr>
<tr>
<td>$g_0$</td>
<td>SS-given</td>
<td>Government spending</td>
<td>779.3754</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>emp</td>
<td>Investment sensitivity to utilisation</td>
<td>0.0023</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>emp</td>
<td>Investment sensitivity to rate</td>
<td>0.001</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\varepsilon_{d1}$</td>
<td>pre-SS</td>
<td>deposit rate mark-up</td>
<td>-0.025</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\varepsilon_{d2}$</td>
<td>free</td>
<td>slope parameter to calculate clearing indicator</td>
<td>0.01</td>
<td>0.0025 - 0.025 (0.0025)</td>
</tr>
<tr>
<td>step</td>
<td>free</td>
<td>mean step-size of banks’ mark-up revision</td>
<td>0.005</td>
<td>0.002 - 0.008 (0.001)</td>
</tr>
<tr>
<td>$\sigma_{step}$</td>
<td>free</td>
<td>standard deviation of banks’ mark-up revision</td>
<td>0.00125</td>
<td>0.0005 - 0.0015 (0.00025)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>emp</td>
<td>Phillips curve slope</td>
<td>8</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>pre-SS</td>
<td>proportion of houses sold annually</td>
<td>0.08</td>
<td>N.A.</td>
</tr>
<tr>
<td>$CAR_t$</td>
<td>pre-SS</td>
<td>target capital adequacy ratio</td>
<td>0.1</td>
<td>N.A.</td>
</tr>
<tr>
<td>$LCR_t$</td>
<td>pre-SS</td>
<td>target liquidity coverage ratio</td>
<td>1</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>pre-SS</td>
<td>CAR risk weight on mortgages</td>
<td>0.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>pre-SS</td>
<td>CAR risk weight on loans</td>
<td>1</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>SS-given</td>
<td>LCR risk weight on deposits</td>
<td>0.03992782</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\chi M$</td>
<td>pre-SS</td>
<td>Mortgage repayment rate</td>
<td>0.05</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\zeta M$</td>
<td>SS-given</td>
<td>component of mortgage default rate</td>
<td>0.04</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\chi L$</td>
<td>SS-given</td>
<td>loan repayment rate</td>
<td>0.16896</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\zeta L$</td>
<td>SS-given</td>
<td>component of loan default rate</td>
<td>0.12</td>
<td>N.A.</td>
</tr>
<tr>
<td>$W_{n}$</td>
<td>SS-given</td>
<td>normal wage rate</td>
<td>14.56015</td>
<td>N.A.</td>
</tr>
<tr>
<td>$u_n$</td>
<td>pre-SS</td>
<td>normal capacity utilisation</td>
<td>0.8</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>SS-given</td>
<td>Housing demand intercept</td>
<td>336.4583</td>
<td>N.A.</td>
</tr>
</tbody>
</table>
Table 2.14 – continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Remark</th>
<th>Description</th>
<th>Value</th>
<th>Sensitivity range (step size)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>emp</td>
<td>Housing demand sensitivity to wealth</td>
<td>0.0008</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>SS-given</td>
<td>Housing demand sensitivity to LTV</td>
<td>606.9444</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>emp</td>
<td>Housing demand sensitivity to rate</td>
<td>570</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\lambda_{10}$</td>
<td>pre-SS</td>
<td>Household portfolio parameter</td>
<td>0.25</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\lambda_{11}$</td>
<td>pre-SS</td>
<td>Household portfolio parameter</td>
<td>4</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\lambda_{12}$</td>
<td>$\lambda_{11}$</td>
<td>Household portfolio parameter</td>
<td>4</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\tau_{au}$</td>
<td>SS-given</td>
<td>Tax rate on YD</td>
<td>0.2147787</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\sigma_{MPC}^1$</td>
<td>emp</td>
<td>Sensitivity of MPC to return rate</td>
<td>11</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\sigma_{MPC}^2$</td>
<td>SS-given</td>
<td>MPC shift parameter</td>
<td>0.08317182</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\sigma_{IB}$</td>
<td>free</td>
<td>Sensitivity of interbank rate to excess supply/demand</td>
<td>0.01</td>
<td>0.0025 - 0.02 (0.0025)</td>
</tr>
<tr>
<td>$i_1$</td>
<td>free</td>
<td>Sensitivity of bank share to rel. rate</td>
<td>0.25</td>
<td>0.15 - 0.35 (0.025)</td>
</tr>
<tr>
<td>$i_2$</td>
<td>free</td>
<td>Sensitivity of bank share to rationing</td>
<td>0.25</td>
<td>0.15 - 0.35 (0.025)</td>
</tr>
<tr>
<td>$AR_{dis}$</td>
<td>free</td>
<td>Persistence of shocks to bank market share</td>
<td>0.825</td>
<td>0.75 - 0.9 (0.025)</td>
</tr>
<tr>
<td>$\sigma_{dis}$</td>
<td>free</td>
<td>Variance of shocks to bank market share</td>
<td>0.0005</td>
<td>2.5e-04 - 7.5e-04 (5e-04)</td>
</tr>
<tr>
<td>$AR_{def}$</td>
<td>emp</td>
<td>Persistence of default shocks</td>
<td>0.98</td>
<td>N.A.</td>
</tr>
<tr>
<td>$s_{def}$</td>
<td>emp</td>
<td>Scale parameter of default distribution</td>
<td>2.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>free</td>
<td>Taylor rule inflation sensitivity</td>
<td>0.25</td>
<td>N.A.</td>
</tr>
<tr>
<td>$CC_{def}$</td>
<td>free</td>
<td>Cross correlation of default shocks</td>
<td>0.5</td>
<td>0.3 - 0.7 (0.05)</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>free</td>
<td>Upper bound for rationing indicator</td>
<td>2</td>
<td>1 - 3 (0.25)</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>free</td>
<td>Sensitivity of rationing indicator</td>
<td>0.25</td>
<td>0.1 - 0.5 (0.1)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>SS-given</td>
<td>Capital to output ratio</td>
<td>1.218925</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

Table 2.15 below shows the aggregate initial values which are needed to initialise the model for the simulations shown in the paper. Variables pertaining to banks (e.g. stocks such as deposits, loans, mortgages etc. but also flows such as interest payments or profits) are set by imposing an initial market share for each bank (assumed equal in all markets) and then distributing each stock and flow according to these shares. The shares assumed here for the twelve banks are 0.13, 0.11, 0.11, 0.1, 0.09, 0.08, 0.07, 0.07, 0.06, 0.06, 0.06 and 0.06. Due to the way the model is set up, all banks offer equal rates on loans and deposits in the initial, deterministic stationary state. Initial values for flows refer to weekly values in all cases.
### Table 2.15: Initial values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Remark</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>pre-SS</td>
<td>Capital stock</td>
<td>300000</td>
</tr>
<tr>
<td>$h$</td>
<td>pre-SS</td>
<td>Housing stock</td>
<td>475000</td>
</tr>
<tr>
<td>$D_h$</td>
<td>SS-given</td>
<td>Household deposits</td>
<td>139932.3</td>
</tr>
<tr>
<td>$M$</td>
<td>SS-given</td>
<td>Mortgages</td>
<td>356250</td>
</tr>
<tr>
<td>$p_h$</td>
<td>pre-SS</td>
<td>House price</td>
<td>1</td>
</tr>
<tr>
<td>$L$</td>
<td>SS-given</td>
<td>Bank loans</td>
<td>150000</td>
</tr>
<tr>
<td>$p$</td>
<td>pre-SS</td>
<td>Price level</td>
<td>1</td>
</tr>
<tr>
<td>$V_h$</td>
<td>SS-given</td>
<td>Household wealth</td>
<td>500563.3</td>
</tr>
<tr>
<td>$lev_h$</td>
<td>pre-SS</td>
<td>ratio of mortgages to housing stock</td>
<td>0.75</td>
</tr>
<tr>
<td>$c$</td>
<td>SS-given</td>
<td>Consumption</td>
<td>2607.1</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>SS-given</td>
<td>MPC out of income</td>
<td>0.8</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>SS-given</td>
<td>MPC out of wealth</td>
<td>0.05</td>
</tr>
<tr>
<td>$H_d^n$</td>
<td>SS-given</td>
<td>Notional housing demand</td>
<td>791.6667</td>
</tr>
<tr>
<td>$u$</td>
<td>SS-given</td>
<td>Capacity utilisation</td>
<td>0.8</td>
</tr>
<tr>
<td>$W$</td>
<td>SS-given</td>
<td>Nominal wage</td>
<td>14.56015</td>
</tr>
<tr>
<td>$lev_f$</td>
<td>SS-given</td>
<td>Firm leverage</td>
<td>0.5</td>
</tr>
<tr>
<td>$y_{fc}$</td>
<td>SS-given</td>
<td>Full-capacity output</td>
<td>5127.47</td>
</tr>
<tr>
<td>$Pr_f$</td>
<td>SS-given</td>
<td>Firm profit</td>
<td>839.3253</td>
</tr>
<tr>
<td>$div_f$</td>
<td>SS-given</td>
<td>Firm dividends</td>
<td>839.3253</td>
</tr>
<tr>
<td>$sav_f$</td>
<td>SS-given</td>
<td>Firm target retained earnings</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_d$</td>
<td>SS-given</td>
<td>Average deposit rate</td>
<td>0.02</td>
</tr>
<tr>
<td>$gb_h$</td>
<td>SS-given</td>
<td>Gov. bonds held by households</td>
<td>59068.45</td>
</tr>
<tr>
<td>$Tax_h$</td>
<td>SS-given</td>
<td>Households’ tax payments</td>
<td>834.8009</td>
</tr>
<tr>
<td>$gb_{cb}$</td>
<td>pre-SS</td>
<td>Gov. bonds held by CB</td>
<td>19689.48</td>
</tr>
<tr>
<td>$r_{gb}$</td>
<td>SS-given</td>
<td>Interest rate on gov. bonds</td>
<td>0.03170631</td>
</tr>
<tr>
<td>$r_{cb}^l$</td>
<td>SS-given</td>
<td>CB lending rate</td>
<td>0.05</td>
</tr>
<tr>
<td>$r_{IB}$</td>
<td>SS-given</td>
<td>Interbank rate</td>
<td>0.045</td>
</tr>
<tr>
<td>$L_s$</td>
<td>SS-given</td>
<td>Supply of loans</td>
<td>715.5</td>
</tr>
<tr>
<td>$M_s$</td>
<td>SS-given</td>
<td>Supply of mortgages</td>
<td>593.75</td>
</tr>
<tr>
<td>$YD$</td>
<td>SS-given</td>
<td>Disposable income</td>
<td>2607.1</td>
</tr>
<tr>
<td>$rr_h$</td>
<td>SS-given</td>
<td>Real return rate on HH assets</td>
<td>0.006930985</td>
</tr>
<tr>
<td>$L_d$</td>
<td>SS-given</td>
<td>Demand for loans</td>
<td>715.5</td>
</tr>
<tr>
<td>$M_d$</td>
<td>SS-given</td>
<td>Demand for mortgages</td>
<td>593.75</td>
</tr>
<tr>
<td>$y$</td>
<td>SS-given</td>
<td>Real GDP</td>
<td>4101.976</td>
</tr>
<tr>
<td>$D_f$</td>
<td>SS-given</td>
<td>Firm</td>
<td>353194.7</td>
</tr>
<tr>
<td>$i_L$</td>
<td>SS-given</td>
<td>Interest payments on loans</td>
<td>393.2578</td>
</tr>
<tr>
<td>$\tau_L$</td>
<td>SS-given</td>
<td>Average rate on loans</td>
<td>0.126</td>
</tr>
<tr>
<td>$\bar{r}_M$</td>
<td>SS-given</td>
<td>Average rate on mortgages</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Table 2.15 – continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Remark</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>SS-given</td>
<td>Stock of reserves</td>
<td>19689.48</td>
</tr>
<tr>
<td>$V_{bb}$</td>
<td>SS-given</td>
<td>Bank capital buffer (aggregate)</td>
<td>32812.5</td>
</tr>
<tr>
<td>$P_{Pr}$</td>
<td>SS-given</td>
<td>Bank profits (aggregate)</td>
<td>461.5912</td>
</tr>
<tr>
<td>$Div_{b}$</td>
<td>SS-given</td>
<td>Bank dividends (aggregate)</td>
<td>461.5912</td>
</tr>
</tbody>
</table>

Appendix 2.C: Sensitivity analysis

I conduct a basic sensitivity analysis on those 12 parameters for which a sensitivity range is shown in table 2.14. This is done by varying the value of each parameter, one by one, along the range and according to the step sizes shown in the table. I simulate each parameter configuration for 100 Monte Carlo repetitions and compare the results to the baseline by inspecting time-series plots as well as the volatility of the time series which were used in calibrating the model. The results for variations in each parameter are discussed below in turn. Results indicate that most of the non-empirically calibrated parameters analysed here have little influence on model dynamics if varied along the ranges considered, suggesting that the choice of parameters for the empirical calibration procedure was broadly appropriate.

$\psi_{ad}$: In contrast to varying only the expectations mechanism of banks, as was done in the experiments above, jointly varying the adaptation parameter in adaptive expectations for all sectors (including banks) at once has a slight effect on macroeconomic volatility. A larger (smaller) value of $\psi_{ad}$ leads tends to increase (decrease) fluctuations as expectations which feed into the determination of various decision-variables become more (less) sensitive to forecast errors.

$\varepsilon_{d2}$: A higher value of $\varepsilon_{d2}$ than in the baseline implies a greater sensitivity of the deposit rates offered by banks to their clearing position. Overall this increases the range of variation in deposit interest rates and also leads to greater short-term fluctuations in deposit rates. This in turn translates into a slight increase in macroeconomic volatility. In the case of a lower value for $\varepsilon_{d2}$ than in the baseline, the opposite applies.

$\sigma_{IB}$: An increase (decrease) in $\sigma_{IB}$, the sensitivity of the interbank interest rate to excess demand or supply on the interbank market obviously increases (decreases) the volatility of the interbank rate.
Beyond this, however, there is no noticeable effect on model dynamics, which is in line with the passive role played by the interbank market in the model.

$\iota_1$ : $\iota_1$ determines the sensitivity of banks’ market shares to interest rate differentials. Consequently, a higher value of $\iota_1$ leads to larger variations in market shares but for the range of values considered does not give rise to persistent monopolisation tendencies. The effects of varying $\iota_1$ on macroeconomic dynamics are slight, with higher (lower) values somewhat increasing (decreasing) the volatility of the price level due to larger variations in bank interest rates as a result of stronger (weaker) price competition.

$\iota_2$ : At the level of individual banks, the effects of variations in $\iota_2$, which determines the sensitivity of banks’ market shares in loans and mortgages to their history of credit rationing, are similar to those caused by varying $\iota_1$. However, there is no significant effect on macroeconomic volatility for the range of values used here.

$\text{AR}_{\text{dis}}$ : An increase or decrease in the persistence of shocks to the distribution of deposits and loan demand between banks does not appear to have any systematic impact on simulation outcomes for the range of values of the parameter which are considered here.

$\sigma_{\text{dis}}$ : $\sigma_{\text{dis}}$ denotes the standard deviation of shocks to the market shares of banks. Similarly to the effect of varying the persistence of these shocks, varying $\sigma_{\text{dis}}$ along the range of values considered here has no significant impact on simulation outcomes.

$\text{CC}_{\text{def}}$ : As one might suspect, an increase (decrease) in the cross-correlation of default shocks among banks significantly increases (decreases) macroeconomic volatility. More systemic fluctuations in defaults produce an increased volatility of interest rates as well as greater correlation in the fluctuations of individual banks’ capital adequacy ratios, both of which feed back on the aggregate sectors and ultimately lead all macro time-series to become more volatile.

$\text{step}$ : $\text{step}$ gives the mean value of the normal distribution which banks use to draw mark-up revisions when changing their interest rates on loans and mortgages. Decreasing or increasing this mean value along the range indicated above has no significant impact on simulation outcomes.
\( \sigma_{\text{step}} \) is the standard deviation of the normal distribution which banks use to draw mark-up revisions when changing their lending rates. Varying the value of this parameter, similarly to what was found for \( \text{step} \), does not significantly alter simulation results.

\( \xi_1 \) gives the upper bound of the rationing indicators on loans and mortgages calculated in equation (2.16), which feed into the distribution of loan and mortgage demand between banks. Varying this parameter has no effect on simulation results, suggesting that the indicators never reach their upper bound in the simulations considered.

\( \xi_2 \) measures the sensitivity of the credit rationing indicators to the intensity with which a bank rationed credit in the past. Varying this parameter along the range indicated above has no significant impact on model dynamics.
Chapter 3

Minsky from the bottom up - Formalising the two-price model of investment in a simple agent-based framework

3.1 Introduction

Since the mid-1980s there has been a steadily growing literature seeking to capture central notions of the economic ideas of Hyman Minsky, in particular his financial instability hypothesis (FIH; e.g. Minsky, 1977, 1978), in formal mathematical models. This literature has grown even more rapidly following the global financial crisis, which has sparked renewed interest in the modelling of real-financial interactions in both mainstream and non-mainstream macroeconomics. There is however comparatively little literature building directly on the two-price model of capital investment (as presented e.g. in Minsky, 1975 and Minsky, 1986). Minsky argues that there are two distinct prices of capital goods, namely a demand price \( P_k \) and a supply price \( P_l \). The demand price is a function of the prospective yield on existing capital goods, whereas the supply price denotes the cost of production and acquisition (financing cost) of an additional unit of capital. Minsky argues that firms will invest until the demand and supply prices are equal, at first using internal funds and then resorting to external finance. Eventually, the demand price will begin to fall away from \( P_k \) with rising investment due to an increased perception of risk on the part of borrowers. The supply price will increase as investment beyond internal funds will incur a financing cost which, due to increased perception of risk on the part of

1This chapter was originally published as Reissl, S. (2020), “Minsky from the bottom up - Formalising the two-price model of investment in a simple agent-based framework”, Journal of Economic Behavior and Organization, 177, pp. 109-142.

2The two-price model hence bears clear resemblance to Tobin’s model of investment based on the q-ratio (Tobin, 1969). However, as pointed out by Wray and Tymoigne (2009), there are important differences related to the role of financing conditions and uncertainty/risk assessment.
lenders, will rise as investment increases further. Increases in aggregate demand and internal finance as a result of aggregate increases in investment (through the Kaleckian profit equation; Minsky, 1986, Ch. 7), together with loose financing conditions and declining margins of safety can lead to a self-reinforcing upward trend in investment which is eventually reversed when expectations about yields are no longer confirmed by actual outcomes and/or financing conditions suddenly deteriorate. This paper presents a simple agent-based stock-flow consistent (AB-SFC) model featuring three strongly simplified aggregated sectors along with an agent-based sector of consumption goods firms which invests in capital goods based on a detailed formalisation of the two-price model as broadly outlined above. It is shown that the resulting model gives rise to cycles in GDP which are driven by the investment and financing decisions of consumption goods firms, with a key role being played by dividend payouts based on a variable retention ratio.

Figure 3.1: The two-price model of investment; adapted from Minsky (1986, p.213)

Nikolaidi and Stockhammer (2017) provide a comprehensive survey of the literature on Minsky models, including existing papers drawing on the two-price model. For instance, Delli Gatti and Gallegati (1990) invoke the two-price model to justify the formulation of an aggregate investment function in which investment is a function of the gap between the demand and supply price as well as available internal financing. Delli Gatti and Gallegati (1992), Delli Gatti et al. (1994a) and Delli Gatti et al. (1994b) - the latter of which includes Minsky himself as a co-author - build on the aforementioned model and include

---

3The issue of what exactly brings about the turning point in the investment boom is somewhat controversial, see e.g. Lavoie (2014, Ch. 4) and Toporowski (2005, Ch. 14).
similar formulations for the aggregate investment function. An investment function along similar lines can also be found in Nasica and Raybaut (2005), and the formalisation of investment demand advanced by Taylor and O’Connell (1985) is also motivated by the two-price model. All of these models are macrodynamic ones without an explicit microeconomic dimension. In a series of papers, Chiarella and Di Guilmi (2011, 2012a,b, 2014, 2017) present a family of simple Minsky models building on the approach of Taylor and O’Connell (1985) but containing an agent-based firm sector. Similarly to the latter paper, the investment behaviour of firms in these models is based on a strongly simplified form of the two-price model, whereby investment is a function of a ‘shadow price’ of capital which depends on animal spirits (derived from a stylised stock market) and the interest rate.

The present paper contributes to the literature in four main ways. Firstly, instead of providing a simplified and reduced formulation, this paper contains a full and detailed dynamic formalisation of the two-price model at the level of individual firms very closely in line with the arguments advanced by Minsky. While none of the individual factors influencing investment in the present model are novel to the literature, I argue that my detailed formalisation leads to a richer investment behaviour than is incorporated in most existing models, determined by the non-linear interactions between multiple factors including firms’ expected revenue, available internal financing and external financing conditions. Many existing papers drawing on the two-price model, such as Taylor and O’Connell (1985) or Delli Gatti et al. (1994b) instead rely on much less complex investment behaviour and more generally a strongly reduced formalisation of the two-price model. I also demonstrate that the inclusion of an agent-based firm sector adds a useful dimension, allowing me to analyse interesting dynamics at the microeconomic level which could not be gleaned from looking exclusively at macroeconomic data. For instance, it is shown that the model gives rise to a realistic firm size distribution. Moreover, the modelling of a disaggregated firm sector allows for the incorporation of the Minskyan trichotomy of hedge, speculative and Ponzi financing units, and it is shown that the distribution of financial stability metrics changes substantially over the cycle, frequently exhibiting multiple peaks. While, as discussed in appendix 3.B, an aggregated version of the model incorporating the same non-linearities as the baseline unsurprisingly also gives rise to cycles, the major point is that even when firms’ decisions do not take place at the aggregate level, their individual uncoordinated investment behaviour is still sufficiently correlated to give rise to smaller but regular cycles. Along with the series of papers by Chiarella and Di Guilmi cited above, the paper hence demonstrates the added value of a heterogeneous agent approach in Minsky models which, as noted by Nikolaidi and Stockhammer (2017), is somewhat underdeveloped.
Secondly, the general framework of the two-price model of capital investment as formalised here is quite flexible in that it allows for the incorporation of a range of behavioural assumptions through slight modifications to what is presented as the baseline model. I demonstrate this in particular through experiments in which the mechanism which firms use to form expectations about the yield on capital goods is varied. An introduction of a simple form of sentiment dynamics and switching behaviour, strengthening strategic complementarities in investment decisions, significantly exacerbates macroeconomic fluctuations, while the introduction of a share of ‘fundamentalist’ firms has a stabilising effect. These experiments highlight the importance of expectations and opinion formation in macroeconomic dynamics, an aspect which is relatively under-represented in the existing Minskian literature. Moreover I shall argue, in line with Fazzari (1992), that Minskyan business cycle theory building on the two-price model contains elements which may allow for a partial bridging of the gap between mainstream (financial accelerator) and non-mainstream theories of real-financial cycles. I show that when formalised in detail, the two-price model contains various elements, such as a variable external finance premium and a key role for fluctuations in the financial solidity of firms, which can be found in nearly equivalent form in the financial accelerator literature. Given the attention paid to expectations formation and bounded rationality (Evans and Honkapohja, 2001; Hommes, 2013) as well as financial factors (Brunnermeier et al., 2013) in the more recent macroeconomic literature, the paper should hence also be of interest to researchers beyond those specifically concerned with possible ways to formalise the FIH.

Thirdly, the paper presents a range of policy experiments aimed at reducing the macroeconomic volatility produced by the baseline model. While the result that fiscal policy can have a strongly stabilising effect in a Minskyan system is hardly novel to the literature, it is also shown that conventional monetary policy can have a beneficial effect. This is interesting insofar as interest rate policy as a stabilisation tool does not feature prominently in Minsky’s own writings on the FIH and only few existing works such as Chiarella and Di Guilmi (2017) and Charpe et al. (2011, Ch. 9) conduct experiments involving interest rate policy. Finally, in an experiment which, - to the best of my knowledge - is novel to the Minskyan literature, I show that the imposition of an upper limit on firms’ dividend payout ratios can have a strongly stabilising effect. This result is very much in line with Minsky’s view of the macroeconomic role of dividends and suggests that restrictions on dividend payments may be a useful additional policy tool in the promotion of economic and financial stability.

Fourthly, using the method of simulated moments, the model presented in this paper is calibrated empirically on a set of statistics drawn from US macroeconomic data, demonstrating that it is capable
of producing plausible macroeconomic dynamics. As noted by Nikolaidi and Stockhammer (2017, p. 1327), the literature on Minsky models is dominated by theoretical exercises and “the vast majority of the authors make no attempt to estimate econometrically the key equations of their models or to calibrate the models in order to produce the patterns observed in the real data”. There exists a small literature of econometric papers testing particular aspects of Minsky’s FIH (e.g. Greenwood-Nimmo and Tarassow, 2013; Nishi, 2019), but there is no systematic connection between these and the theoretical models proposed in the literature. The present paper hence represents a first attempt to bring a relatively simple Minsky model to the data, which contributes to the credibility and relevance of the Minskyan approach to modelling business cycles.

The remainder of this paper is structured as follows. Section 3.2 outlines the model. Section 3.3 describes the calibration procedure and presents the baseline simulation while section 3.4 contains the simulation experiments. Section 3.5 concludes. Parameter and initial values used in the simulations can be found in appendix 3.A. Appendix 3.B contains a sensitivity analysis.

### 3.2 Model outline

Tables 3.1 and 3.2 provide an overview of the aggregate sectoral structure and the accounting relationships depicted in the model.

