Abstract

In this paper we present the modules of a continuous-time model of Keynesian monetary growth, of the variety introduced in Chiarella and Flaschel (2000) and treated in detail in Chiarella, Flaschel and Franke (2005). The model is sufficiently rich with respect to markets, sectors and agents and consistent with respect to budget constraints to capture the important details of actual macro-economies and so to serve as a macro-theoretic basis for larger scale macro models where a variety of Keynesian feedback structures are present. Simulations of this approach provide a persuasive foundation for a basic understanding of the interaction of these various feedback channels known from partial Keynesian reasoning, like the Harrod-Domar theory of the instability of balanced growth, the Goodwin-Rose distributive cycle mechanism, the Dornbusch overshooting exchange rate analysis and the Blanchard analysis of bond and asset markets dynamics. Of primary interest is on this basis the question how the various tax rates, transfer payments and government expenditure parameters of the model can be used to improve the social protection of the sector of worker households, without loosing the efficiency of a well-performing labor market (with its partial Friedmanian supply side aspects), and without neglecting the creation of a sound and sustainable infra-structure” for education, health care and care for the elderly.

Keywords: high order Keynesian macro-dynamics, stability, fiscal and monetary policy, social protection schemes, sustainable balanced growth

JEL CLASSIFICATION SYSTEM: E32, E64, H11.
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CV and Bibliography, Joint Work with Christian R. Proano, Interview, Roads to Social Capitalism
with summary of Marx@200 and outlook after the bibliography as point of departure for future Goodwinian MKS approaches as extension of the KMG approach to the macrodynamics of this paper.
1 Introduction

In this paper we present the modules of a hierarchically structured continuous-time model of Keynesian monetary growth, of the variety introduced in Chiarella and Flaschel (2000) and considered in detail in Chiarella, Flaschel and Franke (2005). We extend this model in this paper to the case of a small open economy by modifying in various ways of approach of Charpe, Chiarella, Flaschel and Semmler (2010, ch.10).

The model is sufficiently rich with respect to markets, sectors and agents and consistent with respect to budget constraints and captures the important details of actual macro-economies and so is able to serve as a macro-theoretic basis for larger scale macro-econometric models of open economies like the Murphy model for the Australian economy.

We describe the model on the level of national accounts and then derive its extensive form dynamics. This dynamic system is based on coherent stock-flow considerations, implies a compact intensive form for its theoretical and numerical investigation and exhibits a locally unique interior balanced growth positions. We use this reference path as starting point for the simulations of its laws of motion which is subjected to a shock in order to generate growth fluctuations. sometimes converging to a different balanced growth path (due to the existence of multiple balanced growth situations when certain externalities of public policy are present).

These simulations provide a persuasive foundation for the basic understanding of the interaction of the various economic feedback channels present in the model, often well-known from partial models of traditional and other Keynesian economic theory, feedback chains as in the Harrod-Domar theory of the instability of balanced growth, of the Goodwin-Rose distributive cycle mechanism, in the Dornbusch overshooting exchange rate analysis and the Blanchard analysis of bond and asset markets dynamics, used there to implement the rational expectations methodology for such an approach to financial markets.

The basic need is here to tame these generally destabilizing forces from a Keynesian perspective by way of suitably chosen fiscal and monetary policy rules, a quite demanding task in view of the 16-18 laws of motion of the considered economic dynamics. We have moreover to cope with stability problems as they originates from the dynamics of the Government Budget Restraint GBR (in addition to the unstable adjustment processes of the private sector), i.e., the debt dynamics that the GBR is giving rise to.

Of primary interest in this paper is however the question how the various tax and transfer schemes (primarily unemployment benefits and pensions payments in the latter case) as well as government expenditure projects can be used to improve the social protection of the sector of worker households, without loosing the efficiency of a well-performing labor market (with its partial modelling of Friedmanian supply side forces), and also without neglecting the creation of a sufficiently rich "infra-structure" for education, health care and care for the elderly, i.e., for the young people, the labor market participants and the retired citizens, an age structure that must be in addition be made sustainable by the policy makers.

One has to realize here however that our model is a macro-model, resting on the usual one-good assumption and – in contrast to my previous ILO project – on a single labor market. It is therefore obvious that the mentioned public goods consist just of a single (aggregated) item, the size of its supply being therefore the only concern here, while its structuring (and also the details of its funding must be left for micro-economic consideration in this paper). The same of course holds for the details of the supply of public services.

Since residential issues are also of great importance for worker households, we moreover add residential services for this type of households which are supplied from the stock of houses created by the housing investment of the other type of households, the asset holders, the sole real asset these asset holders administer in this Keynesian model of monetary growth.

Concerning the topics just enumerated we will provide a range of quantitative answers showing the macro-advantages of an advanced type of "social protection" in a capitalistic accumulation regime, due to public investment into the "infra-structure" of the economy, and based on various
types of income transfers, as well as on anti-cyclical fiscal and monetary policies. We will however also find some obstacles to the creation of what is called a "free lunch" by mainstream economics.

All these aspects are illustrated by numerous simulations of the laws of motion of our macro-dynamical model, in the sections that follow the determination of its reference balanced growth position (which is not necessarily the only one from a global perspective, in particular due to a nonlinearity resulting from public investment into the public capital stock).

2 The real and the financial part of the economy

The following two tables provide a survey of the structure of the economy to be modelled which is related to a certain degree to the description of the Australian economy given in Powell and Murphy (1997). Note in this respect that the aim of the present paper is to establish an integrated continuous-time model, leading to an autonomous system of differential equations, where all sectors are fully specified with respect to their behaviour and their budget constraints from the viewpoint of theoretical models of monetary growth. A bridge will thereby be provided between the Keynes-Metzler type monetary growth models of Chiarella and Flaschel (2000), Chiarella et al. (2005) and the Powell and Murphy (1997) approach.

2.1 The structure of the real part

Let us start with a presentation of the variables that comprise the real part of the economy. Table 1 provides data on the temporary equilibrium position of the economy, based on given prices and expectations and also the real stocks, including their rates of growth.

<table>
<thead>
<tr>
<th></th>
<th>Labor</th>
<th>Non traded Goods</th>
<th>Exports</th>
<th>Imports</th>
<th>Dwellings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers</td>
<td>$L$</td>
<td>$C_w$</td>
<td>$-$</td>
<td>$-$</td>
<td>$C^d_h$</td>
</tr>
<tr>
<td>Asset holders</td>
<td>$-$</td>
<td>$C_c$</td>
<td>$-$</td>
<td>$-$</td>
<td>$C^*_h, I_h$</td>
</tr>
<tr>
<td>Firms</td>
<td>$L^y_f, L^h_f, \tau_f p$</td>
<td>$Y_p, Y, I, \mathcal{I}$</td>
<td>$X$</td>
<td>$J^d$</td>
<td>$-$</td>
</tr>
<tr>
<td>Government</td>
<td>$L^w_g$</td>
<td>$G = C_g + I_g$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>wages, prices</td>
<td>$w, w^u = (1 - \tau_w)w$</td>
<td>$p = (1 + \tau_v)p_y$</td>
<td>$p_x = (1 - \tau_x)sp^*_x$</td>
<td>$p_m = (1 + \tau_m)sp^*_m$</td>
<td>$p_h$</td>
</tr>
<tr>
<td>$\tau'$ taxes</td>
<td>$v=\text{VAT}$</td>
<td>export subsidy</td>
<td>import tax</td>
<td>untaxed</td>
<td>$-$</td>
</tr>
<tr>
<td>Expectations</td>
<td>$\hat{\pi}^e = \pi^e$</td>
<td>$\hat{\pi} = \pi^e$</td>
<td>$\hat{p}_x^* = \pi^*$</td>
<td>$\hat{p}_h^* = \pi^*$</td>
<td>$\pi^e = \hat{\pi}^c$</td>
</tr>
<tr>
<td>Stocks</td>
<td>$L$</td>
<td>$K, N$</td>
<td>$-$</td>
<td>$-$</td>
<td>$K_h$</td>
</tr>
<tr>
<td>Growth</td>
<td>$\dot{L} = \dot{n}$</td>
<td>$\dot{K} = I/K - \delta_h$</td>
<td>$-$</td>
<td>$-$</td>
<td>$\dot{K}_h = \frac{I}{K_h} - \delta_h$</td>
</tr>
</tbody>
</table>

Table 1: The real part of the economy (foreign country data: Inflation rates $\hat{p}_x^*, \hat{p}_m^*$).

Table 1 describes the real sector of the economy. We have a labor market, three commodity markets and the housing market. Domestic production $Y$ concerns one good that is used for all private consumption $C_w + C_c$, all investment $I, I_h, \mathcal{I}$, also in housing, all government consumption $G$ and exports $X$. It uses up all imports $J^d$ as intermediate goods. There is thus only a single domestically produced commodity, apart from the housing services $C^d_h$ demanded by the workers.
Our model exhibits three domestic sectors: households, firms and the government, but with heterogeneous agents in the household sector, workers \( L \) and (pure) asset holders, the former supplying their labor at the wage level \( w \) (which includes taxes \( \tau_w \)), and the latter the housing services \( C^h \) for the workers as far as real flows are concerned. Firms produce as joint output the non-traded and the exported commodity and employ labor \( L^f \) (with varying rates of utilisation \( L^d_f \)) and imports \( J^d \) besides their capital stock \( K \) for these purposes, and they invest in fixed business capital \( I \) and inventories \( I \). The government finally provides public consumption and investment goods \( G \), pays untaxed pensions \( w^r \) and untaxed unemployment benefits \( w^u \) and also employs part of the workforce \( L^w_g \). There are a number of variables needed to describe the laws of motion for the quantities and the prices \( p \) (including value added taxes \( \tau_v \)), and also expectations about their rates of change, which will be explained in detail when we turn to the description of the various equations of the model in Section 4. There is exogenous growth \( \bar{n} \) of the labor force \( L \) (assumed to including productivity growth implicitly), of the capital stock \( K \), and of the stock of housing \( K^h \) (supplied at price \( p^h \) for their residential services) and the also actual change of inventories \( N \) that is different from their desired rate of change if Keynesian aggregate demand is not perfectly foreseen.

### 2.2 The structure of the financial part

Let us next consider the financial part of the economy. The table 2 provides the data valid in the financial stocks of the model, the stocks, corresponding prices, and the growth of stocks in the financial part of the economy.

<table>
<thead>
<tr>
<th></th>
<th>Money</th>
<th>Short-term Bonds</th>
<th>Long-term Bonds</th>
<th>Foreign Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers (w)</td>
<td>( \dot{M}_b )</td>
<td>( B^f_w )</td>
<td>( \dot{B}_c )</td>
<td>( \dot{B}^l )</td>
</tr>
<tr>
<td>Asset holders (c)</td>
<td>( \dot{M}_b )</td>
<td>( B^f_c )</td>
<td>( \dot{B}^l )</td>
<td>( \dot{B}^l )</td>
</tr>
<tr>
<td>Firms (f, corporate bonds)</td>
<td>( \dot{B}_b = \dot{B}^f_b )</td>
<td>( \dot{B}^l )</td>
<td>( \dot{B}^l )</td>
<td>( \dot{B}^l )</td>
</tr>
<tr>
<td>Govern.t+CB (g,Bonds,b,Money)</td>
<td>( \dot{B}_b = \dot{B}^f_b )</td>
<td>( \dot{B}^l )</td>
<td>( \dot{B}^l )</td>
<td>( \dot{B}^l )</td>
</tr>
<tr>
<td>Prices</td>
<td>( b = 1 )</td>
<td>( p_b = 1/\tau_c B^f )</td>
<td>( p_t = 1/\tau_c B^l )</td>
<td>( p_{t*} = 1/\tau_c B_{t*}^l )</td>
</tr>
<tr>
<td>Expectations</td>
<td>( \dot{M}_b )</td>
<td>( \dot{B}_b )</td>
<td>( \dot{B}^l )</td>
<td>( \dot{B}^l )</td>
</tr>
<tr>
<td>Stocks</td>
<td>( M_b )</td>
<td>( B = B^f_w + B^f_c + B^f_b )</td>
<td>( B^l )</td>
<td>( B^l )</td>
</tr>
<tr>
<td>Growth</td>
<td>( \dot{M}_b )</td>
<td>( \dot{B}_b )</td>
<td>( \dot{B}^l )</td>
<td>( \dot{B}^l )</td>
</tr>
</tbody>
</table>

Table 2: The financial part of the economy (foreign country data: Interest rate \( r^* \)).

The first column in the table 2 shows money holdings and their time rate of change which however are hidden in the following behind the assumed working of the interest rate policy rule (as is customary in models of the New Keynesian variety for example). We are focussing here too on the three interest-bearing financial assets of our model that can be held by the (pure) asset owners and by the workers (as shown in the table).

We assume, in order to start with a simple representation of financial flows, that only bonds are issued by the government (in this case also short-term bonds), by firms as corporate bonds (perpetuities, just as in the case of the government), and also offered as such long-term bonds by the foreign asset holders out of their stocks, which are in part traded internationally against the long-term domestic bonds. Financial flows between the sectors of our economy are therefore very narrowly defined (in order to simplify the flow budget restrictions to a sufficient degree).

The laws of motion of the real part of our economy do not yet depend very much on this financial structure of the economy, since, as in Powell and Murphy (1997), we do not use a full portfolio adjustment approach towards the realization of financial stock equilibria. Rather we determine asset prices and asset returns through certain interest-based simple laws of motion, while the new
inflow of financial assets is basically determined from the supply side.¹

Note that we allow for savings out of wages (in a Kaldorian way) and that workers save only in the form of short-term government debt (as interest-bearing perfectly liquid saving deposits). All other assets (plus the excess of short-term government debt over workers’ flow demand) are exclusively held by the (pure) asset holders of our model. Note also that the government sector includes the activities of the central bank, which in our model is formally reduced to the setting of the interest rate on the (only domestically held) short-term government debt according to some type of Taylor rule.

3 The structure of the economy from the viewpoint of national accounting

We consider in this section the production accounts, income accounts, accumulation accounts and financial accounts of the four domestic agents in our economy:² firms, workers, asset holders and the government (including the monetary authority). These accounts, plus the balance of payments, provide basic information on what is assumed for these four sectors as well as which of their activities are excluded from the present theoretical framework. These accounts furthermore serve the purpose of checking that all ex post results of the economy are consistent with each other and showing how the usual basic identity of national accounting (concerning savings and investment) can be derived from them.

3.1 The four sectors of the economy

We start with the accounts of the sector of firms (shown in table 3) that organise production $Y$, employment $L_d^f$ of their workforce $L_w^f$ and gross business fixed investment $I$ and that use (in the present formulation of the model) only corporate bonds $B_f$ as financing instrument (whose interest payments are transferred together with profits as gross profits to asset holders). There are value added taxes $\tau_v$ on consumption goods, import taxes $\tau_m$ and payroll taxes $\tau_{fp}$ with respect to hours worked $L_f$, but no further taxation in the sector of firms, but there are export subsidies by the government.

All accounts are expressed in terms of the domestic currency. Firms build dwellings, which are of the same type as all other domestic production, and sell them to the asset holders (as investors) and thus have no own investment in the housing sector. They sell consumption goods to workers, asset holders and the government, export goods to the world economy, organise fixed gross investments with respect to their capital stock (as well as voluntary inventory changes $I$ with respect to finished non-traded goods) and experience involuntary inventory changes $Y - Y^d$ due to the deviation of aggregate demand $Y^d$ from output $Y$ (which is based on expected sales $Y_e$ and planned inventories $I$).

Firms use up all imports as intermediate goods which thereby become part of the unique homogeneous good that is produced for domestic purposes. They have replacement costs with respect to their capital stock, pay indirect taxes and wages including payroll taxes. Their accounting profit is therefore equal to expected profits (based on sales expectations and paid out as expected gross profits to firm owners) and retained profits (equal to planned inventories). As is obvious from the narrow income account of firms, firms thus only save an amount equal to their intended inventory changes. The accumulation account is self-explanatory as is the financial account which repeats our earlier statement that the financial deficit of firms is financed solely by the issuing of new bonds.

Note that all investment is valued (and performed) without paying value added tax and thus at producer prices $p_y$ in place of the consumer prices $p = (1 + \tau_v)p_y$. All expected profits are distributed.

¹Powell and Murphy (1997) use perfect substitute assumptions, as for example the interest rate parity condition, and rational expectations to describe the behaviour of the asset markets, while we use certain delayed adjustment processes towards such an outcome and thus avoid the use of the jump variable technique for the description of the
Production Account of Firms:

Uses | Sources
--- | ---
Imports \( sp_m J^d \) | Consumption \( pC_w \)
Depreciation \( p_y \delta_k K \) | Consumption \( pC_c \)
Indirect Taxes \( \tau_c p_y (C_w + C_c + G) + \tau_m sp_m^* J^d \) | Consumption \( pG \)
Wages (including payroll taxes) \( (1 + \tau_{fp}) wL^f_j \) | Exports \( (1 - \tau_x) p_x X + \tau_x p_x X \)
Profits \( II = r^g p_y K + p_y \bar{I} \) | Inventory Investment \( p_y \bar{N} \)

Income Account of Firms:

Uses | Sources
--- | ---
interest and net profit \( \rho^{ge} p_y K \) | gross (output-based) profits
savings \( S^n_j = p_y \bar{I} \) | \( S^n_j = p_y \bar{I} \)

Accumulation Account of Firms:

Uses | Sources
--- | ---
Gross Investment \( p_y I \) | Depreciation \( p_y \delta_k K \)
Inventory Investment \( p_y \bar{N} \) | Savings \( S^n_f \)
Financial Deficit \( FD \) (or Windfall Profits)

Financial Account of Firms:

Uses | Sources
--- | ---
Financial Deficit \( FD \) | Corporate Bond Financing \( p_l \hat{B}^f_j \) \[=\] \( p_l \hat{B}^{lf}_c \)

Table 3: The production, income, accumulation and financial accounts of firms.

to asset holders (and taxed in this sector) and there are no taxes on windfall profits (unexpected retained earnings – or losses – of firms that help to finance investment). Note however that the wages \( w \) paid by firms include payroll taxes \( \tau_{fp} w \) (for unemployment insurance, medicare and other social insurance, and pensions) and that wage income \( w \) of workers is taxed at the rate \( \tau_w \). Note finally that the accumulation account of firms is based on realised magnitudes and thus does not refer explicitly to their intended inventory changes.

Consider next the sector of asset-holders (table 4). Investment in housing as well as the supply of housing services has been exclusively allocated to this sector. The production account thus shows the actual sale (not the potential sale) of housing services (= the demand for housing services by assumption) which is subdivided into replacement costs and actual earnings or profits on the uses side of the production account.

The income of asset holders comes from various sources: interest payments on short- and long-term domestic bonds and on long-term foreign bonds (net of tax payments which must be paid abroad), expected gross profits from firms and profits from rents. All domestic profit income is subject to tax payments at the rate \( \tau_c \) and after tax income by definition is divided into the consumption of domestic commodities (including houses, but not housing services) and the nominal financial part of the economy.

2The fifth agent, the foreign economy, is represented by the balance of payments at the end of this section. Relationships with the foreign sector are indexed by \( * \).
Production Account of Households (Asset Owners/Housing Investment):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation $p_y\delta_hK_h$</td>
<td>Rent $p_hC^d_h$</td>
</tr>
<tr>
<td>Earnings $\Pi_h$</td>
<td></td>
</tr>
</tbody>
</table>

Income Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax payment $\tau_cB_c$</td>
<td>Interest payment $rB_c$</td>
</tr>
<tr>
<td>Tax payment $\tau_cB^l_c$</td>
<td>Interest payment $B^l_c$</td>
</tr>
<tr>
<td>Taxes $\tau_c(p_hC^d_h - p_y\delta_hK_h)$</td>
<td>Interest payment $s(1 - \tau^s)cB^l_c$</td>
</tr>
<tr>
<td>Tax payment $\tau_c\rho^c\rho^yK$</td>
<td>Gross profits from firms $\rho^c\rho^yK$</td>
</tr>
<tr>
<td>Consumption $pC_c$</td>
<td>Rent earnings $\Pi_h$</td>
</tr>
<tr>
<td>Savings $S^n_c$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Investment $p_yI_h$</td>
<td>Depreciation $p_y\delta_hK_h$</td>
</tr>
<tr>
<td>Financial Surplus $FS$</td>
<td>Savings $S^n_c$</td>
</tr>
</tbody>
</table>

Financial Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bonds $B_c \subseteq M_b$</td>
<td>Financial Surplus $FS$</td>
</tr>
<tr>
<td>Long-term bonds $p_l\tilde{B}^{l_c}_l \supset p_l\tilde{B}_f$</td>
<td></td>
</tr>
<tr>
<td>Foreign bonds $sp_h\tilde{B}^{l_*}_l$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: The production, income, accumulation and financial accounts of asset owners.

savings of asset owners.

The accumulation account shows the sources for gross investment of asset-holders in the housing sector, namely depreciation and savings, the excess of which (over housing investment) is then invested in financial assets as shown in the financial account. Note here that short-term bonds are fixed price bonds with price 1 (which are perfectly liquid), while long-term bonds have the variable price $p_l = 1/r_l$ (and fixed nominal interest payments of one unit of money per period, i.e., they are all perpetuities (the same holds true for imported foreign bonds, which are of long-term type solely).\(^3\) There is no taxation of financial wealth (held or transferred) in the household sector.

The next set of accounts, the ones of worker households in table 5, are fairly simple and easy to explain. First, there is no production account in this sector. Income of the members of the workforce, which may be employed, unemployed or retired, thus derives from wages, unemployment benefits or pension payments. $L\left[\alpha_wL\right]$ denotes the total number of persons in the current [registered] workforce ($L^w$ the part of the latter that is employed) and $\alpha_w\alpha_rL$ the number of retiree who have access to pension funds ($\alpha_r=\text{const.}\(^4\)) To this we have to add the interest income on saving deposits (short-term bonds) which is taxed at the general rate used for income obtained from financial assets. All wage type incomes are subject to taxation at the rate $\tau_w$ and are again by definition subdivided into nominal consumption (consumption goods and housing services) and savings. Note here that the employment $L^d$ of the employed $L^w$ can differ from their normal employment which is measured by $L^w$, the number of persons who are employed. Note also that wages $w$ are net of payroll taxes (used to finance unemployment benefits, social insurance and pensions in particular).

\(^3\)Due to the assumption of a given nominal rate of interest on foreign bonds, these bonds can be liquidated if this is desired by domestic residents, but they are of course subject to exchange rate risk. Foreign bond purchases by domestic residents will be treated as a residual in the wealth accumulation decisions of the asset holders.

\(^4\)The fraction $\alpha_w$ is set equal to one for reasons of simplicity in section 4.
### Production Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Income Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxes ( \tau_w wL^n + \tau_r rB_w )</td>
<td>Wages ( wL^n )</td>
</tr>
<tr>
<td>Consumption ( pC_w + p_h C_h )</td>
<td>Unemployment benefits ( w^n(L - L^w) )</td>
</tr>
<tr>
<td>-</td>
<td>Pensions ( w^r \alpha_w \alpha_L L )</td>
</tr>
<tr>
<td>Savings ( S^n_w )</td>
<td>( rB_w ), Interest</td>
</tr>
</tbody>
</table>

### Accumulation Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Financial Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bond accum.</td>
<td>Financial Surplus ( FS )</td>
</tr>
</tbody>
</table>

| Table 5: The production, income, accumulation and financial accounts of worker households. |

We assume in the following that workers have a positive savings rate and that they hold their savings in the form of short-term bonds solely, which is mirrored here in the accumulation and financial account in a straightforward way.

There are finally the accounts of the fiscal and monetary authority (see table 6), which due to the many taxation schemes and transfer payments that are assumed are more voluminous than the preceding accounts – at least with respect to the income account. There is a fictitious production account where the supply of public goods is valued at production costs which consist of government expenditures for goods and labor.

The sources of government income consist of taxes on workers’ income, of taxes on profit, interest and rental income (taxed at a uniform rate), payroll taxes, value added taxes and import taxes (we here deduct export taxes which in the simulations are generally export subsidies). Uses of the tax income of the government are interest payments, transfers to the unemployed and retirees, and the costs of the aforementioned government ‘production’. In general all these uses of the tax income of the government will exceed its income so that there will result a negative amount of nominal savings \( S^n_g \) which balances the income account of the government.

There is accumulation of real assets in the government sector, in form of the public capital stock (the infrastructure of the economy). The financial account of the government show moreover how the excess of government outlays over government revenue is financed through short- or long-term debt.

Let us finally list the balance of payments of the economy under consideration. This will be done from the viewpoint of the foreign sector which can be viewed as a fifth agent of the economic structure considered in this paper. The description of the behaviour of this agent will however be confined to steady state behaviour in the subsequent presentation of the structural equations of the model.
Production Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government expenditure for goods $pG$</td>
<td>Costless Provision</td>
</tr>
<tr>
<td>Government expenditure for services $wL_g^w$</td>
<td>of public goods</td>
</tr>
</tbody>
</table>

Income Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest payment $rB$</td>
<td>Wage income taxation $\tau_w wL^w$</td>
</tr>
<tr>
<td>Interest payment $B_l^l$</td>
<td>Profit+interest taxation $\tau_c[p^p y^p K + rB + B_l^l]$</td>
</tr>
<tr>
<td>Pensions $w^w \alpha_w \alpha_r L$</td>
<td>Rent income taxation $\tau_c(p_h C_h^{dl} - p_y \delta_h K_h)$</td>
</tr>
<tr>
<td>Unemployment benefits $w^u(\alpha_w L - L^w)$</td>
<td>Payroll taxes $\tau_f wL_f^h$</td>
</tr>
<tr>
<td>Government expenditures $pG$</td>
<td>Value added tax $\tau_v p_y(C_w + C_c + G)$</td>
</tr>
<tr>
<td>Salaries $wL_g^w$</td>
<td>Import taxes $\tau_m sp_m^m J_d$</td>
</tr>
<tr>
<td>Savings $S_g^n$</td>
<td>Export subsidies $-\tau_x p_x X$</td>
</tr>
</tbody>
</table>

Accumulation Account of the Fiscal Authority:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pK_g$</td>
<td>Resources for Public Investment $\alpha_g pG$</td>
</tr>
<tr>
<td>$S_g^n$</td>
<td>Savings $S_g^n$</td>
</tr>
<tr>
<td>$pK_g$</td>
<td>Financial Deficit $FD$</td>
</tr>
</tbody>
</table>

Financial Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial deficit $FD$</td>
<td>Short-term debt $B$</td>
</tr>
<tr>
<td></td>
<td>Long-term debt $p_l B_l^l$</td>
</tr>
</tbody>
</table>

Table 6: The production, income, accumulation and the financial account of the monetary and fiscal authorities $(\dot{M} = \dot{B}_h)$.

External Account:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 + \tau_m) sp_m^m J_d$</td>
<td>$(1 - \tau_e) sp_e^e X$</td>
</tr>
<tr>
<td>$(1 - \tau_c) B_{c^l}^l$</td>
<td>$(1 - \tau^* c) sB_{c^*}^e$</td>
</tr>
<tr>
<td>$sB_{c^l}^l / r_l^*$</td>
<td>$\dot{B}_{c^*}^e / r_l$</td>
</tr>
</tbody>
</table>

Table 7: The external account

3.2 Savings and investment

On the basis of the uses of the nominal savings of the considered four sectors (see their accumulation and financial accounts), one obtains by their aggregation the result

$$S^n = S_w^n + S_c^n + S_f^n + S_g^n = I^{na} + [\dot{B}_{c^l}^l / r_l - (\dot{B}_l^l - \dot{B}_{c^l}^l) / r_l]$$

with:

$$I^{na} = p_y(I - \delta_h K) + p_y I + p_y(I_h - \delta_h K_h) = p_y(I - \delta_k K) + p_y \dot{N} + p_y(I_h - \delta_h K_h).$$

We here see that total nominal savings are ex post always equal to total nominal net investment plus net capital exports. This important identity of national accounting is based on the four identities that relate the nominal savings of the various sectors to the uses made of these savings.

Having presented the model from the ex post point of view by means of structured tables and the system of national accounts we now turn to the structural form of the model and present in the following section its technological foundations, its behavioural relationships, various definitions
and the budget equations of the four agents of the domestic economy, and its laws of motion for quantities, prices and expectations.

4 The model

In this section we develop the extensive form equations of our model based on the structure laid out in section 3. We significantly reformulate the equations, but not the "philosophy" of the Murphy model for the Australian economy, as presented in Powell and Murphy (1997), from a macro-theoretic perspective, by making it a continuous-time dynamic model of monetary growth, suppressing all discrete lag structures of their quarterly period model in particular.

Our interest in this section is not to fully mirror the dynamical structure and implications of the Murphy model, but to make use of its qualitative understanding of applied Keynesian theory to formulate and to investigate, in a first approximation to this 100 equations approach to macro econometric model building, a set of prominent feedback structures of macrodynamic theory and their role for economic stability analysis, before fiscal, monetary and social policy enter the scene.

This section therefore attempts to build a bridge between empirically motivated work on structural model building (where there generally is no analysis of the mechanisms that are hidden in the formulated structure) and theoretical investigations of reasonably large representation of economies, where the interest is to see what the steady state of such economies will look like, in particular with respect to the share of wages, and its dependence on various forms of taxation, social protection and the stabilizing role of fiscal and monetary policy in the case where destabilizing effects dominate the private sector of the economy.

Others have argued that we are seeing a more fundamental regime change: the third in postwar history, starting with the Keynesian model, from the 1940s to the 1970s; the neoliberal ascendency, from 1978 to 2008; followed by a new regime, which is currently being shaped. Perhaps this new regime will come to be called ‘social capitalism’ or ‘social democratic capitalism’, or simply the term ‘social democracy’ itself. Whatever the nomenclature, the concept is clear: a system of open markets, unambiguously regulated by an activist state, and one in which the state intervenes to reduce the greater inequalities that competitive markets will inevitably generate. (Kevin Rudd, former Australian Prime Minister, February 2009, The Monthly)

4.1 Basic definitions

Let us start with some notation to be used in the structural equations of our approach to Keynesian monetary growth.
Module 1. of the model provides definitions of expected rates of return $\rho^e$, $\rho_h$, based on expected sales in manufacturing (with and without the interest payments $B_f$ of firms on corporate bonds, represented by perpetuities), and for residential services, notation for hourly wages, $w$, including income taxation and later on payroll taxes, prices $p, p_h, p_i$ for goods, residential services and perpetuities, the first including value added taxation, of pension payments to the retired worker of the workforce, $w^r$, and unemployment benefits per unemployed worker (of the workforce), $w^u$, both untaxed.

We use for actual exports the representation $X = x_y Y^e$, for imports $J^d = j_y Y^e$, depending on the real exchange rate $\sigma$, which includes import taxation, and use for the actual employment of the workforce of the firms $L^d = l_y Y$, and augment wages $w$ by payroll fringe costs of firms later on.

Note that the percentages of pensions and unemployment benefits refer to gross wages. They must there take the size of $q^n$ into account when choosing actual parameter sizes and can in any case not be $1(00 \text{ percent})$ of $q^n$.

Module 2 concerns the household sector where two types of households are distinguished, workers and pure asset holders. Of course, these two types of households are only polar cases in the actual distribution of households types. Nevertheless we believe that it is useful to start from such polar household types before intermediate cases are introduced and formalized.5

### 4.2 Households

We consider the behavioral equations of worker households first:

5Powell and Murphy (1997) consider only one type of household sector explicitly (though they briefly refer to effects of income distribution implicitly contained in their formulation of a consumption function), the consumption behavior of which is based on the life cycle hypothesis with respect to wage income and wealth.
2a. Households (Workforce $L$)

$$Y_w^{Dn} = w^n L^h + w^u (L - L^u) + w^r \alpha^r L + (1 - \tau_c) r B_w$$

$$= Y_w^{Dn} + (1 - \tau_c) r B_w$$

$$L^w = L_f^w + L^w_g, \quad \text{employment rate} \ e = L^w / L$$

$$L^h = L^h + L^h_g = L^h + L^h_g$$

$$pC^{d}_{wc} = c_{wc}(q_h) Y_{wcw}^D, \quad c_{wc}(q_h) = c^o_{wc} + c_1(q_h - q_a)$$

$$pC^{d}_{wh} = c_{wh}(q_h) Y_{whw}^D [\text{Goods}], \quad c_{wh}(q_h) = c^o_{wh} - c_1(q_h - q_a)$$

$$S^{n}_{w} = Y_w^{Dn} - pC^{d}_{wc} - q_h pC^{d}_{wh} = \bar{B}_w$$

$$L = L_r(\text{retirees}) = L_k(\text{ids}) = \bar{n}, \quad (L(0), L_r(0), L_k(0) \text{ given})$$

We start the description of workers’ consumption and savings decision by distinguishing between labor income, unemployment benefits and pensions payments (retired persons being given by the number $L^r = \alpha^r L$,\(^6\) as income items behind workers’ consumption plans (plus interest income on their saving deposits which however are simply saved again by them).

In the first equation of this module we provide the definition of the total disposable income of worker households, consisting of wages, unemployment benefits and pension payments, the first after taxes, and of their interest rate income after capital taxation. Next, the total employment of the workforce by firms and the government is defined in terms of the number of employed people. By contrast, the third equation defines hours worked within firms and the government sector, assuming that there is no overtime or undertime work in the government sector. The consumption function of workers, the fourth equation, is based on their disposable work income in the usual linear fashion. Workers consume (measured in terms of real goods) the amount $C^d_{wc}$ as goods and the amount $C^d_{wh}$ as rental services (the price of which therefore is $q_h pC^d_{wh}$). Workers’ savings is the difference between their total income and their actual consumption.

The final two equations define the here still very simple demographic structure of our model with respect to worker households. We assume that all age groups or generations (children and juveniles, potential workforce, retired persons) grow with the same rate. Initial conditions with respect to these three cohorts of worker households are considered as given and determine – due to the assumed uniform rates of growth – the portions of people not yet or no longer in the workforce, below 17 and above 64 for reasons of mathematical simplicity, i.e., giving rise to the population proportions 1:3:1 when 16 years are used as measurement unit. This sector is more advanced than in traditional presentations of differentiated households’ saving habits, since we consider unemployment benefits and pensions explicitly, and the provision of medicare, etc. implicitly, as part of the goods and services provided by the government sector.

Next, we consider the other type of household sector of our model, the (pure) asset owners who desire to consume $C_c$ (goods and houses as supplied by firms through domestic production $Y$) at an amount that is growing exogenously at the rate $\bar{\gamma}$ and which is thus in particular independent of their current nominal disposable income $Y_c^{Dn}$. The consumption decision is thus not an important decision for asset holders. Their nominal income diminished by the nominal value of their consumption $p_c C_c$ is then spent on the purchase of financial assets (three types of bonds and money) as well as on investment in housing supply (residential space for worker households). Note here that the one good view of the production of the domestic good entails consumption goods proper and houses (both at commodity prices $p$) so that asset holders buy houses for their consumption as well as for investment purposes. Investment in the supply of residential services (and that of firms) is not subject to value added taxation.\(^6\)

\(^6\)we assume $\alpha^w = 1$ from now on.
2b. Households (Asset-Holders, flow-consistency assumed):

\begin{align*}
Y_{c}\cdot D_{c} & = (1 - \tau_{c})[\rho(pK + B_{f} + rB_{c} + B_{c}^{g} + \rho_{u}pK_{h}) + s(1 - \tau_{c}^{*})B_{c}^{l}]\cdot B_{c}^{l} = B_{c}^{l} + B_{c}^{l}\cdot (4.18) \\
C_{c} & = 0 \tag{4.19} \\
S_{c}^{n} & = Y_{c}\cdot D_{c} - pC_{c} \tag{4.20} \\
& = M_{c} + B_{c} + p_{1}\cdot B_{c}^{l} + pI_{h} + s\cdot p_{1}^{*}B_{c}^{l}, \quad \dot{B}_{c} = \dot{B}_{c} - \dot{B}_{w} - M_{c}, \quad \dot{B}_{c} = \dot{B}_{c}^{u} + \dot{B}_{c}^{l} \\
C_{h}^{s} & = \alpha_{h}K_{h} \quad [C_{h}^{s} = \ldots \text{ see module 2a.}] \tag{4.21} \\
g_{h} & = \frac{(l_{h})^{d}}{K_{h}} = \alpha_{ph}(\rho_{h} - (r^{l} - \pi^{c})) - \alpha_{rh}(r^{l} - r^{l}_{o}) + \alpha_{ah}(\frac{C_{wh}}{C_{h}^{l}} - \bar{\nu}_{h}) + \bar{n} + \delta_{h} \tag{4.22} \\
\dot{K}_{h} & = g_{h} - \delta_{h} \tag{4.23}
\end{align*}

4.3 Firms

In module 3. of the model we describe the sector of firms, the planned investment demand of which is assumed to be always served, just as all other consumption and investment plans. We thus assume for the short-run of the model that it is of a Keynesian nature, since aggregate demand is never rationed, due to the existence of (sometimes left implicit) excess capacities, inventories, overtime work and other buffers that exist in real market economies. In contrast to the French rationing school, there is thus only one regime possible, the Keynesian one, for the short-run of the model, while supply side forces come to the surface only in the medium- and the long-run of the model. Up to certain extreme episodes in history this may be the appropriate modeling strategy for the macro-level of a market economy.

In contrast to our first working paper for this ILO project, we also leave implicit here that labor productivity grows at the given rate \( m \). This would demand the use of efficiency units measurements which however is but a reformulation of some state variables without any change in their laws of motion and thus neglected here for reasons of simplicity. A high natural rate of growth in the numerical simulation therefore does not mean that population is subject to explosive growth.

3. Firms (Technology, Production, Employment and Investment)

\begin{align*}
Y^{p} & = y^{p}(k_{g})K, \quad y^{p}(k_{g}) = y_{0}^{p} + y_{1}(k_{g} - k_{go}), \quad k_{g} = K_{g}/K \tag{4.24} \\
J^{d} & = j_{\sigma}(\sigma)Y^{e}, \quad j_{\sigma} = j_{o} - j_{1}(\sigma - \sigma_{o}) \tag{4.25} \\
X & = x_{g}Y^{e} \tag{4.26} \\
L_{f}^{h} & = l_{g}(\sigma)Y, \quad l_{g}(\sigma) > 0 \quad \text{hours actually worked} \tag{4.27} \\
u & = Y/Y^{p} \in [0, 1] \tag{4.28} \\
\tilde{L}_{f}^{w} & = \beta_{f}(L_{f}^{h}/L_{f}^{w} - \bar{u}_{w}) + \bar{n}, \quad \text{workforce within firms} \quad \bar{u}_{w} = 1 \tag{4.29} \\
g_{k} & = I/K = \alpha_{pp}(\rho^{g} - (r^{l} - \pi^{c})) - \alpha_{rp}(r^{l} - r^{l}_{o}) + \alpha_{u}(u - \bar{u}) + \bar{n} + \delta_{f} \tag{4.30} \\
Y_{f}^{e} & = Y - Y^{e} = I \tag{4.31} \\
S_{f}^{me} & = p_{b}Y_{f}^{e} \tag{4.32} \\
p_{b}\dot{B}_{c}^{l} & = p(I - \delta_{f}K) + p_{y}(\dot{N} - I) \tag{4.33} \\
I_{a} & = I + \dot{N} \tag{4.34} \\
\dot{K} & = g_{k} - \delta_{f} \tag{4.35}
\end{align*}

We assume for reasons of simplicity a fixed proportions technology\(^7\) with output-employment ratio \( 1/l_{g} \) and potential output-capital ratio \( y^{p} \). Labor productivity \( z = 1/l_{g} \) is in contrast to the first paper constant \( (l_{g}) \) varies inversely to \( \sigma \), so that when imports become more expensive, firms

\(^{7}\)See Chiarella and Flaschel (2000) for the treatment of neoclassical smooth factor substitution in place of such a fixed proportions technology.
try to save such intermediate inputs through additional work efforts). The output level $Y$ actually produced by firms will be provided in a later module by a Metzlerian output-inventory adjustment mechanism. Depending on this output level we define the rate of capacity utilization $u$ and the employment $L^h_f$ of the workforce employed by firms, which in the short-run is assumed to supply any amount demanded by firms through over- or under-time work.

For the adjustment of the workforce of firms we assume as given a normal rate of employment $\bar{u}_w$ of the workforce, which in principle could be set to unity, representing the normal hours worked by the workforce $L^w_f$ currently employed by firms, here represented simply by the benchmark level $\bar{u}_w = 1$, separating over-time from under-time work as caused by fluctuating aggregate demand, expected sales and adjusting output levels. The number of workers $L^w_f$ employed by firms is adjusted by them with speed $\beta_l$ according to the over- or under-time work $L^h_f - \bar{u}_w L^w_f$ they experience, augmented by a term that accounts for trend growth. The rate of investment, finally $I/K$ is driven by three forces, Goodwinian profitability in its deviation from the expected real long-term rate of interest, the deviation of the nominal long-term rate from its steady state position and Harrodian excess capacity utilization.

The payroll tax rate (here only applied to firms) is considered in more detail in module 4. of the model. It will be used as a fiscal instrument in addition to the standard Keynesian fiscal policy rule.

We assume that potential output depends positively on the stock of "infrastructure" $k_g$ and the intermediate input imports $J^d$ of firms negatively on the real exchange rate $\sigma$ (exports $X$ do – for reasons of simplicity – not depend on the real exchange rate, see module 3a. in this respect). In correspondence to this cost savings effect we have an increase in labor needed in this changed production environment, i.e., there is no free lunch for firms in their adjustment to real exchange rate increases.

Next, import and export prices are treated in the simplest way possible by assuming that they are fixed in terms of the foreign currency and thus need only to be multiplied with the exchange rate in order to arrive at domestic producer prices. There is a subsidy on exports $\tau_x$ and a tax rate on imported intermediate commodities of size $\tau_m$. This module of the model basically impacts the profitability of firms as measured by the expected rate of profit $\rho^e$ in the first block of our model.

### 3a. Export and Import Prices in Domestic Currency and Taxation ($s[AUD/USD]$)

\begin{align*}
    p_x &:= (1 - \tau_x)sp^*_x = p_y : \quad \tau_x = 1 - \frac{(1 + \tau_m)(p^*_m/p^*_x)(0)}{\sigma} \quad (4.36) \\
    p_m &:= (1 + \tau_m)sp^*_m = p_y \sigma : \quad (4.37) \\
    \hat{r}_m &= \beta_{nx:rm} \frac{p_y x - p_m j^d}{p_y} = \beta_{nx:rm} y^e(x_y - \sigma_j y(\sigma)), \quad \frac{d(\tau_m)j^d(\sigma(\tau_m))}{d\tau_m} > 0 \quad (4.38)
\end{align*}

### 4.4 The government

In the next module 4. we describe the public sector of the economy in a way that allows for government debt in the steady state and for a monetary policy that adjusts the rate of interest on short-term bonds in view of what happens in the market for long-term bonds compared to the given level of the world rate of interest, the domestic rate of inflation as compared to a target level, an import taxation target, allowing for balanced trade, and the domestic excess activity of firms.
4. Government (Fiscal and Monetary Authority):

\[ T^n = \tau_w w L^h + \tau_{fp} w L^d + \tau_c p_y (C_{uc} + C_c + G) - \tau_x s_p^x X + \tau_m s_p^m J^d \]
\[ + \tau_c \rho^g \rho K + \rho_h p K_h + r B + B^d] - w^u (L - L^w) - w^r \alpha^r L - w L_g^d \] (4.39)

\[ G = g_y (d) \bar{u} y^p (k_g) K - g_p (y^e - y^c) K = G_s - G_p, g_y (d) \bar{u} y^p (k_g) \geq \delta y k_g, k_g = \frac{K_g}{K} \] (4.40)

where \( g_y (d) = g_{yo} - g_1 (d - d_o), \quad d = \frac{B + p_1 B^t}{p Y_e}, \quad d \in [\underline{d}, \bar{d}] : G \above, \bar{d} \below d \):

\[ (1 - \alpha_g) G_s + \delta g K_g - g_p (\cdot), \quad \mu G_s + \delta K_g - g_p (\cdot), \mu > 1 \quad \text{if needed} \]

\[ L_g^w = l_y g_y (d_o) \bar{u} Y_p, \quad L^w = L^w_f + L^w_g \] (4.41)

\[ \dot{r} = \beta_{rr} (r_l - r^s) + \beta_{pr} (\bar{p} - \pi^s) + \beta_{m} \tau_m (\tau_m - \tau_m), \quad dB_b = d M_e \] (4.42)

\[ \dot{r}_{fp} = \beta_{dr} \left( \frac{B + p_1 B^t}{p Y_e} - d_o - \beta_m \tau_{fp} (\tau_{fp} - \tau_{fpa}) \right) \] (4.43)

\[ S^n_g = T^n - p G - (1 + \tau_c) (r B + B^d) \] (4.44)

\[ with \ \tau_c (r B + B^d) = \text{foreign aid expenditure} \] (4.45)

\[ \dot{B} = -\alpha_{fp} S^n_g \] (4.46)

\[ p_1 \dot{B}^d = -(1 - \alpha_{fp}) S^n_g \] (4.47)

\[ \dot{K}_g = \alpha_g G_s - \delta g K_g, \quad \dot{K}_g = 0, \quad \dot{K}_g = \mu \alpha_g G_s - \delta g K_g, \text{respectively} \] (4.48)

In the government sector, wage income taxes are raised with rate \( \tau_w \) on wages \( w \) and there are untaxed unemployment benefits and pension payments, where unemployment benefits \( w^u \) and retirement payments \( w^r \) are in fixed proportion to net wages \( w^n \). The capital income tax rate \( \tau_c \) is applied to the interest income of workers as well as to profit and interest income of asset holders. Finally, the untaxed interest income of the central bank is not – as it is often assumed – transferred back into the government sector. With respect to pension payments we have assumed that the number of retirees is proportional to the size of the workforce: \( \alpha_L, L \), see also the description of worker households (1/3 in the given numerical example). The rate \( \tau_v \) is the value added tax rate, \( \tau_x \) the export tax rate (in fact a subsidy), and \( \tau_m \) the import tax rate. The government uses payroll tax to handle its debt target by way of a rule for \( \dot{r}_{fp} \).

Government expenditures for goods and services are both assumed to be constant fractions of normal output, the former however augmented by an anti-cyclical fiscal policy rule. With respect to the provision of public services we in addition assume – in contrast to the sector of firms – that there is no overtime work in the government sector. From the expenditures for goods and services, the interest payments and the transfers made by the government we obtain the savings of the government sector by deducting the sum of these items from \( T^n \), i.e., the sum of received tax payments.

These savings will in general be negative and thus call for debt financing. Government allocates its debt financing in nominal terms in constant proportions to short- and long-term debt and can always realize its intended debt financing due to the flow consistency requirements of such macro-dynamical models.

Note again that the CB also tries to establish a target rate for import taxation (see module 3a) in order to control the real exchange rate from the perspective of the real economy. The payroll taxation of firms is moreover adjusted towards the establishment of a given debt target, and this in a different way in the case of a rising or falling ratio \( d \). The current debt to (expected) sales ratio \( d \) is assumed to determine the government expenditure ratio \( g_y \) in a negative way. By contrast, we have already assumed that the stock of public capital \( k_g \), where the rate of change is given by the last equation in module 4., exhibits a positive influence on the potential output capital ratio \( y^p \) in particular (and is also moderating the wage bargaining process).

Keynesian business cycle policy is characterized by the minus sign in front of the parameter \( \beta_{fp} \).
Politicians (wisely) reduce government expenditures in good states of the economy and increase them in bad states, where it may depend on the model builder whether they will put the stress on the labor market or on the performance of firms as measured by their rate of capacity utilization. The big question is of course whether undamped private sector business cycle fluctuations can be damped by the intervention of fiscal and monetary policy.