<table>
<thead>
<tr>
<th>Households</th>
<th>C-firms</th>
<th>K-firms</th>
<th>Banks</th>
<th>Gov.</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank Deposits</td>
<td>+D&lt;sub&gt;h&lt;/sub&gt;</td>
<td>+D&lt;sub&gt;f&lt;/sub&gt;</td>
<td>−D&lt;sub&gt;c&lt;/sub&gt;</td>
<td>+D&lt;sub&gt;g&lt;/sub&gt;</td>
<td>0</td>
</tr>
<tr>
<td>Loans</td>
<td>−L</td>
<td>+L</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bonds</td>
<td>+GB</td>
<td>−GB</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firms’ Equity</td>
<td>+E&lt;sub&gt;f&lt;/sub&gt;</td>
<td>−E&lt;sub&gt;f&lt;/sub&gt;</td>
<td>−E&lt;sub&gt;k&lt;/sub&gt;</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Banks’ Equity</td>
<td>+E&lt;sub&gt;b&lt;/sub&gt;</td>
<td>−E&lt;sub&gt;b&lt;/sub&gt;</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>+K</td>
<td>K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Σ</td>
<td>V&lt;sub&gt;h&lt;/sub&gt;</td>
<td>V&lt;sub&gt;f&lt;/sub&gt;</td>
<td>V&lt;sub&gt;k&lt;/sub&gt;</td>
<td>V&lt;sub&gt;b&lt;/sub&gt;</td>
<td>V&lt;sub&gt;g&lt;/sub&gt;</td>
</tr>
</tbody>
</table>

As can be seen, the model consists of five sectors, namely households, capital goods firms (K-firms), banks, a government (which is inactive in the baseline) and consumption goods firms (C-firms), of which only the latter is modelled using an agent-based methodology whilst the rest remains aggregated (the
The C-firm sector is assumed to consist of 50 firms.

### Table 3.2: Transactions Flow Matrix

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>C-firms</th>
<th>K-firms</th>
<th>Banks</th>
<th>Gov.</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>−C</td>
<td>+C</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Investment</td>
<td>−I</td>
<td>+I</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Wages</td>
<td>+W</td>
<td>−W_c</td>
<td>−W_k</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Dividends</td>
<td>+Div</td>
<td>−Div_f_c</td>
<td>−Div_f_k</td>
<td>−Div_h</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Transfer</td>
<td>+T</td>
<td></td>
<td></td>
<td>−T</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Interest Loans</td>
<td>−iL</td>
<td></td>
<td>iL</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Interest Bonds</td>
<td>iGB</td>
<td></td>
<td>−iGB</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Saving</td>
<td>(Sav_h)</td>
<td>(Sav_f_c)</td>
<td>(Sav_f_k)</td>
<td>(Sav_h)</td>
<td>(Sav_g)</td>
<td>0</td>
</tr>
<tr>
<td>∆ Deposits</td>
<td>−∆D_h</td>
<td>−∆D_f_c</td>
<td>+∆D</td>
<td>−∆D_g</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>∆ Loans</td>
<td>+∆L</td>
<td></td>
<td>−∆L</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>∆ Bonds</td>
<td>0</td>
<td></td>
<td>−∆GB</td>
<td>+∆GB</td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Households hold bank deposits and own the firms and banks. They receive all wage and dividend incomes and have expenditures on consumption goods. Consumption goods firms produce consumption goods and invest in capital goods, with part of this investment being financed by loans from the banking sector. They may also hold bank deposits. Capital goods firms produce capital goods but hold no assets or liabilities. Banks give loans, may hold government bonds and receive deposits. The government (which is activated as part of the policy experiments discussed in section 3.4.2) issues bonds, may hold deposits and makes/receives transfer payments to/from the household sector. Despite the presence of a capital stock and investment, the economy depicted here is a stationary one in that labour and capital productivities are constant and there is no long-term growth.

---

4The abstraction from long-term growth in favour of an exclusive focus on business-cycle-like fluctuations is not uncommon in the literature on Minsky models (examples include Delli Gatti and Gallegati, 1990, Kapeller and Schütz, 2014, Di Guilmi and Carvalho 2017 and Jump et al., 2017) and in many other cases, long-term growth simply takes the form of an exogenous trend around which the model fluctuates and which is not linked to the Minskyan elements of the model (as e.g. in Keen, 1995 and Fazzari et al., 2008). In my view the inclusion of such an exogenous trend would add little insight. More broadly, I do not believe that cycles and long-term growth can in fact be fully separated, and the modelling of an explicit link between Minskyan mechanisms and technological change and/or long-term trend growth rates would be very interesting. However, this is beyond the scope of the present paper. Ryoo (2010) presents an interesting example of interactions between short and long-run fluctuations within a Minskyan framework.
3.2.1 Sequence of events

Over the course of one simulation period (where one period is intended to represent one quarter), the following sequence of steps takes place:

1. C-firms calculate capital depreciation and loan payments due at the end of the period. The bankruptcy conditions outlined below are checked; bankrupt firms exit the model and are replaced by new ones.

2. C-firms set their prices for the current period. An aggregate consumption price index is calculated.

3. Households set their consumption budget. Consumption demand is then distributed between C-firms which produce on demand and pay wages.

4. Based on the results of the consumption process, C-firms form their expectations about the prospective yield on capital goods, determine their internal financing and consequently formulate their investment demand and the implied demand for loans. Capital goods are produced on demand. The investment process also determines the interest rates charged to C-firms.

5. C-firms, Banks and K-firms calculate their profits and make dividend payments for the period. All remaining accounting identities are calculated and checked for stock-flow consistency.

The remainder of the section outlines the behavioural assumptions of the model sector by sector.\(^5\)

3.2.2 Households

In order to focus on the behaviour of the agent-based C-firms, all other sectors are deliberately kept as simple as possible. As such, the household sector only contains few behavioural assumptions. Households formulate their demand for consumption goods in real terms according to

\[
c^d = \frac{\alpha_1 Y_{D-1} + \alpha_2 V_{h,-1}}{CPI},
\]

where \(CPI\) is an aggregate price index for consumption goods, calculated as an average of C-firms’ individual prices weighted by their individual revenues, \(YD\) is nominal disposable income and \(V_h\) is nominal household wealth. This is a consumption function which is very commonly used in the literature on SFC models (see Godley and Lavoie, 2007), and also frequently appears in this or roughly equivalent

\(^5\)The government sector, which is completely inactive in the baseline simulations, is described in section 3.4.
forms in the agent-based literature (Dawid and Delli Gatti, 2018). It states that households wish to consume a fixed fraction of their disposable income as well as a fixed fraction of their accumulated wealth. As documented by Dawid and Delli Gatti (2018), various ABM frameworks endogenise the consumption propensities, for instance to implement a buffer-stock saving rule. I intentionally keep the consumption behaviour as simple as possible and hence stick to constant parameter values. Households are assumed to supply any amount of labour demanded by firms at the fixed nominal wage rate \( w \). Any financial savings are accumulated in the form of unremunerated bank deposits.

### 3.2.3 K-firms

Capital firms use a labour-only technology (specifically \( y_k = N_k \) where \( N_k \) is labour employed by K-firms and \( y_k \) is real output of capital goods) to produce capital goods on demand, which are sold at a constant price \( p_k \). They distribute all profits to households such that their equity remains at its initial value, which is set to 0.\(^6\)

### 3.2.4 Banks

Banks take unremunerated deposits from C-firms and households. They make loans to C-firms, supplying any amount demanded at the endogenously determined loan rates outlined below. As such (except for the case of firm bankruptcy which is discussed below) this model does not depict quantity rationing on the credit market which, while important to Minskyan theory in general, is not the object of analysis here. Banks are also implicitly assumed to finance, free of interest, on-demand production of consumption and capital goods, with the consequence that firms are never liquidity-constrained in their production decisions. Due to the instantaneous nature of production and sale in this model, this lending of ‘working capital’ does not show up on any balance sheet as production loans are instantaneously repaid. Banks are assumed to target a fixed capital adequacy ratio, \( \zeta \), given by the ratio of equity to loans, and vary their dividend payouts to households in order hit this target. Consequently, bank dividend payments are given by

\[
Div_b = \max(0, Pr_b + (Eb - \zeta L)),
\]

where \( Pr_b \) is bank profit (interest income on loans minus defaults), \( Eb \) is bank equity and \( L \) is the stock of loans. Banks will hence reduce dividend payouts if their equity falls below target, for instance as a

---

\(^6\)K-firms cannot make losses since the constant \( p_k \) is set to a value greater than the constant direct unit labour cost.
consequence of defaults by C-firms. Through varying dividend payouts, the banking sector is hence able to maintain its financial health even when firms default on their loans, as long as defaults do not cause overall bank profits to become negative. Due to the aggregation of the banking sector into a single entity, it turns out that in the simulations shown below, overall bank profits are in fact always positive even in the case of somewhat larger waves of firm defaults and the banking sector is always able to maintain its capital adequacy ratio on target. In practice, the banking sector is hence not in danger of bankruptcy in the present model, and its financial health does not feed into its lending or interest rate setting decisions. The choice to model the banking sector as an aggregate which in practice does not go bankrupt of course prevents me from depicting any dynamics related bank failures etc. and their feedbacks on the real economy, which also play an important role in Minskyan theory. However, the goal of this paper is to focus on the modelling of the firm sector and in particular to investigate the macroeconomic implications of implementing the two-price model of investment at the level of the agent-based C-firms.

### 3.2.5 C-firms

The capital stock held by C-firms depreciates by a fixed rate $\delta$ in every period. In each period, C-firms also pay interest on their outstanding loans and must repay a fraction $\theta$ of the principal. Each consumption goods firm sets its price as a mark-up over nominal direct unit labour cost, $w$ (labour productivity being normalised to 1). In each period, a random subset of firms is allowed to change its mark-up rates such that on average each firm can change its mark-up every four periods (one simulation period representing one quarter). If a firm is allowed to change the mark-up, the latter is set according to

$$\Omega^j = \frac{2\Omega}{1 + \exp\left(\varepsilon\left(\frac{s_{j-1}}{nF} - 1\right)\right)},$$

with $\varepsilon < 0$, where $\Omega$ is some ‘normal’/conventional mark-up which anchors the price-level, $s_{j-1}$ is a four-period moving average of firm $j$’s market share and $nF$ is the number of C-firms. The rule hence causes firms with a higher market share to increase their mark-up while firms with a relatively lower share will decrease their mark-up. This rule is somewhat similar to the one used by Pedrosa and Lang (2018) as well as Dosi et al. (2010) and represents a simple way to introduce price-competition into the model whilst ensuring that firms’ prices remain relatively similar. Due to the behavioural assumption of mark-up pricing, meaning that each firm $j$ sets its price as $p^j_c = (1 + \Omega^j)w$ and since equation (3.3)
precludes negative mark-ups, firms never set a price below direct unit labour cost, which is constant
due to constant technology and the constant nominal wage rate. Equation (3.3) implies that individual
firms’ mark-ups fluctuate around a fixed level \( \Omega \) as a function of relative market shares. The non-linearity
incorporated into equation (3.3) implies that the average mark-up may fluctuate to some small degree, but
the constancy of the wage rate and technology imply that the model cannot give rise to persistent inflation.
To distribute households’ aggregated demand for consumption goods between firms, two indicators of
relative price and size of each firm are calculated as follows:

\[
\hat{p}_j = \left( \frac{p_j}{p_c} \right)^{\iota_1},
\]

\[
\hat{k}_j = \left( \frac{k_j}{k} \right)^{\iota_2},
\]

(3.4)

where \( p_c \) and \( k \) are the average price and real capital stock, with \( \iota_1 < 0 \) and \( \iota_2 > 0 \). Firm \( j \)’s share is
calculated as

\[
s_j = \lambda_1 s_{j-1} + (1 - \lambda_1) \left( \frac{\hat{p}_j}{2} + \hat{k}_j \right),
\]

(3.5)

which is normalised and then multiplied by an autocorrelated, normally distributed shock with mean 1
and standard deviation \( s_j \sigma \) and normalised once more to ensure that \( \sum_{j=1}^{n_F} s_j = 1 \). A firm with a lower
price will hence tend to attract a greater share of demand, as is the case e.g. in Pedrosa and Lang (2018)
or the K+S model (Dosi et al., 2010). The relative capital stock is included to reflect the idea that larger
firms, regardless of their price, will tend to attract more demand, including because they are more likely
to be able to fulfil it due to having higher productive capacity (cf. Michell, 2014; Pedrosa and Lang,
2018). Having received their shares of the aggregate demand for consumption goods, C-firms produce
according to

\[
y^j_c = \min(c^j_d, \kappa k^j),
\]

(3.6)

where \( \kappa \) is the full-capacity output to capital ratio. C-firms’ labour demand is then given by

\[
N^j_c = y^j_c.
\]

(3.7)

In addition, C-firms are assumed to employ overhead labour equal to a fraction \( \kappa_L \) of full capacity
output, such that the amount of overhead labour is given by \( \kappa_L \kappa k^j \) (cf. Lavoie, 2014, Ch. 5).\(^7\) Overhead

\(^7\)Overhead labour introduces a less variable element into firms’ cost, being more or less fixed in the short term.
labour is paid the same wage as labour used in production, such that each firm pays wages equal to 
\[ W^j_c = w(N^j_c + \kappa_k k^j) \] (\( w \) being the fixed nominal wage rate) to their workers. This in turn gives rise to a nominal net cash flow before loan payments which is given by 
\[ CF^j = y^j_c p^j_c - W^j_c. \] At this point, in order to enable an implementation of the two-price model, it is necessary that firms calculate a measure of available internal financing which can enter into the determination of the demand-price of capital. It is thus assumed that prior to the investment decision, firms decide on a desired level of dividends as a fraction \( \gamma^j \) of their net cash flow. In order to introduce an element of active financing choice into C-firms’ behaviour, \( \gamma^j \) is endogenised as

\[ \gamma^j = \frac{2\gamma}{1 + \exp \left( \tau \left( \frac{iL^j + repl^j}{CF} - \beta_1 \right) \right)}, \] (3.8)

where \( \frac{iL^j + repl^j}{CF} \) is the firm’s debt service to cash flow ratio, a flow-based measure of financial fragility often favoured by Minsky over stock-based measures such as the leverage ratio (see e.g. Minsky, 1986). \( \gamma \) is an intercept and \( \beta_1 \) is a fixed parameter representing some exogenous ‘normal’ value of the debt service to cash flow ratio set equal to the initial stationary state value. Firms with a low debt service to cash flow ratio will hence wish to pay a higher dividend and consequently be prepared, all other things equal, to accept a lower share of internally financed investment as “[t]he ratio of external financing that is acceptable changes over time to reflect the experience of economic units and the economy with debt-financing” (Minsky, 1986, p. 209). In line with Minskyan theory, acceptable financing structures for any firm may hence vary over time, with a firm experiencing low debt service and/or high net cash flows being prepared to take on more risk by going deeper into debt (cf. Minsky, 1977). As often outlined by Minsky (e.g. Minsky 1978; Minsky 1986, Ch. 7; Minsky 1992) based on the Levy-Kalecki profit equation, dividends - or more generally distributed profits and consumption out of such profit income - play an important role in determining firm revenue and hence in validating (or not validating) past commitments to particular financing structures. At the same time, the payment of a dividend itself implies a decision about the financing structure of investment being undertaken by a firm, as any earnings earmarked for dividend payments are not used to finance investment. This ‘dual’ role of dividends is an important aspect of the model which will be further elaborated on in the discussion of the baseline simulation.

This in turn increases the frequency of negative profits which is otherwise fairly low when production takes place on demand and the mark-up takes a realistic value.

\(^8\)C-firms may receive demand which exceeds their productive capacity \( \kappa_k k^j \), in which case consumption is rationed (this effect is very slight in all simulations) and ex-post shares are rescaled to reflect this.
Having calculated desired dividends, C-firms’ available internal funds are given by

\[ \text{int}^j = \max(0, D_f^j + (1 - \gamma^j)CF^j - iL^j - rep_L^j), \quad (3.9) \]

where \( D_f^j \) are any previously accumulated deposits, and \( iL^j \) and \( rep_L^j \) are loan interest payments and required principal repayments respectively. In the next step, C-firms determine the prospective yield of capital goods. In this baseline version of the model, C-firms are assumed to simply extrapolate the current yield (given by current net revenue) over the lifetime of their stock of capital goods, giving rise to

\[ Q^j = \frac{c^j d_p^j - W_r^j}{r_d + \delta}, \quad (3.10) \]

where \( r_d \) is a fixed discount rate which is identical between firms. This formulation appears very much in line with what is described by Minsky (1975), including in that it uses net revenue before interest or dividend payments as a basis for calculation (see Minsky, 1975, p. 106). This provides all the ingredients necessary to formulate a function for the demand price of capital, i.e. the equivalent of the curve \( P_k \) in figure 3.1. As noted by Passarella (2011), the demand price of capital may be regarded as being derived from a q-type ratio, as it is based on the discounted future earnings firms expect to make using their capital stock, relative to the current value of the capital stock, i.e. \( q = \frac{Q^j q_o^j}{p_k q_o^j}. \) 9 If this ratio is larger than 1, C-firms will wish to invest in capital goods. Multiplying both sides by \( p_k \) and defining the demand price as \( p_k^{d,j} = p_k q, \) we get \( p_k^{d,j} = \frac{Q^j}{p_k q}. \) In order to incorporate the notion of borrowers’ risk emphasised by Minsky, i.e. the idea that with increasing investment, the riskiness of further investment perceived by the firm increases, the expression is modified by including an additional term in the numerator, in order to arrive at

\[ p_k^{d,j} = \frac{Q^j - \mu^1 (I^j)^2}{k^j}, \quad (3.11) \]

which makes the demand price of capital for firm \( j \) a decreasing function of firm \( j \)’s current nominal investment \( I^j \) (scaled by the parameter \( \mu^1 \)). In the terminology suggested by Kregel (1992), this relationship can be viewed as representing the ‘first level’ of the two-price approach as it is based on a comparison of the price of current output (as well as the demand for the output) of firm \( j \) and the value of \( j \)’s current capital stock (see also Tymoigne, 2009, Ch. 5). In the present model, investment is cen-

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9 Despite this similarity with the conventional q-ratio, and in contrast with, for instance, the model presented by Chiarella and Di Guilmi (2017), my model does not depict stock market dynamics. Instead, it directly makes the demand-price of capital a function of C-firms’ expectations about future earnings, which appears more closely in line with what is described by Minsky (1975, Ch. 5).
trally driven by firms’ demand expectations (which determine expected future earnings and hence the value derived from additional capital), bringing it close to (post-)Keynesian theories of demand-driven investment.

Note that, in contrast to Minsky’s $P_k$-curve depicted in figure 3.1, my function (depicted in figure 3.2) does not contain a discontinuity whereby the demand price only falls off after all internal funds have been invested. The reason for this is that figure 3.1, along with most of Minsky’s presentation of the two-price model more generally, appears to imply that firms always either invest all available internal funds (and possibly more) or they invest nothing at all. That is, figure 3.1 does not permit the possibility of a case in which $0 < I_j < int^j$, whereas my formulation permits this case and hence appears more general. Indeed, despite presenting and discussing equivalents of figure 3.1, Minsky (1975, p.107) does mention that the demand price may in fact decrease before internal funds have been exhausted (see also Tymoigne, 2009, p. 139). In this case, instead of borrowers’ risk it thus also makes more sense to speak of investors’ risk as $p_{d,j}^k$ is decreasing even in internally financed investment in the present model.

By contrast, as can also be seen in figure 3.2, my schedule for the supply price of capital does contain the discontinuity depicted in figure 3.1, being given by

$$p_{s,j}^k = \begin{cases} p_k & \text{if } I_j \leq int^j, \\ p_k + p_k \left(1 - \frac{int^j}{I_j} \right) \frac{r^j_L}{\sigma + r_d} & \text{if } I_j > int^j, \end{cases}$$

(3.12)

where $r^j_L$ is the nominal loan interest rate charged to firm $j$ so that, $\theta$ being the rate of loan repayment, $p_k \frac{r^j_L}{\sigma + r_d}$ represents the discounted total interest cost of a unit of externally financed investment. Equation
(3.12) hence states that the supply price of capital is equal to $p_k$, i.e. the price charged by capital firms, if investment is purely internally financed. If investment exceeds available internal financing, the supply price will be equal to $p_k$ plus the discounted per-unit interest cost of investment, with $\left(1 - \frac{int_i}{T_F}\right)$ giving the fraction of investment which is externally financed.\(^{10}\)

The banking sector determines the nominal interest rate charged to any firm $j$ as

$$r^j_L = r_0 + \mu_2 \left(\frac{i\tilde{L}^j + \tilde{rep}^j_L}{CF^j}\right) \quad (3.13)$$

The interest rate is hence an increasing function of the debt service to cash flow ratio. $r_0$ is interpreted as a constant nominal base interest rate set exogenously by a monetary authority. The term after the constant $r_0$ is meant to capture the notion of lenders' risk, i.e. that the financing cost of investment increases with the perceived riskiness for any investment exceeding internal funds. Importantly, $i\tilde{L}^j$ and $\tilde{rep}^j_L$ in equation (3.13) refer not to current loan payments (which are determined with certainty by past borrowing) but rather to future ones conditional on the loans being taken out in the current period, i.e. $i\tilde{L}^j$ is a function of $r^j_L$ and $L^j$ (the stock of loans) after current investment, and $\tilde{rep}^j_L$ is a function of $L^j$ after current investment. Equation (3.13) can be solved for $r^j_L$ and then substituted into equation (3.12) to obtain $p^k_{s,j}$ as a function of firm $j$'s investment only. Once the demand- and supply-price schedules have been so determined, C-firms find the point at which the two curves intersect, invest accordingly and (if necessary) take out loans. The resulting amounts of investment and new loans simultaneously also determine $r^j_L$ according to equation (3.13). Investment of each individual firm hence proceeds until the demand and supply prices of capital are equal for the period in question. As the supply price of capital is a function of the price of loans in the form of the interest rate, this process of equalisation may be regarded as related to the second level of the two-price approach which, as set out by Kregel (1992), is based on a distinction between the prices of capital assets and financial liabilities. There is a limitation in that the present model contains neither a tradeable financial asset nor (with the exception of the case of bankruptcy set out below) a secondary market for existing capital assets. Nevertheless I believe that my formalisation, involving both a relationship between output and capital asset prices and one between the prices of capital assets and those of financial liabilities, is able to capture the central elements of the two-price model.

\(^{10}\)The price of a unit of internally financed investment is $p_k$ while that of a unit of externally financed investment is $p_k + p_k r^j_L$. The unit cost of investment when $I^j \geq int^j$ is hence $p_k \left(\frac{int^j}{T_F}\right) + \left(p_k + p_k r^j_L\right) \left(1 - \frac{int^j}{T_F}\right)$ from which the second line of equation (3.12) can easily be derived.
Putting together all elements of the two-price model as formalised here, capital investment becomes an increasing function of expected revenue and available internal financing and a decreasing function of the interest rate on loans, which in turn is increasing in financial fragility as measured by a firm’s debt service to cash flow ratio. While various subsets of these or similar factors also feed into investment in other, more simplified and/or aggregated Minsky models drawing on the two-price model (e.g. Delli Gatti and Gallegati, 1990; Fazzari et al., 2008; Chiarella and Di Guilmi, 2011), in the present framework these factors interact in a highly non-linear way to give rise to firms’ investment decisions at the microeconomic level. Specifically, current investment of each individual firm is given by one particular root of a cubic polynomial obtained by combining the elements of the two-price model outlined above. By contrast, existing simplified formalisations of the two-price model frequently assume much less complex investment functions (e.g. Taylor and O’Connell, 1985; Delli Gatti et al., 1994b; Nasica and Raybaut, 2005). In the agent-based model proposed by Chiarella and Di Guilmi (2011), firms’ investment is a function of ‘animal spirits’ which are determined by a set of equilibrium conditions on capital markets. This makes investment dependent on a host of interacting financial factors such as asset prices, aggregate wealth and the interest rate. The framework is however much more loosely based on the two-price model than the formalisation presented here. Another important difference is that aggregate demand plays a central role in the present model, which seems in line with both Minsky’s work and the post-Keynesian tradition more broadly, whilst in Chiarella and Di Guilmi (2011) output is supply-determined.

Fazzari (1992) argues that New Keynesian macroeconomics, with its relatively strong emphasis (at least compared to New Classical and some neo-Keynesian approaches) on finance and investment in explaining economic fluctuations, may serve to introduce some original insights of both Keynes and Minsky into mainstream discussions. In particular, he contends that the concepts of borrowers’ and lenders’ risk may be recovered in a New Keynesian framework in the presence of asymmetric information. The present formalisation of the two-price model makes it possible to provide even more explicit links to the New Keynesian financial accelerator literature (e.g. Bernanke et al., 1996, 1999). By assumption, the two-price model includes an external finance premium, formalised by the discontinuity in the supply price schedule at the point at which internal financing is exhausted. Moreover, this external finance premium is an increasing function of the amount of external financing required, as formalised in equations (3.12)

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11 Recall that the output of both capital goods and (in the absence of capacity constraints) consumption goods is determined by demand. Moreover, demand for consumption goods, via equation (3.10), is an important determinant of demand for capital goods.

12 The question of whether the presence of this premium is motivated by asymmetric information or fundamental uncertainty (or both) is immaterial for the purpose of this discussion as the outcome is the same in either case.
and (3.13). At the same time, however, it is decreasing in the financial solidity of the borrower, which
here is proxied by the availability of internal financing (rather than net worth as is usually the case in
financial accelerator models). Borrowers’ or investors’ risk, as argued by Fazzari (1992), may arise in an
asymmetric information framework if opportunities for diversification are limited and indeed, the present
model does not allow C-firms to diversify their investment away from the homogeneous capital good.
As I will argue in the discussion of the baseline simulation, these structural similarities allow for the
mechanisms producing the cycles in the model to be closely linked to those producing fluctuations in
financial accelerator models.

Towards the end of each period, C-firms calculate their profits and update their stock of deposits. If
profits are positive, firms pay the desired dividend formulated during the determination of current internal
financing. If profits are negative, C-firms do not pay a dividend. Moreover, C-firms make the interest and
principal payments due on outstanding loans. If, after making dividend payments, C-firms do not possess
sufficient liquidity to make the required loan payments, it is assumed that they may take out additional
loans from the banking sector. If a firm borrows in order to make interest payments and/or to repay the
principal on existing loans, the interest rate charged to that firm is adjusted in line with equation (3.13)
based on the resulting new stock of loans. If on the other hand a firms’ accumulated deposits are still
positive after making dividend and loan payments, it uses the remaining stock of deposits to pay off as
much as possible of its outstanding stock of loans in addition to the mandatory principal payment which
has already been made (i.e. beyond \( rep^j_L \) which is given by \( \theta L^j \)). In this case, too, the interest rate is
adjusted following equation (3.13) based on the newly remaining stock of loans.