Analyzing the dynamics of the government budget constraint

In order to isolate the dynamics of government debt from the rest of the dynamics (and also to show the prime deficit or surplus) we assume that all of its state variables are frozen at the steady state with the exception of the variables \( b = B/pK, b' = B'/pK, \tau \) which describe the evolution of short-term and long-term government debt per value unit of capital together with the adjustments in a taxation rate \( \tau \) that we here assume to take place in view of the deviation of government debt to expected sales ratio from a certain target ratio. The remaining dynamics then can be expressed as follows:

\[
\dot{b} = \alpha g_0 \left[ r_o b + b' + c_0 - \tau c_1 \right] - \left( \bar{n} + \hat{p}_o \right) b \\
\frac{b'}{r_o} = (1 - \alpha g_0) [r_o b + b' + c_0 - \tau c_1] - \left( \bar{n} + \hat{p}_o \right) \frac{b'}{r_o} \\
\dot{\tau} = \alpha \tau \left( \frac{b + b'/r_o}{y_o^c} - \bar{d} \right), \quad d = \frac{b + b'/r_o}{y_o^c}
\]

where \( c_1, c_0 > 0 \) denote certain constants and where \( 1/p_o = r_o \). Note that

\[
\dot{b} = \alpha g_0 \left[ r_o d + c_0 - \tau c_1 \right] - \left( \bar{n} + \hat{p}_o \right) b \\
\frac{b'}{r_o} = (1 - \alpha g_0) [r_o d + c_0 - \tau c_1] - \left( \bar{n} + \hat{p}_o \right) \frac{b'}{r_o}
\]

can be aggregated and give

\[
d = \left( [r_o d + c_0 - \tau c_1] - \left( \bar{n} + \hat{p}_o \right) d \right)/y_o^c
\]

In the steady state there therefore holds:

\[
d_o = \frac{c_0 - r_o c_1}{\bar{n} + \hat{p}_o - r_o} > 0 \quad \text{if} \quad c_0 - r_o c_1 > 0, \quad \bar{n} + \hat{p}_o - r_o > 0,
\]

as should be the case if a primary deficit is in the background of this reduced system and the rate of productivity growth and the inflation target sufficiently high. In such a case it is also obvious that the system is convergent to its steady state position, while the opposite holds if \( r_o \) dominates the denominator of the above fraction.

**Proposition 1** The interior steady state of the government debt dynamics is locally asymptotically stable if there holds:

\[
c_0 - r_o c_1 > 0, \quad \bar{n} + \hat{p}_o - r_o > 0.
\]

Proof: It is easy to see that the obtained 2D dynamics exhibits a positive determinant and a negative trace at the steady state. A global result can also be obtained from an appropriate application of Olech’s Theorem – if wanted.

4.5 Quantity and price adjustment processes on the firm level

We now come to the description of the dynamics of quantities (module 5a) and prices (module 5b). Module 5a of the model basically describes a Metzlerian inventory adjustment process for the
non-traded goods produced by firms.\(^8\) Module 5b then describes the nominal price adjustments in the goods and in the labor market, as well as the adjustment of long-term inflationary expectations \(\pi^c\), measuring some sort of inflationary climate.

\[^{5}\text{a. Quantity Adjustments in the Production of the Domestic Good}\]

\[
Y^e \neq Y^d = C_{wc}^d + C_c + I_h + I + G + X
\]  
(4.49)

\[
I^{na} = pI + pI_h + p_y \dot{N}
\]  
(4.50)

\[
\dot{N}^d = \alpha_n \dot{N}
\]  
(4.51)

\[
\dot{I} = \beta_n (N^d - N) + \gamma N^d
\]  
(4.52)

\[
Y = Y^e + \dot{I}
\]  
(4.53)

\[
\dot{Y}^e = \beta_{y^e} (Y^d / Y^e - 1) + \ddot{n}
\]  
(4.54)

\[
\dot{N} = Y - Y^d
\]  
(4.55)

In this simple Metzlerian approach to goods market disequilibrium, we assume that the output decisions \(Y\) of firms are based on expected sales \(Y^e\) and intended inventory changes \(\dot{I}\). The intended inventory changes in turn are based on the desired inventory level \(N^d\) of firms assumed to be proportional to their expected domestic sales. Inventories are then adjusted according to the discrepancy \(N^d - N\) between desired and actual inventories with speed \(\beta_n\), the inventory accelerator mechanism, again augmented by a term that accounts for trend growth. Actual inventory changes are given by output minus aggregate demand (which in this Keynesian approach is always served). We thus ignore here the possibility that inventories may become exhausted, which would provide a situation of rationing with respect to goods demand. The last equation of this module of the model provides the adjustment mechanism for sales expectations \(Y^e\) which are assumed to follow observed domestic aggregate demand in an adaptive fashion, again augmented by a term that accounts for trend growth.

This module of the model basically represents a refined dynamic multiplier story to the extent that output adjustment towards aggregate demand is not represented by only one – dynamic - equation, but augmented by the inventory adjustments that such a process entails and by the assumption that aggregate demand is not perfectly foreseen. We know that the dynamic multiplier is unstable when the marginal propensity to spend is larger than one, as in the famous Kaldor (1940) trade cycle model, a condition which is here slightly more difficult to establish due to the distinction between output, expected demand and aggregate demand. In addition we may now also have instability due to the Metzlerian inventory adjustment process, which – if sufficiently fast – also establishes a positive feedback chain between output, expected demand and aggregate demand.

These are the basic pure quantity adjustment processes of our Keynesian macrodynamics. A further and final one, the Harrodian mechanism of unstable warranted growth – is shown in figure 1 of the firm sector.

Analyzing the Metzlerian expected sales/inventory dynamics

In order to isolate this mechanism we assume that fixed business investment is given by its trend component solely: \(g_k = \ddot{n} + \delta_k\). In this case we get for the interaction of expected sales \(y^e\) and actual inventories \(\nu\) (both per unit of capital) from the 18D core dynamics of the general model, the equation system

\[
\dot{y}^e = \beta_{y^e} (y^d - y^e)
\]

\[
\dot{\nu} = y - y^d - \ddot{n} \nu
\]

\[
y^d = d_1 y + d_0, \quad \text{with} \quad d_0 > 0, d_1 \in (0, 1)
\]

\[
y = y^e + \beta_n (\alpha_n y^e - \nu) + \ddot{n} \alpha_n y^e.
\]

\(^8\)There are no sales and delivery constraints for traded goods and there is thus no direct need to consider inventory adjustment processes in their case.
These equations provide us with two linear differential equations in the state variables expected sales and actual inventories (per unit of capital) if the above simple textbook version of an aggregate demand function is assumed. It is easy to see that a sufficiently large adjustment parameter value $\beta_n$ (which can approach infinity in continuous time if this is needed) implies that the dependence of $y^d$ on $y$ and thus on $y^e$ obtains a slope that is larger than one, in which case the law of motion for $y^e$ depends positively on the size of $y^e$, or in other words, the entry $J_{11}$ of the Jacobian $J$ of the above dynamics at the steady state becomes positive under these circumstances. We conclude that the trace of $J$ must then become positive if the parameter $\beta_n$ is chosen sufficiently large in addition, since this parameter is not involved in the second component that defines the trace of $J$.

The above equations for the 2D inventory dynamics thus show that output $y$ depends positively on expected sales $y^e$ and this more and more strongly the higher the speed of adjustment $\beta_n$ of planned inventories becomes. The time rate of change of expected sales therefore depends positively on the level of expected sales when the parameter $\beta_n$ is chosen sufficiently large. Flexible adjustment of inventories coupled with a high speed of adjustment of sales expectations are thus bad for obtaining economic stability. There will, however, exist situations (with a low inventory accelerator) where an increase in the latter speed of adjustment may increase the stability of the dynamics.

Next we consider the wage-price spiral of the general model. This type of dynamics represents an important module of the present stage of modeling the details of a small open economy with an integrated treatment of its short-, medium- and long-run behavior. We stress that we do not yet treat imported consumer goods and related price indices and thus the role of import prices in the formation of money wages.

5b. Wage-Price Adjustment Equations and the Inflation Climate:

$$\dot{w} = \beta_{ew}(k_g)(L^w - \bar{e}(k_g)) + \beta_{uw}(k_g)(\frac{L^h}{L^f} - \bar{u}) + \beta_{hw}(\frac{q}{q_o} - 1) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^e \tag{4.56}$$

$$\dot{c} = \beta_{cw}(k_g) > 0, \beta_{uw}(k_g) < 0, \beta_{uw}(k_g) < 0$$

$$\dot{\hat{p}} = \beta_{p}(u - \bar{u}) + J_{0}\hat{s} + \kappa_p \hat{w} + (1 - \kappa_p)\pi^e \tag{4.57}$$

$$\dot{\pi}^c = \beta_{\pi}(\alpha_{\pi}(\hat{p} - \pi^e) + (1 - \alpha_{\pi})(\pi^* - \pi^e)) \tag{4.58}$$

$$\dot{\hat{h}} = \beta_{hp}(\frac{C_{uh}}{C_h} - \bar{h}) + \pi^e \tag{4.59}$$

With respect to gross nominal wages $w$ (which include income taxes, but not yet payroll taxes) we assume that their rate of growth $\dot{w}$ depends positively on the demand pressure on the external labor market, measured by the deviation of the rate of employment from the NAIRU rate of employment $\bar{e}$, and on labor demand pressure within the firms, measured by the degree of over- or undertime work compared to the normal work-time of the employed. Cost pressure for wage earners is measured by two related expression. Firstly, and on the one hand, we assume – in order to show that myopic perfect foresight is not at all a problem for Keynesian macroeconomics – that workers have perfect knowledge of the short-term evolution of price inflation, but use in addition, and on the other hand, on the basis of this knowledge, an inflation rate expression, $\pi^e$, representing the inflationary climate in which the current inflation rate is operating. The inflationary climate variable $\pi^e$ is thus a magnitude that is related to the medium-run and is assumed to be updated in the adaptive fashion as shown in the last equation in this module. Cost pressure for workers is then measured as a weighted average of these two expressions for price inflation $\hat{p}$ and $\pi^e$, implying that workers look beyond the short-run (for $\kappa_w < 1$) and thus take into account also the climate in which current inflation is evolving. This guarantees that the dynamics of the model is not heavily dependent on whether short-term expectations are perfect or not fully correct. We here simply save, by the assumption of myopic perfect foresight, another dynamic law that would describe the evolution of short-term expectations, without much change in the implied dynamics if these expectations are revised sufficiently fast.
The wage bargaining process is mitigated in an economy with a higher level of infrastructure $k_y$, since the role of corporatism is increased thereby.

Turning next to price inflation we here assume (analogously to wage inflation) that it is also based on demand pressure as measured by the rate of capacity utilization $u$ in its deviation from what firms conceive as normal capacity utilization $\bar{u}$. Regarding cost pressure we assume again myopic perfect foresight, now of firms with respect to wage inflation, and form again a weighted average with respect to the inflationary climate $\pi^c$ that is also characterizing the medium-run expectations of firms. Note that we here only use goods price inflation to update the inflationary climate state variable and thus assume that wage and price inflation do not differ very much when averaged over the medium-run by way of the assumed adaptive updating of the climate variable $\pi^c$. This again helps to save one law of motion without implying much change for the structure of the model and its dynamics.

**The Goodwin wage income/insider-outsider labor market feedback chain**

We consider the wage-price spiral in its four possible configurations from a partial 2D perspective. Setting certain adjustment speeds in the wage-price spiral to zero gives as law of motion for real wages $\omega$ under the assumption of a supply driven economy ($y = y^p$):

$$
\dot{\omega} = \beta_{w_e}(e - \bar{\epsilon}) + \beta_{w_u}(u^w_f - \bar{u}^w_f)
$$

so that the demand pressure on outside and inside labor markets are here the sole determinants of the dynamics of real wages (since the short-run inflation rate is fully reflected in the adjustment of money wages). The obtained law of motion for real wages can then be rewritten as

$$
\dot{\omega} = \beta_{w_e}(l^w/l - \bar{\epsilon}) + \beta_{w_u}(l^h_f/l^w_f - \bar{u}^w_f), \quad l^h_f = \hat{l}y^p
$$

with $u^w_f = l^h_f/l^w_f = l_y y^p/l^w_f$ and $e = l^w/l$. The laws of motion of the state variables $l, l^w_f$ variables moreover can be obtained from the equations

$$
\dot{l} = -(\alpha^k_\rho (\rho - \bar{\rho}), \quad \bar{\rho} = \bar{\epsilon} - \beta_\ell(l_y y^p/l^w_f - \bar{u}^w_f) - (\alpha^k_\rho (\rho - \bar{\rho})
$$

with $\rho$ given by $y^p - \delta_k - (1 + \tau_f p)^2 l_y y^p$. After some manipulations, the differential equations for $u^w_f$ and $e$ can be obtained as

$$
\dot{u}^w_f = \frac{\alpha^k_\rho (\rho - \bar{\rho}) - \beta_\ell(l_y y^p/l^w_f - \bar{u}^w_f),}{\rho = y^p - \delta_k - (1 + \tau_f p)\omega l_y y^p}
$$

$$
\dot{\epsilon} = \beta_\ell(u^w_f - \bar{u}^w_f)
$$

These laws of motion reflect the assumed investment behaviour and the employment policy of firms which both influence these dynamics. On the basis of the above assumptions, we therefore obtain a 3D dynamical system in the state variables $\omega, u^w_f, e$, of the real wage, of the rate of employment of the employed, and of the outside rate of employment. For this system we have:

**Proposition 2** The dynamical system for $\omega, u^w_f$ and $e$ has a unique interior steady state given by

$$
\omega_o = \frac{y^p - \delta_k - \bar{\rho}}{(1 + \tau_f p)l_y y^p}, \quad u^w_f, e_o = \bar{\epsilon}.
$$

2. The steady state is locally asymptotically stable if and only if $\beta_{w_u} \bar{u}^w_f > \beta_{w_e} \bar{\epsilon}$ holds true.

3. At the value $\beta_{w_u}^H = \beta_{w_u} \bar{u}^w_f / \bar{u}^w_f$ of the parameter $\beta_{w_u}$ there occurs a Hopf-bifurcation, a cyclical loss of stability, of either subcritical (corridor stability), supercritical (stable limit cycle) or degenerate (center dynamics) type.
All these assertions are easy consequences of the Jacobian of the dynamics at the steady state:

\[
J = \begin{pmatrix}
0 & \beta_w \omega_o & \beta_w \omega_o \\
-\alpha_f^k (1 + \tau_p) l y y^p \bar{w}^w_f & -\beta \bar{u}^w_f & 0 \\
0 & \beta \bar{e} & 0
\end{pmatrix}
\]

Obviously, \(-a_1 = \text{trace } J = -\beta \bar{u}^w_f < 0\) and \(-a_3 = \det J = -\beta_w \omega_o \alpha_p^k (1 + \tau_p) l y y^p \bar{u}^w_f \beta \bar{e} < 0\). For \(a_2\) (the sum of the principal minors) we get \(a_2 = \beta_w \omega_o \alpha_p^k (1 + \tau_p) l y y^p \bar{u}^w_f > 0\). According to the Routh–Hurwitz conditions, see Gantmacher (1959), we have to consider in addition the positivity of

\[
a_1 a_2 - a_3 = \beta \bar{u}^w_f \omega_o \alpha_p^k (1 + \tau_p) l y y^p (\beta_w \bar{u}^w_f - \beta_w \bar{e}).
\]

Hence \(a_1 a_2 - a_3 > 0\) if and only if \(\beta_w \bar{u}^w_f > \beta_w \bar{e}\). The assertion of a Hopf bifurcation at \(\beta_w = \beta_w \bar{e} / \bar{u}^w_f\) is then proved by means of the above expression for \(a_1 a_2 - a_3\).

We thus in particular have that fast inside wage adjustment speeds, \(\beta_w\), are enhancing local asymptotic stability, while the opposite holds true for the adjustment speed of outside wage claims, \(\beta_{wu}\). All other parameters of these dynamics (up to the levels of the employed rates of NAIRU type) do not matter for the stability of this partial dynamics between real wages and the inside and outside rate of employment. This holds in particular for the speed of adjustment \(\beta\) of the hiring and firing policy of the firms.

Figure 1: A limit cycle of the augmented dynamics with the full employment ceiling in operation

Figure 1 provides an example of the dynamics bounded by an extrinsic nonlinearity (a case where the steady state is no longer attracting) where the attracting set is now of a limit cycle type. The figure top left shows the stable limit cycle of the dynamics, while the two cycles that border this figure show its projections into the adjacent planes (and the trajectories on the way to it). The figure bottom right finally shows the time series for the outside and inside rate of employment, with the full employment ceiling for the first rate sometimes in operation and with a rate of employment of inside workers that stays below 130% (and with a fluctuating natural rate of growth \(\bar{n}\) and of capacity utilization \(\bar{u}\) now. When inside employment approaches this level, it is furthermore clearly visible that the rate of growth of labor supply responds to this fact, but as in the case of the inside rate of employment only in a moderate way in order to create the volume of labor supply and its rate of growth that is demanded by firms.

Adding the Rose real wage / goods market feedback chain

In order to sketch the details of this further economic feedback chain which now integrates goods market dynamics into the Goodwinian growth cycle dynamics of the preceding consideration we have
to derive anew the law of motion for real wages from the full wage-price dynamics now. Starting from the equations of the general 18D dynamics we in fact have

\[
\begin{align*}
\dot{w} - \pi^c &= \beta_{w}(l^w/l - \bar{e}) + \beta_{w}(l^p/l - \bar{u}^w) + \kappa_w(\dot{p} - \pi^c), \quad l^p = l_y
\\
\dot{p} - \pi^c &= \beta_p(y/y^p - \bar{u}) + \kappa_p(\dot{w} - \pi^c).
\end{align*}
\]

We can see that these equations form a linear equation system in the two unknowns: \(\dot{w} - \pi^c, \dot{p} - \pi^c\). This system can be uniquely solved if \(\kappa = 1 - \kappa_w \kappa_p \neq 0\) holds true for \(\kappa_w, \kappa_p \in [0, 1]\), if both of these parameter values are not equal to one, meaning that the cost-push terms in both the wage and the price dynamics are not solely based on currently observed price and wage inflation rates. The explicit solution of this equation system is

\[
\begin{align*}
\dot{w} - \pi^c &= \kappa[\beta_{w}(l^w/l - \bar{e}) + \beta_{w}(l_y/l^p - \bar{u}^w) + \kappa_w\beta_p(y/y^p - \bar{u})] + \kappa_p[\beta_{w}(l^w/l - \bar{e}) + \beta_{w}(l_y/l^p - \bar{u}^w)],
\\
\dot{p} - \pi^c &= \kappa[\beta_p(y/y^p - \bar{u}) + \kappa_p(\beta_{w}(l^w/l - \bar{e}) + \beta_{w}(l_y/l^p - \bar{u}^w))],
\end{align*}
\]

which in turn implies for the real wage, \(\omega = w/p_y\) the expression

\[
\dot{\omega} = \kappa[(1 - \kappa_p)[\beta_{w}(l^w/l - \bar{e}) + \beta_{w}(l_y/l^p - \bar{u}^w)) - (1 - \kappa_w)\beta_p(y/y^p - \bar{u})].
\]

The dynamics of the real wage therefore depends positively on the demand pressure in the market for labor and negatively on the demand pressure in the market for goods, while the cost-push terms of the nominal dynamics have neutralised themselves in this relative expression for the wage dynamics. The economic reason for and the meaning of this result is easy to understand, since real wages should generally also depend on what happens in the market for goods. It is therefore astonishing to see that studies of Phillips curves, that integrate labor market phenomena with price inflation, are often built on only one of these demand pressures (the one in the labor market) in the theoretical as well as in the empirically oriented literature.

On the basis of the foregoing discussion we can now describe the feedback chain of real wage increases onto their rate of change implied by the core 18D model of this paper. Increases in real wages will either increase or decrease aggregate demand \(y^d = y\) for the domestic good (per unit of capital) depending among others on the consumption propensity \(c_y\) of workers in comparison to the propensity to invest \(\alpha_p\) that mirrors the influence of the expected profit rate

\[
\rho = y - \delta_k - (1 + \tau_p)\omega l_y
\]

in the investment demand function of firms.

Let us consider firstly the situation where economic activity \(y\) or \(u = y/y^p\) is reduced through this channel by real wage increases. We know from the model that this decreases the employment of the employed and with some time delay also the rate of outside employment \(e\). According to the above dynamical law for real wages we thus get that real wage increases are slowed down if wage flexibility is high and price flexibility is low, since the money wage will then react more strongly than the price level to this reduction in economic activity and will thus dominate the response of real wages to reduced economic activity. In this situation, the interaction between economic activity and real wages is therefore stabilising, since real wage increases are then checked by decreases in economic activity through their impact on real wages. However, in the opposite case of high price flexibility and low wage flexibility real wages will increase in the case of a reduction of economic activity and will thus amplify the initial increase in real wages, which creates a destabilising feedback chain between real wages and economic activity.

Consider next the case where economic activity increases with real wage increases, since consumption demand responds more strongly than investment demand of firms to changes in the real wage. Of course, we then get the opposite conclusions to the cases just considered. Price flexibility will now enhance economic stability, while wage flexibility will be detracting from it. We thus find that the real wage/economic activity interaction is crucially depending on the parameters that characterise the market for goods and for labor.
Either price or wage flexibility must however always be destabilising. The destabilising Rose effect (of whatever type) will be weak if both wage and price adjustment speeds $\beta_{w_o}, \beta_{w_u}, \beta_p$ are low, at least as far as situations of a depressed economy are concerned.

We now go on to consider the situation in which we add growth dynamics to the above considerations, but will now neglect inside employment adjustments ($\beta_{w_u} = 0$) so that increases in the output of firms are immediately transferred to new employment with respect to the external labor market. We thus assume $l^*_f = l^*_f$ and neglect any employment in the government sector. This gives rise to the following growth dynamics

$$
\dot{\omega} = \kappa[(1 - \kappa_p)(\beta_{w_o}l_y(l - \bar{e}) - (1 - \kappa_w)\beta_p(y/y^p - \bar{u})],
$$

$$
\dot{\bar{\omega}} = \gamma + \delta_k - \alpha^k_p(y - \bar{p}) - \alpha^k_a(y/y^p - \bar{u}),
$$

where $\rho = y - \delta_k - (1 + \tau_p)\omega l_y y$ and where the second law of motion is derived as usual from the growth law for the capital stock. Note here that we have included now the third term $\alpha^k_p(> 0)$ of the fixed business investment function, since the rate of capacity utilisation is a variable in the Rose type labor and goods market interactions. Note also that we do not distinguish here between output and the (expected) demand for goods and thus ignore the quantity adjustment process of firms and the details of the formation of aggregate demand. Instead we shall now simply assume that output per capital $y$ is a function of real wages $\omega$ and that this function is increasing if we assume that the impact of real wage changes on $y$ is positive (if consumption demand is more sensitive than investment demand to real wage changes), while it is decreasing in the opposite case.

We thereby arrive at the autonomous nonlinear system of differential equations of dimension 2,

$$
\dot{\omega} = \kappa[(1 - \kappa_p)(\beta_{w_o}l_y(l - \bar{e}) - (1 - \kappa_w)\beta_p(y/y^p - \bar{u})],
$$

$$
\dot{\bar{\omega}} = \gamma + \delta_k - \alpha^k_p(y - \bar{p}) - \alpha^k_a(y/y^p - \bar{u}),
$$

in the two state variables $\omega, l$. We consider here only the case where the rate of profit $\rho = y(\omega) - \delta_k - (1 + \tau_p)\omega l_y y(\omega)$ depends negatively on the real wage $\omega$, namely where the mass purchasing effect of real wage increases is not so large that it outweighs the wage cost effect on the rate of profit. In this case we get for the Jacobian $J$ of the above 2D dynamics at the steady state the following sign structure.

$$
J = \begin{pmatrix}
\kappa[1 - \kappa_p]\beta_{w_o}l_y(y(\omega)/l - (1 - \kappa_w)\beta_p(y/y^p - \bar{u})]
& -\kappa(1 - \kappa_p)\beta_{w_o}l_y y(\omega)/(l^2) \\
-\alpha^k_p(y - \bar{p}) - \alpha^k_a(y/y^p - \bar{u})
& 0
\end{pmatrix}
$$

The sign of $J_{11}$ in the trace is therefore the decisive element that determines the local stability or instability of the Rose real wage mechanism in isolation as well as its interaction with economic growth.

### 4.6 The dynamics of asset prices and expectations

The sixth module lists the dynamic adjustment equations we assume to hold for the asset prices of our model: long-term domestic and foreign bonds, $p_t$, the latter only in so far as the dynamics of the exchange rate is concerned (in view of the given US $-$rate of return on foreign bonds).
### The Blanchard bond market dynamics

Blanchard (1981) has investigated the dynamic adjustment processes in the market for long-term bonds on the basis of myopic perfect foresight and perfect asset substitutability by means of the saddle-path dynamics that is then typically applied in order to get asymptotically stable adjustment processes after the occurrence of unanticipated shocks or changes in the expectations of future events.