Allowing C-firms to borrow in order to make interest payments and/or roll over existing loans also
opens the possibility of classifying them according to Minsky’s well-known distinction between hedge,
speculative and Ponzi units (e.g. Minsky, 1978). According to Minsky, a firm is classified as a hedge
unit if its current free cash flow is sufficient to cover both interest and principal payments on outstanding
loans. It becomes speculative if interest payments can be covered from available cash flow but principal
payments cannot, such that the firm must rely on being able to roll over existing loans. The firm becomes
a Ponzi unit if it must borrow to make both principal and interest payments. In the present model, free
cash flow is defined as net cash flow after dividend payments, i.e.

\[
CF_{\text{free}}^j = CF^j - Div^j. \tag{3.14}
\]
Consequently, C-firms are classified as

\[
\text{Hedge } \quad \text{if } \quad CF_{\text{free},j}^i \geq iL^j + rep_j^L, \\
\text{Speculative } \quad \text{if } \quad CF_{\text{free},j}^i \geq iL^j \quad \land \quad CF_{\text{free},j}^i < iL^j + rep_j^L, \quad (3.15) \\
\text{Ponzi } \quad \text{if } \quad CF_{\text{free},j}^i < iL^j.
\]

C-firms in the model may go bankrupt in two ways. Firstly, if a firm’s equity becomes negative due to persistent losses, it is assumed to go into bankruptcy. In addition, it is assumed that if a firm is a speculative or Ponzi unit, it defaults with a certain probability. In particular, for each C-firm a default probability is calculated as

\[
def_j^i = \max \left(0, \frac{iL^j + rep_j^L - CF_{\text{free},j}^i}{CF_j^i} \right) \quad (3.16)
\]

\[
p(default)_j^i = \tanh(def_j^i \beta_2). \quad (3.17)
\]

These equations (where \(\beta_2 > 0\) is a fixed parameter) imply that if a firm is speculative or Ponzi, it will default with a probability which is increasing in its financial fragility. This bankruptcy condition is similar to the one employed in Chiarella and Di Guilmi (2011) and the follow-up papers building on the model presented therein, in which firms default if their leverage ratio exceeds a certain threshold. Such a bankruptcy condition could be interpreted either as a voluntary/strategic decision of firms which no longer expect their financial health to improve sufficiently to eventually repay their debt, or as a refusal by the banking sector to continue rolling over the loans of increasingly financially fragile firms (i.e. a form of credit rationing).

If either bankruptcy condition is met for a particular firm, all its loans are written off, all its deposits (if any) are removed from the banks’ balance sheet, and a fraction \(1 - \chi\) of its capital is scrapped while the rest is transferred to the banks. The losses to the banking sector from a firm default are hence \(L^j - D_j^i - \chi K^j\).\(^{13}\) A new firm is then assumed to enter the market. This firm receives a transfer of deposits from the households sufficient to purchase the remaining capital stock previously transferred to the banks. This way of modelling bankruptcy is very similar to the ones employed in Caiani et al. (2016) and Pedrosa and Lang (2018).

\(^{13}\)This expression may indeed be positive, in which case it is booked as a profit for the banking sector.
3.3 Calibration & baseline simulation

Appendix 3.A provides a full list of initial and parameter values which must be set prior to simulation of the model. In order to arrive at a satisfactory baseline calibration, I employ a simulated minimum distance estimator, the use of which in ABMs is discussed by Grazzini and Richiardi (2015).

Table 3.3: Empirically calibrated parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\iota_1$</td>
<td>Price sensitivity of demand distribution</td>
<td>-3 : -0.5</td>
</tr>
<tr>
<td>$\iota_2$</td>
<td>Firm size sensitivity of demand distribution</td>
<td>0.5 : 1.5</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Persistence of demand distribution</td>
<td>0.5 : 0.95</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Persistence of demand distribution shock</td>
<td>0.25 : 0.95</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Strength of demand distribution shock</td>
<td>1 : 5</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>Bankruptcy parameter</td>
<td>0.01 : 0.1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Payout ratio intercept</td>
<td>0.3 : 0.5</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Price adjustment parameter</td>
<td>-5 : -1</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Payout ratio adjustment parameter</td>
<td>-60 : -20</td>
</tr>
</tbody>
</table>

3.3.1 Calibration

In particular, I apply the method of simulated moments (see e.g. Gilli and Winker, 2003; Franke and Westerhoff, 2012; Schmitt, 2020) to calibrate 9 of the model’s parameters by attempting to minimise the distance between a set of 9 moments/statistics generated by the model and their empirical counterparts. The list of parameters which are empirically calibrated in this manner is shown in table 3.3. For the empirical calibration procedure I obtain the following quarterly macroeconomic time-series for the United States for the period Q4 1951 to Q3 2019:

1. Real Gross Private Domestic Investment (Federal Reserve Economic Data, series identifier GPDIC1)
2. Nominal Gross Domestic Product (Federal Reserve Economic Data, series identifier NA000334Q)
3. Nonfinancial corporate business; net interest and miscellaneous payments (Federal Reserve Economic Data, series identifier BOGZ1FA106130003Q)
4. Nonfinancial corporate business; debt securities and loans; liability (Federal Reserve Economic Data, series identifier BCNSDODNS)

Following seasonal adjustment where required, series 2-4 are used to calculate the ratios of firms’ interest payments and debt to nominal GDP. To remove trend components from the empirical time series (real

14 All files necessary to initialise and parametrise the model and to reproduce the simulations shown in the paper can be found under https://github.com/SReissl/Minsky.
investment, corporate debt to GDP and corporate interest payments to GDP), I employ the recently proposed ‘Hamilton filter’ (Hamilton, 2018; Schüler, 2018), running the following regression model on all empirical time series:\footnote{The specification of the regression model is the same as that recommended by Hamilton (2018) for quarterly data.}

\[
\ln(x_t) = \alpha + \sum_{i=8}^{11} \beta_i \ln(x_{t-i}) + \epsilon_t. \tag{3.18}
\]

The residuals from this regression are interpreted as the cyclical component, which is used to calculate the moments/statistics used in the calibration procedure, while the predicted values represent the trend component. Using the de-trended series I calculate the following moments/statistics:

- The standard deviations of real investment, firm debt to GDP and firm interest payments to GDP (\(sdI\), \(sdDebt\) and \(sdInt\))
- The one-lag autocorrelations of the aforementioned series (\(acI\), \(acDebt\) and \(acInt\))
- The contemporaneous cross-correlation between real investment and firm debt to GDP as well as that between real investment and firm interest payments to GDP (\(ccIDebt\) and \(ccIInt\))
- A measure of cycle frequency, given by the average number of times the de-trended time series of real investment changes sign within 100 periods (\(freq\))

As becomes clear from both the choice of parameters to be calibrated as well as the moments/statistics to be reproduced, the focus of this calibration procedure is on the behaviour of C-firms and the aggregate dynamics this gives rise to. The reason for this is that the rest of the model is extremely simplified and I would not expect the model to be capable of replicating a broader set of moments (indeed it is not constructed for the purpose of doing so). Denoting the vector of parameters to be calibrated by \(\Theta\), the vector of empirical moments/statistics by \(m^e\) and the vector of simulated moments/statistics by \(m\), I aim to minimise the loss function

\[
\mathcal{L}(\Theta) = (m(\Theta) - m^e)'W(m(\Theta) - m^e), \tag{3.19}
\]

i.e. I aim to find

\[
\Theta^* = \arg \min_{\Theta} \mathcal{L}(\Theta). \tag{3.20}
\]

Following Franke and Westerhoff (2012), the weighting matrix \(W\) is the inverse of the variance-covariance matrix of the empirical moments/statistics, obtained using bootstrapping. In calculating \(\mathcal{L}\), the variance of the chosen moments/statistics is hence taken into account. The calibration proceeds by sampling the
parameter space delimited by the initial ranges shown in table 3.3 using latin hypercube sampling (Salle and Yildizoglu, 2014) and simulating the model 100 times with different, reproducible seeds for each sampled parameter combination. Sampling is then repeated around points of the parameter space which appear promising in terms of the value of the loss function. This procedure continues until further sampling no longer produces noticeable reductions in the value of the loss function.\(^16\) The values of the parameters thus calibrated are given in appendix 3.A. Table 3.4 compares the moments/statistics produced by the calibrated model (calculated on %-deviations from the post-transient mean of the stationary simulated time series) to their empirical counterparts.

### Table 3.4: Comparison of empirical & simulated moments/statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Empirical</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>sdI</td>
<td>0.12656</td>
<td>0.12558</td>
</tr>
<tr>
<td>sdDebt</td>
<td>0.05252</td>
<td>0.06574</td>
</tr>
<tr>
<td>sdInt</td>
<td>0.14689</td>
<td>0.14404</td>
</tr>
<tr>
<td>ac(_1)I</td>
<td>0.89383</td>
<td>0.92075</td>
</tr>
<tr>
<td>ac(_1)Debt</td>
<td>0.89356</td>
<td>0.97007</td>
</tr>
<tr>
<td>ac(_1)Int</td>
<td>0.88574</td>
<td>0.98239</td>
</tr>
<tr>
<td>ccIDebt</td>
<td>-0.13073</td>
<td>-0.08832</td>
</tr>
<tr>
<td>ccIInt</td>
<td>-0.29597</td>
<td>-0.08356</td>
</tr>
<tr>
<td>freq</td>
<td>9.57854</td>
<td>9.08023</td>
</tr>
</tbody>
</table>

It can be seen that by and large, the model is able to reproduce the observed statistics reasonably closely. This should be regarded as a satisfactory result in particular given the simplicity of the model beyond the fairly detailed C-firm sector. If the sector of C-firms modelled here were embedded in a more detailed macroeconomic framework, it should be possible to improve the fit to an even greater degree.

The full initialisation protocol of the model proceeds by imposing an initial stationary state (in which all firms are identical) on the model.\(^17\) This is done by imposing restrictions on parameters and the initial values of stocks and flows until a sufficient number of degrees of freedom have been removed for all remaining initial values and some parameter values to be implied by these restrictions (see appendix 3.A for a more detailed discussion). Some parameters the values of which are set using the empirical calibration procedure outlined above are ‘free’ in the sense that their value does not have an impact on the determination of other parameters or initial values. Others, such as \(\gamma\) (the intercept of C-firms’

\(^{16}\)Of course, due to the large size and high dimensionality of the parameter space, it is impossible to determine whether the parameter combination produced by this procedure does in fact represent the global minimum of the loss function. The results obtained do however seem satisfactory in terms of the model’s ability to reproduce the empirical statistics under the obtained calibration.

\(^{17}\)This stationary state is a purely analytical device which conveniently ensures that all initial values are mutually consistent, including from an accounting perspective. It should not be interpreted as possessing any empirical relevance or being some sort of fundamental attractor.
dividend payout ratio) do feed back on the rest of the initialisation protocol as, for instance, the values of \( \mu_1 \) and \( \mu_2 \) (the borrowers’ and lenders’ risk parameters respectively) consistent with a stationary state are a function of \( \gamma \).

Despite the empirical calibration of a subset of parameter values and the use of the model’s SFC structure to reduce the available degrees of freedom, I am still left with some discretion in initialising and parametrising the model. Whenever this is the case, I choose empirically plausible values where possible. Tables 3.7 and 3.9 in appendix 3.A provide a full list of initial and parameter values needed for the baseline model. Appendix 3.B provides a simple sensitivity analysis carried out on initial values and parameters which are not empirically calibrated. The calibrated model is simulated for a total of 1000 periods (quarters). Simulations are carried out for 100 Monte Carlo repetitions with different, reproducible seeds. Following the beginning of the simulation with stochastic elements, the model diverges from the initial stationary state and after a transition period settles into the dynamics described below. The plots below show the model behaviour from simulation period 500 onwards.

### 3.3.2 Macroeconomic dynamics

Figure 3.3 depicts the dynamics of real GDP and consumption for a single, representative run of the model. It can be seen that the model economy experiences persistent and fairly regular cycles in output and consumption. Figure 3.4 shows that the aggregated investment expenditure of the agent-based C-firms similarly exhibits regular cycles.

**Figure 3.3: Real GDP and consumption**

![Graph showing real GDP and consumption](image)
The left panel of figure 3.5 shows that the average leverage ratio of C-firms (defined as loans over capital stock) is pro-cyclical. The right panel shows that turning points in the average debt service to cash flow ratio (which feeds into the determination of C-firms’ loan interest rates and dividend payout ratios) immediately precede turning points in GDP. The mechanism driving the cycles can be summarised as follows: As shown in the right panel of figure 3.5, the debt service to cash flow ratio peaks during the downturn and subsequently begins to fall as a consequence of de-leveraging and firm defaults. In accordance with equation 3.8, this in turn leads the dividend payout ratio to bottom out and then gradually increase.

Figure 3.5: Average leverage ratio and debt service to cash flow ratio plotted against GDP
Moreover, following equation 3.13, loan interest rates peak and begin to fall, as shown in figure 3.6. The combination of de-leveraging, declining loan rates and initially low dividend payout ratios serves to stabilise investment both by decreasing the supply price (though falling interest rates) and increasing the demand price of capital (through increasing availability of internal financing). Figure 3.7 shows that at the trough of the cycle, almost all capital investment taking place is internally financed.

![Figure 3.6: Average loan rate](image)

![Figure 3.7: Aggregate share of externally financed investment](image)

With a decreasing debt service to cash flow ratio, dividend payout ratios gradually begin to rise again, leading to an increase in firm dividends. The combination of increasing investment and firm dividend
payments produce an increase in household disposable income and subsequently aggregate demand for consumption goods and the expected yield on capital goods. During the upswing, there is a positive feedback mechanism between C-firms’ financing and investment choices and the resulting revenues; investment rises and is increasingly externally financed, leading the leverage ratio to eventually increase, but high resultant cash flows validate the increasingly risky strategies taken by firms, keeping interest rates and debt service to cash flow ratios low (indeed the ratio even declines through much of the boom).

Figure 3.8: Shares of Hedge, Speculative and Ponzi firms

This positive feedback loop is eventually broken by a combination of mechanisms. For some firms, dividend payout ratios eventually reach the upper bound incorporated into equation (3.8) and do not continue growing. In addition, despite debt service still being low relative to cash flows, some firms begin to enter speculative and Ponzi territory (see figure 3.8) due to their high dividend payouts and aggressive use of external finance eroding margins of safety, raising the probability of some firms going into bankruptcy.

Finally, loan interest rates eventually begin to rise as the proportion of externally financed investment becomes so high that growth in cash flows is no longer sufficient to keep the anticipated post-investment debt service ratio incorporated into equation (3.13) low. A combination of stagnating dividend payouts (slowing growth in disposable income and consumption and hence growth in the expected yield of cap-

18 Though it is difficult to tell from the graph, the number of speculative firms is weakly pro-cyclical. In general, firms tend to quickly transition from speculative to Ponzi or hedge status such that the number of speculative firms does not vary as strongly as those of hedge and Ponzi ones.
ital goods), some initial firm bankruptcies, and rising loan interest rates precipitates the turning point of the cycle. The positive feedback mechanism which fuelled the upswing now goes into reverse. With falling dividend payouts and investment, disposable income and aggregate demand begin to fall. This in turn lowers C-firms’ cash flows, causing them to become more financially fragile and cut dividends and investment further. Interest rates continue to rise and more firms enter bankruptcy. Just as increasingly risky strategies are validated and hence encouraged by outcomes during the upswing up until the turning point, more conservative strategies and attempts to de-leverage during the downturn themselves generate the conditions necessitating further cutbacks. Eventually, dividend payments bottom out, interest rates stop increasing and a sufficient number of firms have de-leveraged (including through bankruptcy). Hence the trough of the cycle is reached and the dynamic described above begins anew.

Without going too deeply into the debate, the results of the present model can be related to a longstanding point of contention in the Minskyan literature, namely that raised by Lavoie and Seccareccia (2001). They argue that Minsky’s FIH contains a fallacy of composition in that, due to the Kaleckian profit equation, increases in aggregate investment involving collective attempts by firms to increase their leverage may lead to a decline in the aggregate leverage ratio (see also Lavoie, 1995), a phenomenon termed the ‘paradox of debt’ (Lavoie, 2014, Ch. 1). As ‘Minsky cycles’ are frequently held to imply or even depend on a pro-cyclical leverage ratio in the literature (Nikolaidi and Stockhammer, 2017; Stockhammer, 2019), this insight may be taken as an argument against the likelihood of Minskyan dynamics at the macroeconomic level. By now there is a fairly sizeable literature discussing the conditions under which the paradox of debt is likely to emerge, and how this impacts the arguments of Minsky (e.g. Passarella, 2012; Ryoo, 2013; Michell, 2014; Kemp-Benedict, 2015). As shown in the left panel of figure 3.5, the baseline simulation of the present model does produce a pro-cyclical (and slightly lagging) aggregate leverage ratio, but this is of little consequence as the leverage ratio does not feed into any behavioural equations. The debt service to cash flow ratio, which is the key indicator of financial fragility feeding into C-firms’ behaviour, is in fact counter-cyclical as shown in the right panel of figure 3.5, with peaks (throughs) in the ratio occurring slightly before troughs (peaks) in GDP. Nevertheless, the dynamics of this ratio play the key role in generating the Minskyan dynamics of the model. For much of the upswing, cash flow grows faster than debt service, leading to a declining debt service ratio (and a

\[^{19}\text{As noted by Passarella (2012) and Yilmaz and Stockhammer (2019), an endogenous retention rate is one possible way to produce a pro-cyclical leverage ratio.}\]

\[^{20}\text{Indeed, as show in table 3.4, other possible indicators of financial fragility are also counter-cyclical in both the simulated and empirical data in the sense of being negatively correlated with aggregate investment.}\]
declining loan interest rate) which, however, encourages a more aggressive use of external finance and lower retention ratios. This in turn creates an underlying weakness in the system as with growing stocks of debt, firms increasingly come to rely on decreasing debt service ratios and interest rates such that as soon as the ratio and loan rates begin to increase even from low levels, GDP growth slows down and eventually enters the downswing. Similarly, during the downturn, GDP only begins to grow once the debt service ratio and the interest rate have peaked.

Overall, the above discussions demonstrate that the cycles produced by the present model are centrally driven by the financing decisions and investment strategies of firms which strongly depend on financing conditions and investors’ expectations about yields, making it legitimate to call them ‘Minsky cycles’. As emphasised in the introduction, none of the factors and mechanisms determining investment in the present model are new to the literature, but the discussion of the baseline simulation in my view shows that the investment dynamics generated are behaviourally richer than those produced by many of the more simplified and aggregative models discussed by Nikolaidi and Stockhammer (2017). The cycles generated by the model presented in Fazzari et al. (2008) are somewhat similar insofar as internal financing as an argument in the investment function and endogenous fluctuations in the interest rate play a key role, though the exact mechanisms are of course different (for instance, changes in the interest rate are driven by expected inflation in Fazzari et al., 2008), and their model differs from the present one in various other respects (e.g. the absence of an explicitly modelled banking sector). In contrast to papers such as Nikolaidi (2014), Lojak (2018) or Jump et al. (2017), the cycles shown here are not driven by evolving leverage targets (indeed firms in the present model have no explicit targets whatsoever regarding their financial ratios) or switching behaviour in sentiment or financing strategies of firms (although the computational experiments presented below do explore sentiment dynamics and strategy switching behaviour). In many Minsky models (e.g. in Fazzari et al. 2008 or in Keen 1995), there is no explicit financing choice on the part of firms; rather, debt is a residual determined by the difference between investment and profit. In the current model, by contrast, firms vary their retention rate in order to increase or decrease the proportion of investment that is internally financed. Yilmaz and Stockhammer (2019) is an example of another Minsky model with an endogenous retention rate while in Chiarella and Di Guilmi (2011), firms actively choose between debt and equity financing.

Indeed, further simulations suggest that, along with several other features of the model, the endogenous payout ratios are key in producing the observed cycles. Setting the parameter $\tau$ in equation (3.8) to zero and hence making the payout ratio exogenous leads to a disappearance of the endogenous cycles,
with investment showing only small stochastic fluctuations around a constant level. The model can also be recalibrated such that \( \mu_1 \), the borrowers'/investors' risk parameter in equation (3.11) becomes equal to zero. In this case, the demand price of capital will become a constant for any expected yield \( Q^j \) and existing capital stock \( k^j \) instead of declining with increasing investment. With this modification, the model quickly diverges far away from the initial stationary state to a pattern of small irregular fluctuations around a very high level of GDP and the regular cycles exhibited by the baseline disappear. These simulations are reported in appendix 3.B. Moreover, results of the sensitivity analysis carried out in appendix 3.B (especially the results discussing changes in C-firms' initial leverage ratios) suggest that when the model calibration is changed such that the value of \( \mu_2 \), the lenders' risk parameter incorporated into equation (3.13), becomes very small (meaning that loan rates react more weakly to changes in financial fragility), the model also ceases to produce cycles. This suggests that both borrowers'/investors' and lenders' risk play an important role in giving rise to the observed dynamics, particularly in keeping levels of investment bounded within a particular interval (in the case of \( \mu_1 \)) and in bringing about cyclical turning points (in the case of \( \mu_2 \)).

As indicated in section 3.2, it is also possible to draw parallels between the mechanisms generating the cycles and the financial accelerator literature. As discussed, movements in the loan interest rate, i.e. in the endogenous 'external finance premium' play a key role in driving the fluctuations generated by the present model, as do changes in cash flows and internal financing (which, as argued above, may be viewed as playing a role equivalent to that of firm net worth in financial accelerator models). Increases in cash flows and internal financing have a positive impact on desired investment, which during the upswing is amplified by decreases in the external finance premium as cash flows grow faster than debt service; in the downturn, this dynamic is reversed with increases in the interest rate amplifying decreases in desired investment resulting from declining cash flows. These mechanisms are exactly equivalent to those incorporated into financial accelerator models such as Bernanke et al. (1996) in which movements in the external finance premium amplify the effects of shocks to net worth. In this respect the model is similar to other Minskyan frameworks which incorporate a financial accelerator mechanism, such as Delli Gatti et al. (2006, 2010). In comparison to the present model, the aforementioned papers discuss models with a much more simplified formalisation of firm behaviour with no connection to the two-price model and a focus on the network effects of firm bankruptcies. Bargigli et al. (2016) present a macroeconomic ABM featuring heterogeneous firms and banks in which a financial accelerator mechanism working through an endogenous external finance premium plays a key role in shaping dynamics.
3.3.3 Microeconomic dynamics

Turning to a more detailed look at simulation data generated at the microeconomic level, figure 3.9 gives an example of the dynamics of investment and the loan interest rate for one individual firm. It can be seen that investment of the individual firm is considerably more volatile than aggregate investment and includes fluctuations at much higher frequencies. Similarly the loan interest rate of the individual firm fluctuates at a much higher frequency - as well as a higher amplitude - than the aggregate one.\textsuperscript{21} The strategic complementarities in investment decisions arising via the impact of investment on aggregate demand and vice-versa lead to a sufficient correlation between the independent individual investment and financing decisions of single firms to still give rise to regular cycles at the aggregate level. Nevertheless, as is shown in appendix 3.B, similar aggregate patterns of fluctuation can also be generated by an aggregate model incorporating the same non-linearities as the baseline shown here. While it is interesting to observe that cycles can also arise in a model in which the firm sector is disaggregated such that the decisions of individual firms are only indirectly linked to aggregate variables, there is of course a trade-off in that the use of an agent-based firm sector introduces additional complexity to the model.

While I believe that in the present model, the choice of using an agent-based methodology is justified (including by the additional insights from microeconomic data presented below), there is of course a case to be made for the use of aggregated, non-linear macrodynamic models which have traditionally dominated the literature on Minsky models.

\textsuperscript{21}The steep decline observed between periods 400 and 500 represents a period in which the firm in question goes bankrupt, the subsequent trajectory depicting the behaviour of the firm replacing it.
The microeconomic data generated by the model can be used to discern patterns which could not be observed in a purely aggregated model. Figure 3.10 shows the result of fitting theoretical distributions to the right tail of the size-distribution of C-firms in terms of both capital and sales. The plot is constructed by taking a snapshot of the distribution of capital and sales at period 800 across all 100 Monte Carlo repetitions.\footnote{Because cycles do not coincide temporally across Monte Carlo runs, the plot does not contain a cyclical element.} It can be seen that the model gives rise to considerable heterogeneity among C-firms in terms of firm size, and that both the log-normal and logistic distributions (for the former of which there is some empirical support see e.g. Caiani et al., 2016) appear to provide a decent fit. The size distributions hence clearly exhibit heavy tails.

Figure 3.10: Distribution of firm size

![Q-Q plot (Capital)](image1)

![Q-Q plot (Sales)](image2)

\[ \circ \text{Log-normal} \quad \times \text{Logistic} \]

\[ \circ \text{Log-normal} \quad \times \text{Logistic} \]

Figure 3.11 gives an idea of the evolution of the distribution of financial fragility (defined as the value of equation (3.16)) over the cycle. It differentiates between boom, peak, downturn and trough of the cycle. Peak \((t^p)\) and trough \((t^t)\) periods are identified by finding the maximum and minimum values of post-transient GDP in each Monte Carlo run respectively. Boom and downturn periods are then defined as \(t^p - 6\) and \(t^t - 6\) respectively.\footnote{The plots shown are robust to varying this definition by a few periods.} Data for the financial fragility of each firm during these periods is recorded and pooled together across Monte Carlo repetitions to produce the histograms shown in the figure.\footnote{Qualitatively identical histograms are obtained when Monte Carlo averages instead of pooled observations are used. However, the small number of observations in the case of averages make the characteristics discussed below less easily visible.} It can be seen that the distribution of financial fragility among C-firms shifts substantially over the cycle. During the booms, there are two peaks; one around which relatively robust firms are centered and one of an emerging group of financially fragile firms. By the time of the peaks this latter
group grows substantially such that a majority of firms are either speculative or Ponzi. De-leveraging and firm bankruptcies lead to a reduction of this concentration during the downturns until at the troughs of the cycles a vast majority of firms have become hedge financing units, only for the cycle to begin anew with some firms beginning to accumulate debt and become increasingly financially fragile. The histogram for the boom period is particularly interesting as the double-peaked distribution reveals a build-up of financial fragility among one group of firms. This would be masked by looking at an average or ‘representative’ firm which would still appear relatively robust.

Once more using Monte Carlo averages, along with locally fitted polynomials, figure 3.12 demonstrates that the relationship between C-firms’ cash flow and their financial fragility differs significantly between peak and trough periods. It shows that at the peak there is a positive relationship between the cash flow and financial fragility, meaning that those firms which attract a higher share of demand are also those which have been driven to take more risk by relying more heavily on external finance. This is in line with the mechanisms described above, whereby high cash flows validate and encourage financially risky strategies up until the peak of the cycle. At the trough it can be seen that, while overall financial fragility and cash flows have decreased as well, the relationship between cash flow and financial fragility has become u-shaped. Many of the previously highly fragile firms have de-leveraged substantially over the
course of the downturn, making the relationship negative up to a point. However, a small number of firms has managed to maintain relatively high cash flows thus being able to maintain a higher burden of debt service through the downturn.

The above analysis of the microeconomic simulation data gives an idea of the added value of the agent-based approach taken in this model as compared to the aggregative approaches which have traditionally been employed in the Minskyan literature. Having presented the baseline simulation, I move on to conduct some experiments.

3.4 Simulation experiments

The experiments carried out on the model are divided into two sets; one which focuses on C-firms’ expectations and one which is concerned with stabilisation policy. All experiments are implemented by first letting the baseline model run for 499 periods and subsequently imposing the modification from period 500 onwards, continuing the run until period 1000. The values of parameters introduced to implement the experiments can be found in table 3.8 in appendix 3.A.

3.4.1 Expectations & sentiment dynamics

In the first simulation experiment, I introduce a variation in the way in which C-firms form opinions about the prospective yield of capital goods. Instead of assuming that they simply use current revenue as a basis to calculate the demand-price of capital as in equation (3.10), I instead allow them to take either
an optimistic or pessimistic attitude and adjust their investment strategy accordingly. This approach is similar to that taken by Lojak (2018), who introduces a sentiment variable directly affecting sales expectations and hence investment into an aggregate Kalecki-Minsky model, finding that the non-linearity incorporated into the sentiment dynamics itself can give rise to cycles.\footnote{Another paper using a strategy switching approach in a Minsky model is Jump et al. (2017) in which firms switch between different strategies for the target debt to income ratio based on the mechanism proposed by Brock and Hommes (1997).} In the present experiment, a ‘sentiment’ variable for each firm is calculated as

\[
\text{sentiment}_j = \frac{1}{1 + \exp(\eta(\hat{\pi}_j - \bar{\pi}))},
\]

with \( \eta < 0 \). \( \hat{\pi}_j \) is a weighted average of firm \( j \)’s current profit rate and the current average profit rate prevailing in the C-firm sector. \( \bar{\pi} \) is the average profit rate in the C-firm sector over the 300 periods preceding the implementation of the modification to firm behaviour.\footnote{The results presented here and for the other experiments below are robust to a change in the length of the 300-period window used to calculate this and the other averages introduced below.} A firm’s ‘sentiment’ is hence increasing both in its own and the economy-wide profit rate, capturing an element of social influence. Following calculation of the sentiment variable, a random number is drawn from a uniform distribution for each firm. If \( \text{sentiment}_j \) is greater than the random number drawn for \( j \), the firm in question takes an optimistic attitude (\( \text{attitude} = 1 \)), otherwise, it is a pessimist (\( \text{attitude} = 0 \)). Equation (3.10) is replaced by

\[
Q_j = \begin{cases} 
(c_j d_j p_j - W^j_\delta)_{\text{opt}} + \delta & \text{if attitude} = 1 \\
(c_j d_j p_j - W^j_\delta)_{\text{pes}} + \delta & \text{if attitude} = 0,
\end{cases}
\]

where \( \text{opt} > 1 \) and \( \text{pes} < 1 \). The demand price curve for an optimistic firms will hence shift upward and consequently they will tend to invest more, while the opposite is the case for pessimist firms.

Figures 3.13 and 3.14 summarise the effects of this modification. Figure 3.13 shows that the cycles arising in the baseline model are preserved and investment becomes significantly more volatile. The standard deviations of output, investment and consumption increase by 28, 68 and 18 percent relative to the baseline respectively. The explicit link from aggregate profitability to the sentiment of individual firms contained in equation (3.21) serves to strengthen the existing strategic complementarities in firms’ investment decisions by producing waves of optimism and pessimism as shown in figure 3.14, which in turn amplify cycles at the macroeconomic level. Further analysis shows that a lower level of \( \rho \), implying a larger degree of co-movement in C-firms’ sentiment (i.e. a higher degree of social influence), leads to
additional increases in macroeconomic volatility while a higher $\rho$ exerts a stabilising influence relative to figures 3.13 and 3.14. Similarly, a lower (higher) level of $\eta$, making sentiment more (less) sensitive to changes in profitability increases (decreases) volatility. The same effect can also be obtained by increasing (decreasing) the value of $op$ and simultaneously and symmetrically decreasing (increasing) the value of $pes$.

Figure 3.13: Capital investment with sentiment dynamics

![Figure 3.13: Capital investment with sentiment dynamics](image)

Figure 3.14: Average attitude of C-firms

![Figure 3.14: Average attitude of C-firms](image)

This experiment highlights the central role played by investors’ expectations and beliefs in the two-price model. Building on the work of Keynes (1936), Minsky frequently emphasises the importance
expectations and changing attitudes and beliefs for his theory of financial fragility, but this aspect has not always featured very prominently in formal Minsky models, including those making explicit reference to the two-price model. There are papers, such as Taylor and O’Connell (1985) or Chiarella and Di Guilmi (2011), in which expectations play an important role in principle, but besides the paper by Lojak (2018) discussed above, Fazzari et al. (2008) appears to be the only existing work in the related literature examining expectations formation in particular detail.\(^{27}\) Note that the modification introduced by the experiment discussed above leaves the basic building blocks of the two-price model unchanged; just as the next experiments described below it merely alters the way in which one of the inputs into the formation of the demand price is calculated. As emphasised in section 3.1, the two-price model as formalised here hence flexibly allows for the incorporation of various behavioural assumptions. Here I demonstrate this by varying one particular assumption, but one could conceivably also experiment with other variations. For instance, equations (3.8) and/or (3.13) could be altered to make the dividend payout ratio and/or the loan interest rate functions of ‘Minskyan’ variables other than the debt service to cash flow ratio while leaving the basic structure of the model unchanged.

I conduct two further experiments to illustrate the central role of beliefs and expectations in the model. In these experiments, C-firms are assigned a type at the beginning of each simulation, namely ‘baseline’ and ‘fundamentalist’. Baseline-type firms calculate the demand-price of capital exactly as in the baseline, using equation (3.10) to derive their belief about the prospective yield. In the first experiment, fundamentalist-type firms use the average yield prevailing in the 300 periods preceding the beginning of the experiment as their estimate in the belief that this proxies some ‘fundamental’ value of the yield to which the actual one will revert. Apart from this, they behave exactly as do the baseline firms. In the second experiment fundamentalist firms additionally alter their dividend payout behaviour. Instead of each fundamentalist firm \(j\) paying out a fraction \(\gamma^j\) of current cash flow, they instead pay a fraction \(\gamma^j\) of the average cash flow prevailing in the 300 periods preceding the beginning of the experiment. They hence still alter their financing choices by varying their dividend payout ratios according to equation (3.8) in order to manage their financial robustness, but they pay out a fraction of what they believe to be the fundamental value of their cash flow, hence introducing an element of dividend smoothing. In these experiments there is no switching behaviour; firms retain the type they are assigned at the beginning of

\(^{27}\) Fazzari et al. (2008) examine the robustness of their results to a change from naive expectations to least squares learning along the lines described by Evans and Honkapohja (2001), finding that in their model Minsky cycles also arise under the more sophisticated specification of expectations and that naive expectations are roughly model-consistent.
the simulation for the entire duration of the run. If a fundamentalist firm goes bankrupt and is replaced by a new firm, this new firm is also assigned the fundamentalist type. The experiments are conducted by repeatedly running the modified versions of the model with increasing shares of fundamentalist firms. The share of fundamentalist firms \( s_f \) is increased from 0.1 to 1 in steps of size 0.1. Each parametrisation is run for 100 Monte-Carlo repetitions. At the beginning of each run, types are assigned randomly such that on average, the share of fundamentalist firms is equal to the value of \( s_f \).

Table 3.5 shows the standard deviations of GDP generated by the modified versions of the model for different values of \( s_f \), relative to the baseline with \( s_f = 0 \). It can be seen that in both cases the standard deviation of GDP decreases monotonically and quite strongly with an increasing number of fundamentalist firms, reaching values of 51 and 27 percent of the baseline respectively when all firms behave as fundamentalists.

<table>
<thead>
<tr>
<th>( s_f )</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
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<tr>
<td>Type 2</td>
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<td>0.67</td>
<td>0.58</td>
<td>0.49</td>
<td>0.43</td>
<td>0.38</td>
<td>0.34</td>
<td>0.30</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Figure 3.15: Dynamics of GDP in the presence of fundamentalist firms

Figure 3.15 illustrates the changes to the dynamics of GDP introduced by the two types of fundamentalist firms, with each panel showing GDP for the baseline, the case of 50% and that of 100% fundamentalist firms. The reduction of volatility can be observed in both graphs, as can a reduction in the regularity
of the macroeconomic cycles. This is due to fluctuations in investment becoming much less regular in the presence of fundamentalist firms as the incidence of herding behaviour present in the baseline is reduced. The addition of dividend smoothing to the fundamentalist firms’ behaviour leads to a reduction in the volatility of dividend payments, which both stabilises disposable income (and therefore aggregate demand) and leads to a reduction in financial fragility among C-firms, hence reinforcing the stabilising effect exerted by the fundamentalist expectations about the yield on capital goods. These two experiments further emphasise the central role of firms’ beliefs and expectations in Minskyan investment cycles. Together with the first experiment, they show that depending on how they are specified, beliefs about the prospective yield of capital goods can act as a type of coordination device which may produce herding behaviour that exacerbates economic fluctuations, or they may exert a stabilising effect on the economy. Moreover, the experiments also further highlight the important and dual role of dividend payments in the present model which, as argued above, is in line with the work of Minsky. The next experiments explore possibilities to stabilise the baseline model through policy interventions.

3.4.2 Policy experiments

To begin with, I present experiments aimed at stabilising the model using monetary and fiscal policy. As with the previous experiments, the policy experiments below are activated after 500 simulation periods in each run. Based on his views about the inherent instability of financialised capitalist economies, Minsky argued strongly in favour of public sector interventions, emphasising the necessity of ‘big government’ and ‘big bank’ to stabilise the economy (Minsky, 1986; Wray, 2016). ‘Big government’ is intended to stabilise the economy by maintaining aggregate demand and cash flows to the private sector through fiscal policy interventions which must be of a sufficient size relative to the overall economy. ‘Big bank’, i.e. a powerful central bank, must act as a lender of last resort to the financial sector, maintain the payments system and regulate financial markets. Importantly, Minsky argued that public policy must be designed to limit both the downward and the upward instability of the system, i.e. in addition to intervening during recessionary periods it must also prevent the emergence of unsustainable booms which eventually result in a period of crisis. As far as central bank policy is concerned, the present model only allows for the implementation of conventional (interest rate) monetary policy, rather than the more comprehensive form of central bank policy envisioned by Minsky under the heading of ‘big bank’. I assume that the exogenous nominal base interest rate $r_0$, which serves as the intercept for the loan interest rate in equation (3.13) is endogenised according to
\[ r_0 = \max(0, \phi_{b2} r_{0,-1} + (1 - \phi_{b2}) (\bar{r}_0 + \phi_{b1} \hat{y})) \]  

where \( \bar{r}_0 \) is set equal to the level of the exogenous \( r_0 \) in the baseline model. \( \phi_{b2} \) governs the adjustment speed of the interest rate. \( \hat{y} \) is the growth rate of real GDP, meaning that the central bank increases the base rate when growth is positive and decreases it when growth is negative, with the strength of the adjustment depending on the value of \( \phi_{b1} \) (> 0). The implicit goal is to keep growth close to zero, i.e. to keep the economy close to a stationary state. Since, as outlined above, the present model does not include factors producing long-term growth the central bank hence aims to reduce the endogenous fluctuations around a stationary level of GDP which are produced by the model. Drawing a parallel to the New Keynesian literature, this may be regarded as similar to a central bank which adjusts interest rates in response to changes in the output gap (e.g. Galí, 2015). Though Minsky himself did not advocate the use of interest rate policy for macroeconomic stabilisation, the structure of the two-price model in which the loan interest rate plays a key role in determining the supply price of capital does suggest that interest rate policy should have some effect.

Fiscal policy is implemented through the activation of the government sector which, as pointed out in section 3.2, is included in the model but remains inactive in the baseline simulations. Fiscal policy takes a very simple form, namely that of transfer payments to the household sector which may be either positive or negative (in which case they act like a tax). The transfer payment in any period is given by

\[ T = \phi_{g}(\bar{y} - y) \]

where \( \bar{y} \) is the average value of real GDP in the 300 periods preceding the start of the policy experiment. Fiscal policy is hence explicitly aimed at adjusting households’ disposable income through transfer payments (or taxes in the case of negative payments) in order to keep GDP close to the average value prevailing prior to the experiment. A positive transfer payment implies that the government is running as explained in section 3.2, the model does not incorporate persistent inflationary forces and indeed, aggregate price level fluctuations observed in simulations due to fluctuations in the average mark-up are extremely minor and short-term. As such, the monetary policy rule does not incorporate inflation. Both monetary and fiscal policy are hence functions of GDP, but specified somewhat differently. It was found that monetary policy works best when reacting to growth rates of GDP in the present model, while fiscal policy works best when it reacts to changes in the level of GDP relative to some target value. If monetary policy instead reacts to deviations (in level or percent) from some target value of GDP this may lead to large fluctuations in the base interest rate which are themselves destabilising. Conversely, if fiscal policy reacts to growth rates, it is still stabilising but may result in GDP stabilising around a level below the mean prevailing in the baseline as the transfer payment is not anchored by some fixed value as \( \bar{r}_0 \) in equation (3.23).

\[28\] As explained in section 3.2, the model does not incorporate persistent inflationary forces and indeed, aggregate price level fluctuations observed in simulations due to fluctuations in the average mark-up are extremely minor and short-term. As such, the monetary policy rule does not incorporate inflation.

\[29\] Both monetary and fiscal policy are hence functions of GDP, but specified somewhat differently. It was found that monetary policy works best when reacting to growth rates of GDP in the present model, while fiscal policy works best when it reacts to changes in the level of GDP relative to some target value. If monetary policy instead reacts to deviations (in level or percent) from some target value of GDP this may lead to large fluctuations in the base interest rate which are themselves destabilising. Conversely, if fiscal policy reacts to growth rates, it is still stabilising but may result in GDP stabilising around a level below the mean prevailing in the baseline as the transfer payment is not anchored by some fixed value as \( \bar{r}_0 \) in equation (3.23).
a deficit. To cover this deficit, it sells bonds to the banking sector which pay an interest rate equal the exogenous base interest rate. If the government runs a surplus, these bonds are repaid and if the stock of bonds goes to zero the government begins to accumulate unremunerated deposits with the banking sector, which it can in turn run down if it begins to run a deficit again. For simplicity, government bonds on the banks’ balance sheet do not enter into the calculation of the capital adequacy ratio, being considered risk-free.

Figure 3.16 summarises the effects of the two policy regimes on the dynamics of GDP, showing that both fiscal and monetary policy lead to a considerable degree of stabilisation of GDP around roughly equal levels. Fiscal policy is able to reduce the standard deviations of GDP and consumption by over 82 and 80 percent relative to the baseline respectively, while monetary policy achieves reductions by over 63 and 66 percent respectively. While both policies can hence very considerably reduce volatility in output and consumption, they differ starkly with regard to their efficacy in stabilising aggregate investment. Fiscal policy reduces the volatility of investment by over 57 percent while with interest rate policy the decrease is only around 8.5 percent. Effects of similar magnitudes are also achieved when the baseline model is augmented by the sentiment dynamics outlined in the first simulation experiment above and then subjected to the policy treatments.

As argued by Minsky, fiscal policy stabilises aggregate demand and private sector cash flows, thereby indirectly exerting a stabilising influence on investment decisions not only through maintaining aggregate demand at the possible onset of downturns but also through curtailing booms which would eventually
lead to increases in financial fragility. It is thereby able to stabilise both consumption and investment, leading to a large overall reduction in the volatility of GDP. A stabilising effect of countercyclical fiscal policy is a common result in the Minskyan literature (e.g. Nikolaidi, 2014; Dafermos, 2018), though Keen (1995) finds that in his model, fiscal policy can only prevent a breakdown but not eliminate cyclical fluctuations. Increasing (decreasing) the value of the parameter $\phi_g$ by $\pm 25\%$ relative to the value used here monotonically increases (decreases) the stabilising effect of fiscal policy. A countercyclical fiscal policy as specified here also does not lead to persistent deficits or exploding government debt. On average over time and across all Monte Carlo repetitions, the government budget balance shows a deficit of only 0.003% of nominal GDP, which on average leads to a very small positive stock of outstanding government bonds equal to 0.7% of GDP at the end of a simulation run.

Monetary policy acts directly on the investment decision by increasing (decreasing) the supply price of capital when growth is high (low). At the same time has an effect on the payment commitments of borrowers as well as the interest income of banks and hence the latters’ dividend payments. While adjustments in the base interest rate are able to eliminate the regular cycles in investment characterising the baseline simulation, they also lead to a considerable increase in short-term fluctuations in investment even when the interest rate adjusts fairly slowly, explaining why the volatility of investment declines only slightly. The volatility of consumption and hence GDP still declines quite strongly however as longer-term fluctuations in investment and their effects on disposable income are eliminated and firm dividends become somewhat less volatile. Both an increase in $\phi_b$ (strengthening the reaction of the base interest rate to growth) and a decrease in $\phi_b$ (making the base rate adjust more rapidly) relative to what is shown here can in fact increase the short-term fluctuations in investment caused by monetary policy to such a degree that the stabilising effect demonstrated here is negated. While interest rate policy may hence be stabilising, it must be carefully calibrated in order to not react too aggressively.

The result of the monetary policy experiment hence does not fully contradict the Minskyan literature which often regards interest rate policy as a potentially destabilising factor,\textsuperscript{30} as changes in the base interest rate are themselves a source of short-term fluctuations in investment here. Nevertheless, the overall stabilising effect of conventional monetary policy shown here may appear somewhat unusual since interest rate policy as a potential stabilisation tool has not played a prominent role in the Minskyan literature\textsuperscript{31}

\textsuperscript{30}In particular, interest rate policy has been posited as a possible factor bringing about the turning point at the peak of a cycle (cf. Nikolaidi and Stockhammer, 2017).

\textsuperscript{31}There are some exceptions to this rule. Chiarella and Di Guilmi (2017) examine an augmented Taylor rule incorporating a measure of financial fragility. They find that variations in the parameters of this policy rule have rather complex effects and that attempts by the central bank to target financial stability directly through interest
and, as pointed out above, Minsky himself focused on other aspects of central bank policy. Nevertheless, considering the specification of the two-price model in which the financing cost of investment plays a central role, and given the close link between central bank rates and bank lending rates (abstracting from possible issues of interest rate pass-through) it appears intuitive that interest rate policy might be stabilising if appropriately specified. More generally, the experiment shows that with its inherent feedback mechanisms for changes in interest rates (or financing conditions more broadly), the two-price model is a very suitable framework for analysing the effects of central bank policy in Minskyan systems. An interesting extension of the framework presented here might hence involve the introduction of factors which could reduce the effectiveness of conventional monetary policy (as seems empirically plausible particularly in the case of major finance-driven cycles). A more detailed model could then also explore the types of central bank policy advocated by Minsky, including attempts at financial regulation.

The important and dual role of firms’ dividend payout decisions as a factor in determining both disposable income of households (and hence aggregate demand) and the financing structure of capital investment has been outlined in section 3.2. Given this important role of dividends both in the present model and in the work of Minsky himself, the final policy experiment is concerned with the implementation of a limit on C-firms’ dividend payout ratios. For this experiment it is assumed that from simulation period 500, a regulatory authority places a uniform upper limit on the dividend payout ratio $\gamma^j$ of each firm. This upper limit adjusts endogenously and is given by

$$\gamma_{max} = \min(2\gamma, 2\gamma - \phi_{div} \frac{y - \overline{y}}{\overline{y}}),$$  \hspace{1cm} (3.25)$$

where $2\gamma$ is the upper bound already incorporated into equation (3.8) and $\overline{y}$ is once again the average of real GDP prevailing in the 300 pre-experiment periods. The upper bound on the dividend payout ratio is hence adjusted downward whenever GDP exceeds $\overline{y}$ (but is not adjusted upward in the opposite case). In line with the dual role of dividends emphasised above, the implicit goal of this policy is twofold. Firstly, it is intended to prevent firms from relying too heavily on external financing and hence to limit the emergence of financial fragility. Secondly, it may contribute to limiting booms in demand as it rate policy may have destabilising effects. Charpe et al. (2011, Ch. 9) analyse the effects of conventional monetary policy on a high-dimensional macrodynamic model of debt deflation, finding that it can have a stabilising effect. Fazzari et al. (2008) do not consider active interest rate policy, but in their model nominal interest rates move one for one with expected inflation (while the real rate is constant). These endogenous interest rate dynamics play an important role in the dynamics of their model, contributing to the increase in debt service which eventually triggers downswings in their Minsky cycles. Keen (1995) finds that in his model, a level of the (exogenous and fixed) interest rate beyond a certain threshold gives rise to instability.
places a limit on the growth of income from firm dividend payments. Figure 3.17 summarises the effect of this policy on the dynamics of GDP. It can be seen that the policy has a strongly stabilising effect. Despite limiting the movements of dividend payments only in an upward direction, it also eliminates the deep downturns present in the baseline model by preventing the build-ups of financial fragility occurring during booms. The standard deviations of GDP, consumption and investment decline by over 62, 36 and 63 percent respectively.

Figure 3.17: Effect of dividend policy on GDP

Table 3.6 compares the average number of firm bankruptcies per run in the baseline and under the three policy regimes (the numbers in brackets being the 95% confidence intervals from a Wilcoxon signed rank test). It shows that, in contrast to monetary and fiscal policy, dividend policy leads to a significant reduction in the number of firm bankruptcies. While under monetary and fiscal policies firm bankruptcies become idiosyncratic as the regular cycles disappear, individual firms still become fragile enough to go into bankruptcy at least as frequently as in the baseline. By contrast, dividend policy is able to limit the financial fragility of individual firms, hence reducing the number of bankruptcies. Increasing the value of \( \phi_{\text{div}} \) from 0 to the value of used in the simulation shown here results in a monotonic decline of macroeconomic volatility. Further increases in \( \phi_{\text{div}} \), however, may lead to dividend policy itself becoming a destabilising factor as large and high frequency impositions of restrictions on dividend payments and the subsequent lifting thereof may themselves cause large fluctuations in demand and income. Just like monetary policy, dividend policy must hence be appropriately calibrated.

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Table 3.6: Average firm bankruptcies under different policy regimes

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Fiscal Policy</th>
<th>Monetary Policy</th>
<th>Dividend Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcies</td>
<td>120</td>
<td>121.4</td>
<td>124.5</td>
<td>99.5</td>
</tr>
<tr>
<td></td>
<td>(118; 122)</td>
<td>(119.5; 124)</td>
<td>(122.5; 126.5)</td>
<td>(98; 101)</td>
</tr>
</tbody>
</table>

This last experiment, which to my knowledge is novel to the Minskyan modelling literature, shows that limitations on firm dividend payouts may be a useful policy tool in promoting economic and financial stability which merits further research. Similar proposals have been advanced with regard to bank dividend payments to improve the stability of the financial sector (e.g. Goodhart et al., 2010; Muñoz, 2019). In a large-scale macroeconomic ABM based on the Eurace@Unibi framework (see Dawid et al., 2018a), van der Hoog (2018) finds that restrictions on the dividend payments of banks have only weakly stabilising effects, but suggests that similar restrictions on firms might strengthen the result. Cincotti et al. (2010), using the version of the Eurace framework maintained in Genoa report that higher dividend payout ratios increase the amplitude of the endogenous cycles produced by their model. In a somewhat more parsimonious macroeconomic ABM setting, Riccetti et al. (2016) find that higher firm and bank dividend payouts lead to an increase in macroeconomic instability. More broadly, the finding presented here mirrors common arguments from the literature on financialisation which regards increases in dividend payouts as a destabilising factor (e.g. Tori and Onaran, 2018). While in the previous financial crisis the key role was played by household debt, a subject which the Minskyan literature had originally not strongly focused on, the recent growth in the market for leveraged loans (Adrian, 2019) signals a return of the firm sector as a locus of financial instability. As such, Minsky models with a traditional focus on the firm sector such as the present one continue to hold relevance for the analysis of financialised capitalist economies.

3.5 Conclusion

The purpose of this paper has been to provide a formalised dynamic version of Hyman Minsky’s two-price model of investment in a simple agent-based setting and to examine the emergent macroeconomic dynamics. Implementing the two-price model for a sector of agent-based consumption goods firms interacting with a set of strongly simplified stock-flow consistent aggregate sectors, the resulting model

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32As noted by Nikolaidi and Stockhammer (2017) there has however been rapid growth in the literature on Minsky models incorporating household debt in particular in the aftermath of the global financial crisis.
was calibrated empirically, showing that as far as the aggregate results of firms’ behaviour are concerned, it is capable of producing plausible macroeconomic dynamics. The model gives rise to persistent investment-driven cycles at the aggregate level which emerge from the uncoordinated investment decisions and financing choices of the agent-based firms. This demonstrates that the microeconomic framework of Minsky’s two-price model can indeed serve as a suitable basis to depict dynamics in line with the Financial Instability Hypothesis.