We instead assume in the structural form of our model that rate of return differentials are not instantaneously removed, but give rise to somewhat delayed adjustments in asset prices. We also argue that there are always heterogeneous expectations present, of asset holders who fall into two groups - ambitious agents who devote significant parts their time (and resources) to the effort of forming perfect anticipations and less ambitious ones who behave in an adaptive fashion. We have argued furthermore that the market share of the latter agents, despite their less accurate predictions of asset price dynamics, does not tend to zero due to the fact that all asset owners have a life cycle profile that lets them act in an ambitious fashion when they are young and in a less ambitious fashion when they become old (due to changes in their preference relations). Though ambitious agents have more profitable investments their influence is bounded since they become less ambitious later on.

For the laws of motion for the price of long-term bonds and expectations about its rate of change we have assumed

\[
\dot{p}_l = \frac{\beta_{p_l}}{1 - \beta_{p_l}(1 - \alpha_s)} \left[ \frac{1}{p_l} + \alpha_s \pi_{ls} - r^* \right],
\]

\[
\dot{\pi}_{ls} = \beta_{\pi_{ls}} (\hat{p}_l - \pi_{ls}).
\]

Note that the short-term rate of interest \( r^* \) is considered as given in this partial analysis of the market for consols. Insertion yields

\[
\hat{\pi}_{ls} = \beta_{\pi_{ls}} [\frac{\alpha_s \beta_{p_l}}{1 - \beta_{p_l}(1 - \alpha_s)} - 1] \pi_{ls} + \frac{\beta_{p_l}}{1 - \beta_{p_l}(1 - \alpha_s)} \frac{1}{p_l} + \text{const.}].
\]

We see that the trace of the Jacobian \( J \) of the 2D dynamical system at the steady state can be made as positive as desired. This is so, since the parameter \( \beta_{p_l} \) can always be chosen to make \( J_{22} \) positive, then \( \beta_{\pi_{ls}} \) can be chosen so as to scale up \( J_{22} \) in the trace to an arbitrarily large value without changing the other coefficient \( J_{11} \) of the trace. It is therefore easy to show that the determinant of the Jacobian \( J \) of the full 2D dynamics shown above is always positive and that the system switches from stable nodes and thereafter stable foci to unstable foci and them to unstable nodes when the adjustment speed of expectations of less ambitious agents is increased from zero towards infinity. Therefore all local stability scenarios – apart from saddle-point dynamics – are possible, depending on the adjustment speed of adaptively formed expectations.

In sum, the foregoing analysis implies that there is a tendency for the price dynamics of long-term bonds to become at least locally explosive when the adjustment speed of bond prices becomes
sufficiently large and when the expectations adjustment speed of less ambitious asset owners approach infinity. We stress that the bond rate dynamics influences investment behaviour of firms and of asset holders and thus will transfer its instability to the real part of the 18D dynamical system.

The Dornbusch exchange rate dynamics mechanism

The evolution of the exchange rate and expectations about its behaviour can be reduced to an independent 2D subsystem of the general 18D dynamics if the data concerning bond price dynamics are considered as given for the time being. In that case the dynamics of \( s \) and \( \pi_{ss} \) read

\[
\dot{s} = \frac{\beta_s}{1 - \beta_s(1 - \alpha_s)}[r_l^* + \alpha_s \pi_{ss} - (r_l + \pi_l)],
\]

\[
\dot{\pi}_{ss} = \beta_{\pi ss}(\hat{s} - \pi_{ss}).
\]

To study the resulting dynamics in isolation we thus assume that the other asset market situation are frozen at their steady state values which fixes the expression \((r_l + \pi_l)\) involved in the above equations to \( r^* \). From this we thus derive as law of motion for the change in exchange rate expectations of the less ambitious agents

\[
\dot{\pi}_{ss} = \beta_{\pi ss}(\frac{\beta_{ss} \alpha_s}{1 - \beta_{ss}(1 - \alpha_s)} - 1)\pi_{ss},
\]

which clearly provides (trivial) monotonically explosive dynamics if the parameters in the fraction are chosen such that it becomes larger than one. Increasing the parameter \( \beta_{\pi ss} \) beyond any bound then makes this process as explosive as desired and thus will significantly contribute to local instability of the full 18D dynamics. Compared to the isolated dynamics for long-term bonds we therefore here find a particularly simple representation of the centrifugal forces that surround asset market dynamics.

Increasing the parameters \( \beta_{ss} \) for exchange rate flexibility increases the positive influence of the expected exchange rate changes \( \pi_{ss} \) on the actual rate of change of the exchange range without any bound. For positive \( \alpha_{ss} \) we get in this way a positive feedback of exchange rate expectations on their time rate of change which becomes the more destabilising the faster these expectations are adjusted.

In order to stabilize such a market the Central Bank should attempt to control the long-term rate of interest on domestic government bonds as well as its rate of growth through appropriate means (demanding more then just a control of the short-term rate of interest).

\[
\dot{s} = \frac{\beta_s}{1 - \beta_s(1 - \alpha_s)}[r_l^* + \alpha_s \pi_{ss} - (c_1 r_l(s - s_0) - c_2 \hat{r}_l(s - s_o))],
\]

\[
\dot{\pi}_{ss} = \beta_{\pi ss}(\hat{s} - \pi_{ss}).
\]

In the here considered situation the condition \( 1 - \alpha_s < c_1 + c_2 \) would then be sufficient for local asymptotic stability if the functions \( r_l, \hat{r}_l \) are chosen in appropriate ways.

4.7 Foreign country data

Module 7. finally provides the data needed from the 'foreign' economy in the simplest form possible. It is assumed that the modeling of the foreign economy is based on the same qualitative principles we used for the description of the domestic economy, but that this economy is fixed in its steady
state position.

7. ROW Data (exogenous):

\[ r_i^* = r^* \text{ interest rate, } \quad \pi_l^* = 1/r_i^*, \quad \hat{p}_l^* = 0 \quad (4.66) \]
\[ \pi^* = \text{ inflation rate} \quad (4.67) \]
\[ p_y^* = \text{ price level of the export good } \quad \hat{p}_y^* = \pi^* \quad (4.68) \]
\[ p_m^* = \text{ price level of the import good } \quad \hat{p}_m^* = \pi^* \quad (4.69) \]

This closes the description of the extensive or structural form of the model of our small open economy.

5 The implied intensive form of the model

In order to study the dynamics of our stylized disequilibrium growth model analytically and numerically to intensive or per (value) unit of capital form. To simplify subsequent presentations of the dynamics of the model and also its steady state solution somewhat, we assume in the remainder of this paper for the consumption of asset owners \( C_c(0) = 0 \). Moreover, we set \( \delta = \delta_f = \delta_h = \delta_g \).

These two assumptions do not restrict the dynamical behavior of the system in any important way. We will also use the abbreviations \( \bar{q} w = q^*(1 - \tau_w - \tau_{wp}) w, \quad \bar{q}^w = q^u(1 - \tau_w - \tau_{wp}) w \) in the following intensive form of the model.

The laws of motion

The Quantity Dynamics of Firms:

\[ \ddot{y} = \beta_ey' (y' - y) + \bar{n} - (g_k - \delta) \quad (5.1) \]
\[ \dot{\nu} = y - y' - (g_k - \delta) \nu \quad (5.2) \]
\[ \ddot{u}_f = \beta_{uf} (u_f^w - \bar{u}_f^w) + \bar{n} - (g_k - \delta), \quad u_f^w = \nu_{f}^w / \nu_f^w \quad (5.3) \]

The Dynamics of the Industrial Wage Share \( u_f = \frac{wL_f}{pY_f} \) and of the Rate of Inflation:

\[ \dot{v}_f = \frac{(1 - \kappa_p)[\beta_{ew}(\frac{\nu}{T} - \bar{e}) + \beta_{uw}(u_f^w - \bar{u}_f^w) + \beta_{hw}(\frac{q_h}{q_o} - 1)] - (1 - \kappa_p)[\beta_p(u - \bar{u}) + j_o \hat{s}]}{1 - \kappa_p \kappa_w} \quad (5.4) \]
\[ \dot{p} = \frac{\beta_p(u - \bar{u}) + j_o \hat{s} + \kappa_p[\beta_{ew}(\frac{\nu}{T} - \bar{e}) + \beta_{uw}(u_f^w - \bar{u}_f^w) + \beta_{hw}(\frac{q_h}{q_o} - 1)]}{1 - \kappa_p \kappa_w} + \pi^c \]
\[ \ddot{\pi}^e = \beta_{\pi^e}(\alpha_{\pi^e}(\dot{p} - \pi^e) + (1 - \alpha_{\pi^e})(\pi^* - \pi^e)) \quad (5.5) \]
\[ \ddot{q}_h = \beta_h(\frac{\alpha_{wh}}{\alpha_h k_h} - \bar{u}_h) + \pi^c, \quad i.e. \]
\[ \dot{q}_h = \beta_h(\frac{\alpha_{wh}}{\alpha_h k_h} - \bar{u}_h) - \beta_p(u - \bar{u}) - j_o \hat{s} - \kappa_p[\beta_{ew}(\frac{\nu}{T} - \bar{e}) + \beta_{uw}(u_f^w - \bar{u}_f^w) + \beta_{hw}(\frac{q_h}{q_o} - 1)] \quad (5.6) \]
Asset Prices and Medium-run Expectations:
\[
\hat{p}_t = \frac{\beta_{p_t}}{1 - \beta_{p_t}(1 - \alpha_s)}[r_t + \alpha_s\pi_{ls} - \hat{r}_t] = -\hat{r}_t \tag{5.7}
\]
\[
\pi_{ls} = \beta_{\pi_{ls}}(\hat{p}_t - \pi_{ls}) \tag{5.8}
\]
\[
\hat{s} = \frac{\beta_{ss}}{1 - \beta_{ss}(1 - \alpha_s)}[r_t^* + \pi_t^* + \alpha_s\pi_{ss} - (r_t + \pi_t)], \text{i.e.} \tag{5.9}
\]
\[
\hat{\sigma} = \frac{\beta_{ss}}{1 - \beta_{ss}(1 - \alpha_s)}[r_t^* + \pi_t^* + \alpha_s\pi_{ss} - (r_t + \pi_t)] + \pi^* - \hat{p} \tag{5.10}
\]
\[
\pi_{ss} = \beta_{\pi_{ss}}(\hat{s} - \pi_{ss}), \quad \pi_l = \alpha_s\pi_{ls} + (1 - \alpha_s)\pi_{lc}, \quad \pi_{lc} = \hat{p}_l
\]

Growth Dynamics:
\[
\hat{l} = \bar{n} - (g_k - \delta) \tag{5.11}
\]
\[
\hat{k}_h = g_h - \delta - (g_k - \delta) \tag{5.12}
\]
\[
\hat{k}_g = \alpha_g g_y(d)\bar{u}y^p - g_k \hat{k}_g, \quad \text{if } d \in [d, \overline{d}], \quad \alpha_g = 0/\mu \alpha_g \quad \text{otherwise} \tag{5.13}
\]

Monetary and Fiscal Policy Rules:
\[
\dot{r} = \beta_{\tau_f}(r_t - r^*) + \beta_{\tau_m}(\tau_m - \bar{\tau}_m) + \beta_{pr}(\hat{p} - \pi^*) \tag{5.14}
\]
\[
\dot{\tau}_{fp} = \beta_{d:fp}(\frac{b + p_t b^f}{g^p} - d_o) - \beta_{\tau_{fp}}(\tau_{fp} - \tau_{fp0}), \quad b = \frac{B}{pK}, b^f = \frac{B^f}{pK} \tag{5.15}
\]
\[
s_g = t - g - (rg + b^f), \quad s_{go} = t_o - g_o - (r^*b_o + b_{o}^f), \quad g_o = g_y(d_o)\bar{u}y^p
\]
\[
t = (1 - q^n)v_{fy} + \tau_{fp}v_{fy} + \tau_{v}(\rho_{c} + c + g) - \tau_{x}q^x x + \tau_{m}g_{m}y^d + \tau_{c}[\rho_{eg} + \rho_{h}k_h] - [(\frac{1 - \bar{e}}{\bar{e}} + \bar{q}^{\alpha_x}1)g + g_y(d)\bar{u}y^p]\]
\[
g = g_s - g_p(x), (1 - \alpha_g)g_s - g_p(x), \mu g_s - g_p(x), g_s = g_y(d)\bar{u}y^p
\]
\[
\tau_x = 1 - \frac{(1 + \tau_{m})p_{m}(0)}{p_{m}(0)\sigma} < 0
\]
\[
\hat{\tau}_m = \beta_{\alpha\tau_m}x_g(x_y - \sigma j_y(\sigma)) \tag{5.16}
\]

Government Bonds = Government Debt Accumulation Dynamics \( (b = \frac{B}{K}, b^f = \frac{B^f}{K}, d = (b + p_t b^f)/y^p) \):
\[
b = -\alpha_{fp} g_y - (\hat{p} + g_k - \delta)b \tag{5.17}
\]
\[
b^f = -\alpha_{fp} g_y - (\hat{p} + g_k - \delta)b^f \tag{5.18}
\]
Supplementing Static Relationships for the Laws of Motion of the Baseline Model:

\[ y^d = c^d_{wc} + g_k + g_hk_h + g + x \]

\[ y^D_{ww} = (q^n l^w/l_y + \tilde{q}^w (l/l_y - l^w/l_y) + \tilde{q}^x l/l_y) v_f, \quad l^w/l_y = (1 + g_y(d)) y, \quad l = l^w/e, \quad v_f = \frac{w^D_f}{p} = l^w w/p \]

\[ y = y^f + \beta_n (\alpha_{w} (y^f - x) - \nu) + \tilde{n} \alpha_{w} (y^f - x) \]

\[ v_w = y^D_{ww}/y \quad [\neq v_f] \quad \text{the total wage share (net of interest)} \]

\[ x = x y^e \]

\[ f^d = j_y(\sigma) y^e \]

\[ c^d_{wc} = c_{wc}(q_h) y^D_{ww} \]

\[ c^d_{wh} = c_{wh}(q_h) y^D_{ww} \]

\[ \rho^e = (y^e - x)/(1 + \tau_d) + q^e x - (1 + \tau_m) v_f y - q_m f^d - \delta \]

\[ \rho_h = q_h c^d_{wh}/k_h - \delta \]

\[ q^x = \frac{sp^*_m}{p} = \frac{p^*_m(0) \sigma}{p^*_m(0) (1 + \tau_d)(1 + \tau_m)} \]

\[ q^m = \frac{sp^*_m}{p} = \frac{\sigma}{(1 + \tau_d)(1 + \tau_m)} \]

\[ d = b + p b^d \]

\[ g_k = \alpha_{pk} (\rho^e - (r^d - \pi^e)) - \alpha_{rk} (r^d - r^d) + \alpha_{uk} (u - \tilde{u}) + \tilde{n} + \delta \]

\[ g_h = \alpha_{ph} (\rho_h - (r^d - \pi^e)) - \alpha_{rh} (r^d - r^d) + \alpha_{wh} (\frac{c^d_{wh}}{\alpha_{wh} k_h} - \tilde{u}_h) + \tilde{n} + \delta \]

6 Steady State Analysis

In this section we show that there is a uniquely determined and, up to the level of nominal variables, economically meaningful balanced growth path or steady state solution of our model which provides us with a useful reference path for the dynamical evolutions implied by the model, which may or may not converge to this steady state solution.

The calculation of this interior, economically meaningful, steady state of the full model is in many respects simple due to the given growth rate of the world economy and the given foreign interest rate (on consols). Note that we only consider expressions for the total supply of domestic bonds in the following, and not their distribution at home and abroad, which however can be easily obtained from the savings decisions of workers and pure asset owners.

The first set of steady state conditions presented below concerns the growth rates of our small open economy:

\[ g_{ko} = g^d_{ko} = \tilde{n} + \delta \quad (6.1) \]

\[ g_{ho} = g^d_{ho} = \tilde{n} + \delta \quad (6.2) \]

These equations state that capital (and thus also output) will grow with the rate \( \tilde{n}(k_{go}) \).

The next set of steady state conditions concerns inflation and expected inflation –for all prices and capital gains that exist in our model which are all equalized, except for wage rates. This also holds for the various rates of interest and the profit rates of our model. Note again that all starred variables (of the foreign economy) are given for the study of this small open economy and that only
state variables of the model are numbered in the following set of steady state equations:

\[
\begin{align*}
\pi^* &= \pi^c_o = \hat{p}_o = \hat{p}_{yo} = \hat{p}_{ho} \\
\tau_o &= \tau_{lo} = 1/p_{lo} \\
&= r^* \\
\pi_{ls} &= 0 \\
\pi_{sa} &= 0 \\
r^* &= \rho^o g + \pi^* = \rho_{ho} + \pi^* \\
\end{align*}
\]

(6.3) (6.4) (6.5) (6.6) (6.7) (6.8)

The next block concerns the steady state determination of various quantities and the steady state ratio of government debt to aggregate demand:

\[
\begin{align*}
k_{go} &= \text{from } \alpha g_y(d)\bar{uy}_o = (\bar{\bar{n}} + \delta)k_{go}, \text{ i.e.:} \\
k_{go} &= \frac{\alpha g_y(d)\bar{uy}_o}{\bar{\bar{n}} + \delta} \\
y_o &= \bar{uy}_o \\
l^h_{fo} &= l^w_{fo} = y_o y \\
l^w_{go} &= g_y(d)y_o y \\
l^w_o &= l^w_{fo} + l^w_{go} = (1 + g_y(d_o))y_o y \\
l_o &= l^w_o/e_o \quad [e_o = \bar{e}(k_{go})] \\
\end{align*}
\]


Further steady state relationships on the side of quantities are:

\[
\begin{align*}
y^g_o &= \frac{y_o}{1 + \bar{n} \alpha o(1 - x_y)} \\
\nu_o &= \alpha o(1 - x_y)y^g_o \\
q_h &= \frac{r^* + \delta}{\alpha h \bar{u}_h} \text{ from } \rho_h = \frac{q_h e^d_{wh}}{k_h} - \delta = r^*, \quad c^d_{wh} = \bar{u}_h \alpha_h k_h \\
y^d_o &= c^o_{wc} y^D_{wwo} + \bar{n} + \delta + (\bar{n} + \delta)\frac{c^o_{wh} y^D_{wwo}}{\bar{u}_h \alpha_h} + g_y y_o + x_y y^e_o = y^e_o \rightarrow \\
y^D_{wwo} &= \frac{(1 - x_y)y^g_o - (\bar{n} + \delta + g_y y_o)}{c^o_{wc} + (\bar{n} + \delta)\frac{c^o_{wh}}{\bar{u}_h \alpha_h}} \in (0, y^e) \\
v^D_{wwo} &= \left[\tilde{q}^y \frac{1 - e_o}{e_o} + \bar{q}^y \alpha^r \frac{1}{e_o} \right] (1 + g_y y_o) + \tilde{q}^y (1 + g_y y_o) v_{fo} \rightarrow \\
v^D_{wwo} &= \frac{y^D_{wwo}}{(1 + \tau_v) v_{fo} y_o - \delta, \text{ see (6.26) for the balanced trade condition}} \\
\rightarrow (1 + \tau_{fo}) v_{fo} y_o = y^g_o (1 - x_y)/(1 + \tau_v) - \delta - r^* \\
k_{ho} &= \frac{c^o_{wh} y^D_{wwo}}{(\bar{u}_h \alpha_h)} \\
\end{align*}
\]

And for the aggregate of government bonds we finally get:

\[
g_o - t_o = (\check{\bar{n}} + \check{p}_o - r^*)(b_o + b_o/y_o^c), \quad \text{i.e.} \quad r^* < \check{\bar{n}} + \check{p}_o \rightarrow d_o > 0 \text{ if } g_o > t_o \quad (6.22)
\]
\[
d_o = \frac{b_o + \check{p}_o b_o^c}{y_o^c} = \frac{(g_o - t_o)/y_o^c}{\check{\bar{n}} + \check{p}_o - r^*} \quad \text{i.e.} \quad r^* < \check{\bar{n}} + \check{p}_o \rightarrow d_o > 0 \text{ if } g_o > t_o
\]
\[
t_o - g_o = y_o^c(1 - x_y)/(1 + \tau_v) - \delta - r^* + \tau_v(c_{wc}(qho)y_{wwo}^d + g_o) + \check{\tau}_m q_{mo} j_o^d - \tau_{xo} q_{xo} x_o
\]
\[
+ \tau_v r^*(1 + k_{ho}) - (1 + g_{yo}) y_o^c \left[ \check{q}^u \frac{1 - e_o}{e_o} + \check{q}^o \frac{1 - e_o}{e_o} + q^n v f_o - g_o \right], \quad \text{i.e.}
\]
\[
g_o - t_o = g_{yo} \check{y}_{yo}^p + \frac{(1 - x_y) y_o^c - (\check{\bar{n}} + \delta + g_{yo} y_o^c)}{e_{wc} + (\check{\bar{n}} + \delta) \frac{\check{\tau}_{mo}}{\tau_{mo}}} + \delta + r^* + \tau_{xo} q_{xo} x_o
\]
\[
- y_o^c(1 - x_y)/(1 + \tau_v) - \check{\tau}_m q_{mo} j_o^d - \tau_v(c_{wc} y_{wwo}^d + g_o) - \tau_v r^*(1 + k_{ho}) \quad (6.23)
\]

from which the individual distribution of bonds can be derived if desired (between workers, capitalists and the foreign economy). To avoid the possibility of speculative attacks on the country in view of its deficit, the above restriction on it seems to be reasonable.

Next, one can determine the following rates and ratios from setting their time rate of change equal to zero. Note however that all nominal domestic values are indetermined in our model, due to the interest rate policy of the central bank.

\[
\tau_{mo} = \check{\tau}_m \quad (6.24)
\]
\[
q_{mo} = \frac{\sigma_o}{(1 + \tau_v)(1 + \tau_{mo})} \quad (6.25)
\]
\[
q_{xo} = \frac{p_o^*(0)}{(1 + \tau_v)(1 + \tau_{mo})} \quad (6.26)
\]
\[
\sigma_o = \text{ from } \tau_m = 0 : x_y = \sigma_o j_o \quad (6.27)
\]
\[
\tau_{xo} = 1 - \frac{(1 + \tau_{mo}) p_o^*(0)}{p_m^*(0)} \quad (6.28)
\]

This concludes the calculation of the balanced growth reference position of the dynamics which are now to be simulated in detail.

7 Some basic simulations of the stability properties of the laws of motion (the case of an inflation-free steady state)

The collected laws of motion

Note that we now have only 16 laws of motion, since we decided that the central bank should act directly on the long end of the bond markets (on \( p_l \)) and should do this in view of government debt and balanced trade. Concerning inflation targeting, a central bank can control at best the average inflation rate and can do this in any case only through its actions on financial markets. Adding an inflation targeting portion to the law of motion for \( r_l = 1/p_l \) would of course not be a big issue in the present form of the model and its general implications.