Beyond the use of the empirical calibration procedure, which is novel to the Minskian literature, the paper contributes to the literature by providing a detailed formalisation of a central but relatively neglected aspect of Minsky’s work. This gives rise to a behaviourally rich depiction of firms’ investment and financing decisions which is not present in many of the more simplified aggregate models dominating the literature. The paper also emphasised the value added by an agent-based approach in generating a useful additional layer of analysis. It was shown that even when the firm sector is disaggregated, firms’ individual investment decisions are still sufficiently correlated to give rise to cyclical aggregate dynamics. Moreover, the micro-level data generated by the model can be used to gain a range of interesting insights which could not be gleaned from purely aggregate data.

The first set of simulation experiments showed that the incorporation of sentiment dynamics and strategy switching behaviour can exacerbate the macroeconomic fluctuations present in the baseline model, while the introduction of firms which behave as fundamentalists with regard to their expected yield has a stabilising effect. This served to emphasise the importance of investors’ beliefs and expectations in the two-price model, but also highlighted the flexibility of the general framework for the incorporation of various behavioural assumptions. The paper hence also adds to the small literature on Minsky models in which expectations formation plays a major role in producing the observed dynamics. Moreover, it was argued that the formalisation of the two-price model provides a useful way to relate Minskyan dynamics to the mechanisms driving fluctuations in more conventional models of real-financial interactions, especially the financial accelerator literature.

Finally, the model also allowed for the introduction of stabilisation policy. It was shown that both conventional monetary (interest rate) policy and fiscal policy can lead to a strong reduction in the volatility produced by the model. The finding regarding monetary policy is interesting in that interest rate policy has played a rather limited role as a stabilisation tool in the Minskian literature, but it was argued that the assumptions underlying the two-price model do suggest a potentially stabilising role for interest rate policy which could be further explored in extensions of the present framework. In a third policy experi-
ment which is novel to the literature, it was shown that restrictions on firms’ dividend payouts can have a stabilising effect, suggesting a possible additional policy lever to promote financial and macroeconomic stability which merits further research.
Appendices

Appendix 3.A: Initialisation and parameter values

Table 3.7 shows the values of parameters used in the baseline simulations. It also gives information on whether a given value is empirically calibrated (“emp’), using the procedures and data sources detailed in section 3.3.1, imposed to produce an initial, stock-flow consistent stationary state (“pre-SS”), implied by that stationary state (“SS-given”), or free (i.e. independent of the initial stationary state). All pre-SS and free parameters are subjected to a sensitivity analysis below.

Table 3.7: Parameters (Baseline model)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Remark</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_f )</td>
<td>free</td>
<td>Number of C-firms</td>
<td>50</td>
</tr>
<tr>
<td>( w )</td>
<td>pre-SS</td>
<td>Nominal wage</td>
<td>1</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>pre-SS</td>
<td>Normal mark-up</td>
<td>0.5</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>free</td>
<td>MPC out of income</td>
<td>0.8</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>SS-given</td>
<td>MPC out of wealth</td>
<td>0.04</td>
</tr>
<tr>
<td>( \theta )</td>
<td>pre-SS</td>
<td>Loan repayment rate</td>
<td>0.015</td>
</tr>
<tr>
<td>( \delta )</td>
<td>pre-SS</td>
<td>Capital depreciation rate</td>
<td>0.015</td>
</tr>
<tr>
<td>( r_d )</td>
<td>pre-SS</td>
<td>Discount rate</td>
<td>0.0125</td>
</tr>
<tr>
<td>( p_k )</td>
<td>SS-given</td>
<td>Price of capital goods</td>
<td>1.5</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>pre-SS</td>
<td>Banks’ target capital ratio</td>
<td>0.12</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>SS-given</td>
<td>Borrowers’ risk parameter</td>
<td>3.829714</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>SS-given</td>
<td>Lenders’ risk parameter</td>
<td>0.03259977</td>
</tr>
<tr>
<td>( r_0 )</td>
<td>pre-SS</td>
<td>Base interest rate</td>
<td>0.0125</td>
</tr>
<tr>
<td>( \iota_1 )</td>
<td>emp</td>
<td>Demand distribution price sensitivity</td>
<td>-1.9</td>
</tr>
<tr>
<td>( \iota_2 )</td>
<td>emp</td>
<td>Demand distribution size sensitivity</td>
<td>0.8</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>emp</td>
<td>Persistence of demand distribution</td>
<td>0.76</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>emp</td>
<td>Persistence of demand distribution shock</td>
<td>0.12</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>emp</td>
<td>Strength of demand distribution shock</td>
<td>2.8</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>pre-SS</td>
<td>Full-capacity output to capital ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>( \kappa_L )</td>
<td>pre-SS</td>
<td>Overhead labour cost ratio</td>
<td>0.065</td>
</tr>
<tr>
<td>( CAR )</td>
<td>pre-SS</td>
<td>Banks’ capital adequacy ratio</td>
<td>0.12</td>
</tr>
<tr>
<td>( \chi )</td>
<td>free</td>
<td>Haircut applied to capital</td>
<td>0.75</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>SS-given</td>
<td>Stationary-state debt service to cash flow ratio</td>
<td>0.4528492</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>emp</td>
<td>Bankruptcy parameter</td>
<td>0.06</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>emp</td>
<td>Price adjustment parameter</td>
<td>-1.1</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>emp</td>
<td>Payout ratio intercept</td>
<td>0.44</td>
</tr>
<tr>
<td>( \tau )</td>
<td>emp</td>
<td>Payout ratio adjustment parameter</td>
<td>47</td>
</tr>
</tbody>
</table>

Table 3.8 gives the values of parameters which are added to the model in order to implement the simulation experiments regarding expectations formation and policy discussed in section 3.4. All these parameters are free as their value can be set independently of the initial stationary state, and the con-
sequences of varying them are described in the discussions of the respective simulation experiments in section 3.4.

Table 3.8: Parameters (Simulation experiments)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Sentiment index weighting parameter</td>
<td>0.5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Sentiment adjustment parameter</td>
<td>-50</td>
</tr>
<tr>
<td>$\text{opt}$</td>
<td>Scaling parameter (optimistic firms)</td>
<td>1.1</td>
</tr>
<tr>
<td>$\text{pes}$</td>
<td>Scaling parameter (pessimistic firms)</td>
<td>0.9</td>
</tr>
<tr>
<td>$\phi_g$</td>
<td>Fiscal policy strength</td>
<td>1</td>
</tr>
<tr>
<td>$\phi_{b1}$</td>
<td>Monetary policy strength</td>
<td>0.6</td>
</tr>
<tr>
<td>$\phi_{b2}$</td>
<td>Monetary policy adjustment speed</td>
<td>0.9</td>
</tr>
<tr>
<td>$\phi_{div}$</td>
<td>Dividend ceiling adjustment</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 3.9 below shows the aggregate initial values which are needed to initialise the model for the simulations shown in the paper. Agent-based C-firms are initialised to be exactly identical and each initial aggregate stock and flow is divided between firms according to their initial market share, $\frac{1}{n_f}$. As with table 3.7, the table also gives information as to whether initial values are set to impose the initial steady state (“pre-SS”) or implied by the initial steady state (“SS-given”). All pre-SS initial values are subjected to a sensitivity analysis below. The initialisation and parametrisation proceeds by sequentially imposing (freely set or empirically calibrated) values for initial conditions and parameters such that values of other initial conditions and/or parameters are implied by the former under the assumption of an initial stationary state and the occasional use of accounting identities implied by the SFC structure of the model. For instance, given an aggregate real capital stock $k$, capacity utilisation $u$, a depreciation rate $\delta$ and a full capacity output to capital ratio $\kappa$, real output $y$, real consumption $c$ and real investment $i$ are determined. These can then, in conjunction with other imposed values, be used to find additional initial conditions and/or parameter values. The full initialisation and parametrisation protocol of the model, including the equations used to find the SS-given parameter and initial values, is available at https://github.com/SReissl/Minsky.

Table 3.9: Initial values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Remark</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_b$</td>
<td>SS-given</td>
<td>Bank equity</td>
<td>1944</td>
</tr>
<tr>
<td>$u$</td>
<td>pre-SS</td>
<td>Initial capacity utilisation</td>
<td>0.8</td>
</tr>
<tr>
<td>$D_{hf}$</td>
<td>SS-given</td>
<td>Household deposits</td>
<td>14256</td>
</tr>
<tr>
<td>$E_{fk}$</td>
<td>pre-SS</td>
<td>K-firm equity</td>
<td>0</td>
</tr>
<tr>
<td>$YD$</td>
<td>SS-given</td>
<td>Disposable income</td>
<td>5400</td>
</tr>
<tr>
<td>$V_h$</td>
<td>SS-given</td>
<td>Household wealth</td>
<td>27000</td>
</tr>
<tr>
<td>$k$</td>
<td>pre-SS</td>
<td>Real capital stock</td>
<td>18000</td>
</tr>
</tbody>
</table>
Table 3.9 – continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Remark</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Lev$</td>
<td>pre-SS</td>
<td>C-firms’ initial leverage</td>
<td>0.6</td>
</tr>
<tr>
<td>$L$</td>
<td>SS-given</td>
<td>Stock of loans</td>
<td>16200</td>
</tr>
<tr>
<td>$r_L$</td>
<td>SS-given</td>
<td>Loan interest rate</td>
<td>0.02726278</td>
</tr>
<tr>
<td>$D_{fc}$</td>
<td>SS-given</td>
<td>C-firms’ deposits</td>
<td>0</td>
</tr>
<tr>
<td>$C'$</td>
<td>SS-given</td>
<td>Nominal consumption</td>
<td>5400</td>
</tr>
<tr>
<td>$E_{fc}$</td>
<td>SS-given</td>
<td>C-firms’ equity</td>
<td>10800</td>
</tr>
<tr>
<td>$p_c$</td>
<td>SS-given</td>
<td>Consumption goods price level</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 3.10 provides a list and description of all endogenous which appear in the paper.

Table 3.10: List of endogenous variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{h/fc/g}$</td>
<td>Bank deposits (of households/C-firms/government)</td>
</tr>
<tr>
<td>$L$</td>
<td>Loans</td>
</tr>
<tr>
<td>$GB$</td>
<td>Government bonds</td>
</tr>
<tr>
<td>$E_{b/fc/fk}$</td>
<td>Bank/firm equity</td>
</tr>
<tr>
<td>$K$</td>
<td>Capital</td>
</tr>
<tr>
<td>$V_{b/fc/fk/b/g}$</td>
<td>Net worth of sector</td>
</tr>
<tr>
<td>$C$</td>
<td>Consumption</td>
</tr>
<tr>
<td>$I$</td>
<td>Investment</td>
</tr>
<tr>
<td>$W$</td>
<td>Wages</td>
</tr>
<tr>
<td>$D_{ivh/fc/fk}$</td>
<td>Bank/firm dividends</td>
</tr>
<tr>
<td>$T$</td>
<td>Transfer payments/Taxes</td>
</tr>
<tr>
<td>$iL^j$</td>
<td>Interest on loans (paid by C-firm $j$)</td>
</tr>
<tr>
<td>$iGB$</td>
<td>Interest on government bonds</td>
</tr>
<tr>
<td>$YD$</td>
<td>Nominal household disposable income</td>
</tr>
<tr>
<td>$CPI$</td>
<td>Price index for consumption goods</td>
</tr>
<tr>
<td>$y_k$</td>
<td>Real output of capital goods</td>
</tr>
<tr>
<td>$N_k$</td>
<td>Labour demand of K-firm</td>
</tr>
<tr>
<td>$Pr_b$</td>
<td>Bank profits</td>
</tr>
<tr>
<td>$c_{d(j)}$</td>
<td>Real consumption demand (of C-firm $j$)</td>
</tr>
<tr>
<td>$\Omega^j$</td>
<td>Mark-up of C-firm $j$</td>
</tr>
<tr>
<td>$s^j$</td>
<td>Market share of C-firm $j$</td>
</tr>
<tr>
<td>$p_c^j$</td>
<td>Price of C-firm $j$</td>
</tr>
<tr>
<td>$k^j$</td>
<td>Real capital stock of C-firm $j$</td>
</tr>
<tr>
<td>$\hat{p}_c^j$</td>
<td>Relative price of C-firm $j$</td>
</tr>
<tr>
<td>$\hat{k}^j$</td>
<td>Relative real capital stock of C-firm $j$</td>
</tr>
<tr>
<td>$\bar{p}_c$</td>
<td>Average price of consumption goods</td>
</tr>
<tr>
<td>$\bar{k}$</td>
<td>Average real capital stock</td>
</tr>
<tr>
<td>$y_c^j$</td>
<td>Real output of C-firm $j$</td>
</tr>
<tr>
<td>$N_c^j$</td>
<td>Labour demand of C-firm $j$</td>
</tr>
<tr>
<td>$W_c^j$</td>
<td>Total wages paid by C-firm $j$</td>
</tr>
<tr>
<td>$CF^j$</td>
<td>Cash flow of C-firm $j$</td>
</tr>
<tr>
<td>$\gamma^j$</td>
<td>Payout ratio of C-firm $j$</td>
</tr>
<tr>
<td>rep$_L^j$</td>
<td>Loan repayments of C-firm $j$</td>
</tr>
<tr>
<td>int$_L^j$</td>
<td>Available internal financing of C-firm $j$</td>
</tr>
<tr>
<td>$Q^j$</td>
<td>Expected future cash flow of C-firm $j$</td>
</tr>
</tbody>
</table>
Table 3.10 – continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{d,j}$</td>
<td>Demand price of capital of C-firm $j$</td>
</tr>
<tr>
<td>$p_{s,j}$</td>
<td>Supply price of capital of C-firm $j$</td>
</tr>
<tr>
<td>$r^j$</td>
<td>Loan interest rate charged to C-firm $j$</td>
</tr>
<tr>
<td>$CF^j_{free}$</td>
<td>Free cash flow of C-firm $j$</td>
</tr>
<tr>
<td>$def^j$</td>
<td>Default index of C-firm $j$</td>
</tr>
<tr>
<td>$p(default)^j$</td>
<td>Default probability of C-firm $j$</td>
</tr>
<tr>
<td>$sentiment^j$</td>
<td>Sentiment variable of C-firm $j$</td>
</tr>
<tr>
<td>$\hat{\pi}^j$</td>
<td>Perceived profitability variable of C-firm $j$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Average profit rate of C-firms</td>
</tr>
<tr>
<td>$\hat{y}$</td>
<td>Growth rate of real GDP</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>Average of real GDP</td>
</tr>
</tbody>
</table>

Appendix 3.B: Sensitivity analysis

Figure 3.18 shows the time series for GDP when the agent-based C-firms are replaced by one single large firm. This also implies removing all stochastic elements from the model, along with all bankruptcy conditions, as well as any kind of price adjustment since the latter is based on market shares in the baseline model (see equation (3.3)). It can be seen that the model (when initialised slightly off the initial stationary state) still produces endogenous cycles, though they are shaped somewhat differently and are naturally much more regular than in the stochastic model featuring firm heterogeneity. Moreover, the cycles have a much higher amplitude than those produced by the heterogeneous agent model, and the time-series fluctuates around a higher mean value.

![Figure 3.18: GDP without firm heterogeneity](image-url)
The overall effect of the presence of firm heterogeneity hence appears to be somewhat stabilising. This is unsurprising insofar as the case of a single firm is equivalent to having 50 individual but completely identical firms which all take the same decisions. In the heterogeneous agent model, the investment and finance-driven cycles are more damped and less regular as any individual firm may or may not experience high demand and revenues and/or low interest rates at any particular time and hence may or may not engage in high capital investment and financially risky strategies. Naturally, the issue of potentially heterogeneous decisions does not arise in the single firm model, leading to cycles of higher amplitude. The absence of firm bankruptcy in the single agent firm also prolongs booms and downturns, but at the same time is also the reason why the single firm model converges to a higher mean value of GDP (as the absence of bankruptcy also implies that no capital is scrapped as in the heterogeneous firm model). Given that the deterministic version of the model with only a single C-firm incorporates the same non-linear behavioural equations as the heterogeneous agent version, it is unsurprising that the former, too, gives rise to endogenous cycles. However, an important point of the above analysis was to demonstrate that even when decisions are taken individually by heterogeneous firms at the microeconomic level, rather than based directly on macroeconomic variables, cycles can still arise. Moreover, it was shown that the addition of a microeconomic dimension can add interesting insights about distributional dynamics, including the distribution of financial fragility. The cost of additional complexity introduced through the use of an agent-based methodology is hence in my view justified in the case of the present model. More broadly, however, the simulation result shown above indicates that depending on the research question, an aggregated non-linear macrodynamic model may be an appropriate tool and researchers should consider carefully what additional insights are gained by the use of a more disaggregated approach.

Figures 3.19 and 3.20 demonstrate the results of setting $\tau$ and $\mu_1$ to 0 respectively. As was mentioned in section 3.3.2, both modifications lead to the disappearance of the regular cycles observed in the baseline model. In the case of $\tau = 0$, there is a deep recession at the beginning of the simulation caused by a decline in investment, after which the model converges to a stable level of GDP at which all investment is internally financed and there are no cycles. In the case of $\mu_1 = 0$, implying that the demand price of capital does not decrease with investment, it can be seen that the model converges to an extremely high value of GDP and only irregular fluctuations arise. When the demand price does not decline with investment, the model is hence displaced far from the initial stationary state as investment in the initial periods of the simulation increases very strongly. This suggests that the presence of $\mu_1$ which makes the demand price decline with increasing investment plays a central role in creating the dynamics observed.
I carry out a simple sensitivity analysis on some of the initial values and parameters which are not calibrated empirically, in particular those labelled ‘pre-SS’ in tables 3.7 and 3.9. The values of these parameters and initial values can be set independently, but they themselves feed into the determination of the values of various others (‘SS-given’) in finding the initial stationary state. For instance, since in a stationary state we must have $c^d CPI = C = YD$ (as otherwise household wealth would not be
constant), the values of $\alpha_1$ and $\alpha_2$ in equation (3.1) are not mutually independent. Hence, varying the value of $\alpha_1$ also implies a change in the value of $\alpha_2$. The analysed parameters and initial values are varied one by one along the ranges and in steps of the sizes shown in table 3.11. For each resulting parameter combination, the model is simulated 25 times with different reproducible seeds. The figures below provide a summary of the outcomes of these simulations. For each parameter and initial value analysed, they plot mean real GDP - $\mu(y)$ - as well as the standard deviations of real GDP, real consumption, real investment and the number of Ponzi firms relative to their respective means - $\sigma(y)$, $\sigma(c)$, $\sigma(i)$ and $\sigma(Ponzi)$ - against the values of the parameter or initial value. The dashed line in each plot indicates the baseline value of the respective parameter or initial value.

### Table 3.11: Sensitivity analysis

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Range</th>
<th>Step</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>MPC out of income</td>
<td>0.65 – 0.95</td>
<td>0.025</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Haircut applied to capital</td>
<td>0.6 – 0.85</td>
<td>0.025</td>
</tr>
<tr>
<td>$n_f$</td>
<td>Number of C-firms</td>
<td>40 – 60</td>
<td>5</td>
</tr>
<tr>
<td>$w$</td>
<td>Nominal wage</td>
<td>0.75 – 1.25</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Normal mark-up</td>
<td>0.4 – 0.6</td>
<td>0.025</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.01 – 0.0225</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Loan repayment rate</td>
<td>0.01 – 0.0225</td>
<td>0.0025</td>
</tr>
<tr>
<td>$r_d$</td>
<td>Discount rate</td>
<td>0.0075 – 0.02</td>
<td>0.0025</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Base interest rate</td>
<td>0.0075 – 0.02</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Full-capacity output to capital ratio</td>
<td>0.2 – 0.35</td>
<td>0.025</td>
</tr>
<tr>
<td>$\kappa_L$</td>
<td>Overhead labour cost ratio</td>
<td>0.04 – 0.09</td>
<td>0.005</td>
</tr>
<tr>
<td>$k$</td>
<td>real capital stock</td>
<td>15000 – 21000</td>
<td>1000</td>
</tr>
<tr>
<td>$Lev$</td>
<td>C-firms’ initial leverage</td>
<td>0.4 – 0.8</td>
<td>0.05</td>
</tr>
<tr>
<td>$u$</td>
<td>Initial capacity utilisation</td>
<td>0.7 – 0.9</td>
<td>0.025</td>
</tr>
<tr>
<td>$CAR$</td>
<td>Banks’ capital adequacy ratio</td>
<td>0.07 – 0.17</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Regarding $\alpha_1$, the propensity to consume out of disposable income, figure 3.21 shows that an increase in the value of the parameter, meaning that consumption reacts more strongly to movements in disposable income, leads to a monotonic increase in the volatility displayed by the model (as summarised by $\sigma(y)$, $\sigma(c)$, $\sigma(i)$ and $\sigma(Ponzi)$). The mean value of GDP follows an inverted U-shape, meaning that both a high and a low value of the parameter decrease mean GDP relative to the baseline. With a low value, the model converges to a pattern of relatively small irregular fluctuations around a lower mean value of GDP as consumption does not react strongly enough to changes in income to encourage sufficient investment and at the same time produce regular cycles. With a high value of $\alpha_1$ cycles are so strongly amplified that the volume of firm bankruptcies leads to a lower value of mean GDP.

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33 This smaller number of Monte Carlo repetitions is sufficient to get a good qualitative idea of the results of these variations.
The effects of varying $\chi$, the proportion of capital retained when a firm goes bankrupt, are rather straightforward. As shown in figure 3.22, increasing the value of this parameter monotonically increases both the volatility of time series and the mean value of GDP. A smaller proportion of capital being written off in cases of bankruptcy implies smaller losses of productive capacity, explaining the higher value of GDP. At the same time, however, these losses in productive capacity dampen the cycles displayed by the model as they increase the heterogeneity among firms in terms of size (and hence demand), leading to less uniform behaviour at the aggregate level when $\chi$ is lower.

Varying the number of firms $n_f$ contained in the model along the range considered here strongly affects the volatility displayed by the model, as illustrated in figure 3.23. This is due to the standard deviation of the microeconomic shocks to individual firms’ market shares themselves being a function of lagged market shares, as outlined in the model description. A higher overall number of firms implies, on average, a smaller market share for each individual firm. This in turn means that microeconomic shocks will be smaller, decreasing the degree of firm heterogeneity which, as explained in the discussion of the single agent model above, leads to amplified cycles. The number of firms also has a weakly negative effect on mean GDP which is due to more frequent firm bankruptcies when cycles are amplified.

Increasing the value of the exogenous nominal wage rate $w$, as summarised by figure 3.24, leads to a monotonic increase in mean GDP driven by higher consumption. The volatility of GDP (as well as that of the number of Ponzi firms) hardly changes. While the volatility of consumption increases slightly, this is counterbalanced by a slight decrease in the volatility of investment as an increase in $w$ with unchanged mark-ups decreases the range of variation in retained earnings.

Increasing $\Omega$, the parameter which anchors C-firms’ mark-ups, significantly increases financial stability in the model, as shown in figure 3.25. With higher mark-ups, C-firms are less reliant on external financing and firm bankruptcies are less frequent, decreasing the volatility of investment and the number of Ponzi firms and increasing mean GDP. The volatility of consumption at first increases due to higher variations in dividend incomes but then stabilises along with the rest of the model, the volatility of GDP following suit.

The model turns out to be highly sensitive to a variation in the capital depreciation rate $\delta$, shown in figure 3.26, especially when the latter is varied on its own (without concurrently changing interest, discount and loan repayment rates). With higher depreciation, the model converges to a lower mean GDP as investment does not always suffice to replace depreciating capital. At the same time, the cyclical fluctuations of
the model are unabated, but given that the model fluctuates around lower mean values, relative volatility is increased.

A higher value of the loan repayment rate $\theta$ leads to lower values of both $\mu_1$ and $\mu_2$, the borrowers’ and lenders’ risk parameters, in the initialisation protocol for the model. These lower values encourage high investment, leading to an increase in mean GDP as is illustrated by figure 3.27. GDP and consumption become more volatile in both absolute and relative terms while investment volatility relative to its own mean value actually declines, even though it increases in absolute terms. The higher loan repayment rate also implies higher payment commitments, somewhat increasing the volatility of the number of Ponzi firms.

The model is also strongly affected by a variation of the discount rate $r_d$, summarised by figure 3.28. Increasing $r_d$ leads to a decrease in the value of $\mu_1$ in the initialisation protocol, which encourages an increase in investment raising mean GDP. The volatilities of GDP, consumption and investment all increase in absolute terms, but only that of investment rises more than proportionately with its own mean value while the other two decrease in relative terms. A lower relative volatility in consumption also leads to a stabilisation of cash flows, somewhat decreasing fluctuations in the number of Ponzi firms.

Varying the value of the exogenous base interest rate $r_0$ also has a fairly pronounced effect on the model, illustrated by figure 3.29. With a higher base interest rate, C-firms will tend to be more financially fragile, all other things equal. This in turn produces stronger finance and investment driven cycles and, due to more frequent firm bankruptcies, a somewhat lower value of mean GDP.

A higher full-capacity output to capital ratio $\kappa$, all other things equal, leads to a higher value of mean GDP and a lower share of investment in GDP. As shown in figure 3.30, this lower investment share in turn strongly decreases the aggregate volatility displayed by the model.

Increasing the ratio of overhead labour cost $\kappa_L$, as demonstrated by figure 3.31, has a slight positive effect on mean GDP as it increases the flow of a comparatively constant stream of income for households. The volatilities of GDP and consumption are largely unaffected, while investment and the number of Ponzi firms become slightly more volatile due to an increased reliance of C-firms on external finance.

Changing the initial value of the real capital stock $k$, summarised in figure figure 3.32, has a strong effect only on the mean value of GDP. The volatilities of GDP, consumption and the number of Ponzi firms
are almost unchanged while the volatility of investment decreases very slightly but monotonically with increases in \( k \). This suggests that in the nonlinear system determining investment according to the two-price model, the scale of the existing capital stock (which enters into the determination of the demand price) has a slight (negative) influence on the volatility of investment.

A low value of C-firms’ initial leverage ratio \( \text{Lev} \) strongly increases the value of the parameter \( \mu_2 \) in the initialisation protocol. The results of this are shown in figure 3.33. Due to the determination of the loan interest rate through the non-linear two-price model a low value of \( \text{Lev} \) leads to frequent very large spikes in loan interest rates which make the model converge to a path characterised by a low value of mean GDP, small fluctuations in GDP and consumption but relatively large and irregular jumps in investment. Increasing \( \text{Lev} \) makes the model converge to trajectories similar to those shown in the baseline until, for very high values, the regular cycles once more disappear and the model displays only minor fluctuations due to a very low value of \( \mu_2 \).

Increasing the initial value of C-firms’ capacity utilisation \( u \) strongly decreases the volatility displayed by the model, as illustrated by figure 3.34. The reason for this is that with higher capacity utilisation, individual firms are capacity-constrained more frequently, limiting the possible range of fluctuation in the model. Beyond a certain threshold, these rationing effects also decrease the mean value of GDP.

Increasing the value of the banks’ target capital adequacy ratio \( \text{CAR} \), as summarised by figure 3.35, gives rise to a slight increase in mean GDP and a small reduction in the fluctuations produced by the model as it slightly decreases the volatility of household wealth which somewhat stabilises consumption demand.
Figure 3.21: Sensitivity analysis - Propensity to consume out of income

Figure 3.22: Sensitivity analysis - Capital retained under bankruptcy

Figure 3.23: Sensitivity analysis - Number of C-firms

Figure 3.24: Sensitivity analysis - Nominal wage rate
Figure 3.29: Sensitivity analysis - Base interest rate

Figure 3.30: Sensitivity analysis - Full capacity output to capital ratio

Figure 3.31: Sensitivity analysis - Ratio of overhead labour cost

Figure 3.32: Sensitivity analysis - Real capital stock
Figure 3.33: Sensitivity analysis - Initial leverage ratio

Figure 3.34: Sensitivity analysis - Initial capacity utilisation

Figure 3.35: Sensitivity analysis - Target capital adequacy ratio
Chapter 4

From CATS to CAOS: Fiscal multipliers and agents’ expectations in a macroeconomic agent-based model

4.1 Introduction

Over the past decade, various different macroeconomic agent-based modelling (ABM) frameworks have been developed and refined by different research groups, and there is a growing literature applying these frameworks to a range of policy questions. Despite these advances, the role of agents’ beliefs and expectations formation on simulation results, and especially their impact on the outcomes of policy experiments, is to date not well-investigated in the ABM literature (cf. Dosi et al., 2017a). While agents in conventional macroeconomic models are arguably too rational and forward-looking, agents in ABMs frequently form expectations in a way which implies behaviour that seems excessively naive. While forward-looking behaviour in the way it is inserted into conventional models typically cannot be implemented in an agent-based context, it nevertheless appears desirable to work towards the incorporation of somewhat more sophisticated and in some sense forward-looking behavioural rules. This is particularly so since, despite their various advantages over more conventional and aggregative modelling paradigms, ABMs are potentially susceptible to the Lucas critique (Lucas, 1976). Although, as argued by Haldane and Turrell (2018, p. 2), “no model is fully Lucas critique-proof; it is a matter of degree”, it nevertheless appears reasonable to argue that agents in a macroeconomic model should be capable of reacting to policies and policy announcements in a systematic and sensible manner. Moreover, the construction of agent-based frameworks which are relatively similar to their mainstream counterparts, or even attempts

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1 Indeed, even the models which according to Lucas and his followers should be robust to the critique are in certain ways vulnerable to it (see e.g. Kirman, 1992; Altissimo et al., 2002).
at merging the paradigms (cf. Assenza and Delli Gatti, 2013; Gobbi and Grazzini, 2019) can be helpful in aiding comparability of results and in gaining a better understanding of the fundamental differences between modelling approaches.

The focus of the present paper is on the consumption behaviour of households and its effect on government expenditure multipliers. Beginning from a variant of the ‘complex adaptive trivial systems’ (CATS) family of models, namely the empirically estimated macroeconomic ABM framework presented by Delli Gatti and Grazzini (2019) (based on Assenza et al., 2018b), the paper provides a thorough investigation of the government expenditure multiplier. It is shown that in the unmodified framework this multiplier is significantly greater than one and that the model gives rise to regime-dependent multipliers. In a next step, the consumption behaviour of households is modified through the implementation of a framework in which households intertemporally optimise consumption subject to an estimated budget constraint, giving rise to what may, somewhat provocatively, be termed a ‘complex adaptive optimising system’ (CAOS). This modification is found to lead to a significant improvement in the welfare households derive from consumption. It is also demonstrated that under the alternative consumption behaviour, government expenditure multipliers are significantly smaller than in the baseline model, although they are still positive and still exhibit regime-dependence, particularly due to the presence of coordination failures on the goods market and liquidity-constrained households. Extending the modified consumption behaviour, heterogeneous and ex-ante exogenous beliefs of households about the effects of government expenditure shocks on their budget constraints are introduced. These beliefs may change endogenously as agents learn either through adaptation or switching behaviour following an actual shock. In both cases it is shown that beliefs may be temporarily self-fulfilling in that their presence may increase or reduce the magnitude of the fiscal multiplier. In a final extension, households use a least squares learning algorithm in order to form an estimate of the effect of fiscal policy shocks. The addition of expectation and learning dynamics allows for an incorporation of announcement effects of fiscal policy whereby agents react systematically to announcements of future changes in government expenditure. Moreover, it is shown that the expectations resulting from least-squares learning appear to be broadly correct on average.

The paper contributes to the literature on agent-based macroeconomic models in three main ways. Firstly, it provides a thorough assessment of the government expenditure multiplier and the factors affecting it in a canonical framework. Such an analysis is novel to the macro-ABM literature. Secondly, the paper shows how more forward-looking consumption behaviour closer to common mainstream formulations can be introduced into a relatively simple macro-ABM for comparative purposes. Finally, the paper
contributes to a partial addressal of the Lucas Critique in an agent-based framework through the incorporation of learning and announcement effects of fiscal policy, enabling agents to react systematically to policy announcements.

The paper is structured as follows: section 4.2 provides a motivation of the present paper, placing it within the wider literature. Section 4.3 provides a brief overview of the pre-existing model used. Section 4.4 explores the effects of government expenditure shocks in the baseline model. Section 4.5 introduces the modification in households’ consumption behaviour and investigates its effect on the government expenditure multiplier both in the absence and presence of beliefs. Section 4.6 presents the least squares learning experiment. Section 4.7 summarises and discusses the results of the previously shown exercises. Section 4.8 concludes. Parameter values can be found in appendix A. Appendix B tests some assumptions underlying the alternative consumption model. The code necessary to replicate all simulations shown below is available at https://github.com/SReissl/CATS.

4.2 Policy, expectations and learning in agent-based models

Dawid and Delli Gatti (2018) provide an excellent survey of the major existing macroeconomic ABM frameworks including the Keynes + Schumpeter model (Dosi et al., 2010), the Eurace models (Cincotti et al., 2010; Dawid et al., 2012) as well as the CATS family of models (Delli Gatti et al., 2011; Assenza et al., 2015), a version of which is utilised in the present study. They compare the institutional features and behavioural assumptions of the respective baseline models while identifying a range of common elements across the existing model families. While the agent-based approach allows for a realistic depiction of macroeconomic dynamics and their micro- or mesoeconomic origins, it also makes macroeconomic ABMs very complex and their output difficult to analyse. One important area of work has therefore been to address issues of transparency and reproducibility (Dawid et al., 2019), along with attempts to establish techniques and standards for the empirical validation of models, a process which is still ongoing (Windrum et al., 2007; Guerini and Moneta, 2017). Related to this, various authors have worked on possible ways to empirically calibrate or estimate macroeconomic agent-based models (Gilli and Winker, 2003; Grazzini and Richiardi, 2015; Grazzini et al., 2017) and on techniques to deal with the typically very high-dimensional parameter spaces and the high computational cost involved (Salle and Yildizoglu, 2014; Barde and van der Hoog, 2017; Lamperti et al., 2018).

At the same time, the major macroeconomic ABMs surveyed by Dawid and Delli Gatti (2018) are also
being applied to an increasing range of policy questions (see Fagiolo and Roventini, 2017 for a survey).
For instance, versions of the framework being used in the present paper have been applied to the study of monetary policy as well as the effects of prudential regulation. Delli Gatti and Desiderio (2015) show that a central bank following a Taylor rule can significantly abate the effects of a negative productivity shock compared to a scenario in which the central bank rate is fixed. Assenza et al. (2018a) show that a Taylor rule can decrease the incidence of crises in simulated time-series and that a maximum leverage ratio for banks can also have a positive effect on macroeconomic stability. Various other aspects of monetary policy, including the appropriate choice of inflation targets (Ashraf et al., 2016), variations in the central bank’s mandate (Dosi et al., 2015; Chiarella and Di Guilmi, 2017) and the interactions between or joint effects of monetary policy and prudential regulation (Krug, 2018) have been addressed using other models. Considerations of appropriate financial regulation and prudential policy to decrease the risk of serious financial crises have also been explored by Popoyan et al. (2017) and van der Hoog and Dawid (2019). Other areas in which macroeconomic ABMs have been applied to policy questions include labour markets and income distribution, with various studies comparing the results emerging from different degrees of labour market flexibility (e.g. Seppecher, 2012; Dosi et al., 2017b), analysing regional disparities in labour market regimes (Dawid et al., 2018b), or exploring possible links between inequality and economic growth (Caiani et al., 2019).

As one would expect, fiscal policy, which is the focus of the present paper, has also featured prominently in the ABM literature. Assenza et al. (2018b) use the same model which is utilised here to compare different fiscal policy regimes (including one in which a Stability and Growth Pact-type rule is implemented) showing that in many cases, the model will converge to an unemployment rate consistent with a balanced government budget, allowing policy-makers to target quasi-equilibrium unemployment rates through varying fiscal policy parameters. Indeed, fiscal rules such as the Stability and Growth Pact or the Fiscal Compact have been examined in a range of studies (e.g. Dosi et al., 2015; Teglio et al., 2019) which generally conclude that such rules which artificially constrain fiscal manoeuvring space have negative effects on economic performance. Caiani et al. (2018) study the effects of changes in fiscal rules in the setting of a monetary union, concluding that more restrictive rules will have a negative effect on GDP, employment and productivity. Dawid et al. (2018c) similarly consider a context akin to that of the European Monetary Union, showing that fiscal transfers from the core can alleviate crises in the periphery region. Harting (2015) focuses on the effects of different forms of fiscal policy during economic downturns, showing that a fiscal policy which promotes technological innovation not only augments economic
growth but also stabilises cyclical fluctuations. Napoletano et al. (2017) present what is to my knowledge the only existing paper calculating fiscal multipliers in a macroeconomic ABM. Building a strongly simplified ABM of an endowment economy with time-varying financial constraints, they examine the effects of a bankruptcy shock under different fiscal policy regimes. They find that fiscal multipliers in their model are state-dependent and can comfortably exceed one depending on the exact specification of the fiscal policy intervention.

In general it can hence be said that fiscal policy tends to be highly effective in existing macroeconomic ABMs. One reason for these results is that agents in ABMs, in contrast to most of their counterparts in conventional macroeconomic models, tend to act according to heuristics and rules of thumb based on bounded rationality. Expectations are typically formed using naive, adaptive or extrapolative rules and agents’ planning horizons are rather limited. The heuristics used by agents may be quite sophisticated (as e.g. some which can be found in Dawid et al., 2012) or the model may purposely rely on fairly simplistic behavioural rules (as tends to be the case e.g. with models building on Delli Gatti et al., 2011) but in all cases the implied degree of rationality is below that which can be found in dynamic general equilibrium models. One reason for this is certainly methodological; even the simplest macroeconomic ABMs are typically too complex to allow for a rational expectations solution to be found. On the other hand, the use of bounded rationality and heuristics is also frequently justified on theoretical and epistemological grounds, building on the arguments of authors such as Simon (1982) or Gigerenzer (2008). Firstly, simple heuristics may be quite successful and ‘rational’ in complex environments - a result which does appear to bear out e.g. in the analysis of Dosi et al. (2017a) and the simulations conducted in chapter 2. Secondly, even if it were possible to discover a rational expectations solution in complex ABM settings, it would still appear more realistic to assume that in a highly complex setting, agents would rely on more or less sophisticated heuristics based on bounded rationality. Nevertheless, the use of behavioural rules which do not allow agents to react systematically and in a forward-looking manner to policy interventions does make macroeconomic ABMs vulnerable to the Lucas critique.2

While an implementation of forward-looking and rational behaviour in the same sense as it is used in conventional models will not be possible in most macroeconomic ABMs, it nevertheless appears sensible to allow agents to plan and pro-actively adapt their behaviour to what they believe the effects of policy

2 As already pointed out in the introduction, this statement does not imply that conventional models are immune to the Lucas critique in a broad sense. One particular weakness of such models is that the representative agent may not be robust to policy interventions, potentially invalidating the results of welfare analyses (Jerison, 1984; Geweke, 1985).
may be. One way to do so is by allowing agents to learn about their environment and the effects of policy in some fashion and to let them endogenously adapt their behaviour as necessary. In a series of papers, Salle and co-authors (Salle et al., 2013; Salle, 2015; Salle et al., 2019) emphasise the themes of expectations, learning and adaptation in the context of policies and policy changes, using a macroeconomic ABM framework designed to be comparable to a conventional New Keynesian macroeconomic model. The model includes feedback channels for inflation expectations on households’ reservation wages and consumption propensities, and agents are engaged in learning behaviour to update their behavioural rules. This allows the authors to examine the effects of different central bank communication strategies for managing inflation expectations and the associated impacts on the effectiveness of monetary policy. Salle (2015) places a particular focus on the issue of robustness, letting households use artificial neural networks to form inflation expectations based on data generated by the model and information communicated by the central bank, allowing them to adapt endogenously to changes in central bank policy. This series of papers is close in spirit to the present work, the latter part of which will focus on allowing agents to form expectations about the impacts of fiscal policy and to adapt their behaviour accordingly. In contrast to the aforementioned works, the present paper makes use of an existing, fairly well-known framework rather than a purpose-built model and shows a possible way to bring the behaviour of agents as close as possible to that of their counterparts in conventional models. This in turn also provides for an improved understanding of the role of heuristics in driving policy effects.

4.3 Model overview

The model I choose to carry out the analysis is the one presented by Delli Gatti and Grazzini (2019), who estimate the model of Assenza et al. (2018b) using the Bayesian estimation algorithm for macroeconomic ABMs outlined in Grazzini et al. (2017). Parameter values for the baseline are set equal to those calibrated and estimated by Delli Gatti and Grazzini (2019). The model is one of an economy with no long-term growth and features five sectors, namely households (capitalists/firm owners and workers), consumption goods firms, capital goods firms, a public sector and a single, representative bank. Figure 3

Indeed, the use of learning algorithms is closely linked to the complexity and agent-based economics literature, and various techniques such as genetic algorithms (Dawid, 1999), evolutionary learning (Arifovic, 2000) and classifier systems (Holland, 1975) have been proposed as possible ways to model learning in an ABM. In the more conventional literature, least squares learning algorithms as outlined by Evans and Honkapohja (2001) have been widely applied.

For a more detailed summary of the model than can be provided below, the reader is referred to Assenza et al. (2018b) and Assenza et al. (2015).
4.1 provides an overview of the model in terms of sectoral balance sheets and transactions, while figure 4.2 depicts the different markets on which agents interact in the model.

Households, as indicated above, can be divided into workers and capitalists, the latter of which own the consumption and capital goods firms (there is one capitalist for each firm). Capitalist households also jointly own the single bank in the model. Both types of households hold unremunerated deposits with the banking sector as their only assets and by assumption do not borrow for any purpose. Workers inelastically offer one unit of labour to firms at a uniform wage determined by an aggregate Phillips-curve type equation with downward stickiness. If they are unemployed, they receive a certain fraction of the current wage in benefits and visit a fixed number of randomly drawn firms in each period looking for a job. Wage income is taxed at a fixed rate. Capitalists receive dividend income from the firms they own as well as from the bank, in the case that profits are positive. Dividend income is not taxed. Each household determines a level of desired consumption using a weighted average of past incomes along with a fixed fraction of its current wealth. It then visits a fixed number of randomly drawn firms and demands consumption goods, starting from the cheapest firm.

---

5This assumption will become important below since it implies that households may be liquidity-constrained in their consumption decisions.
C-firms produce a perishable consumption good using a Leontieff production function. They form expectations about demand and set their prices using an adaptive process based on their current relative price as well as past excess demand or supply as shown in figure 4.3. For instance, a firm in quadrant $a$, which has experienced excess demand and the previous price of which is below the average price, will increase its price. A firm in quadrant $c$ with a higher than average price and excess demand will increase its production. Firms hire or fire workers based on their current production plans and existing labour force. They demand capital goods based on essentially Kaleckian investment behaviour, reacting to trends in a weighted average of past values of capacity utilisation in order to keep utilisation close to an exogenous target. C-firms are allowed to undertake investment at stochastic intervals and, as on all other markets, each C-firm visits a fixed number of randomly drawn K-firms to shop for capital goods. C-firms demand loans from the banking sector if their liquidity is insufficient to finance production and investment. Liquidity reserves are held in the form of deposits.

K-firms are highly similar to C-firms, the major differences being that they produce using a labour-only technology (and hence do not invest) and that their output is not instantly perishable but stored as a gradually depreciating inventory. If a C-firm’s or K-firm’s equity turns negative, it is declared bankrupt. A new firm then enters the market (new C-firms receive the capital stock of the bankrupt firm) and receives an injection of equity and liquidity from the capitalist household which owns it.
The Government levies a constant rate tax on labour income and pays unemployed workers a benefit equal to a fixed fraction of the current wage rate. Deficits are covered by the issuance of bonds at a fixed risk-free interest rate. It is assumed that all bonds are purchased by the bank. In the baseline, the government does not undertake any spending on firms’ output. Such spending will be introduced as a fiscal shock below.

The Bank in the model takes deposits from firms and households and makes loans to the firm sector. In addition it purchases all bonds issued by the government. The bank provides all loans demanded by firms provided that the total exposure of the bank to any individual firm remains below a given fraction of its equity. This exposure is calculated based on a perceived bankruptcy probability of the firm, which is an increasing function of its leverage. The bankruptcy probability also feeds into the interest rate offered by the bank to each borrowing firm, with more risky firms paying a higher rate, i.e. a higher mark-up over the risk-free rate (see Assenza et al., 2015, 2018b for details). Importantly for the present paper, it should be noted that the model does not feature monetary policy as there is an exogenous and fixed risk-free interest rate, such that there is no interaction between monetary and fiscal policy.

Overall, model dynamics are strongly driven by coordination failures in goods, labour and credit markets stemming from both incomplete and imperfect information of agents. Firms will frequently experience periods of excess demand or supply, they may be unable to undertake planned production or investment
due to capacity constraints, matching failures on the labour market or credit rationing, and consumers may be unable to fully exhaust their consumption budget due to the limited number of firms they visit. Nevertheless, as is the case with all other existing major macro-ABM frameworks, the model is essentially demand-driven, although there exist a range of potential supply-side constraints including limited productive capacity and a finite labour force. The fundamentally demand-driven nature of the model unsurprisingly gives rise to a positive macroeconomic effect of expansionary government expenditure shocks, as is outlined in the next section. Macroeconomic dynamics are characterised by persistent and irregular fluctuations of GDP and its components around a long-term stochastic quasi-steady state. In order to reserve space for the thorough analysis of the effects of government expenditure shocks carried out below, the reader is referred to Assenza et al. (2015), Assenza et al. (2018b) and Delli Gatti and Grazzini (2019) for an extensive presentation of simulations as well as validation exercises of the model.

4.4 Fiscal policy shocks in the baseline model

While the analysis presented by Assenza et al. (2018b) also focuses on changes in taxes and transfer payments in addition to government consumption expenditure, and does not investigate the size of fiscal multipliers, my analysis solely considers the introduction of government expenditure on consumption goods with a focus on the size of the resulting multiplier. This focus on a single policy tool enables me to carry out a thorough analysis, running a range of different experiments. Rather than looking at different policy regimes (e.g. the combination of a certain tax rate, together with a certain replacement ratio and some permanent, exogenous level of government spending on consumption goods for the entire duration of a simulation), as do Assenza et al. (2018b), I consider only increases in government expenditure from the baseline level of 0, for a single period (one period being one quarter), at a certain point during the simulation. Unless otherwise stated, the additional government demand for consumption goods is always equal to 10% of real GDP in the period prior to the shock.\footnote{The shocks are hence admittedly quite large, but especially in the absence of any persistence, large shocks are necessary to clearly discern the effects of the policy change in the noisy simulation data. Delli Gatti and Grazzini (2019) use quite large shocks in their paper for the same reason.}

Government expenditure on consumption goods enters the model in a simple fashion. As with all AB macro models, each simulation period proceeds in sequential fashion, with a series of distinct events taking place one after the other rather than simultaneously. At the beginning of the period, C-firms plan their production and it is assumed that in the period in which the shock is introduced, C-firms which
wish to adjust their production relative to the previous period (in accordance with figure 4.3) take into account the government demand they will receive, as calculated by equation (4.1) below. Government expenditure on consumption goods is introduced after the closing of the consumption goods market on which C-firms and households interact. A desired level of real government consumption \( g_d \) is distributed to each C-firm according to a share calculated as

\[
s_f^g = \frac{R_f}{\sum_{f=1}^{F} R_f}
\]  

(4.1)

where \( R_f \) is the (previous) revenue of firm \( f \). The government then purchases from each firm \( f \) the minimum between its desired expenditure \( s_f^g g_d \) and the remaining output produced earlier in the period which has not already been sold to households. The effect of government expenditure shocks is analysed on the one hand through the calculation of cumulative multipliers and through the construction of ‘robust impulse-response functions’ similar to those proposed by Delli Gatti and Grazzini (2019). The cumulative multiplier at time \( t \) is defined as the cumulative change in output relative to the baseline divided by the actual government consumption expenditure induced by the shock (which may differ from the desired government expenditure). In Delli Gatti and Grazzini (2019), the robust IRFs are constructed by calculating and plotting the average response (in percentage terms relative to the baseline) of a macro time-series generated by the model (e.g. GDP) to a shock across all Monte Carlo repetitions. Since the model displays a strong tendency to return to a stochastic stationary state practically identical to the previous one following transitory shocks, the construction of such functions gives useful information about the magnitude and duration of the effects of temporary fiscal policy shocks.

Since the shocks considered in the present paper are not persistent and, at least compared to those shown by Delli Gatti and Grazzini (2019), relatively small, their effects are at times difficult to discern from looking at Monte Carlo averages only. I therefore draw on Dawid et al. (2018b) in using spline regressions (see e.g. Ruppert et al., 2003) in order to smooth out and depict the effects of the fiscal shocks. In particular, for each outcome variable \( X \) (such as GDP or consumption), I estimate the model

\[
X_{t,p,i} = s(t) + I_{\text{shock}=1}s_{\text{shock}}(t) + \varepsilon_{t,p,i}
\]  

(4.2)

where \( t \) is the simulation period, \( i \) is the MC repetition and \( p \) indicates the presence or absence of the policy shock. \( s(t) \) is the baseline spline, to which the policy spline \( s_{\text{shock}}(t) \) is added through the dummy \( I \) if the run in question includes a government expenditure shock. The policy spline then depicts
the smoothed reaction of the model to the policy shock (in the figures below reactions are expressed as \% of baseline levels), hence serving the same purpose as the robust IRFs suggested by Delli Gatti and Grazzini (2019). Both the baseline model and the model including the expenditure shock are simulated for 1500 periods, each 200 times with different reproducible seeds, and the shock is introduced in the same period in each individual run. In this way, possible cyclical effects (which are investigated below) are averaged out as booms and downturns do not coincide temporally across different runs.

Figures 4.4 and 4.5 summarise the effect of a government expenditure shock in the baseline model in terms of the cumulative multiplier and responses of key aggregate model variables. In the case of cumulative multipliers, the black and grey lines represent the median and 90\% confidence intervals from a Wilcoxon signed rank test across Monte Carlo repetitions respectively. In the case of the policy splines the black line represents the estimated spline while the grey lines are standard error bands.

[Figure 4.4: Cumulative government expenditure multiplier in the baseline model]

It can be seen that the model generates a small but positive multiplier on impact,\textsuperscript{7} which increases to eventually become significantly larger than one. Figure 4.5 plots the policy splines for real GDP, real consumption, real consumption demand, as well as gross fixed capital formation. It shows that the model

---

\textsuperscript{7}The impact multiplier is smaller than one since it is defined as the \textit{additional} output produced relative to the baseline, divided by the amount of actual government expenditure. Due to the algorithm by which firms adjust their production as well as the various constraints to which they may be subject at any given time, firms will usually not increase their production by the full amount of government demand which they receive, and some of the output purchased by the government will be goods which would have been produced even in the absence of the shock.
reaction to the fiscal shock is mainly driven by consumption whilst capital investment does not react strongly to the non-persistent shock. Real GDP increases on impact as firms increase their output in response to the additional government demand. Actual consumption increases in the first instance due to a reduction in rationing on the consumption goods market as a consequence of the additional output made available by firms which reduces the incidence of coordination failures on the market, as well as through a loosening of liquidity constraints on households. Consumption demand (i.e. desired consumption) reacts somewhat more sluggishly and persistently, causing a hump-shaped response of actual consumption in later periods.

Another interesting experiment can be conducted by investigating whether government expenditure multipliers are affected by cyclical conditions, i.e. whether they are larger during downturns and smaller during booms. Following the global financial crisis and the associated policy responses, a great deal of evidence in favour of such regime-dependent multipliers has been presented (see e.g. Auerbach and Gorodnichenko, 2012a; Fazzari et al., 2014; Qazizada and Stockhammer, 2014; Gechert and Rannenberg, 2018). There are good reasons to suppose that such cyclical effects might be present in the model used here. Firstly, during downturns the unemployment rate will be high and C-firms’ capacity utilisation
may be low, hence removing two of the potential supply-side constraints which can limit the effectiveness of fiscal policy. Secondly, recall that the model by construction includes a liquidity constraint on households. During recessions, this constraint may be binding for a greater number of households and may be eased by the introduction of government expenditure as an additional source of demand, an effect which has frequently been invoked to explain the empirical regime-dependence of multipliers (Mittnik and Semmler, 2012; Warmedinger et al., 2015). At the same time however, credit constraints on firms may be more stringent during downturns, possibly preventing them from expanding production. To test for a cyclical effect on the government expenditure multiplier, I identify the post-transient periods of minimum and maximum GDP in each of the 200 baseline runs without the policy shock, and then run two experiments, one in which the shock is introduced in the period of minimum GDP in each run and one in which it is introduced at the maximum.