Note also that we do not yet pay much attention to factual parameter sizes (up to the application to crude rules of thumb). When experimenting with the stability model we had to learn that empirical restrictions should be build into the model in a stepwise fashion in order to see how this will be improve stability along this way. A more important task for the moment is to get reasonable sizes for the balanced growth path, where we here neglect inflation still. Due to the many parameters of the model and more than 16 important steady state values one has to adjust the parameters of the model for quite a while. This is so to speak the most basic calibration exercise one can think of, see the example provided after the second set of parameter values of this section.
\[
\dot{y}^e = \beta_y (y^d/y^e - 1) + \tilde{n} - (g_k - \delta) \tag{7.1}
\]
\[
\dot{v} = y - y^d - (g_k - \delta) \nu \tag{7.2}
\]
\[
\dot{l}_f^w = \beta_{lf} (\frac{p^l}{l_f^w} - \bar{u}_f^w) + \tilde{n} - (g_k - \delta) \tag{7.3}
\]
\[
\dot{\nu} = (1 - \kappa_p) [\beta_{ew}(\frac{l^w}{\nu} - \bar{e}) + \beta_{uw}(\frac{p^l}{l^w} - \bar{u}^w) + \beta_{hw}(\frac{q^h}{q^w} - 1)] - (1 - \kappa_w) [\beta_p (\frac{q^p}{q^w} - \bar{u}) + j_o \delta] \tag{7.4}
\]
\[
\hat{q}_h = \beta_h (c^d_{wh} - \bar{u}_h) - (\hat{p} - \pi^e) \tag{7.5}
\]
\[
\tilde{\pi}^c = \beta_{x^c}(\hat{p} - \pi^e) - (1 - \alpha_{x^c}) \pi^e \tag{7.6}
\]
\[
\hat{\sigma} = \beta_{ss} \left[ \frac{r^* + \alpha_s \pi_{ss} - (r_l - \hat{r}_l)}{1 - \beta_{ss}(1 - \alpha_s)} \right] - \hat{p} \tag{7.7}
\]
\[
\hat{\pi}_{ss} = \beta_{\pi_{ss}} (\hat{s} - \pi_{ss}) \tag{7.8}
\]
\[
\hat{l} = \tilde{n} - (g_k - \delta) \tag{7.9}
\]
\[
\hat{k}_h = g_h - \delta - (g_k - \delta) \tag{7.10}
\]
\[
\hat{k}_g = \alpha_g \tilde{u}_g - (g_k - \delta) \tag{7.11}
\]
\[
\hat{r}_l = \beta_{dr_l} (\frac{b + p_{bl}^l}{y^e} - d_o) + \beta_{\tau_{rl}} (\tau_m - \tilde{\tau}_m) - \beta + r \tau (r_l - r^*), \quad r = r^* \tag{7.12}
\]
\[
\dot{\tau}_{fp} = \beta_{d \tau_{fp}} (\frac{b + p_{bl}^l}{y^e} - d_o) - \beta_{\tau_{fp}} (\tau_{fp} - \tau_{fpo}) \tag{7.13}
\]
\[
\hat{\tau}_m = \beta_{nx \tau_m} y^e (x_y - \sigma_j y (\sigma)) \tag{7.14}
\]
\[
\hat{b} = -\alpha_{fp} s_g - (g_k - \delta) b \tag{7.15}
\]
\[
\hat{b}^l = \frac{(1 - \alpha_{fp}) s_g}{p_l} - (g_k - \delta) \bar{b}^l \tag{7.16}
\]
Supplementing relationships

\begin{align*}
\dot{p} &= \beta_p \left( \frac{D_p}{y_p} - \bar{u} \right) + j_o s + \kappa_p [\beta_{cw} \left( \frac{D_w}{r} - \bar{e} \right) + \beta_{uw} \left( \frac{D_w}{r} - \bar{u} \right) + \beta_{hw} \left( \frac{D_w}{q_o} - 1 \right)] + \pi^c \\
\dot{s} &= \frac{\beta_{ss}}{1 - \beta_{ss}(1 - \alpha_s) \pi_{ss} - (r_l - \hat{r}_l)}, \quad r_l = 1/p_t, \quad r = r^* \\
y^{D}_{ww} &= (q^w \nu^w / l_y + q^a (l/l_y - (l^w/l_y)) + q^d \alpha^d (l/l_y))v_f, \quad l^w = l^f + l_y g_y(d) \bar{u}y^p \\
t &= (1 - q^a) v_f y + \tau_{fp} v_f y + \tau_{e} (c_{uc} + c_c + g) - \tau x q_x x + \tau_m q_m j^d - \tau_c [\rho^{cg} + \rho h_k h] \\
\rho h &= q_h \frac{c_{wh}}{k_h - \delta} \\
g_k &= \alpha_x (\rho^{cg} - (r^d - \pi^c)) + \alpha_r (r^d - r^*) + \alpha_u (\frac{y}{y_p} - \bar{u}) + \bar{n} + \delta \\
g_h &= \alpha_{ph} (\rho h - (r^d - \pi^c)) - \alpha_{rh} (r^d - r^*) + \alpha_{uh} (\frac{c_{wh}}{k_h} - \bar{u}h) + \bar{n} + \delta \\
y^d &= c_{uc} + g_k + g_h k_h + g + x \\
y &= y^e + \beta_{n} (\alpha_{n} (y^e - x) - \nu) + \bar{n} \alpha_{n} (y^e - x) \\
l^w &= l^f + l_y g_y(d) \bar{u}y^p \\
l^d_f &= l_y y \neq l^f, \quad \text{outside the steady state} \\
\tau_x &= 1 - \frac{(1 + \tau_m) p_m^s(0)}{p^*_x(0) \sigma} \\
x &= x y y^e \\
j^d &= j(y^e) y^e \\
q^x &= \frac{sp^*_x}{p} = \frac{p^*_x(0) \sigma}{p_m^s(0)(1 + \tau_c)(1 + \tau_m)} \\
q^m &= \frac{sp^*_m}{p} = \frac{\sigma}{(1 + \tau_c)(1 + \tau_m)} \\
y^e_o &= \frac{y}{1 + \bar{n} \alpha_{n} e (1 - x_y)} + \bar{n} \alpha_{n} e, \quad q_o = \frac{r^* + \delta}{\alpha h \bar{u} h}, \\
\tau_{fpo} &= \frac{y^f_o (1 - x_y) / (1 + \tau_e) - v_f o y_o - \delta - r^*}{v_f o y_o} \\
d_o &= \frac{g_o - \bar{t}_o}{\bar{n} - r^*} \\
y^p &= y^p (k_g), \quad \bar{e} = \bar{e}(k_g) \\
\beta_{cw} = \beta_{cw} (k_g), \quad \beta_{uw} = \beta_{uw} (k_g), \quad g_y (d) = g_y (d) - g_1 (d - d_o)
\end{align*}

/* PARAMETERS */
betqw = 0.5; betp = 0.3; betpic = 0.3;
bets = 0.2; betpiss=0.5;
betye = 15; betn = 0.1; alpnd = 0.05;
betq = .3; betlf = 0.5; bartau = 0.1;
betd = 1; betdd = 0.1; bettau = 0.2;
betdrl = 1.3; betrnl = 1; bettaumrl = 0.2;

alpp = 0; alppic = 0.5; alppiss = 0.3; alpr = 0.2;
alpuk = 0.5; alprhok = 0.73; alprk = 0.5; del = 0.05;
alpuh = 0.3; alprhoh = 0.2; alprh = 0.5; alph = 1;

kapp = 0.5; kapw = 0.5; ly = 1.5;
xyexport = 0.3; j0 = 0.3; j1 = 0.2; pfmpfx0 = 1;
baru = 1; baru = 0.95; baruw = 1; barn = 0.07;

tilqu = 0.55; tilqr = 0.45;
gp = 1; yp0 = 1; yp1 = 0; rf = 0.04;
gy0 = 0.3; alpg = 0.5; gy1 = 1;
tauc = 0.5; tauv = 0.2; tauw = 0.35;

betuw0 = 0.75; betuw1 = 0;
betew0 = 0.75; betew1 = 0;
bare1 = 0; bare0 = 0.96; /* to avoid supply bottlenecks */
cw0 = 0.6; cw1 = 0;
ch0 = 1; ch1 = 0;
Figure 2: Profit-led / labor-market led case close to explosiveness (which in fact starts soon below the sensitivity to profitability parameter value $\alpha = 0.73$ of the investment function, even before the unstable situation of a wage-led regime has been reached). Public debt and primary deficit top-left/right. Goodwin’s distributive cycle and the payroll-tax controlled debt $\tau - d$ cycle in the middle, and the $d - g, h - k$ cycles at the bottom. Note that the level of infra-structure exceeds the one of the capital stock, while the debt level is roughly 1/3 of it (primary deficit = 1/100). The Goodwin cycle is shown with unit labor costs on the horizontal and total employment $e$ on the vertical axis – with $(1 - e)L$ as the expression that inside workers fear to become a member of.
Figure 3: Profit-led / labor-market led case ($\alpha_{pk} = 0.8$), i.e., increased profit-led goods demand, after a 1% shock increase in the wage share (the unit wage costs) $v_f$ of firms at time $t=1$. Same choice of graphs as before.
Figure 4: "Pension to Net wages’ ratio changes \( \tilde{q} \rightarrow 1.02\tilde{q} \rightarrow 0.98\tilde{q} \) at times \( t=400 \) and \( t=600 \) (top-right: no change in the steady primary deficit \( t_{go} \), the series at the top top-right, top-left government debt). Same cycles as before, the graph in the middle on the right shows in its lower section the return to the steady state for \( \alpha_{pk} = 0.9 \), with firms’ employment rate now on the vertical axis of the distributive cycle bottom-left.
Figure 5: We further support the profit-led / labor-market led case by choosing $\alpha_{\rho_k} = 0.9$ and consider two "unemployment benefits to net wages" ratio changes $\tilde{q}^u \to 1.02\tilde{q}^u \to 0.98\tilde{q}^u$ at times $t=400$ and $t=600$. We consider the same cycles as before, with firms' employment rate now on the vertical axis of the distributive cycle. The time series show total public debt and the primary and total measure of the deficit of the government.
Figure 6: Government expenditure changes $g \rightarrow 1.02g \rightarrow 0.98g$ at time $t=40$ and $t=60$ ($\alpha_{ok} = 0.9$). Endpoint $TH=80$. All considered implications are expansionary in the first case, but are – for reasons of comparison – confronted with the opposite government action at time $t=60$. 
Figure 7: Profit-led / labor-market led: Shocks $\bar{n} \to 1.02\bar{n} \to 0.98\bar{n}$ at time $t=400$ and $t=600$ $\alpha_{pk} = 0.9$ through changes in migration policy (debt top-left, deficits top right). Goodwin cycle, moves up and left by the first shock, while the opposite occurs with the infra-structure cycle. Deficits react now on an average, since they depend on $n$. 
Figure 8: The given value $\alpha^r = 0.3$ now shrinks between 40 and 60 by assuming there that $\tilde{\alpha}^r = -0.02$ holds until $\tilde{\alpha}^r = 0$ again from 60 onwards, due to a changed migration policy of the government, or other policy measures, concerning worker households (of age between 18 – 65), see the shown "parameter-diagram" for $\alpha^r$ top-right, with a better primary deficit top-left now. Lower unit wage costs and higher employment rate of firms, slow return to the old infra-structure ratio with smaller payroll contributions of firms and cheaper, but smaller housing to capital stock ratio are some of the consequences.
8 Policy experiments

We start with the observation that the parameter $l_y$, relating firm’s production and government’s consumption with the corresponding employment can differ between the two agents, since government can choose less ‘cost-oriented’ magnitudes in the provision of public services, for example in hospitals, due to the care it has to take for its citizens in a non-profit oriented environment. In the following simulations of the model it is important to keep in mind that total workforce employment (per unit of capital) is composed of a dynamically endogenous an a statically endogenous variable, while employment in hours is given by $l_j + l_y g_y(d)u_y^p = (1 + g_y(d))l_y u_y^p$, and thus statically endogenously determined throughout through the three expression $l_y, g_y, y$.

In the steady state we moreover have $lwstst = (1 + g_y0)ly * baru * yp0$ and on this basis $lstst = lwfstdbart0, lstst = alpr * lstst, lustst = lstst - lwstst$ for the workforce, the pensioners and the unemployed in terms of the notation of the following program listing. These are then all given magnitudes as long as we do not vary the parameters $g_y0, yp0, alpr$ for example. Using a measure like the level of pensions $ywwr$ per unit of capital in subsequent simulations is therefore also providing information on how pensions per head $ywwr/(\alpha r l)$ will be behaving in the longer run – as long as the parameters in the numerator (behind $l$) are not changed.

Program listing (the plotted shares are defined just before the output and graphics section of the listing, see below)

```plaintext
/* OPEN ECONOMY CASE */
new; library pgraph;
pl=10; j=10; TH=100; /*plots every pl iteration, j*pl iterations */
hstep=1/(j*pl); /* per time unit for a time horizon of TH */ /* shocks at time t=1 */
vfshock=1; qrshock=1; tilqr1shock=1; /* vf, tilqr, tilqr1 multiplicative shocks */
xz=ones(TH*j,18); /* initializing the output matrices xx,y0 */
y1=zeros(TH*j,9);
y2=zeros(TH*j,9);
y0=zeros(TH*j,18);
/* PARAMETERS */
betqw = 0.5; betp = .2; betpic = 0.3; /* Adjustment Speeds */
bets = 0.2; betpiss=0.5;
betye = 15; betn = 0; alpnd = 0;
betq = .3; betlf = .5; bartaul=0.1;
betd=1; betdd=.1; bettau = 0.2;
betrr= 1.30; betrrl = 1.00; bettauarl=.20;
alpfp = 0; alppic = 0.5; alppiss=.3; /* Proportions */
alph=1;
alpha = 0.3;
alphu = 0.5; alphrhok = .9; alphrhok=0.5; del = 0.05; /* Behavior */
alphu=0.3; alphrh=0.2; alphrh=0.5; alph=1;
kapp = 0.5; kapw = 0.5; ly=3/2;
xyexport = 0.3; j0=0.3;ji=0.2; pfmpfx0=1; /* Trade */
baruh = 1; baru = 1; baruw=1; /* Normal Utilization Rates */
tilqu = 0.55; tilqr=0.45; /* Policy Parameters */
yp0=1;yp1=0; rf =0.04; barn = 0.07; /* Includes Inflation and Productivity Growth */
gy0=0.3; alpg=.5; gy1=1; gp=4;
```

Program listing (the plotted shares are defined just before the output and graphics section of the listing, see below)
tauc=0.25; taur=0.2; tauw=0.35; /* including payroll taxes when of this size */

/* FUNCTIONS: */

betuw0 = 0.75; betuw1 = 0; /* Infrastructure and Debt Effects still excluded */
betew0 = 0.75; betew1 = 0;
bare0 = 0.92; bare1 = 0;
cw0 = 0.6; cw1 = 0;
ch0 = 1; ch1 = 0;

/* BALANCED GROWTH */

yestst = baru*yp0/(1+barn*alpnd*(1-xyexport));
climpistst = 0;
rlstst = rf;
pissstst = 0;
nustst = alpnd*(1-xyexport)*yestst;
kgstst = alpg*gy0*baru*yp0/(barn+del);
lwfstst = baru*yp0*ly;
lwstst = lwfstst + gy0*baru*yp0*ly;
lstst = lwstst/(alpw*bare0);
ywo = ((1-xyexport)*yestst-(barn+del+gy0*baru*yp0))/(cw0+(barn+del)*ch0/(baru*alph));
qhstst = (rf+del)/(alph*baru);
vfstst = ywo/(((tilqu+alpw+tilqr*alpr)/(alpw*bare0)+((1-tauw)-tilqu)*((1+gy0)*baru*yp0);

/* INITIALIZING VALUES */

yeold = yestst;
uold = nustst;
lwfold = lwfstst;
lold = lstst;
vfold = vfstst;
qhold = qhstst;
khold = khstst;
kgold = kgstst;
bold = bstst;
blold = blstst;
climpiold = climpistst;
siggiold = siggistst;
rlold = rlstst;
taumold = taumstst;
taupold = taufpstst;
pissold = pissstst;
alprold=alpr;

/* THE FOLLOWING CODE ITERATES */
t=0; it=0; tt=0; /* it = ITERATION STEPS*/
do while t < TH - (1/j)/2; /* TIME = 1,2,... < TH */
if it==10*j*pl; vfold=vfshock*vfold;endif;
if it==40*j*pl; tilqr=qrshock*tilqr; endif;
if it==60*j*pl; tilqr=qrshock*tilqr; endif;

/* FUNCTIONS: */
betewfkgbetew0-betew1*(kgold-kgstst);
betuwfkgbetuw0-betuwx1*(kgold-kgstst);
barekg=bare0+bare1*(kgold-kgstst);
ypfkgyf0-yp1*(kgold-kgstst);
gyfd=gy0-gyx1*((bold+blold/rlold)/yeold-dstst);
cwcfqhcw0+cw1*(qhold-qhstst);
cwhfqhcw0+cw1*(qhold-qhstst);
jd=(j0+j1*(siggiold-siggistst))*yeold;
qm=siggiold/(1+tauv)*(1+taumold));
qx=qm/pfmpfx0;

/* ALGEBRAIC EQUATIONS */
export=xyexport*yeold;
ysupply=yeold+betn*(alpnd*(yeold-export)-nuold)+barn*alpnd*(yeold-export);

lhf=ysupply*ly;
ywwdef=((1-tauw)-tilqu)*(lwfold/ly+gyfd*baru*ypfk)
+(tilqu*alpw*tilqr*alpr)*lold/ly)*vfold;
wcg=cwcfqh*ywwdef;
wch=cwhfqh*ywwdef;

rhoeg=yeold*(1-xyexport)/(1+tauv)-(1+taufpold)*vfold*ysupply
+qx*export-qm*jd-del;

ef=lwfold/(alpw*lold);

ew=ef+(gyfd*baru*ypfk*ly)/(alpw*lold);

hatph=betq*(wch/(alph*khold)-baru)+climpiold;

hats=bets/(1-bets*(1-alppiss))*
(rf+alppiss*pissold-
(rlold
-(betdrl*(((bold+blold/rlold)/yeold-dstst)
+bettaumrl*(taumold-bartaum)
-betrrl*(rlold-rf))
/(rlold)
)
)
);

hatp=(betp*(ysupply/ypfkg-baru)+j0*yestst*hats+kapp*(betewfkg* (ew-barekg)+betuwfkg*(lhf/lwfold-baruw) 
+betqw*(qhold/qhstst-1))/(1-kapp*kapw)+climpiold;
gov=gyfd*baru*ypfkg-gp*(yeold-yestst);
gk=alprhok*(rhoeg-(rlold-climpiold))-alprk*(rlold-rf) 
+alpu*(ysupply/ypfkg-baru) 
+barn+del;

gh=alprhoh*(qhold*wch/khold-del-(rlold-climpiold)) 
-alprh*(rlold-rf) 
+alpuh*(wch/(alph*khold)-baruh)+barn+del;

uwf=lhf/lwfold;

ut=ysupply/ypfkg;

taux=1-(1+taumold)*pfmpfx0/siggiold;

yd=wcg+gk+gh*khold+gov+export;

rhoh=qhold*wch/khold-del;

/* DIFFERENTIAL EQUATIONS */

yenew=yeold+hstep*yeold*(betye*(yd/yeold-1)+barn-(gk-del));
nunew=nuold+hstep*(ysupply-yd-(gk-del)*nuold);
lwfnew=lwfold+hstep*lwfold*(betlf*(lhf/lwfold-baruw)+barn-(gk-del));
lnew=lold+hstep*lold*(barn - (gk-del));

vfnew=vfold+hstep*vfold*(((1-kapp)*((1-kapp*(betewfkg*(ew-barekg) 
+betuwfkg*(lhf/lwfold-baruw)+betqw*(qhold/qhstst-1)) 
-(1-kapw)*(betp*(ysupply/ypfkg-baru)+j0*yestst*hats)/(1-kapp*kapw)));

climpinew = climpiold+hstep*betpic*(alppic*(hatp-climpiold) 
+(1-alppic)*(-climpiold));

qhnew=qhold+hstep*qhold*(betq*(wch/(alph*khold)-baruh) 
- hatp + climpiold);

rlnew=rlold; /* hstep*(betdrl*(((bold+blold/rlold)/yeold-dstst) 
+bettaumrl*(taumold-bartaum)-betrrl*(rlold-rf)); */
sigginew = siggiold + hstep*siggiold*(bets/(1-bets*(1-alppiss))*(rf+alppiss*pissold-
(rlold-(rlnew-rlold)/(hstep*rlold)))-hatp);
pissnew = pissold + hstep*betpiss*(hats-pissold);
khnew = khold + hstep*khold*((gh-del)-(gk-del));
kgnew = kgold + hstep*(alpg*gyfd*baru*ypfgk-gk*kgold);
taumnnew = taumold + hstep*taumold*betaum*(xyexport*yeold-siggiold*jd);
sngo = tgo - (rf*bstst+blstst);

tgt = (1-(1-tauw))*vfold*ysupply + taufpold*vfold*ysupply + tauv*(wchg+gov) - taux*qx*xyexport*yeold+taumold*qm*jd + tauc*(rhoeg+rhohe*khold) - (tilqu*vfold*(alpw*lold/ly-1wfold/ly-gyfd*baru*ypfgk) + tilqr*vfold*alpr*lold/ly+(1-tauw)*vfold*gyfd*baru*ypfgk) - gov;
sng = tgt - (rf*bold+blold);

bnew = bold + hstep*(-alpfp*rlold*sng - (hatp+gk-del)*bold);
blnew = blold + hstep*-(1-alpfp)*rlold*sng - (hatp+gk-del)*blold);
taufpnew = taufpold + hstep*(betd*((bold+blold/rlold)/yeold-dstst) - betdd*(taufpold-taufpoldst));

pled = cw0*(((1-tauw)-tilqu)*(lwfold/ly-1wfold/ly-gyfd*baru*ypfgk)+(tilqu+tilqr*alpr*lold/ly)*vfold - alprhok*(1+taufpold)*vfold*ysupply;

ywru = tilqu*(lold/ly-1wfold/ly-gyfd*baru*ypfgk)*vfold;
ywrr = tilqr*alpr*(lold/ly)*vfold;
ywbn = ((1-tauw)*(lwfold/ly)+(1-tauw)*gyfd*baru*ypfgk)*vfold;

tshare = (tauw*vfold*ysupply + tauv*vfold*gyfd*baru*ypfgk + taufpold*vfold*ysupply + tauv*(wchg+gov) - taux*qx*xyexport*yeold + taumold*qm*jd + tauc*(rhoeg+rhohe*khold+rf*bold+blold))/yeold;
pshare = (1-tauc)*(rhoeg+rhohe*khold+rf*bold+blold)/yeold;

share = (cw0*ywwdef+qhold*ch0*ywwdef)/yeold;
isshare = (gk+gh*khold)/yeold;
gshare = gy0*baru*yp0/yeold;
xshare = xyexport;
if it \% pl == 0; tt=tt+1; /* every pl’s iteration */
xx[tt,\ldots] = t*xx[tt,\ldots]; /* tt’th row = (t,t,t,t,t,t,t,t,t) */

/* and */
y1[tt,\ldots]=vfold~ywwn~ywwr~ywwu~tshare~pshare~ishare~ew~kgold;
y2[tt,\ldots]=taufpold~gshare~qhold~cshare~sng~tgt~pled~bold+blold/rlold;
y0[tt,\ldots]=y1[tt,\ldots]~y2[tt,\ldots];

/* OUTPUT TO SCREEN: */
"Time, ywwo" t~vfold~ywwn~ywwr~ywwu~cshare~tshare~pshare~ishare~kgold~taufpold
~qhold~yeold~ypfkg~tgt~pled~bold+blold/rf;
"","",""; t=t+1/j; endif;

/* UPDATE OF VARIABLES: */
yeold = yenew;
nuold = nunew;
lwfold = lwfnew;
lold = lnew;
vfold = vfnew;
climpiold = climpinew;
qhold = qhnew;
siggiold = sigginew;
pissold = pissnew;
khold = khnew;
kgold = kgnew;
taumold = taumnew;
bold = bnew;
blold = blnew;
taufpold = taufpnew;
it=it+1; endo;

/* FOR GRAPHICAL REPRESENTATIONS */
graphset; /* _pqgedit = 1; */
_paxht = .5; _pnumht = .2; _plegctl = {1,6}; _plegstr = "tilqr-shock";
begwind;
window(4,2,0);
setwind(1);
xlabel("Time");
ylabel("Pensions");
let v2=3; xxx=submat(xx,0,v2); yy=submat(y0,0,v2);
XY(xxx,yy);
nextwind;
xlabel("Time");
ylabel("U-benefits");
let v2= 4; xxx=submat(xx,0,v2); yy=submat(y0,0,v2);
XY(xxx,yy);
nextwind;
xlabel("Time");
ylabel("P-share");
let v2= 6; xxx=submat(xx,0,v2); yy=submat(y0,0,v2);
XY(xxx,yy);xlabel("Time");
nextwind;
ylabel("I-share");
let v2=7; xxx=submat(xx,0,v2); yy=submat(y0,0,v2);
XY(xxx,yy);
nextwind;
xlabel("Time");
ylabel("W-share");
let v2= 2; xxx=submat(xx,0,v2); yy=submat(y0,0,v2);
XY(xxx,yy);
nextwind;
xlabel("Time");
ylabel("P-Debt");
let v2= 18; xxx=submat(xx,0,v2); yy=submat(y0,0,v2);
XY(xxx,yy);
nextwind;
xlabel("U-W-Costs");
ylabel("E-rate");
XY(y0[.,1],y0[.,8]);
nextwind;
xlabel("Time");
ylabel("P-capital");
let v2= 9; xxx=submat(xx,0,v2); yy=submat(y0,0,v2);
XY(xxx,yy);
endwind;

Balanced Growth Ratios:
----------------------

<table>
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<tr>
<th>UW-costs</th>
<th>TotalNetWshare</th>
<th>Pensionshare</th>
<th>U-benefits share</th>
<th>cons-share</th>
<th>taxshare</th>
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<td>0.354</td>
<td>0.315</td>
<td>0.068</td>
<td>0.022</td>
<td>0.270</td>
<td>0.460</td>
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<th>profitshare</th>
<th>gross</th>
<th>invest.share</th>
<th>k_g</th>
<th>taufp</th>
<th>qh</th>
<th>k_h</th>
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<tbody>
<tr>
<td>0.071</td>
<td>0.167</td>
<td>1.250</td>
<td>0.393</td>
<td>0.090</td>
<td>0.389</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sng</th>
<th>tgt</th>
<th>profit-led</th>
<th>debt/capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.069</td>
<td>-0.023</td>
<td>-0.211</td>
<td>0.985</td>
</tr>
</tbody>
</table>
Figure 9: Convergence after a 1% Unit-Wage-Costs shock (with pension share top-left, unemployment benefits share top-right, next profit and investment share, the wage share – all measured relative to \( y^e = y = y^p \) here.) On the right follows public debt and public capital per unit of capital and bottom=left we finally show again the Goodwin distributive cycle in the case of damped oscillations, the only case we consider relevant for the topics addressed by the paper.
Figure 10: A 5% positive unemployment benefits ratio $q^u$ shock at $t=40$ and $t=60$. Note that the profit share includes all interest payments, also to workers (all net of taxes). Continuing the simulation run to a time horizon $TH=20000$ shows that the distributive cycle converges to a new steady state position below and to the right of the old one where the shock occurred at $t=1$. The steady state is therefore not uniquely determined from a global perspective. With only the $v_f$ shock however, the system would return to the old steady state, see the previous figure. Since UWCs increase, the employment rate suffers, but the total wage share (net of interest), the investment share and the infrastructure to private capital stock ratio improve.
Figure 11: A 5% $\tilde{q}$ shock at $t=40$ and $t=60$. The shocks are directly displayed top left in the pension ratio diagram. The overall outcome is similar to the previous shock. The effects on pensions (of course), investment and the overall wage share are more pronounced now. Overall it seems however advisable to look for indirect effects which improve the absolute position of the pensioners, since $\tilde{q}$ appears – as a negative influence – in the denominator of the unit-wage-costs steady state value, but not in the disposable wage income of all worker households, its numerator.
Figure 12: A 5% negative $\tau_w$ shock at $t=40$ and $t=60$ (a decrease in wage taxation, occurring isolated from unemployment benefits and pensions) and no initial unit wage costs increase). Looking from the purely theoretical view at the very long run, we see that the pensioners (also immediately) and the unemployed benefit from this increase in net wages (the latter clearly, when the old rate of employment has been reestablished), while the profit share is unaffected in the new steady state position. The share of investment planned by firms however increases as does the public capital stock, coupled with a lower debt to capital ratio, once fluctuations have settled down. Note that we have not yet included the positive effect of $k_g$ on the potential output of firms which would soon come into being in the present case (if included, see the next section of the paper). Speeding up convergence, and applying smaller shocks more often, may improve the situation further.
Figure 13: A 5% negative $\tau_v$ shock at $t=40$ and $t=60$. The unemployed and the pensioners benefit from the cut in value added taxes in the long-run, as does the total wage share, government debt and the public infrastructure. The shock shows up directly in the profit rate and thus in the profit share and the investment share, in the former with no long-run effects however. The steady state position in the distributive cycle shifts to the right and also somewhat down – even in the very long run – with no obvious explanation from the calculated steady state values, indicating again the existence of multiple steady state solutions from the global perspective. We see that partial reasoning can be misleading in a macro-dynamic framework with elaborated Keynesian feedback channels and various multiplier effects.
Figure 14: A 5% negative $\bar{n}$ shock at $t=40$ and $t=60$ provides a mixed impression concerning the implied macro-economic performance, definitely concerning the evolution of the overall rate of employment. Note that public debt must fall, since the steady state primary deficit to capital ratio is decreased directly through the decrease in the rate $n$, just as the rate of investment.
Figure 15: 5% contractive $g_{0}$ shock at $t=40$ and $t=60$. This case is of the type of a "Greek Tragedy" as far as the wage related incomes are concerned. The figures show that this policy is supporting the profit share, and it indeed reduces public debt (at a given moderate rate of interest), with very bad results concerning the public infrastructure however. The employment rate is increasing in the longer run along the way of the economy to a new steady state position where unit wages costs are much lower than they were beforehand.
Figure 16: The Keynesian case of a positive 5% government expenditure shock. Pensions, unemployment benefits and net wage income are now measured per head and not as share of $y^e$, since the increase in government expenditure increases the employed workforce in fact instantaneously, while it received no immediate shock beforehand when the unit wage costs of firms were increased at $t=1$ or when the different shocks at $t=40$ and $60$ were considered. We now also consider the net wage income per head and the total consumption and tax share (in place of the wage and the profit share and the debt to capital ratio which is increased) and get positive results of this shock – with the exception of the overall employment rate which suffers from the increase of the wage share of firms as the distributive cycle bottom-left shows.
Textbook Keynesian theory is teaching that an increase in the private propensity to save – instead of decreasing the rate of interest – can give rise to the so-called savings paradox, i.e., savings stays at its previous level, while income decreases in order to adjust increased desire to save to the given amount of investment of firms. In the present model we have however that \( y^{D}_{w0} \) is increasing (in order to clear the goods market) in the steady state, when the parameter \( cw0 \) is decreased so that there is some hope for a better outcome than in the simple textbook story. The question of the chosen time horizon becomes here a particularly crucial one, in particular regarding the consumption share.

So far we have always shown the same order of figures (with the exception of the \( gw0 \) increase and \( cw0 \) decrease, the latter by 2 %). We now consider the implications of migration into the
labor market, used to relax the too high pensioner proportion in the society and therefore make the portion of pensioners endogenous now.

Figure 18: A plus 1% growth rate $\bar{n}$ workforce migration shock at $t=20$ and $t=40$

The outcome is not really convincing, despite a decrease in the proportion $L^*/L$ from 30 to 25 percent. This situation will therefore be investigated anew in the next section. The present section is closed with a number of simulation which partly indicate already what we want to consider in more detail in the next section.
Figure 19: Decreasing the import taxation target in the FX-market policy of the Central Bank.
Figure 20: Increasing public expenditure and the public investment rate within this expenditure.
Figure 21: Increasing labor market flexibility and unemployment benefits.
Figure 22: Increasing the pension and benefit rate simultaneously.
9 Social protection: Productive capacity, income generation and public "infra-structure" investment

Let us first briefly provide an example that our model can generate Goodwin like cycles though aggregate demand is wage-led (here to the degree:0.0088349484). We first show the time series of debt and the deficit, than Goodwin’s distributive cycle and the payroll-debt cycle (where the former adjusts to the latter) and finally debt vs. the public capital stock and long-term interest vs. the real exchange rate.

Figure 23: A wage-led economy with a clockwise distributive cycle.

However it seems that this is always based on an increase in volatility when one moves from an initially profit-led situation step by step towards a wage-led one. Nevertheless the point where profit-led switches to wage-led by increasing the corresponding parameter in the investment function further is generally not a bifurcation point concerning the dampedness of the business fluctuations now need the behavior of the economy change immediately to a qualitatively new type. This is plausible, since the wage/profit lead distinction is based just on the aggregate demand function and thus not even on the many laws of motion of this fairly comprehensive macro-economic model.
Next we turn to the more stable profit-led case (with coefficient -0.081222412 in the per unit of capital expressed aggregate demand function). However we now find that in the case where potential output is made a positive function of the public capital stock that it – as expected – increases with it, see top-right below, but that as the same time the degree of its utilization falls, see the third figure to the left in the following sequence of figures, so that the generated income is not really increasing, in particular not actual wage income and pensions, see again the figures. This seems to be due to a trade-off effect between a higher employment rate $e$ and a lower capacity utilization rate $u$ in the long run, where the former is also coupled with a slightly lower level of unit wage costs in firms (the wage share they provide, when inverted, see bottom-left).

Figure 24: Increasing public expenditure and public investment (two 5% shocks in gy0 at t=20, 40).

We thus have to modify the model a bit in order to suppress this trade-off in order to restore the given steady state values for $ew$ and $ysupply/yp0$. This modification is shown in the following partial programm listing, see the previous section for the full version of it. The modified rule for the payroll taxes paid by firms now simply reads:

$$\dot{\tau}_{fp} = \beta_{\tau} \cdot \frac{y}{y^p} - \bar{u} - \beta_{fp} \cdot \tau_{fp} (\tau_{fp} - \tau_{fpo})$$

It states that payroll taxes are now changed along the business cycle in an anti-cyclical way, in addition to the anti-cyclical fiscal policy rule we have already assumed. On this basis, the parameters for the plots shown below are the following ones.
Figure 25: Stabilizing the rate of capacity utilization of firms and the total employment rate (after the initial wage costs shock and the 5% shocks in gy0). Pensions and the total wage share now clearly improve.

Top-right we can see that expected sales (which follow aggregate demand) are now also following potential output with some time delay, since we have assumed for simplicity that the normal rate of capacity utilization is given by the benchmark level 1.) To the left of this figure we see the time series for the relative price of rental services and below these two figures the evolution of pensions as well as actual wage income which are now both clearly increasing. Capacity utilization and the public debt per unit of capital are plotted next, the latter clearly rising in the considered situation as is the public infrastructure below it. There is finally the Goodwin distributive cycle bottom-left
which ends monotonically at a point where employment is slightly lower than in the steady state and the wage share equal to the unit wage costs of firms increased as compared to the start in the calculated steady state situation.

The outcome shown in the above plots is similar to the ones in the previous figures. In the utilization rate diagram, third on the left, the two positive shocks in the ratio $y_0$ at times $t=20$ and $40$ are clearly visible. And in the Goodwin distributive cycle diagram we now better see (besides the horizontal unit wage costs shock and the recession that follows it) how the rate of employment is recovering to the right of the shocked starting situation and unit wage costs are increasing along this way. Our general conclusion is that we need improvements on the supply side in order to get noticeable results for income distribution, private consumption and investment. The basic mechanism in our view is public investment in the public capital stock (which in a "one-good" macro-model can be nearly everything: traffic investment, schools, hospitals, residential homes for the elderly, etc.) and public services which increase the potential output-capital ratio by improving the environment in which firms are operating.

Figure 26: Activating the long-term interest rate policy of the central bank.
Figure 27: A stronger performance of the interest rate policy of the CB.

In figure 28 we increase the reaction of the CB to the deviation of the public debt to sales ratio of the government from its target level. This mitigates the amplitude of the shown business fluctuations top-right and thus adds further stability to the dynamics besides the one enforced by the fiscal policy rules for government expenditures (and the one for payroll taxes we have just introduced).
Figure 28: Increasing the taxes on wages, occurring in isolation from unemployment benefits and pensions), profits and the value added tax simultaneously.

In the plots 6 we have increased simultaneously value added taxes, wage taxation and asset income taxes each by 50%. We have chosen such strong increases in order to have fairly visible consequences, but see only quantitative changes, but no qualitative ones. But also the quantitative effects are fairly minor, up to the level of public debt itself, and in the case of pensions even positive. We mention here briefly that this level is measured by $qr^*alpr^*(lold/ly)*vfold$ in the program where old refers to the previous value in the numerical simulations so that the increase in the wage share of firms is the basis of the improvement of pensions (which are measured in terms of goods, since the number of retirees per unit of capital is multiplied with the value of labor productivity $1/ly$).
Figure 29: Increasing in addition unemployment benefits and pensions by ten percent simultaneously.

Similar observations concern the addition increase of the ratios of pensions and unemployment benefits to net wages, qr and qu. Pensions indeed rise, while net wages fall a bit. Due to the order of magnitudes here involved the stock of public debt does not change significantly here, though the assumed percentage between workers and pensioners is 30 percent.
We add a reallocation between infrastructure investment and other public expenditure in favor of the former by 1 percent. The implied change is enormous and by and large very positive for the economy, though the utilization rate of firms is somewhat lowered thereby, but not the long-term employment rate. However the economy goes through two long phases of depressed labor market situation (but only one on the market for goods). This suggests that policy measures must be carefully investigated due to their possibly very long negative consequences and counteracting measures may be needed in addition.

Figure 30: Reallocating public investment and consumption
As a final case study we add to the scenario of figure 28 a change in the speeds of adjustment which causes the system to switch to a frequency which is of business cycle type. The initial increase in unit wage costs has a very short expansionary demand effect, but causes thereafter a significant recession in the given profit-led environment with subsequent booms and recessions. In the second recession the expansionary $g_Y$ shock happens and immediately stops the recession, see the figure top-left below. The positive $g_Y$ shock thereafter brings the economy then clearly to a higher level of potential output.

Figure 31: Moving the system towards business cycle frequency.
10 Social Protection and Social Capital Formation. Supply-side Shocks through the Pensioners to Workforce Ratio and more

This section considers proposals for an endogenization of the proportion $\alpha_r$ between pensioners and the workforce, a.) by a flexible migration policy for a period of 48 years (which takes the "child production" of the new permanent residents – which become pensioners after three 16 years periods of work-life – into account), b.) by incrementally varying the retirement age over a time span of 20 years, c.) by a family policy which induces a baby boom for a time span of 48 years and d.) by incrementally varying the participation rate of the workforce over a time span of 20 years.

As point of departure we always assume as family structure the usual standardized one: two parents, two children and independently of that the pensioners with a given life span of 16 years, one third of the work-life of the standardized parents of the economy. This uniform population structure of 20 percent children, 60 percent potential workforce and 20 percent pensioners serves the purpose to simplify the programming routines for the above 4 types of supply side changes in the here considered open economy.

The following figures portrays this basic structure of the population on the basis of which production, income generation, consumption and investment is to be conducted.

![Figure 32: The population structure of the economy and the four possibilities to change the pensioner/worker ratio.](image)

We consider in all 4 situations the pure working of such supply side shocks first, then add a unit-wage-costs shock at $t=5$, in order to get business cycle movements, before the $\alpha_r$ shocks start at $t=20$ and finally consider Keynesian policy reactions to the implications of these supply side shocks, in addition to the Keynesian policies we always use in order to get damped oscillations from the laws of motion of the model.
Policy parameters:

\[ q_u = 0.7: \text{unemployment benefits (no taxation);} \]
\[ q_r = 0.6: \text{pensions (no taxation);} \]
\[ g_p = 2: \text{anti-cyclical Keynesian fiscal policy reaction parameter;} \]
\[ y_{p1} = 0.2: \text{reaction parameter of potential output ypfg to the size of the public capital stock kg;} \]
\[ r_f = .04: \text{interest rate peg;} \]
\[ g_y0 = 0.3: \text{public expenditure / output ratio;} \]
\[ a_{lp} = .5: \text{public investment / public expenditure ratio;} \]
\[ g_y1 = 1: \text{negative reaction of public expenditure to the Maastricht debt ratio;} \]
\[ t_{au} = .35: \text{tax rate for interest related income;} \]
\[ t_{au} = 0.2: \text{value added tax rate;} \]
\[ t_{au} = 0.2: \text{tax rate on wages (wage factor for net income calculations: qn=0.8).} \]

We consider the case of migration first. We always assume that the investment behavior of firms remains the given one for the considered supply side shocks so that we need not modify the initially given steady state from which the dynamics is always started (in order to see its balanced path for a while, until such a shock hits the economy).

In the first set of three figures (34 – 36) that follow, the top-left time series shows that the ratio "pensioners to workforce" (employed and unemployed) is decreased from 30 percent to about 26.6 percent through a controlled migration process (as it is for example typical for Australia). The 7 other diagrams on this page then show some implications of this migration policy of the government.

In the second set of diagrams, figure 35, we add the unit-wage-costs shock of 5 percent (at t=5), we have always been using in order to generate the (deterministic) business fluctuations of the model.

In order to avoid a recession in the thereby generated simulation runs, we assume in the third figure 36 that the government accompanies its migration policy by a policy which shifts the proportion of public investment in total public expenditures upwards by 10 percent.

The choice of the 7 implied diagrams nearly always remains the same. We see that the chosen investment policy of the government is improving the working of the macro-economy, with the exception of (part of) the period of the migration process itself.

Note that the first migrants (17-32 years of age) will become pensioners in the fourth 16 years period that follows their immigration which then leads to a partial increase in the ratio \( \alpha_r \) again (see figure top-left). Top-right we see the evolution of the potential output/capital ratio due to the ongoing public infrastructure investment. Below and on the left, we show pensions and unemployment benefits (dotted) per head, on the right the total work related income share (including pensions and benefits). And further below on the left the output-capital ratio \( y = Y/K \) (dotted) and the capacity utilization rate \( u = y/y'(k_g) \) of firms and on the right public debt per unit of capital. And in the last row, on the left the distributive cycle (or what is left of it) and to the right the public capital stock per unit of private capital.
Figure 33: Moving the fraction of pensioners $\alpha_r$ down (until the first generation of the three generations of migrants become pensioners themselves) by increasing the growth rate $\bar{n}$ of the workforce: an adjusted controlled immigration policy (see top-left figure)

The relatively small loss in the rate of employment seems to be the only problematic outcome of this migration policy which is improving the public and private supply-side conditions very much.
Figure 34: Moving the fraction of pensioners $\alpha_r$ down by increasing the growth rate $\bar{n}$ of the workforce plus a unit-wage-cost shock at $t=5$.

The situations of the figures 34 thus is not changed by too much through the addition of the 5 percent shock to the unit-wage-costs of firms, though – of course – the initial recession becomes deeper in this profit-led economy. Note that the system converges to a new steady state for capacity utilization and the employment rate in particular, which therefore must have effects on $\bar{v}_f$ which must balance each other in the new steady state (which for example can be due to the significant non-linearity in the term $y/y^p(k_g)$ in particular). In the next figure we apply the extra policy shocks:

```c
if it==50*j*pl; alpg=1.1*alpg; endif;
if it==80*j*pl; alpg=1.1*alpg; endif;
```
The next three figures 37 – 39 consider the same for an extension of the work-life of workers from 65 to the age of 70 years (assumed to keep ratios simple). We do this in a stepwise fashion over a time-span of 20 years. In figures 37 and 38 capacity utilization remains fairly stable somewhat below 1 so that the time series top right of potential output per unit of capital $y_p f k g$ as a function of the public capital stock provides the information on the actual output capital ratio quite well already.
Figure 36: Moving the fraction of pensioners $\alpha_r$ down by increasing the retirement age to 70 (see top-left figure). Below we have that pensions per head become larger than benefits per head after some time and below that we see an increasing output-capital ratio $y = Y/K$ despite a falling rate of capacity utilization $y/y^p(k_g)$, just as in the next set of diagrams.

Such a policy seems to work better than the previous one. It will be supplemented in figure 40 by the assumption that these additions to the workforce are absorbed and paid by the public sector in order to ease the harder work conditions they have been subjected when working in the private sector of the economy when still below 65. Of course, remaining as experienced workers in the private sector is possible, but a matter of the microeconomics behind our macro-model.
Figure 37: Moving the fraction of pensioners $\alpha_r$ down by increasing the retirement age to 70 (see top-left figure) plus a unit-wage-cost shock at $t=5$.

The unit-wage-costs change seems to be even less important in the case of the present labor market policy than in the previously considered case.
Figure 38: Moving the fraction of pensioners $\alpha_r$ down by increasing the retirement age to 70 (see top-left figure) plus a change in public investment.

The accompanying policy shocks are of a more complicated and redistributive nature here with however by and large positive implications:

```plaintext
if it==20*j*pl; gy0=1.65*gy0; endif;
if it==30*j*pl; gy0=1.1*gy0; qr=1.65*qr; endif;
if it==40*j*pl; gy0=1.1*gy0; qr=1,1*qr; endif;
if it==50*j*pl; tilqr= 1.1*qr; endif;
```
Figure 39: Moving the fraction of pensioners $\alpha_r$ down by increasing the retirement age to 70 (see top-left figure) plus a change in public investment – and the public (social) employment of all workers above the age of 65.

The next three figures consider the same for a policy induced baby boom. The depicted situations look less attractive than the previous ones and are also much more volatile in their reaction to the unit-wage costs shock.
Figure 40: Moving the fraction of pensioners $\alpha_r$ down by inducing a baby boom and its long run consequences (see top-left figure)
Figure 41: Moving the fraction of pensioners $\alpha_r$ down by inducing a baby boom and its long run consequences (see top-left figure) plus a unit-wage-cost shock at $t=5$.

In the next figure we apply the policy shocks:

if it==50*j*pl; alpg=1.1*alpg; endif;
if it==80*j*pl; alpg=1.1*alpg; endif;
Figure 42: Moving the fraction of pensioners $\alpha_r$ down by inducing a baby boom and its long run consequences (see top-left figure) plus a change in public investment.

The next three figures consider the same by assuming an initial participation rate of 70 percent (instead of 1) and a yearly increase in this participation rate by one percent over a time span of 20 years, see the diagram top-left for the conducted change in the participation rate of the potential workforce. Without an accompanying support from policy makers the changes in participation in the labor market look somewhat problematic, see figures 44 and 45. This situation changes if the policy mix of the retirement age policy is added in figure 46.
Figure 43: Moving the fraction of pensioners $\alpha_r$ down by increasing the participation rate of the workforce, starting from a rate of 70 percent (see top-left figure). Below is benefits per head as dotted line and further below we see capacity utilization increasing, while the output/capital ratio is falling, both for some time.

Overall, the outcome of this pure supply side shock is clearly a negative one. This situation is not changed very much if the unit-wage-costs shock is added again as the subsequent figure shows.
Figure 44: Moving the fraction of pensioners $\alpha_r$ down by increasing the participation rate of the workforce, starting from a rate of 70 percent (see top-left figure) plus a unit-wage-cost shock at $t=5$. 
Figure 45: Moving the fraction of pensioners $\alpha_r$ down by increasing the participation rate of the workforce, starting from a rate of 70 percent (see top-left figure) plus a change in public investment.

Again, the accompanying policy shocks are of a more complicated and redistributive nature here with however by and large very positive implications, so that the situation is now a drastically changed one as compared to the previous situation – due to the Keynesian measures added to the initially given pure supply-side shocks.

if it==20*j*pl; gy0=1.1*gy0; endif;
if it==30*j*pl; gy0=1.1*gy0; qr=gshock*qr; endif;
if it==40*j*pl; gy0=1.1*gy0; qr=gshock*qr; endif;
if it==50*j*pl; qr=gshock*qr; endif;
11 Conclusions and outlook

In this paper we have presented a macro-model of Keynesian monetary growth, of the variety introduced in Chiarella and Flaschel (2000) and considered in detail in Chiarella, Flaschel and Franke (2005) and Charpe, Chiarella, Flaschel and Semmler (2010). The model was formulated sufficiently rich with respect to markets, sectors and agents and consistent with respect to budget constraints in order to allow to capture the important aspects of actual macro-economies.

We described the model on the level of national accounts and derived on this basis its extensive form dynamics. Our model allowed for coherent stock-flow considerations, a compact intensive form for its theoretical and numerical investigation and a locally unique interior balanced growth position which we used as starting point and thus reference point for our simulations of its laws of motion.

These simulations provided a persuasive foundation for a basic understanding of the interaction of its various economic feedback channels, known by partial reasoning from Keynesian economic theory, like the Harrod-Domar theory of the instability of balanced growth, the Goodwin-Rose distributive cycle mechanism, the Dornbusch overshooting exchange rate investigation and the Blanchard analysis of bond (and asset) markets dynamics.

The basic task was in this respect to tame these generally destabilizing forces by way of suitably chosen fiscal and monetary policy rules, a quite demanding task already in view of the 18 laws of motion of the model. We had moreover to cope with the stability problems caused by the GBR (government budget constraint). We had to use Keynesian fiscal and monetary policy here too, in order to control the debt dynamics that the GBR was giving rise to.