Figure 4.6: Cumulative government expenditure multipliers during downturns (solid) and booms (dashed)

Figure 4.6 shows that there is indeed a difference between multipliers during booms and downturns, particularly so during the initial periods following the shock. Impact multipliers differ as firms are more capable of quickly increasing output during downturns. As firms produce additional output to meet the demand from the government, they hire additional workers, leading to the liquidity constraint

---

8The way in which the liquidity constraint is modelled here differs somewhat from that typically used in the New Keynesian DSGE literature (e.g. Gali et al., 2007) in that in the present model, while households are not allowed to borrow, they may save if willing and able to do so, i.e. they are not hand-to-mouth consumers.
being binding for fewer households, which can consequently increase their consumption. Just like the model analysed by Napoletano et al. (2017), the framework used here hence gives rise to state-dependent fiscal multipliers. The analysis of government expenditure shocks in the baseline model hence already reveals some interesting and arguably realistic effects. The following sections investigate to what extent the effects of government expenditure shocks depend on households’ consumption behaviour, including their expectations and beliefs about the effects of fiscal policy.

4.5 Time for CAOS - agent-based intertemporal optimisation

In order to more closely investigate the influence of consumption behaviour on fiscal multipliers, the baseline model is modified with respect to Delli Gatti and Grazzini (2019). In the baseline version of the model, households formulate a plan for desired consumption based on a weighted average of current and past incomes (dividends for the owners of firms and wages/unemployment benefits for worker households), i.e. each household $i$ calculates

$$Y^i_t = \xi Y^i_{t-1} + (1 - \xi)Y^i_t,$$

(4.3)

(where $Y^i_t$ is their income in the current period) and then determines their desired consumption according to

$$C^i_d = \overline{Y^i} + \chi D^i$$

(4.4)

where $D^i$ are their deposits with the bank. This rule serves to impart a degree of persistence to consumption demand, as is empirically warranted, inducing agents to react less than one-for-one to transitory shocks to disposable income. $\overline{Y^i}$ is understood as “a proxy for future expected income” (Assenza et al., 2015, p. 9). The modification of the baseline model proposed here alters this modelling of future expected income to introduce a behavioural rule which is somewhat more forward-looking and brings households’ consumption behaviour closer to conventional permanent income/life-cycle models of consumption (Hall, 1978).

In various respects which are important for the purpose of this paper, the structure of the baseline model is very simple relative to some other well-established macro-ABM frameworks. Workers are all identical with regard to their productivity and there is a uniform wage determined via a Phillips-curve equation at the aggregate level. Unemployed workers accept the first open position they find during the search
and matching process on the labour market, and if a firm is firing workers, it does so in a random order. All unemployed workers receive the same level of benefits (which are a fixed fraction of the current wage). Similarly, any consumption goods firm is structurally identical to all others in the sense that they all produce the same good using the same technology (and the same is true for capital goods firms), although any particular firm may at any given time of course be in a particularly good or bad financial situation. All firms (as well as the representative bank, which is owned in equal parts by all capitalists) have very simple dividend heuristics whereby they pay out a fixed fraction of profits if the latter are positive. In addition, workers’ and capitalists’ real incomes do not exhibit long-term trends but minor fluctuations around a stochastic stationary state, as do all other model variables in the real dimension (cf. Assenza et al., 2015).

Making use of the above observations, households’ behaviour is modified by assuming that they base the estimation of their future income on transition probabilities between different states and a long-term weighted average of the associated real incomes. For instance, given the parsimonious modelling of the labour market, it appears to be at least a reasonable approximation to represent workers’ employment status over time by a Markov chain with a transition matrix $T$ of the form

\[
\begin{pmatrix}
E & U \\
E & \pi_{EE} & 1 - \pi_{EE} \\
U & 1 - \pi_{UU} & \pi_{UU}
\end{pmatrix}
\]

Transition data is collected centrally during the simulation and used to estimate the transition probabilities, using data of the past $H$ periods. It is assumed that all workers have access to this aggregated information so that for the moment, they all have a common estimate of the transition probabilities. Moreover, it is assumed that workers calculate an unweighted average of the $H$ past values of the uniform real wage and the real unemployment benefit, resulting in a payoff vector $P = \begin{bmatrix} \bar{w} \\ s \cdot \bar{w} \end{bmatrix}$. Given that the real wage shows no long-term upward or downward trends in model simulations, this appears to be a reasonable way to forecast the long-run average future real wage and benefit. $T$ and $P$ can in turn be used to provide an estimate of future income, as they describe the likelihood of future state transitions along with an estimate of the associated incomes. In addition to being broadly familiar from various economic applications, this way of formulating the values of agents’ states draws closely on the machine learning

\footnote{The validity of this claim is assessed through simulation experiments presented in appendix B.}
literature, specifically that on reinforcement learning (Sutton and Barto, 1998). In the case of capitalists, the matrix $T$ represents the transitions between the states of receiving and not receiving a dividend (again calculated centrally and uniform across firms), and $P$ contains a long-run average of an individual firms’ dividends as its first element and zero as the second (i.e. there is a $P$ for each individual firm owner). The transition probabilities and payoff vectors form the basis of an alternative consumption behaviour involving intertemporal optimisation.

Consider the simplest case of a consumer holding a stock of assets and earning an income who attempts to maximise utility from consumption over a horizon $H$. Their intertemporal budget constraint can be written as

$$
\sum_{t=0}^{H} R^{-t} c_t \leq d_0 + \sum_{t=0}^{H} R^{-t} y_t,
$$

where $d_0$ is the real value of their assets in period 0 and $R$ is a gross interest rate. Based on the transition matrices and payoff vectors calculated as described above, an estimate of income in some future period $t$ from the perspective of period 0 can be calculated as

$$
y_t = \prod_{i=1}^{t} T_i P_t,
$$

where $y_t$ is a vector with a number of rows equal to the number of states. The estimated sum of future incomes, conditional on a series of expected transition matrices $T_i$ and payoff vectors $P_t$,\(^\text{10}\) is then given by

$$
\sum_{t=0}^{H} y_t = P_0 + \sum_{t=1}^{H} \left( \prod_{i=1}^{t} T_i P_t \right),
$$

so that

$$
\sum_{t=0}^{H} R^{-t} y_t = P_0 + \sum_{t=1}^{H} \left( R^{-t} \prod_{i=1}^{t} T_i P_t \right) = YP.
$$

By iterating forward from $t = 0$ to $t = H$ (in the simplest case assuming that both $T$ and $P$ are constant), the transition matrices and payoff vectors hence allow for the derivation of an estimated budget constraint (or, more precisely, a set of budget constraints with cardinality equal to the number of possible states contained in $T$), giving an estimate of the net present value of the sum of resources available for consumption over the future $H$ periods. In a slight abuse of terminology, I call this sum ‘permanent income’, $YP$. The consumer can then optimise, maximising a utility function subject to this estimated budget constraint. The consumer will hence face the problem

\(^{10}\)Neither $T$ nor $P$ need necessarily be constant. Households may well expect transition probabilities or payoffs to change in some future period and incorporate this expectation into their calculations.
\[
\max_{c_t} \sum_{t=0}^{\mathcal{H}} \beta^t u(c_t)
\] (4.9)

\[
\text{s.t. } \sum_{t=0}^{\mathcal{H}} R^{-t} c_t \leq d_0 + P_0 + \sum_{t=1}^{\mathcal{H}} R^{-t} \left( \prod_{i=1}^{t} T_i \mathbb{P}_i \right),
\]

where \( \beta \) is a discount factor. The solution to this problem will be a set of consumption paths with cardinality equal to the number of states contained in \( T \) or, to put it differently, there will be one maximisation problem for each possible current state. Writing the Lagrangian and taking derivatives gives a set of first order conditions of the form

\[
u'(c_t) = (\beta R)^{-t} \lambda.
\] (4.10)

The case in which \( \beta R = 1 \) gives rise to the familiar result that the household plans to consume a fixed fraction of their estimated total budget in each period, as in the simplest versions of the life-cycle or permanent income models of consumption (Friedman, 1957; Modigliani and Brumberg, 2005). If \( \beta R \neq 1 \), \( u(c_t) \) must be specified in order to solve for \( c_t \). In the case of the present model, \( R = 1 \) by construction since, while there is a base interest rate in the model, households cannot earn it as their only asset are \textit{unremunerated} bank deposits. \( \beta \) on the other hand is calibrated below to a value producing sensible results, meaning that in the present framework, \( \beta R \neq 1 \). Utility from consumption is assumed to be given by

\[
u(c_t) = \frac{c^{(1-\theta)} - 1}{1 - \theta},
\] (4.11)

such that the first order condition, given that \( R = 1 \), produces

\[
c_t = \left( \frac{\lambda}{\beta^t} \right)^{-\frac{1}{\theta}}.
\] (4.12)

Substituting this into the budget constraint to solve for \( \lambda \) and letting \( B = \sum_{t=0}^{\mathcal{H}} \beta^t \), the consumption demand of a households \( j \) becomes

\[
c_{jt} = \left( \beta^t \left( \frac{d_0^j + \sum_{t=0}^{\mathcal{H}} y_t^j}{B} \right)^{-\theta} \right)^{-\frac{1}{\theta}},
\] (4.13)

\[11\text{For instance, in the context of the model presented above, one path for a worker who is employed in } t = 0 \text{ and one for a worker who is unemployed in } t = 0.\]
which in period $t = 0$ simplifies to

$$c^j_0 = \frac{d^j_0 + \sum_{t=0}^{\mathcal{H}} y^j_t}{B}.$$  (4.14)

This consumption behaviour is followed by both worker and capitalist households, with the $T$’s and $P$’s being re-estimated in each period in order to gain an updated estimate of the budget constraint and re-optimize. Note that since the $T$’s and $P$’s are identical for all worker households, the only source of heterogeneity in the consumption demand of worker households stems from heterogeneity in asset holdings $d$, which in turn reflects worker households’ idiosyncratic employment history. Firm owner households, on the other hand, also each have an individual $P$ vector as firm dividends are less uniform than worker earnings (recall that there is a uniform wage rate).

The purpose of this modification is to introduce what might be viewed as an agent-based version of the permanent income or life-cycle model of consumption within a relatively simple macroeconomic framework. The values of the variables entering into equation (4.7) are of course still calculated based on past data and in this sense the formulation is backward-looking. Nevertheless, the consumption behaviour thus induced can be regarded as more forward-looking than the baseline one in that agents take into account their current state as well as the estimated likelihood of future ones and formulate a consumption path conditional on this expectation. Moreover, while in the simplest case, $T$ and $P$ are assumed constant and equal to the most recent estimate for each agent over the horizon $\mathcal{H}$, the framework also allows for the incorporation of expected future variations in transition probabilities and payoffs which would have an immediate effect on consumption demand.

Before moving on to an analysis of the effects of this modification on the government expenditure multiplier, two more points should be emphasised. Firstly, the alternative consumption behaviour outlined above does not eliminate the presence of the liquidity constraint on households, i.e. I assume that they are not allowed to borrow against their estimated future income. Secondly, fiscal policy will have an impact on agents’ consumption demand to the extent that it impacts their current state, the estimated transition probabilities, and their estimate of the income vectors. There is no notion of Ricardian equivalence in this model, i.e. agents do not, for instance, conjecture a future rise in income tax following a temporary shock to government expenditure (and indeed such rises do not take place as the tax rate is fixed in all simulations).\footnote{An analysis of simulation data shows that the brief positive government expenditure shocks considered in this paper in fact do not give rise to a significantly higher level of government debt in the long run.}

In all experiments shown below, the discount factor $\beta$ is set to a value of 0.9999.\footnote{This discount factor is admittedly very high, even compared to the DSGE literature in which high discount}
the length of the time-window used to estimate income vectors and transition probabilities, and the hori-
zon over which households optimise consumption, is set to 400 (implying a horizon of 100 years). $\theta$ is
set to a value of $\frac{2}{3}$, implying an intertemporal elasticity of substitution equal to 1.5, which is in line with
values commonly used in the macroeconomic literature (e.g. Fernández-Villaverde and Rubio-Ramírez,
2005; Smets and Wouters, 2007; Del Negro et al., 2015). The results presented below are qualitatively ro-
bust to changes in these parameters, though changes to one of them necessitate concurrent changes in the
others to obtain sensible results. The parameters are calibrated such that on average, the level of house-
holds’ consumption demand is roughly equal to that under the baseline consumption behaviour. Relative
to the baseline model, the implementation of the ‘permanent income’ and optimisation mechanism sig-
nificantly reduces the volatility of consumption and (to a somewhat lesser extent) GDP in simulations.
Table 4.1 shows the standard deviations of consumption demand, actual consumption and output in the
modified model as a ratio of those obtained from the baseline, demonstrating the reduction in volatility.

Table 4.1: Comparison of standard deviations in the modified model and the baseline

<table>
<thead>
<tr>
<th></th>
<th>Consumption Demand</th>
<th>Consumption</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
<td>0.3820</td>
<td>0.6586</td>
<td>0.7133</td>
</tr>
</tbody>
</table>

The introduction of a utility function for households also opens up the possibility of conducting a simple
form of welfare analysis. Recall that instantaneous utility from consumption for some household $j$ is
assumed to be given by

$$u(c^j_t) = \left(\frac{c^j_t}{(1-\theta)}\right) - \frac{1}{1-\theta}.$$  (4.15)

A measure of the welfare of an individual household $j$ over a simulation can be constructed as

$$U^j = \frac{\sum_{t=0}^{T} u(c^j_t)}{T},$$  (4.16)

where T is the length of the simulation and $t = 0$ is the first post-transient simulation period. An
factors are relatively common (see e.g. Smets and Wouters, 2007; Gerali et al., 2010), although the mainstream
literature does at times consider the case of $\beta = 1$ (Ascari and Rossi, 2012, p.1123). This issue could be overcome
by the introduction of an interest rate on the households’ assets. However, this would represent an additional
modification of the original baseline model I wish to avoid for ease of comparability between the baseline and the
model featuring the modified consumption behaviour. The focus of the present paper lies exclusively on exploring
fiscal multipliers and their relationship with households’ consumption behaviour rather than on extending the
baseline model itself.
aggregate measure of welfare from consumption can then be calculated as

\[ W = \sum_{j=1}^{J} U_j, \]  

(4.17)

where \( J \) is the totality of all households in the model. Table 4.2 shows a comparison of welfare from consumption between the baseline model (where consumption following the baseline heuristic is inserted into the same utility function used in the modified framework) and the model featuring the alternative consumption behaviour. Differences in utility are calculated agent-by-agent and then summed up. Line one shows the Monte-Carlo average difference in welfare, both for consumption demand (which is the object being optimised by agents in the modified model) and actual consumption (which may differ from consumption demand as outlined above), along with 95% confidence intervals. Line two expresses the difference in welfare in terms of consumption units\(^{14}\) and the third lines gives this difference as a percentage of average per-period consumption/consumption demand in the baseline model.

| Table 4.2: Difference of welfare between the modified model and the baseline |
|-----------------------------------------|-----------------------------------------|
| **Welfare** | **Consumption Demand** | **Actual Consumption** |
| 6.51384 | (5.84885; 7.17046) | 8.98573 | (8.63143; 9.32912) |
| 4.24096 | (3.81849; 4.66100) | 4.90486 | (4.72348; 5.08150) |
| 0.8314 | | 1.0511 |

As can be seen, the implementation of the optimising behaviour leads to a small but significant increase in welfare from actual consumption which is equivalent to roughly 1% of aggregate consumption in every period. Interestingly, the increase in welfare derived from consumption is even greater than that implied by the increase in welfare calculated on desired consumption. This suggests that the optimising behaviour, which leads to a significant reduction in macroeconomic volatility, is also able to somewhat reduce welfare losses from coordination failures on the consumption goods market. Since the optimisation behaviour is purposely calibrated such that the level of average consumption under optimisation is almost equal to that under the baseline heuristic, the gain in welfare is indeed due to the reduction in the volatility of consumption demand and actual consumption. Despite the effects outlined above, the model is qualitatively unaffected by the modification to consumption behaviour insofar as model variables still

\(^{14}\)This is calculated for each agent as \( u^{-1}(U^m_j) - u^{-1}(U^b_j) \) where \( m \) signifies the modified model and \( b \) the baseline. The differences are then summed up across agents.
exhibit fluctuations around a real stationary state, the difference being that these fluctuations tend to be smaller.

Figure 4.7: Cumulative multipliers in the baseline (solid) and modified (dashed) model

Figure 4.8: Cumulative government expenditure multipliers during downturns (solid) and booms (dashed)
Nevertheless, the reaction of the altered model to a government expenditure shock, shown in figure 4.7, is substantially different from that of the baseline model. It can be seen that in the modified model, the cumulative multiplier - particularly in later post-shock periods - is much lower than in the baseline version and that the median always remains well below one. Figure 4.8 demonstrates that under the modified consumption behaviour, the effects of cyclical conditions on the government expenditure multiplier remain intact.

Figure 4.9 compares the policy splines produced by both versions of the model, showing that the more muted response of the modified model is driven by a smaller response of consumption, and that the duration of the impact is shorter than in the baseline model. Desired consumption reacts to a fiscal shock in a very muted fashion. The reaction is chiefly due to the induced increase in employment, which has a direct impact on workers’ estimated budget constraint as the latter is state-dependent (i.e. there is one budget constraint for each possible current state), as well as to state-transitions of firm owners. The estimated transition probabilities and payoff vectors are based on a large amount of past data and are hence hardly affected by the one-period shock to government expenditure. Despite the implementation of the alternative consumption behaviour, households may still be liquidity-constrained and this constraint may be eased by the fiscal shock, leading to an increase in actual consumption greater than that observed in desired consumption.

![Figure 4.9: Comparing quasi-IRFs](image)

(a) Baseline model  
(b) Modified model

The above experiment demonstrates the importance of households’ income expectations in driving the model’s response to government expenditure shocks and in particular highlights that the longer-term reaction of the baseline model to such shocks appears to be substantially driven by the relatively simple heuristic households draw on for their consumption decision in the baseline model. A major effect of the
introduction of the alternative consumption behaviour is to substantially strengthen the degree to which households aim to smooth consumption which in turn contributes to a smaller cumulative multiplier. The present experiment, however, does not yet fully exhaust the possibilities afforded by the alternative modelling of consumption behaviour as expectations of future income do not explicitly account for the effects of policy. The following experiments, therefore, retain the modified consumption behaviour introduced above but aim to show how it can be extended to investigate the role of households’ beliefs and expectations about the effects of fiscal policy.

As a first experiment, I suppose that households hold initially exogenous and heterogeneous beliefs about the effects of a government expenditure shock. In particular, I assume that households may take either an optimistic or a pessimistic view of the prospective effects of a positive government expenditure shock on their estimated budget constraint. One class of households which I term ‘Keynesian’ or optimist believe that the effect of the shock will be expansionary, whilst another class which I term ‘Classical’ or pessimist may be thought of as holding a strong belief in favour of ‘expansionary fiscal consolidations’ (Giavazzi and Pagano, 1990; Alesina et al., 2019) such that they expect the effect of the shock to be negative. Importantly, it is assumed for the moment that agents cannot switch between the optimistic and pessimistic types; rather they are assigned a type at the beginning of the simulation which remains constant, but the strength of their belief adapts according to post-shock developments. Recall that households’ estimates of their budget constraint or ‘permanent income’ are based on two components, namely their assessments of the transition probabilities (from employed to unemployed, dividend paid to no dividend paid, etc.) and their pay-off vectors (containing long-run averages of the real wage, firm dividends etc.), which in turn they project forward using equation (4.7). It is now assumed that from the period in which the shock is introduced, households believe that for the following 40 periods, the transition probabilities and payoff vectors are going to differ from their previous estimate by some factor, the value of which depends on their type. For instance, the expectations of a worker household \( j \) are modified as follows:

\[
\begin{align*}
\hat{\pi}_{EU,t}^j &= \pi_{EU,t} \cdot \iota_{EU,t}^j \\
\hat{\pi}_{EE,t}^j &= 1 - \hat{\pi}_{EU,t}^j \\
\hat{\pi}_{UU,t}^j &= \pi_{UU,t} \cdot \iota_{UU,t}^j \\
\hat{\pi}_{UE,t}^j &= 1 - \hat{\pi}_{UU,t}^j \\
\hat{\omega}_t^j &= \omega_t \cdot \iota_{\omega,t}^j
\end{align*}
\]  

(4.18)
where $\pi_{EU,t}$ is the estimate (based on past data) of the transition probability (from employed to unemployed in this example) at $t$, $\hat{\pi}_{EU,t}^j$ is worker $j$’s belief about the post-shock probability, and so on. The initial values of the $\iota$’s are all identical across households of the same type, with the value depending on whether the household is Keynesian (in which case $\iota_{EU,t}^j < 1$, $\iota_{UU,t}^j < 1$ and $\iota_{w,t}^j > 1$) or Classical ($\iota_{EU,t}^j > 1$, $\iota_{UU,t}^j > 1$ and $\iota_{w,t}^j < 1$). An equivalent set of beliefs about changes in transition probabilities and payoffs is introduced for firm owners. During the period in which the shock is introduced, I hence impose on each household an exogenous belief which either increases or decreases the estimate of its future income and hence modifies its budget constraint and consumption behaviour. As indicated above, each household initially believes that this change will have a duration $\ell$ of 40 post-shock periods15 and adapts the projection of its expected future income accordingly. Over the periods following the shock, the household observes whether and to what extent the actual transition probabilities and payoffs, calculated using only data from post-shock periods, have in fact changed as implied by its initial beliefs. The $\iota$’s are then adjusted adaptively. For instance, for each worker household $j$ they change according to

$$
\begin{align*}
\iota_{EU,t}^j &= \psi \cdot \iota_{EU,t-1}^j + (1 - \psi) \frac{\pi_{EU,t}^p}{\hat{\pi}_{EU,t-1}^j}, \\
\iota_{UU,t}^j &= \psi \cdot \iota_{UU,t-1}^j + (1 - \psi) \frac{\pi_{UU,t}^p}{\hat{\pi}_{UU,t-1}^j}, \\
\iota_{w,t}^j &= \psi \cdot \iota_{w,t-1}^j + (1 - \psi) \frac{\pi_{w,t}^p}{\hat{\pi}_{w,t-1}^j},
\end{align*}
$$

(4.19)

where the $p$-superscripts denote values calculated using only post-shock data as inputs. The adaptive process ceases when the $\iota$’s reach a value of 1, at which point the household no longer holds any belief and may be regarded as neutral. To gain an understanding of the effects of these exogenously imposed beliefs on the fiscal multiplier, I re-run the fiscal policy experiment for different shares of Keynesian households (with the rest being Classical). In all experiments, the initial values of the $\iota$’s for worker households are set as follows: $\iota_{EU}^j = 0.9$ (Keynesian), $\iota_{EU}^j = 1.1$ (Classical); $\iota_{UU}^j = 0.9$ (Keynesian), $\iota_{UU}^j = 1.1$ (Classical) and $\iota_{w}^j = 1.1$ (Keynesian), $\iota_{w}^j = 0.9$ (Classical). The corresponding $\iota$’s for firm owners are set to the same values. The value of $\psi$ is set to 0.95 implying some persistence in beliefs.16

15 The choice of 40 periods is made for presentational reasons and affects the strength of the belief-effect discussed below quantitatively but not qualitatively.

16 Note that these values, together with the assumed duration $\ell$ of 40 periods, imply fairly extreme and persistent beliefs considering that the shock itself is neither exceptionally large nor persistent. Qualitatively similar results of shorter duration are obtained for smaller values of the respective parameters but for expositional reasons the specification shown above is chosen here to emphasise the differences produced by the model under different beliefs.
Figure 4.10: Cumulative multipliers for the case of adaptive beliefs

Figure 4.10 demonstrates how the impact of the government expenditure shock is affected by the presence of beliefs, showing the cumulative multiplier for different shares of ‘Keynesian’ households in the economy. It can be seen that for high shares of ‘Keynesian’ or optimist households, the cumulative multiplier is much higher than in the absence of beliefs. In the case of only 50% optimist households, with the rest being pessimist, the cumulative multiplier is almost identical to that obtained in the baseline without types, meaning that the two beliefs appear to cancel each other out. High shares of pessimist or ‘Classical’ households, on the other hand, can in fact give rise to negative cumulative multipliers after some time.\footnote{The presence of this effect however depends on the strength of the initial belief and the speed of adaptation.} Beliefs about the impact of government expenditure shocks can hence to some degree be self-reinforcing in the present model, even being potentially able to change the sign of the effect of shocks on aggregate output. The fact that cumulative multipliers stabilise after some time (rather than tend to $±\infty$) shows that beliefs do return to neutrality eventually however, meaning that the self-reinforcing dynamic is only temporary.

The introduction of belief dynamics as described above also allows for the incorporation of a simple announcement effect of government expenditure, as expectations about future changes in transition probabilities and payoff vectors can enter into households’ present estimates of future income. Suppose for
instance that at time $t$ the government announces a one-period increase in government consumption from 0 to some positive level in period $t + x$. As long as $x < \mathcal{H}$ (the projection horizon used by households), and assuming that households believe the announcement to be credible, households’ beliefs about how the shock in $t + x$ will affect their income from $t + x$ onwards will be incorporated immediately into their estimated budget constraint. Figure 4.11 illustrates this announcement effect, for simplicity using the case in which all households are Keynesian. It can be seen that desired and actual consumption increase as soon as the shock is announced (period 0 in the plots), with output rising in response. There is then a further increase in output and consumption as the actual shock occurs, since additional output is being produced by firms in response to anticipated government demand.