Of primary interest in this paper was however the question how its many tax-, transfer- and government-expenditure-parameters could be used to improve ”social protection” for the sector of worker households, without loosing the efficiency of a well-performing labor market, and without neglecting the creation of a sufficiently rich ”infra-structure” for the educational system, medicare and care for the elderly, an age structure which needed to be made sustainable by the policy makers.

One has to stress here however that our model is a macro-model, resting on the usual one-good assumption. It is therefore obvious that public goods consist just of a single (aggregated) item, the size of its supply therefore being the only concern in this paper, while its structuring (and also the details of its funding must be left for micro-economic reasoning here).

Since residential issues are a matter of great importance for the worker households, we have also added a supply function for residential services for these households, generated from a stock of houses created through the past investment expenditures of the asset holders of the model, the sole real asset they administer in our approach.

All these aspects were illustrated by numerous simulations of the laws of motion of the formulated macro-dynamics in the sections that followed the determination of its reference balanced growth position, which however turned out to be non-unique from the global perspective, due to the externalities, we assumed to result from the public investment into the public capital stock in particular.

Concerning the four types of labor market adjustments considered in section 10 of the paper (meant to move the pensioner/workforce ratio into a lower sustainable position we finally obtained the conclusion that a socially safe-guarded, not necessarily dis-utility creating, increase of the work-life of workers and thus of their retirement age, combined with an increase in the participation rate of the potential workforce, may represent the best foundation for an increase of all wage-oriented incomes, when grounded on increases in the public ”infrastructure” and the resulting improvements of the output potential of firms (per unit of their capital stock, \( y^p = Y^p/K \)), i.e., of the productive capacity of the economy.

In sum, the message of this paper is that the Keynesian way of stabilizing the macro-economy is working very well in our high-dimensional macro-model. Moreover, this model allows to implement a kind of ”social protection”, where the investigated country can be considered as being ”rich” from the perspective of the provision of public goods and services, and well-equipped for the provision of output and incomes, on the basis of which the ”social protection” of the worker households – and
also the care for the other agents of the economy – can then indeed be solved in successful ways.

Increasing the public capital stock through increased public investment however often leads to an increase in the "Maastricht" debt to GDP ratio. However, we would expect – if this does not happen in an extreme fashion – that the resulting "rich" infra-structure of the country in question and its increased and in fact utilized output and income potential will protect it against speculative attacks as we, for example, have experienced then in the case of Greece in the recent past. Nevertheless, this topic needs further exploration in future research.

Another important assumption, which we implicitly needed to neutralize the international competitive pressure on unit wage costs, was that we made use of an export subsidy – instead of allowing international competition to put a restraint on domestic wage negotiations – to equalize the domestic price level with the international price level (measured in the domestic currency) for the exported commodities. The increases in unit wage costs that we observed in a number of cases were therefore not yet a problematic outcome, but are maybe welcome if they induce increases in the economy-wide state of labor productivity. Our suggestion here is that progress in generally taking place – as far as process innovations are concerned – through rising wages and not so much in the opposite case of an international downward spiral in national wage levels where it may become profitable again (in the considered manufacturing sector) to provide labor with very regressive means of production. Again, these are topics which must be left for future research here.

Finally, the role of financial markets definitely needs further consideration, regarding their actual functioning as well as regarding their sophisticated (international) regulation. This topic is of course to be treated differently in a small open economy from what applies to the USA, the Eurozone or China in this matter.
Appendix: Notation

The following list of symbols contains only domestic variables and parameters. Foreign magnitudes are defined analogously and are indicated by an asterisk (*). To ease verbal descriptions we shall consider in the following the ‘Australian Dollar’ as the domestic currency (A$) and the ‘US Dollar’ ($) as a representation of the foreign currency (currencies).

A. Statically or dynamically endogenous variables:

$Y$  
Output of the domestic good

$Y^d$  
Aggregate demand for the domestic good

$Y^p$  
Potential output of the domestic good

$Y^{dp}$  
Normal sales of the domestic good

$Y^*$  
Normal output of the domestic good

$Y^{Dn}$  
Expected sales for the domestic good

$Y^{Wn}$, $Y^{Cn}$  
Nominal disposable income of workers and asset-holders

$Y_f$  
Income of firms

$L_1$  
Population aged 17 – 65

$L_2$  
Population aged 66 – ...

$L_0$  
Population aged 0 – 16

$L$  
Total employment of the employed

$L_{wf}$  
Total government employment (= public work force)

$L_{w}$  
Work force of firms

$L_{wp}$  
Total active work force

$e^{w} (V^{w})$  
(Normal) Employment rate of those employed in the private sector

$e = L^w / L$  
Rate of employment (V the employment–complement of the NAIRU)

$C_{w}(C^{w})$  
Real (equilibrium) goods consumption of workers

$C_{c}(C^{c})$  
Real (equilibrium) goods consumption of asset owners

$C = C_{w} + C_{c}$  
Total goods consumption

$c_{s}$  
Supply of dwelling services

$c_{d}$  
Demand for dwelling services

$I$  
Gross business fixed investment

$I_h$  
Gross fixed housing investment

$I^{a}(I^{na})$  
Gross (net) actual total investment

$T$  
Planned inventory investment

$N$  
Actual inventories

$N^{d}$  
Desired inventories

$r$  
Nominal short-term rate of interest (price of bonds $p_b = 1$)

$r^{l}$  
Nominal long-term rate of interest (price of bonds $p_b = 1 / r^{l}$)

$\pi_{b} = \hat{p}_{b}$  
Expected appreciation in the price of long-term domestic bonds

$S^{a} = S_{p}^{a} + S_{f}^{a} + S_{g}^{a}$  
Total nominal savings

$S^{a}_{p} = S_{w}^{a} + S_{c}^{a}$  
Nominal savings of households

$S_{f}^{a}$  
Nominal savings of firms (= $p_{y} Y_{f}$, the income of firms)

$S_{g}^{a}$  
Government nominal savings

$T^{a}(T)$  
Nominal (real) taxes

$G$  
Real government expenditure

$\rho^{e}$  
Expected short-run rate of profit of firms

$\rho^{a}$  
Actual short-run rate of profit of firms

$\rho^{l}$  
Expected long-run rate of profit of firms

$\rho_{h}$  
Actual rate of return for housing services

$\rho_{h}^{l}$  
Expected long-run rate of return for housing services

$K$  
Capital stock

$K_h$  
Capital stock in the housing sector

$w$  
Nominal wages before taxes

$w^{u}$  
Unemployment benefit per unemployed

$w^{r}$  
Pension rate

$p$  
Price level of domestic goods including value added tax

$p_{y}$  
Price level of domestic goods net of value added tax

$p_{x}$  
Price level of export goods in domestic currency

$p_{m}$  
Price level of import goods in domestic currency including taxation

$p_{h}$  
Rent per unit of dwelling

$\pi^{c} = \hat{p}^{c}$  
Expected rate of inflation (inflation climate)
Exchange rate (units of domestic currency per unit of foreign currency: AUD/USD)

Expected rate of change of the exchange rate

Labor supply

Stock of domestic short-term bonds (index d: stock demand)

Short-term debt held by workers

Short-term debt held by asset owners

Stock of domestic long-term bonds, of which $B_l^d$ are held by domestic asset-holders (index d: demand) and $B_l^c$ by foreigners (index d: demand)

Foreign bonds held by domestic asset-holders (index d: demand)

Natural growth rate of the labor force and labor productivity

Exports

Imports

tax rate on wages

Tax rates on imported commodities

actual rate of growth of the capital stock $K$

actual rate of growth of the housing capital stock $K_h$

Actual public debt / output ratio

Depreciation rate of the capital stock of firms

Depreciation rate in the housing sector

All $\alpha$-expressions (behavioral or other parameters)

All $\beta$-expressions (adjustment speeds)

Steady growth rate in the rest of the world

Normal rate of capacity utilization of firms

Normal rate of capacity utilization in housing

Weights of short- and long-run inflation ($\kappa_w\kappa_p \neq 1$)

Steady state values

Real variables in intensive form

Nominal variables in intensive form

Government Budget Restraint

Time derivative of a variable $x$

Growth rate of $x$

Steady state values

Real variables in intensive form

Nominal variables in intensive form
Neo-liberal” Barebone Capitalism or ”Keynesian” Socially-Protected Capital Accumulation. A Graphical Summary for Closed KMG Economies

Abstract

In this appendix to the subject of ’Socially-protected’ capitalism we present the modules of a general, hierarchically structured continuous-time model of Keynesian monetary growth, of the variety introduced in Chiarella and Flaschel (2000) and considered in detail in Chiarella, Flaschel and Franke (2005). The model improves and modifies the preliminary 18D format used in Charpe, Chiarella, Flaschel and Semmler (2010, ch.7), here applied to the case of a closed economy. The model is sufficiently rich with respect to markets, sectors and agents and complete with respect to budget constraints or – as one now prefers to say – is stock-flow consistent.

We describe the model at the level of national accounts and then introduce against this background its extensive or structural form. Its laws of motion and supplementing static equations are derived next on the basis of which a balanced growth path can be obtained and investigated from a comparative dynamic perspective, in particular with respect to a rich set of taxation and transfer schemes between households, firms and the government. Stability is discussed in terms of various feedback channels which characterize the private sector of the economy and which in sum tend to destabilize it if fiscal and monetary policy remain passive.

Due to the size of the model, the set of fully interacting feedback channels can be studied only numerically, while isolated feedback chains can be investigated theoretically as in earlier work. The model allows for two contrasting limit cases, the Keynesian case of a ’socially-protected’ form of capital accumulation as against a Neo-liberal ’barebone’ form of capitalism where among certain conditions some credit support is given to firms in order to allow for an acceptable form of income distribution that provides worker households with the necessary income in a capitalist environment where public goods or transfers are completely absent.
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**Notation**
1 Introduction

In the following sections, we present in various ways the modules of a hierarchically structured continuous-time model of Keynesian monetary growth, of the variety introduced in Chiarella and Flaschel (2000) and considered in detail in Chiarella, Flaschel and Franke (2005). The model is sufficiently rich with respect to markets, sectors and agents and consistent with respect to budget constraints to capture the important details of actual macro-economies. We describe the model on the level of national accounts and then derive its extensive form dynamics. Our model allows for coherent stock-flow considerations, a compact intensive form for its theoretical and numerical investigation and a locally unique interior balanced growth positions which we use as starting and reference point for our simulations of its laws of motion.

These simulations provide a persuasive foundation for a basic understanding of the interaction of its various economic feedback channels some well-known known by partial reasoning from Keynesian economic theory, like the Harrod-Domar theory of the instability of balanced growth, the Goodwin-Rose distributive cycle mechanism, and the Blanchard analysis of bond and asset markets dynamics. The basic task here is to tame these generally destabilizing forces from a Keynesian perspective by way of suitably chosen fiscal and monetary policy rules, a quite demanding task in view of the many laws of motion of the considered macro-dynamics.

Of primary interest is however the question how the many tax-, transfer- and government-expenditure parameters of the full model can be used to improve the social protection of the sector of worker households, as compared to the barebone version of the macro-dynamics, without loosing the efficiency of a well-performing labor market (with its partial modelling of Friedmanian supply side forces), but also without neglecting the creation of a sufficient ”infra-structure” for education, health care and care for the elderly, i.e., for the young people, the labor market participants and the retired. We therefore compare in this paper a capitalist system, where public goods and transfers to worker households are absent, with the situation where social protection is given to them, as workers as well as as retirees (and also for the rest of the society).

We can show that the case with social protection will indeed be advantages in basically all respects, as compared to the case of bare bone capitalism, leading to more prosperity and a better income distribution on the basis of one important positive externality, which is the increase in the potential production of firms per unit of their capital following an increase in the public capital stock, the ”infrastructure of the society”. Such an infrastructure covers a lot of details, represented by a single public good on the macro-level, ranging from schools, hospitals, public transportation facilities to old-age homes and more. Of course, public services must supplement such public goods to make them productive ones.

Since residential issues are also of great importance for the worker households, we finally add residential services to this households sector, services which are supplied from of a stock of houses created by the housing investment of the asset holders in our economy, the sole real asset these asset holders administer in this Keynesian model of monetary growth.

Concerning the topics just enumerated we will provide a range of answers showing the macro-advantages of an advanced type of ”social protection” through public investments into the ”infra-structure” of the economy, through various types of income transfers, and through Keynesian fiscal and monetary policy, but we will also find some obstacles which prevent the creation of what is called a ”free lunch” by mainstream economics. These aspects are illustrated by some simulations of the laws of motion of our macro-dynamical system in the sections that follow the determination and study of its balanced growth path.
2 The real and the financial part of the economy

The following two tables provide a survey of the structure of the economy to be modelled in this paper. The aim of the paper is to establish an integrated continuous-time dynamical model, leading to an autonomous system of differential equations, where all sectors are fully specified with respect to their behaviour and their budget constraints from the viewpoint of complete theoretical macro-models of monetary growth. A bridge will thereby be provided between the Keynes-Metzler type monetary growth models of ?, ?, and the applied ? approach for the Australian economy, where we however use only a closed economy perspective here. The open economy case will be considered in a companion paper to the present one.

2.1 The structure of the real part

Let us start with a presentation of the variables that comprise the real part of the economy. Table 1 provides the data for the temporary equilibrium position of the economy, based on given prices and expectations and it also shows the real stock variables of the model and their rates of growth.

<table>
<thead>
<tr>
<th></th>
<th>Labor</th>
<th>Goods</th>
<th>Dwellings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers</td>
<td>$L^w = \alpha_w L$</td>
<td>$C_w$</td>
<td>$C_d^h$</td>
</tr>
<tr>
<td>Asset holders</td>
<td>$-$</td>
<td>$C_c$</td>
<td>$C_d^h, I_h$</td>
</tr>
<tr>
<td>Firms</td>
<td>$L_f^d, L_f^w$</td>
<td>$Y_p, Y, I, I$</td>
<td>$-$</td>
</tr>
<tr>
<td>Government</td>
<td>$L_g^d = L_g^w$</td>
<td>$G$</td>
<td>$-$</td>
</tr>
<tr>
<td>Taxes</td>
<td>$\tau_w, \tau_{wp}, \tau_c$</td>
<td>$\tau_v, \tau_f, p_f$</td>
<td>$-$</td>
</tr>
<tr>
<td>Wages, Prices</td>
<td>$w, w^n, w^e, w^{ua}$</td>
<td>$p = (1 + \tau_v)p_y$</td>
<td>$p_h$</td>
</tr>
<tr>
<td>Expectations</td>
<td>$\pi^c$</td>
<td>$\pi^c$</td>
<td>$\pi^c$</td>
</tr>
<tr>
<td>Stocks</td>
<td>$L$</td>
<td>$K, N$</td>
<td>$K_h$</td>
</tr>
<tr>
<td>Growth</td>
<td>$\dot{L} = \gamma - m$</td>
<td>$\dot{K} = I/K - \delta_f$</td>
<td>$\dot{K}_h = I_h/K_h - \delta_h$</td>
</tr>
<tr>
<td></td>
<td>$\dot{N} = Y - Y_d$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 1: The real part of the economy.

We use a superscript d for demand and (sometimes) s for supply. The symbol $\alpha_w$ denotes the participation rate of the labor force $L$, the employment of which in the sector of firms is given by $L_f^w$. The symbol $L_f^d$ denotes the employment of the employed workforce in hours and $w$ the hourly gross wage. The symbols used for net wages, unemployment benefits and pension payments should self-explaining.

Payroll taxes are shared between workers and firms. We denote by $p$ the price level that includes value added taxes at the rate $\tau_v$. The expression $\pi^c$ will be used to describe the expected medium-run inflation climate of the economy. The stock of inventories of firms is denoted by $N$. Finally, labor force growth is determined from outside the household sector through the trend rate in investment minus the growth rate of labor productivity $m$, towards which the growth rate of the labor force growth rate is adjusting. This assumption will be reconsidered later.


2.2 The structure of the financial part

Let us next consider the financial part of the economy which we will keep less advanced as compared to the real part of this macro-economy.

<table>
<thead>
<tr>
<th></th>
<th>Money</th>
<th>Short-term Bonds</th>
<th>Long-term Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Workers</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Asset holders</td>
<td>$M$</td>
<td>$B_c$</td>
<td>$\dot{B}_c$</td>
</tr>
<tr>
<td>Firms</td>
<td>–</td>
<td>–</td>
<td>$\dot{B}_f$</td>
</tr>
<tr>
<td>Government</td>
<td>–</td>
<td>$\dot{B}_g, \dot{B}_b$</td>
<td>$\dot{B}_g$</td>
</tr>
<tr>
<td>Prices</td>
<td>1</td>
<td>$1 [r]$</td>
<td>$p_l = 1/r_l$</td>
</tr>
<tr>
<td>Expectations</td>
<td>–</td>
<td>–</td>
<td>$\pi_l = \bar{p}_l$</td>
</tr>
<tr>
<td>Stocks</td>
<td>–</td>
<td>$B_g = B_c + B_b + B_{B_c}$</td>
<td>$B_g^l + B_f^l = B_c^l$</td>
</tr>
<tr>
<td>Growth</td>
<td>–</td>
<td>$\dot{B}_g$</td>
<td>$\dot{B}_g^l$</td>
</tr>
</tbody>
</table>

Table 2: The financial part of the economy.

We use the subscripts $g, c, w$ for government, pure asset holders and workers respectively and assume as usual in a continuous-time stock-flow model (where stocks and flows have different economic dimensions) the existence of flow consistency by assuming the inflow of new stocks are just accepted by asset holders (here $c$ and $w$) (while the central bank can change the money supply through the purchase of short-term bonds solely ($\dot{M} = \dot{B}_b$)).

This trivial Walras’ law of flows is to be supplemented by a dynamic Walras’ law of stocks within which dynamic reallocations of the stocks held by pure asset holders take place, in particular the enforced inflow of new assets from the government and the central bank. We will do this in a very simple way here in order to allow the interest rate policy $r$ of the CB operating on the interest rate of fixed-price short-term bonds $B$ and to have in this way an indirect impact effect on the long-term rate of interest $r_l = 1/p_l$ of the perpetuities $B^l$, we use here as the only risky type of asset on the financial markets.

3 The structure of the economy from the viewpoint of national accounting

We consider in this section the production accounts, income accounts, accumulation accounts and financial accounts of the four internal agents in our economy. These accounts provide notation and basic information on what is assumed for these four sectors as well as which of their activities are excluded from the present theoretical framework. These accounts also serve the purpose of checking that ex post results of the economy are consistent with each other.

3.1 The four sectors of the economy

We start with the accounts of the sector of firms (shown in Table 5.3) which organise production $Y$, employment $L_{df}$ of their workforce $L_w$ and gross business fixed investment $I$ and which use (in the present formulation of the model) only corporate bonds $B_f^l$ as financing instrument (no debt in the form of bank loans nor equities issued by firms). There are value added taxes $\tau_v$ on consumption goods and payroll taxes $\tau_{fp}$ with respect to hours worked $L_{df}$, but no further taxation in the sector of firms and there are no subsidies (up to an exceptional numerical example).

Firms build and sell dwellings, which are of the same aggregate type as all other domestic production, and sell them to the asset holders (as investors) and thus have no own investment in the housing sector. They sell consumption goods to workers, asset holders and the government, organise fixed gross investments with respect to their capital stock $K$ (as well as voluntary inventory $I_v$).
Production Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>Consumption $p_w C_w$</td>
</tr>
<tr>
<td>Depreciation $p_y \delta_f K$</td>
<td>Consumption $p_w C_c$</td>
</tr>
<tr>
<td>Indirect Taxes $\tau_y p_y (C_w + C_c + G)$</td>
<td>Consumption $p_w G$</td>
</tr>
<tr>
<td>Wages (including payroll taxes) $\tau_{wp} w L_f^d$</td>
<td>Gross Investment $p_y I$</td>
</tr>
<tr>
<td></td>
<td>Durables (Dwellings) $p_y I_h$</td>
</tr>
<tr>
<td>Profits $\Pi = \rho^g p_y K + p_y I = \rho p_y K + p_y \dot{N}$</td>
<td>Inventory Investment $p_y \dot{N}$</td>
</tr>
</tbody>
</table>

Income Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit payments (including interest paid by firms) $\rho^g p_y K$</td>
<td>Profits $\Pi$</td>
</tr>
<tr>
<td>Savings $S_f^n = p_y I$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Investment $p_y I$</td>
<td>Depreciation $p_y \delta_k K$</td>
</tr>
<tr>
<td>Inventory Investment $p_y \dot{N}$</td>
<td>Savings $S_f^n$</td>
</tr>
<tr>
<td></td>
<td>Financial Deficit $FD$</td>
</tr>
</tbody>
</table>

Financial Account of Firms:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Deficit $FD$</td>
<td>Bond Financing $p_l B_f^l$</td>
</tr>
</tbody>
</table>

Table 3: The production, income, accumulation and financial accounts of firms.

Changes $I$ with respect to finished goods) and experience involuntary inventory changes $Y - Y^d$ due to the deviation of aggregate demand $Y^d$ from output $Y$ (which is based on expected sales $Y^e$ and planned inventories $I$).

Firms have replacement costs with respect to their capital stock and pay wages including payroll taxes. Their accounting gross profit (including interest payments $B_f^l$ on their perpetuity issue) is equal to expected profits (based on sales expectations. As is obvious from the narrow income account of firms, firms thus only save an amount equal to their intended inventory changes. The accumulation account is self-explanatory as is the financial account.

Note that all investment is traded without value added taxes and thus at producer prices $p_y$ in place of the domestic consumer prices $p = (1 + \tau_v) p_y$, which include indirect taxes (value added taxes). Such taxes thus only apply to consumption activities, not to gross investment, and also not on housing investments and the inventory investment of firms. All expected profits are distributed to asset holders (including the interest payments of firms to them). Note however that the wages $w$ paid by firms are augmented by payroll taxes $\tau_{wp} w$ (for unemployment benefits, medicare and other social insurance, as well as pensions) and that wage income $w$ of workers is taxed at the rate $\tau_w$. Note finally that the accumulation account of firms is based on realised magnitudes and thus does not refer explicitly to their intended inventory changes.
Production Account of Households (Asset Owners/Housing Investment):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation $p_y \delta_h K_h$</td>
<td>Rent $p_h C^d_h$</td>
</tr>
<tr>
<td>Earnings $\Pi_h$</td>
<td></td>
</tr>
</tbody>
</table>

Income Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax payment $\tau_c r B_c$</td>
<td>Interest payment $r B_c$</td>
</tr>
<tr>
<td>Tax payment $\tau_c B^l_c$</td>
<td>Interest payment $B^l_g$</td>
</tr>
<tr>
<td>Taxes $\tau_c(p_h C^d_h - p_y \delta_h K_h)$</td>
<td>–</td>
</tr>
<tr>
<td>Tax payment $\tau_c \rho^{eg} p_y K$</td>
<td>Profits and interest from firms $\rho^{eg} p_y K$</td>
</tr>
<tr>
<td>Consumption $p_c C_c$</td>
<td>Earnings $\Pi_h$</td>
</tr>
<tr>
<td>Savings $S^n_c$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Investment $p_y I_h$</td>
<td>Depreciation $p_y \delta_h K_h$</td>
</tr>
<tr>
<td>Financial Surplus $FS$</td>
<td>Savings $S^n_c$</td>
</tr>
</tbody>
</table>

Financial Account of Households (Asset Owners):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bonds $B_c$</td>
<td>Financial Surplus $FS$</td>
</tr>
<tr>
<td>Long-term government bonds $B^l_l$</td>
<td>–</td>
</tr>
<tr>
<td>Corporate bonds $B^l_f$</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 4: The production, income, accumulation and financial accounts of asset owners.

Consider next the sector of asset-holders (see Table 4). Investment in housing as well as the supply of housing services has been exclusively allocated to this sector. The production account thus shows the actual sale (not the potential sale) of housing services (equal to the demand for housing services by assumption) which is divided into replacement costs and actual earnings or profits on the uses side of the production account.

The income of asset holders comes from various sources: interest payments on short- and long-term bonds, interest payments of firms (as part of their expected profit) and profits from housing services. All domestic profit income is subject to tax payments at the rate $\tau_c$. After tax income by definition is subdivided into the consumption of domestic commodities (including houses, but not housing services) and the nominal savings of asset owners.