A further variation on the belief dynamics introduced above is the incorporation of switching behaviour between types (see e.g. Franke and Westerhoff, 2017). In a modified version of the model, households are still initially classified as ‘Keynesian’ or ‘Classical’, but instead of adapting the strength of their belief according to equation (4.19), they instead switch between types with fixed beliefs based on the value of a switching index. The value of the latter is a function of the post-shock development of estimated ‘permanent income’ as well as the relative concentration of types within the population of households. For instance, for some household $j$ which is currently an optimist, the switching index is calculated as

\[
\text{deviation}_t^j = \frac{Y_{t}^j - \hat{Y}_t^j}{Y_{t}^j} \\
\text{switch}_t^j = \epsilon \cdot \text{deviation}_t^j + (1 - \epsilon)(\text{opt}_{t-1} - (\text{neut}_{t-1} + \text{pes}_{t-1})) \\
\text{index}_t^j = \frac{1}{1 + \exp(\sigma_1 \text{switch}_t^j + \sigma_2)}
\]  

(4.20)

\[18\] The present paper completely abstracts from the issue of credibility of policy announcements, but this would be an interesting dimension to explore in further research.
where $\hat{Y}_P^J_t$ is $j$’s expected ‘permanent income’ (incorporating both their belief and pre-shock data), $Y_P^{P,t}$ is their ‘permanent income’ estimated on post-shock data only and $opt$, $pes$ and $neut$ are the shares of optimists, pessimists and neutral households in the system. The values of all parameters are given in appendix A. $index^J_t$, which is bounded between 0 and 1, gives the probability that household $j$ will switch from being an optimist to being neutral. In the following period, $j$ reassesses their opinion and may switch from being neutral back to being an optimist or to becoming a pessimist, using appropriately adapted versions of equation (4.20) and depending on the developments of $Y_P^{P,t}$ relative to estimates incorporating pre-shock data and the new relative numbers of the different types within the population. The model incorporating switching behaviour is run for different initial shares of optimist and pessimist households (it is always assumed that there are initially no neutral households), in each case recording the model reaction to a government expenditure shock.

![Cumulative Multipliers for different initial shares of 'Keynesian' households](image)

**Figure 4.12: Cumulative multipliers for the case of switching**

Figure 4.12 shows that the effect on cumulative multipliers is even stronger than that shown in figure 4.10 for the case of adaptive beliefs without switching. The absence of adaptation in the switching case appears to make beliefs more persistent, producing more pronounced differences in multipliers and more strongly negative multipliers for low initial shares of ‘Keynesian’ households. As with the case of adaptive beliefs, the case of switching of course also allows for the introduction of an announcement.
effect of policy. Both cases, however, suffer from the deficiency that households’ initial beliefs about the effects of the government expenditure shock are completely exogenous rather than being based on a quantitatively reasonable assessment of what the effect of the shock might be. For the moment, there has also not been any attempt to assess to what extent households’ beliefs are in some sense ‘correct’, though results suggest that beliefs which are strong enough can become at least partly and temporarily self-fulfilling. For these reasons, the next section explores a variant of the model in which households attempt to learn about the effects of government expenditure on their budget constraint from repeated shocks over time using least squares learning.

4.6 Learning about policy

The application of least squares learning algorithms as presented by Evans and Honkapohja (2001) is a relatively long-standing component of the macroeconomic literature (e.g. Marcet and Sargent, 1989; Bullard and Mitra, 2002; Branch, 2004) but has featured only sparsely in the agent-based literature (see Chapter 2 and Dosi et al., 2017a). The goal of the present exercise is similar to applications of least squares learning in conventional models, although (due to the complexity of the underlying framework) its scope is much more limited. The question I wish to answer is simply whether households are able, on average, to correctly learn the impact effect\(^{19}\) of a government expenditure shock on their budget constraint.

Recall once more that households base their estimates of future income on state transition matrices and income vectors calculated using \(H\) periods of past simulation data (where \(H = 400\) in the simulations shown in this paper). As argued above, the estimates of these transition matrices and income vectors hardly change in response to one-period government expenditure shocks due to the large amount of past data used to calculate them. Instead, the impact of government expenditure shocks on consumption demand stems chiefly from households temporarily moving to a different state as a consequence of the shock, which in turn alters their budget constraint as, for instance, \(YP^E > YP^U\), i.e. the ‘permanent income’ of a currently employed worker household is greater than that of a currently unemployed household.\(^{20}\) Ignoring the negligible impacts of government expenditure shocks on households’ estimates of transition probabilities and income vectors, an approximation of the expected impact-effect of a shock

\(^{19}\)That is, the effect only in the period in which the shock is applied.

\(^{20}\)This is simply due to the fact that the wage is higher than the unemployment benefit, such that a difference in the sums produced by equation (4.7) is preserved as long as \(H < \infty\).
on the aggregate value of households’ ‘permanent income’ (let this be denoted by $Y_{P}^{agg}$) can hence be expressed as

$$\Delta Y_{P}^{agg} = \Delta E(Y_{P}^{E} - Y_{P}^{U}) + \Delta D(Y_{P}^{D} - Y_{P}^{ND}) + \Delta D_k(Y_{P}^{Dk} - Y_{P}^{ND_k}), \quad (4.21)$$

where $E$, $D$ and $D_k$ denote the numbers of employed workers, C-firms paying dividends and K-firms paying dividends respectively and the bars over the $Y_P$’s represent averages across agents. In order to obtain a numerical estimate of this impact, values for $\Delta E$, $\Delta D$ and $\Delta D_k$ must be estimated, which is done using the least squares learning algorithm. Estimation is performed on aggregate data, i.e. learning does not take place at the level of individual agents (for computational reasons). The three estimated models are

$$E = \gamma_1^E E_{-1} + \gamma_2^E c + \gamma_3^E g_d + \epsilon^E, \quad (4.22)$$

where $c$ is aggregate real private consumption and $g_d$ is aggregate real demand for consumption goods coming from the government,

$$D = \gamma_1^D + \gamma_2^D D_{-1} + \gamma_3^D g_d + \gamma_4^D w + \epsilon^D, \quad (4.23)$$

where $w$ is the real wage rate, and

$$D_k = \gamma_1^{D_k} + \gamma_2^{D_k} D_{k,-1} + \gamma_3^{D_k} g_d + \epsilon^{D_k}. \quad (4.24)$$

The goal is to gain a ‘correct’ estimate of the $\gamma_3$’s in the three models, which can in turn be used to predict the changes in employment and the number of additional firms paying dividends as a consequence of a government expenditure shock. In particular, if the estimates are broadly correct, it should be the case that

$$\Delta E = \lceil \gamma_3^E g_d \rceil, \quad (4.25)$$

and equivalently for dividends.\(^{21}\) When a government expenditure shock is announced, and recalling the structural similarity between different individual workers (as well as firms) in the model, an individual worker household $j$, for instance, would hence expect its budget to change by

$$\frac{\lfloor \gamma_3^E g_d \rfloor}{W} (Y_{P}^{E,j} - Y_{P}^{U,j}) \quad (4.26)$$

\(^{21}\)Recall that in non-shock periods, $g_d = 0$. 

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in the period in which the shock takes place, where \( W \) is the number of worker households in the economy. This expected change, in discounted form, is added to households’ estimated budget constraint in the periods after the shock has been announced and before it actually takes place, and subtracted in the period in which the shock takes place (in order to avoid a double-counting of the effect). This in turn affects households’ consumption behaviour as the perceived budget constraint entering their optimisation problem is altered. Agents hence react systematically to the announcement of a government expenditure shock and, provided that the estimates of the \( \gamma_3 \)’s are broadly correct, do so in a model-consistent fashion. If this is the case, households’ behaviour in reaction to a government expenditure shock becomes at least partly robust to the Lucas critique, though due to the complexity of the model I limit myself on to the impact effect of the shock, disregarding any lagged effects which occur in following periods.

In order to implement least-squares learning about the effect of government expenditure shocks, the simulation set-up must be changed slightly. In particular, agents must be exposed to repeated expenditure shocks in order to collect data that can be used for estimation. For the present experiment, the model is hence simulated 200 times for a duration of 5000 instead of 1500 periods. After the transient and up to period 2500, the model is exposed to a series of random one-period increases in government expenditure from the baseline level of 0, which occur on average every 10 periods. The resulting data is collected as input for the learning algorithm. Prior to period 2500, however, the results of the estimations are not incorporated into households’ behaviour in the way described above, in order to allow the estimated coefficients to converge to reasonable values before allowing for feedback effects. From period 2500 onwards, shocks are applied at regular intervals of 10 periods and each shock is announced 5 periods before it takes place (i.e. both the timing and the size of the shock are disclosed). Households then immediately incorporate this information into their consumption behaviour as described above, using the results of the learning algorithm which in turn continue to be updated, now incorporating any potential feedback effects from households’ altered behaviour.

Parameters are estimated using ordinary least squares. The dataset consists of a rolling window of the past \( H \) periods of simulation data out of which only shock periods and periods immediately preceding shocks are used for estimation, such that estimates are only updated following shocks.\(^{22}\) Due to the limited amount of data used for each estimation, the algorithm is hence similar to a constant-gain one (cf. Evans and Honkapohja, 2001, Ch. 3). In order to assess to what extent parameter estimates take ‘correct’ values, given that the actual law of motion of employment and dividend payouts is unknown, I

\(^{22}\) Using the full set of \( H \) observations appears to lead to a downward bias in the estimated parameters as in non-shock periods \( g_{td} \) always equals zero.
use a relatively simple protocol. For the 500 final periods of each individual run, I calculate the actual
changes in employment and dividend payouts following shocks, i.e.

\[
\begin{align*}
\Delta E_t &= E_t - E_{t-1} \\
\Delta D_t &= D_t - D_{t-1} \\
\Delta D_{k,t} &= D_{k,t} - D_{k,t-1},
\end{align*}
\]

where \( t \) is the shock period, as well as the predicted changes given by

\[
\begin{align*}
\hat{\Delta} E_t &= \lfloor \gamma_{3,t-x} g_{d,t} \rfloor \\
\hat{\Delta} D_t &= \lfloor \gamma_{3,t-x} g_{d,t} \rfloor \\
\hat{\Delta} D_{k,t} &= \lfloor \gamma_{3,t-x} g_{d,t} \rfloor,
\end{align*}
\]

where \( t - x \) is the period in which the shock is announced. I then compare both the within-run means
and confidence intervals of the respective actual and predicted changes, as well as those calculated across
Monte Carlo repetitions. The results of this are summarised in table 4.3. Lines one and two show the
MC average actual and predicted effects along with the associated 95% confidence intervals, whilst line
three gives the percentage of runs in which the within-run confidence intervals of the predicted and actual
effects overlap. It can be seen that the estimation of the effects of government expenditure shocks on both
employment and C-firm dividend payouts appears to be quite accurate. Results are less accurate for K-
firm dividend payouts, though both actual and predicted effects are centred around zero and regressions
on the entire dataset suggest that the impact effect of government expenditure on K-firm dividend payouts
is both small and statistically insignificant, meaning that an estimated effect close to zero appears broadly
correct.

Table 4.3: Actual and estimated effects of government expenditure shocks

<table>
<thead>
<tr>
<th></th>
<th>Employment (MC average)</th>
<th>Dividends (C-firms)</th>
<th>Dividends (K-firms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>10.98235 (9.45099; 12.51371)</td>
<td>5.88915 (5.06797; 6.71032)</td>
<td>−0.03323 (−0.03787; −0.02860)</td>
</tr>
<tr>
<td>Predicted (MC average)</td>
<td>11.49983 (9.89632; 13.10335)</td>
<td>6.18203 (5.32002; 7.04404)</td>
<td>−0.01058 (−0.01205; −0.00910)</td>
</tr>
<tr>
<td>% overlap</td>
<td>97.5</td>
<td>100</td>
<td>37</td>
</tr>
</tbody>
</table>

Overall agents are hence able to successfully learn the impact effect of government expenditure shocks

162
on their budget constraint and incorporate this information into their optimisation problem. It should be noted, however, that relative to the exogenously imposed beliefs discussed in section 4.5, the estimated effects calculated according to equation (4.28) are quite small. Figure 4.13 compares the cumulative multiplier in the presence of learning effects to that produced by the basic ‘permanent income’ model without learning. It can be seen that under learning the multiplier tends to be slightly lower in early periods but these differences are neither statistically significant nor persistent. Consequently, there is also no significant change in households’ welfare under least squares learning.

Figure 4.13: Cumulative multipliers under learning (solid) compared to the absence of learning effects (dashed)

The reason for this result is that, as already outlined in section 4.5, the majority of the impact of government expenditure shocks on GDP in the ‘permanent income’ version of the model (beyond the additional output consumed by the government itself) stems from the easing of liquidity constraints and the reduction of coordination failures on the goods market. Desired consumption as derived from households’ optimisation problem only reacts very weakly to non-persistent government expenditure shocks such that the difference produced by the learning effect is very small.
4.7 Discussion

Overall, the results presented in the preceding sections emphasise the important role of households’ consumption behaviour in determining the effects of government expenditure shocks. Positive government expenditure shocks were shown to be strongly expansionary in the baseline featuring a relatively simple, backward-looking consumption heuristic. Whilst still expansionary, their effect is much weaker if households use a greater amount of past data to formulate a forward-looking optimal consumption plan subject to an estimated budget constraint. A similar dichotomy can be drawn with respect to explicit expectations about the effects of government expenditure shocks on future income. While exogenously imposed beliefs can strongly increase the magnitude of the multiplier, its size hardly changes if agents can learn from repeated shocks and beliefs become broadly ‘rational’. In general, the size of the multiplier hence appears to strongly depend on the degree of rationality incorporated into agents’ beliefs and decision-making processes. Nevertheless it was argued that even exogenously imposed (i.e. ex-ante non-rational) optimistic or pessimistic beliefs can be partly self-fulfilling, meaning that governments can potentially increase the impact of their fiscal policy measures if they are able to credibly manage agents’ expectations and generate confidence about the effectiveness of fiscal policy.

As discussed in section 4.5, a simple welfare analysis shows that households are able to significantly increase their welfare from consumption by engaging in optimising behaviour, producing a welfare gain equivalent to around one percent of average per-period consumption. Due to the smaller overall effect on consumption, however, the welfare gain households experience from expansionary government expenditure shocks is somewhat lower in the ‘permanent income’ version of the model. In the baseline, the average welfare gain of each agent from an expansionary government expenditure shock equal to 10% of real GDP in the 20 periods following the shock is equivalent to around 0.9 consumption units per period, whereas under the modified consumption behaviour the welfare gain is equivalent to only around 0.39 consumption units. While an expansionary government expenditure shock is hence welfare-improving under both specifications, as it leads to a reduction in coordination failures and an easing of liquidity constraints, the effect is stronger if households are not optimising their consumption plans.

Beyond emphasising the dependence of the multiplier on consumption behaviour, beliefs and expectations, the paper also showed that the model gives rise to state-dependent multipliers, i.e. multipliers that are larger during recessions and smaller during booms. This empirically well-documented finding is robust under both versions of households’ consumption behaviour. The model used in this study highlights a large range of potential channels for this effect, including liquidity constraints of households,
credit constraints of firms, capacity constraints, and labour market tightness. Some of these may merit additional attention in empirical studies as an improved understanding of what precisely drives the state-dependence of multipliers could be important in designing fiscal policy interventions to be as effective as possible.

The results presented in this paper may be viewed as being broadly in line with the existing literature. As indicated in section 4.2, fiscal policy tends to be highly effective in macroeconomic ABMs, and the fairly large cumulative multiplier produced by the baseline model is certainly in line with this general tendency. It should be noted, however, that expenditure multipliers of similar or even greater magnitude can also be obtained in suitably specified DSGE models, particularly when monetary policy does not react to changes in fiscal policy as is indeed also the case in the present model (Auerbach and Gorodnichenko, 2012b). By contrast, the cumulative multiplier produced by the modified model, which remains well below one and in later periods fluctuates around 0.5, is closer to those produced in RBC-type frameworks (Gechert and Will, 2012; Mitra et al., 2019) and consistent with the lower end of the range of available empirical estimates (cf. Batini et al., 2014). As documented by Gechert and Will (2012), the range of empirical and model-based estimates for all types of fiscal multipliers is relatively large, such that there is no consensus in the existing literature. Some of the most widely cited empirical estimates for government expenditure multipliers, such as those presented in Barro (1981), Hall (2009) and Ramey (2011a,b), fall roughly within the interval bounded by the multiplier in the baseline model and that in the modified one, although both frameworks could of course be calibrated to produce smaller or larger multipliers than those shown here. The general conclusion in this regard should hence be that both the baseline model featuring strongly simplified and heuristics-based consumption behaviour and the modified framework featuring optimising and forward-looking households are capable of producing realistic reactions to government expenditure shocks.

The pure focus on positive government expenditure shocks and households’ consumption behaviour in the present paper, while enabling a very detailed analysis, clearly implies some limitations. For instance, it is likely that the effects of negative government expenditure shocks would not be precisely symmetrical to positive ones due to the various constraints and non-linearities incorporated in the model. Moreover, it would be interesting to examine how government expenditure can be optimally distributed among firms to maximise its impact, and to what extent fiscal multipliers are dependent on the type of instrument (as the present model would also allow for tax changes and transfers) or the type of expenditure (the

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23 In fact, for particular specifications of utility, multipliers may be infinite (Auclert and Rognlie, 2017).
government could conceivably also purchase capital goods for instance). It should also be noted that,
deepth the implementation of the alternative consumption behaviour, the majority of the modified model
is still driven by relatively simple behavioural heuristics such that the gap to conventional frameworks
remains wide. In addition, the absence of active monetary policy and the complete abstraction from
considerations of Ricardian equivalence limits the comparability between the model used here and the
DSGE literature. Finally, the least squares learning exercise considered in this paper is very limited in
that households only attempt to learn the impact effect of government expenditure shocks on their budget
constraints rather than dynamic effects, not to mention the law of motion of all other state variables rele-
vant to them. Despite the limited nature of both the optimising behaviour and the learning dynamics, their
incorporation into a pre-existing macroeconomic ABM was fairly roundabout and implied a large mod-
ification. Nevertheless I believe that the paper represents an interesting addition to the literature which
may serve to inspire similar approaches in other frameworks, contribute to an improvement in compara-
bility across modelling paradigms, and provide a first step in addressing some common criticisms raised
against macroeconomic ABMs.

4.8 Conclusion

This paper has provided an investigation of the effects of government expenditure shocks and the de-
pendence of such effects on household consumption behaviour in a canonical macroeconomic ABM
framework. Cumulative government expenditure multipliers were demonstrated to strongly depend on
the way households form their demand for consumption goods; in particular it was shown that the in-
clusion of intertemporal utility maximisation behaviour subject to an estimated budget constraint leads
to a multiplier that is significantly smaller than its baseline counterpart. Furthermore, the magnitude of
cumulative multipliers can be increased or decreased through the introduction of exogenously imposed
beliefs about the effect of government expenditure shocks on future income, highlighting the potentially
important role of expectation management in determining the effects of government policy. In a final
experiment, it was shown that under least squares learning, households are able to form broadly correct
expectations about the impact effect of government expenditure shocks on their budget constraint and
incorporate this expectation into their behaviour.

Beyond the thorough analysis of government expenditure multipliers, which is novel in the field of
macroeconomic ABMs, the paper contributes to the literature by demonstrating how more forward-
looking behavioural rules can be introduced into a well-known existing model. This both contributes to a better understanding of the role of heuristics in determining simulation results and may facilitate comparisons between macroeconomic ABMs and their mainstream counterparts. Finally, the combination of forward-looking behaviour with beliefs or learning makes it possible to incorporate announcement effects of fiscal policy into the model. Since agents are made to react systematically (and, under least squares learning, broadly correctly), to government expenditure shocks, the paper demonstrates a potential way in which the Lucas critique as applied to macroeconomic ABMs can be addressed at least partially.
Appendices

Appendix 4.A: Parameter values

The parameter values of the baseline model used in this paper are identical to those provided by Delli Gatti and Grazzini (2019). Table 4.4 below lists all parameters together with an explanation and their value.

Table 4.4: Model parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
<td>Number of workers</td>
<td>1000</td>
</tr>
<tr>
<td>$F$</td>
<td>Number of C-firms</td>
<td>100</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of K-firms</td>
<td>20</td>
</tr>
<tr>
<td>$z_c$</td>
<td>Number of C-firms visited by consumers</td>
<td>2</td>
</tr>
<tr>
<td>$z_e$</td>
<td>Number of Firms visited by unemployed</td>
<td>5</td>
</tr>
<tr>
<td>$z_k$</td>
<td>Number of K-firms visited by C-firms</td>
<td>2</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Memory parameter for baseline ‘permanent income’</td>
<td>0.7382</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Propensity to consume out of wealth</td>
<td>0.0172</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>Quantity adjustment parameter</td>
<td>0.7301</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Price adjustment random parameter</td>
<td>0.1649</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Bank’s gross mark-up</td>
<td>1.007</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>Capital depreciation</td>
<td>0.03</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Probability to invest</td>
<td>0.3260</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Bank’s leverage parameter</td>
<td>0.0024</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Debt repayment rate</td>
<td>0.0328</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Memory parameter for capacity utilisation</td>
<td>0.1591</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labour productivity</td>
<td>0.5</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Capital productivity</td>
<td>1/3</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Dividend payout ratio (firms)</td>
<td>0.2</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>Dividend payout ratio (bank)</td>
<td>0.3</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Normal/target capacity utilisation</td>
<td>0.85</td>
</tr>
<tr>
<td>$\eta_i$</td>
<td>Inventory depreciation</td>
<td>0.0781</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Bank’s risk evaluation parameter (C-firms)</td>
<td>-15</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Bank’s risk evaluation parameter (C-firms)</td>
<td>13</td>
</tr>
<tr>
<td>$\lambda_{k1}$</td>
<td>Bank’s risk evaluation parameter (K-firms)</td>
<td>-5</td>
</tr>
<tr>
<td>$\lambda_{k2}$</td>
<td>Bank’s risk evaluation parameter (K-firms)</td>
<td>5</td>
</tr>
<tr>
<td>$r$</td>
<td>Risk-free interest rate</td>
<td>0.01</td>
</tr>
<tr>
<td>$s$</td>
<td>Unemployment benefit ratio</td>
<td>0.5</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Income tax rate</td>
<td>0.0594</td>
</tr>
<tr>
<td>$u_{up}$</td>
<td>Upward wage adjustment parameter</td>
<td>0.1</td>
</tr>
<tr>
<td>$u_{down}$</td>
<td>Downward wage adjustment parameter</td>
<td>0.01</td>
</tr>
<tr>
<td>$u_T$</td>
<td>Unemployment threshold</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 4.5 contains the parameters which are introduced into the original model by the modifications and subsequent experiments described in the main body of the paper.
Table 4.5: Parameters used in experiments

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{H} )</td>
<td>Length of estimation window and projection horizon</td>
<td>400</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.9999</td>
</tr>
<tr>
<td>( \frac{1}{\tau} )</td>
<td>Intertemporal elasticity of substitution</td>
<td>1.5</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>Initial values of the exogenous belief shock</td>
<td>1.1; 0.9</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Belief adaptation parameter</td>
<td>0.95</td>
</tr>
<tr>
<td>( \ell )</td>
<td>Ex-ante duration of belief shock</td>
<td>40</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Weighting parameter for switching index</td>
<td>0.95</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>Switching parameter (workers)</td>
<td>20</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>Switching parameter (owners)</td>
<td>15</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>Switching parameter</td>
<td>4</td>
</tr>
</tbody>
</table>

Appendix 4.B: Testing the alternative consumption model

Recall that in section 4.5, households’ consumption behaviour is modified by assuming that they maximise utility from consumption subject to an estimated budget constraint. This budget constraint is calculated based on two components, namely unweighted long-run averages of income flows (wages and dividends) and state transition probabilities (e.g. from employed to unemployed or from receiving to not receiving a dividend). Sequences of state transitions are represented by \( 2 \times 2 \) transition matrices. For instance, the matrix for workers is given by

\[
\begin{pmatrix}
E & U \\
E & \pi_{EE} & 1 - \pi_{EE} \\
U & 1 - \pi_{UU} & \pi_{UU}
\end{pmatrix}
\]

This formulation implicitly assumes either that the stochastic process described by the matrix above satisfies the Markov property (i.e. that, for instance, the probability of being employed in the next period depends only on the state in the present period), or else that agents use this representation as an approximation of a process which is in fact not Markov. In either case it appears appropriate to test whether transition probabilities produced by the model can be sensibly represented in this form. Model simulation data can be used to test to what extent the stochastic state transition processes of workers and firm owners satisfy the Markov property. If the property holds, it should be true, for instance, that

\[
p(E|EE) = p(E|EU) = p(E|E),
\]

i.e. that the probability of an agent to be employed, given that they were employed in the previous
two periods \((EE)\) is equal to the probability given that they were employed in the previous period and unemployed two periods ago \((EU)\). Simulated transition data across 200 Monte Carlo runs of the model is used to calculate the respective probabilities which are then compared. The results of this exercise are summarised by figures 4.14 to 4.16 in which boxplots are used to compare the probabilities.

![Worker Households](image1)

**Figure 4.14: Transition probabilities of worker households**

![C-Firm Owners](image2)

**Figure 4.15: Transition probabilities of C-firm owner households**
It can be seen that in all cases the differences between the calculated probabilities are very small and in almost all cases there is a strong overlap between the distributions. This suggests that while the Markov property may not hold exactly in all cases, the representation of probabilities using matrices of the form given above is a reasonably good approximation.

Recall that in the model, households use aggregate transition data to estimate the transition probabilities rather than looking only at their own history of transitions (meaning that in the absence of exogenously imposed beliefs, they all hold a common estimate). Figures 4.17 to 4.19 show the distributions of individual transition probabilities together with the population mean and the estimates resulting from the backward-looking calculation of aggregate transition probabilities. While the mean and the aggregate estimate are very close to each other in all cases, the distributions do show some amount of dispersion among agents’ individual transition probabilities. This might be taken as an argument against the use of aggregate transition probabilities as an input into the behaviour of individual agents. As outlined in the model description, however, agents of the same class (workers, C-firm owners and K-firm owners) are identical in all factors which determine their transition probabilities. If agents are assumed to be informed enough to be aware of this fact, the rational implication would be for them to ignore their own idiosyncratic transition history as they should expect their own transition probabilities to converge to the population mean in the limit.
Figure 4.17: Distribution of individual worker transition probabilities

Figure 4.18: Distribution of individual C-firm owner transition probabilities
Figure 4.19: Distribution of individual K-firm owner transition probabilities
Bibliography