The accumulation account shows the sources for gross investment of asset-holders in the housing sector, namely depreciation and savings, the excess of which (over housing investment) is then invested in financial assets as shown in the financial account. Note here that short-term bonds are fixed price bonds $p_b = 1$ (which are perfectly liquid), while long-term bonds have the variable price $p_b = 1/r_l$ (with fixed nominal interest payments of one unit of money per period and bond), so-called consols or perpetuities.

There is no taxation of financial wealth (held or transferred) in the household sector. Furthermore, though asset holders will consider expected gross rates of return on financial markets in their investment decision, there is no taxation of capital gains on these markets.
Production Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Income Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxes ((\tau_w + \tau_{wp})wL^d + \tau_d B_w)</td>
<td>Wages (wL^d)</td>
</tr>
<tr>
<td>Consumption (p_v C_w + p_h C_h^d)</td>
<td>Unemployment benefits (w^u(\alpha_w L - L^w))</td>
</tr>
<tr>
<td>Financial Surplus (S^n_w)</td>
<td>(r B_w) Interest payments</td>
</tr>
</tbody>
</table>

Accumulation Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Surplus (FS)</td>
<td>Savings (S^n_w)</td>
</tr>
</tbody>
</table>

Financial Account of Households (Workers):

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term bond accumulation (B_w)</td>
<td>Financial Surplus (FS)</td>
</tr>
</tbody>
</table>

Table 5: The production, income, accumulation and financial accounts of worker households.

explain. First, there is no production account in this sector. Income of the members of the workforce, which may be employed, unemployed or retired, thus derives from wages, unemployment benefits or pension payments where \(\alpha_w L\) denotes the total number of persons in the currently registered workforce \((L^w\) the part that is employed) and \(\alpha_r L\) the number of retirees who get pension income \((\alpha_w = \text{const.} \ \text{the participation rate of the potential workforce} \ L)\). To this we have to add the interest income on saving deposits (short-term bonds) which is taxed at the general rate used for financial asset income. All and only wage income is subject to taxation at the rate \(\tau_w\) and total wage related income is again by definition subdivided into nominal consumption (consumption goods and housing services) and savings. Note here that the employment \(L^d\) of the employed \(L^w\) can differ from their normal employment which is just measured by \(L^w\), the number of persons who are employed. Note also that wages \(w\) are net of payroll taxes (used to finance unemployment benefits, social insurance and pensions in particular).

We assume in the following that workers have a positive savings rate and that they hold their savings in the form of short-term bonds solely, which is mirrored here in the accumulation and financial account in a straightforward way.

There are finally the accounts of the fiscal and monetary authority (see Table 6), which due to the many taxation rules and transfer payments that are assumed are more voluminous than the preceding accounts – at least with respect to the income account. There is first however a fictitious production account where the supply of public goods is valued at production costs which consist of government expenditures for goods and labor.

The sources of government income consist of taxes on workers’ income (taxed at a uniform rate plus their payroll tax contribution), of taxes on the various forms of profit, interest and rental income (again taxed at a uniform rate), payroll taxes from firms and value added taxes. Uses of the tax income of the government are interest payments, transfers to the unemployed and retirees, and government 'production'. In general all these uses of the tax income of the government will exceed its income so that there will result a negative amount of nominal.
"Production Account" of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>Costless Provision</td>
</tr>
<tr>
<td>Government expenditure for services</td>
<td>of public goods</td>
</tr>
<tr>
<td>$wL^d_g$</td>
<td></td>
</tr>
</tbody>
</table>

Income Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest payment $rB$</td>
<td>Wage income taxation $(\tau_w + \tau_{wp})wL^d$</td>
</tr>
<tr>
<td>Interest payment $B^l$</td>
<td>Profit/interest taxation $\tau_c[p^{eg}p_yK + rB_g + B^l_g]$</td>
</tr>
<tr>
<td>Pensions $w^r\alpha_rL$</td>
<td>Rent income taxation $\tau_r(p_hC^d_h - p_y\delta_hK_h)$</td>
</tr>
<tr>
<td>Unemployment benefits $w^u(\alpha_wL - L^w)$</td>
<td>Payroll taxes from firms $\tau_{fp}wL^d$</td>
</tr>
<tr>
<td>Government consumption $p_vG$</td>
<td>Value added tax $\tau_vp_y(C_w + C_c + G)$</td>
</tr>
<tr>
<td>Salaries $wL^d_g$</td>
<td></td>
</tr>
<tr>
<td>Savings $S^n_g$</td>
<td></td>
</tr>
</tbody>
</table>

Accumulation Account of the Fiscal Authority:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings $S^n_g$</td>
<td></td>
</tr>
<tr>
<td>Financial Deficit $FD$</td>
<td></td>
</tr>
</tbody>
</table>

Financial Account of Fiscal and Monetary Authorities:

<table>
<thead>
<tr>
<th>Uses</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial deficit $FD$</td>
<td>change in short-term debt $B$</td>
</tr>
<tr>
<td></td>
<td>change in long-term debt $p_tB^l$</td>
</tr>
</tbody>
</table>

Table 6: The production, income, accumulation and financial accounts of the monetary and fiscal authorities.

savings $S^n_g$ which balances the income account of the government.

There is however accumulation of real assets in the government sector, called "infrastructure" for briefness, which is part of their total expenditure on the one good of the economy. This means that we only have to look into the financial account of the government to see how the excess of government outlays over government revenue is financed through short- or long-term debt. Note that there is central bank money in the economy used in the background of their interest rate policy rule.

4 The model

In this core section 4 of the paper we develop the extensive form equations of the general model, based on the accounting structure we presented in sections 2 and 3. We significantly reformulate the equations of CCFS (2010), but not the "philosophy" of their chapter 7 model, there derived from the Powell and Murphy (1997) model for the Australian economy, by making it a continuous-time macro-dynamical theoretical model of monetary growth, where all discrete lag structures of the originally quarterly model are depressed.
Our interest is not to fully mirror the dynamical structure and implications of the "Murphy" model, but to make use of such qualitative understanding of applied Keynesian theory to formulate and to investigate, in a first approximation of this 100 equations approach to macroeconometric model building, a set of prominent feedback structures of macro-dynamic theory and their role for economic stability analysis, before fiscal, monetary and social policy will enter the scene.

We thereby attempt to build a bridge between empirically motivated work on structural macro-dynamic model building (where there generally is no analysis of the mechanisms that are hidden in the formulated structure) and theoretical investigations of a reasonably large representation of such economies, where the interest is to see how the balanced growth path will look like, in particular with respect to the share of wages, and its dependence on various forms of taxation, social protection and fiscal and monetary policy.

4.1 Basic definitions

Let us start with some notation to be used in the structural equations of our approach to Keynesian monetary growth.

1. Definitions (Rates of Return, Wages and Prices)

\[ \rho^e = \frac{p_y Y^e - B_f^l - (1 + \tau_{fp})wl_f Y - p\delta_f K}{pK} \]  
\[ \rho^{eg} = \frac{p_y Y^e - (1 + \tau_{fp})wl_y Y - p\delta_f K}{pK} \]  
\[ \rho_h = \frac{p_h C_w^d - p\delta_h K_h}{pK_h} \]  
\[ w^n = (1 - \tau_w - \tau_{wp})w = q^n w \]  
\[ w^u = q^u w^n, q^u < 1 \]  
\[ w^r = q^r w^n, q^r < 1 \]  
\[ p_t = 1/\tau_l \]  
\[ p = (1 + \tau_v) p_y, \quad p(0) = p_{lo} \quad \text{as initial condition} \]  
\[ p_h = q_h p \]

Module 1 of the model provides definitions of expected rates of return \( \rho^e, \rho_h \), based on expected sales in manufacturing (with and without the interest payments \( B_f^l \) of firms on corporate bonds, given by perpetuities), and for residential services, notation for hourly wages, \( w \), including income taxation, and augmented by payroll taxes, prices \( p, p_h, p_t \) for goods, residential services and for perpetuities, the first including value added taxation, of untaxed pension payments to the retired worker of the workforce, \( w^r \), and untaxed unemployment benefits per unemployed worker (of the workforce), \( w^u \).

Module 2 concerns the household sector where two types of households are distinguished, workers and pure asset holders. Of course, these two types of households are only polar cases in the actual distribution of households types. Nevertheless we believe that it is useful to start from such polar household types before intermediate cases are introduced and formalized.

4.2 Households

We consider the behavioral equations of worker households first:
consumption plans (all based on the participation rate labor income, unemployment benefits and pensions payments as income items behind workers’ by trend investment and its implicit workforce plus family recruitment activities) and that only juveniles, potential workforce, retired persons) grow with the same rate (in this paper determined with respect to worker households. We assume that all age groups or generations (children and Workers’ savings is the difference between their total income and their actual consumption. equation, is based on their perception of disposable work-related income in the usual linear fashion. defines hours worked within firms and the government sector, assuming that there is no overtime government is defined in terms of the number of employed people. By contrast, the third equation Initial conditions with respect to these three cohorts of worker households are considered as given pension payments when retired (i.e., $\tilde{\alpha}^r \alpha_w \alpha^r$ being the number entitled for receiving pension payments, plus interest income on their saving deposits (which however is simply saved again by them).

We start the description of workers’ consumption and savings decision by distinguishing between labor income, unemployment benefits and pensions payments as income items behind workers’ consumption plans (all based on the participation rate $\alpha^w$), retired persons being given by $L^r = \alpha^r L$, with $\tilde{\alpha}^r L, \tilde{\alpha}^r = \alpha_w \alpha^r$ being the number entitled for receiving pension payments, plus interest income on their saving deposits (which however is simply saved again by them).

In the first equation of this module we provide the definition of the disposable income of workers, wages, unemployment benefits and pension payments, the first after taxes, and of their interest rate income after capital taxation. Next, the total employment of the workforce by firms and the government is defined in terms of the number of employed people. By contrast, the third equation defines hours worked within firms and the government sector, assuming that there is no overtime or undertime work in the government sector. The consumption function of workers, the fourth equation, is based on their perception of disposable work-related income in the usual linear fashion. Workers’ savings is the difference between their total income and their actual consumption.

The final two equations define the here still very simple demographic structure of our model with respect to worker households. We assume that all age groups or generations (children and juveniles, potential workforce, retired persons) grow with the same rate (in this paper determined by trend investment and its implicit workforce plus family recruitment activities) and that only the proportion $\alpha_w$ of the potential workforce $L$ belongs to the actual workforce $L$ and thus gets pension payments when retired (i.e., $\tilde{\alpha}^r = \alpha_w \alpha^r$ gives the basis for the payments of pensions). Initial conditions with respect to these three cohorts of worker households are considered as given and determine, together with the assumed uniform rates of population growth, the time profile of the portions of the people not in the potential workforce, since already too old. This sector is more advanced than traditional presentations of differentiated households and saving habits, since we consider unemployment benefits and pensions explicitly (and for example the provision of health care implicitly as part of the goods and services provided by the government sector), but needs further elaboration later on.

Next, we consider the other type of household of our model, the (pure) asset owners who desire to consume $C_c$ (goods and houses as supplied by firms through domestic production $Y$) at an amount that is growing exogenously at the rate $\tilde{\gamma}$ and which is thus in particular independent of their current nominal disposable income $Y_c^{Dn}$. The consumption decision is thus not an important decision for pure asset holders. Their nominal income diminished by the nominal value of their consumption $pC_c$ is then spent on the purchase of financial assets (two types of bonds and money) as well as on investment in housing supply (for worker households). Note here that the one good view of the production of the domestic good entails consumption goods proper and houses (both at nominal values) as supplied by firms through domestic production, and further calls for the provision of both goods. We start the description of workers’ consumption and savings decision by distinguishing between

$$\begin{align*}
Y_w^{Dn} &= w^n L^d + w^n (\alpha^w L - L^w) + w^r \tilde{\alpha}^r L + (1 - \tau_c) r B_w \\
Y_{ww} &= (1 - \tau_c) r B_w \\
L^w &= L_f^w + L_g^w \\
\alpha^g &= \alpha^w \alpha^r \\
pC_{wc}^g &= c_{wc}(q_h) Y_{ww}^g, \quad c_{wc}(q_h) = c_{wc}^o + c_1(q_h - q_{ho}),
\end{align*}$$

$$q_h = p_h/p = \text{residential services, price ratio}$$

$$\begin{align*}
pC_{wh}^g &= c_{wh}(q_h) Y_{ww}^g, \quad c_{wh}(q_h) = c_{wh}^o - c_1(q_h - q_{ho}), \\
pC_{wh} &= c_{wh}(q_h) Y_{ww}^g, \quad c_{wh}(q_h) = c_{wh}^o - c_1(q_h - q_{ho}), \\
L &= \hat{L}_r = \hat{L}_{k(ids)} = \tilde{\gamma} - m > 0, \quad (L(0), L_r(0) = \alpha^r L(0), L_{k(ids)}(0) \text{ given})
\end{align*}$$
2b. Households (Asset-Holders, flow-consistency assumed):

\[
Y_{c}^{Dn} = (1 - \tau_c)[\rho^p K + B_f + rB_c + B'_c + \rho_h pK_h], \quad B'_c = B_f + B'_g
\]  
(4.17)

\[
\dot{C}_c = \ddot{\gamma}
\]  
(4.18)

\[
S^m_c = Y_{c}^{Dn} - pC_c
\]  
(4.19)

\[
C^s_h = \alpha_h K_h \quad [C^d_h = \ldots \; \text{see module 2a}]
\]  
(4.20)

\[
g_h = \left( \frac{I_h}{K_h} \right)^d = \alpha_{ph}(\rho_h - (r_l - \pi^c)) - \alpha_{rh}(r^l - r^*) + \alpha_{uh}\left( \frac{C^d_{wh}}{C^s_h} - \bar{u}_h \right) + \ddot{\gamma} + \delta_h
\]  
(4.21)

\[
\hat{K}_h = g_h - \delta_h
\]  
(4.22)

4.3 Firms

In module 3 of the model we describe the sector of firms, whose planned investment demand is assumed to be always served, just as all other consumption and investment plans. We thus assume for the short-run of the model that it is of a Keynesian nature since aggregate demand is never rationed, due to the existence of excess capacities, inventories, overtime work and other buffers that exist in real market economies. There is thus only one regime possible, the Keynesian one, for the short-run of the model, while supply side forces come to the surface only in the medium and the long run of the model. Up to certain extreme episodes in history this may be the appropriate modeling strategy for the macro-level of a market economy.

Firms (Technology, Production, Employment and Investment)

\[
Y^p = y^p(k_g)K, \quad y^p(k_g) = y^p_0 + y_1(k_g - k_{g0}), k_g = K_g/K
\]  
(4.23)

\[
\dot{\bar{l}}_y = -m, \quad l_y = L^d_f/Y, \; m > 0
\]  
(4.24)

\[
u = Y/Y^p \in (0, 1)
\]  
(4.25)

\[
\ddot{L}_f = \beta_{lf}(L^d_f/L^w_f - \bar{u}_f^w) + \bar{\gamma} - m, \quad \bar{u}_f^w \in (0, 1)
\]  
(4.26)

\[
g_k = I/K = \alpha_p(\rho^g - (r_l - \pi^c)) + \alpha_u(u - \bar{u}) + \ddot{\gamma} + \delta_f
\]  
(4.27)

\[
Y^e_f = Y - Y^e = \mathcal{I}
\]  
(4.28)

\[
S^me_f = p_g Y^e_f
\]  
(4.29)

\[
p_l \dot{B}_f^l = p_l - p\delta_f K + p_g(\dot{N} - \mathcal{I})
\]  
(4.30)

\[
I^a = I + \dot{N}
\]  
(4.31)

\[
\dot{K} = g_k - \delta_f
\]  
(4.32)

We assume for reasons of simplicity a fixed proportions technology with output-employment ratio $1/l_y$ and potential output-capital ratio $y^p$. Labor productivity $z = 1/l_y$ is growing at the constant rate $m$, which together with the given potential-output capital ratio suggests that technological change in this model is exogenous and of neutral Harrod type.

Note however that Kaldor’s stylized fact of a steady output-capital ratio in our view is based on statistics which neglects product innovation, i.e., that for example "hardisks" have become smaller and smaller in size and weight, but larger and larger in their capacity. Implicitly, the above assumption on potential output therefore contains the possibility that this ratio is rising significantly over time if quality changes were measured in such a quantity in an appropriate way. The model therefore can cover process as well as product innovation in its empirical applications.
Adaptive Revisions of Expectations

Production of Firms
Aggregate Goods Demand
Change in Expected Sales of Firms
Harrodian Fixed Investment Accelerator

Figure 1: Harrod’s Investment Accelerator: Multiplier-accelerator instability.

The output level $Y$ actually produced by firms will be provided in a later module by a Metzlerian output-inventory adjustment mechanism. Depending on this output level we define the rate of capacity utilization $u$ and the employment $L_f^d$ of the workforce employed by firms, which in the short-run is assumed to supply any amount demanded by firms through over- or under-time work.

For the adjustment of the workforce of firms we assume as given a normal rate of employment $\bar{u}_w$ of the workforce, which in principle could be set to unity, representing the normal hours worked by the workforce $L_f^w$ currently employed by firms. Due to absenteeism however, the hours supplied by the employed under normal conditions will be less than the norm $L_f^w$, and is here represented explicitly by the benchmark level $\bar{u}_w \in (0, 1]$, separating over-time from under-time work caused by fluctuating aggregate demand, expected sales and adjusting output levels. The number of workers $L_f^w$ employed by firms is adjusted by some sort of Okun’s Law with speed $\beta_l$ according to the over- or under-time work $L_f^d - \bar{u}_w L_f^w$ they experience, augmented by a term that accounts for trend growth (always set equal to the trend growth rate $\bar{\gamma}$ in firms’ investment decision – to which all other trend growth terms adjust with infinite speed here). The intended rate of investment, finally $I^d/K$ is driven by two forces in this module of the model, Goodwinian profitability and Harrodian capacity utilization. We assume that potential output depends positively on the stock of "infrastructure" $k_g$.

4.4 The government

In the next module 4 we describe the public sector of the economy in a way that allows for government debt in the steady state and for a monetary policy that adjusts the rate of interest on short-term bonds in view of the level of the long-term rate of interest, the domestic rate of inflation as compared to a target level and the excess activity of firms.
4. Government (Fiscal and Monetary Authority):

\[
T^n = (\tau_w + \tau_{wp})wL^d + \tau_{fp}wlyY + \tau_{pv}g(C_{wc} + C_c + G)
\]

\[
+ \tau_c[p^g pK + \rho_h pK_h + rB + B^l] - w^u(\alpha^w L - L^w) - w^v \hat{\alpha}^v L - wL^d
\]

\[
G = g_y \bar{u}Yp(k_y) - g_p(Y_e - Y_e^o) = G_i - G_{bc},
\]

where \( G_i = g_y \bar{u}Yp = g_y K, \) \( g_y = g_y0 - g_y1(d - do), \) \( d = \frac{B_g + p_t B^l_t}{Y_e} \)

\[
L^w_g = l_y g_y \bar{u}Yp
\]

\[
\dot{r} = \beta_{rt}(r_t - r^*) + \beta_{pr}(\hat{p} - \pi^*) + \beta_{ur}(u - \bar{u}), \quad \dot{B}_h = \dot{M}_c
\]

\[
\dot{\tau}_{fp} = \beta_{d\tau_{fp}}(\frac{B + p_t B^l}{Y_e} - d_o) - \beta_{d\tau_{fp}}(\tau_{fp} - \tau_{fpo})
\]

\[
S^n_g = T^n - pG - (1 + \tau_c)(rB + B^l), \quad \tau_c(rB + B^l) = "public aid" \text{ expenditure}
\]

\[
\dot{B} = -\alpha_{fp}S^n_g
\]

\[
p_t \dot{B}^l = -(1 - \alpha_{fp})S^n_g
\]

\[
\dot{K}_g = \alpha_g g_y \bar{u}Yp - \delta_y K_g
\]
to the provision of services we in addition assume – in contrast to the sector of firms – that there is no overtime work in the government sector. From the expenditures for goods and services, the interest payments and the transfers made by the government we obtain the savings of the government sector by deducting the sum of these items from $T^n$, the sum of the received tax payments. These savings will in general be negative in our investigations and thus imply debt financing. Government allocates its debt financing needs in nominal terms in constant proportions to short- and long-term debt and can always realize its intended debt financing due to the flow consistency requirements of such macro-dynamical models.

The contribution of firms to payroll taxation is adjusted towards the objective of a given debt target, and this if needed in a different way in the case of a rising or falling ratio $d$. The current debt to (expected) sales ratio $d$ determines the government expenditure ratio $g_y$ in a negative way. By contrast, we have already assumed that the stock of public capital $k_g$, where the rate of change is given by the last equation in this module, exhibits a positive influence on the potential output capital ratio $y^p$.

Keynesian business cycle policy is characterized by the minus sign in front of the parameter $\beta_{fp}$. They (wisely enough) reduce government expenditures in good states of the economy and increases them in bad states (in order to neutralize the pro-cyclical behavior in the private sector), where it may depend on the model builder whether stress will be put on the labor market or on the performance of firms as measured by their rate of capacity utilization. The big question is of course whether undamped private sector business cycle fluctuations can be damped by the intervention of the assumed fiscal and monetary policy rules.

### 4.5 Quantity and price adjustment processes on the firm level

We now come to the description of the dynamics of quantities (module 5a) and prices (module 5b). Module 5a of the model basically describes a Metzlerian inventory adjustment process for the goods produced by firms (an advanced form of a Keynesian multiplier dynamics). Module 5b then describes the nominal price adjustments in the goods and in the labor market, as well as the adjustment of long-term inflationary expectations $\pi^c$, understood as measuring the inflationary climate in which the economy is operating.

#### 5a. Quantity Adjustments in the Production of the Domestic Good

\[
\begin{align*}
Y^e &\neq Y^d = C^d_{wc} + C_c + I_h + I + G \\
I^{na} &= pI + pI_h + p_y \dot{N} \\
N^d &= \alpha_n d Y^e \\
I &= \beta_n (N^d - N) + \gamma N^d \\
Y &= Y^e + I \\
\dot{Y}^e &= \beta_{yp} (Y^d / Y^e - 1) + \dot{\gamma} \\
\dot{N} &= Y - Y^d
\end{align*}
\]

In this simple Metzlerian approach to goods market disequilibrium we assume that the output decisions $Y$ of firms are based on expected sales $Y^e$ and intended inventory changes $I$. The intended inventory changes in turn are based on the desired inventory level $N^d$ of firms assumed to be proportional to their expected sales. Inventories are then adjusted according to the discrepancy $N^d - N$ between desired and actual inventories with speed $\beta_n$, the inventory accelerator mechanism, again augmented by a term that accounts for trend growth. Actual inventory changes are given by output minus aggregate demand (which in this Keynesian approach is always served). We ignore here the possibility that inventories may become exhausted, which would provide a situation of rationing with respect to goods demand. The last equation of this module of the model provides the adjustment mechanism for sales expectations $Y^e$ which are assumed to follow observed domestic...
aggregate demand in an adaptive fashion, again augmented by a term that accounts for trend growth.

This module of the model basically represents a refined dynamic multiplier story to the extent that output adjustment towards aggregate demand is not represented by only one dynamic equation, but augmented by inventory adjustments that such a process entails and by the assumption that aggregate demand is not perfectly foreseen. We know that the dynamic multiplier is unstable when the marginal propensity to spend is larger than one, as in the famous Kaldor (1940) trade cycle model, a condition which is here slightly more difficult to establish due to the distinction between output, expected demand and aggregate demand. In addition we may now also have instability due to the Metzlerian inventory adjustment process, which if sufficiently fast also establishes a positive feedback chain between output, expected demand and aggregate demand.

These are the basic pure quantity adjustment processes of our Keynesian macrodynamics. A further and final one, the Harrodian mechanism of unstable warranted growth – is shown in figure 1 of the firm sector and is working through the investment function of our model.

Next we consider the wage-price spiral of the model. This type of dynamics represents an important module of the present stage of modeling of an integrated treatment of short-, medium- and long-run behavior.

5b. Wage-Price Adjustment Equations, Expectations:

\[
\dot{w} = \beta_{ew}(k_g)\left(\frac{L^w}{\alpha_w L} - \bar{e}(k_g)\right) + \beta_{uw}(k_g)\left(\frac{L_f^d}{L_f^e} - \bar{u}^w_f\right) + \beta_{hw}\left(\frac{q_h}{q_o} - 1\right) - \beta_v(u - v_o) + \kappa_w\hat{p} + (1 - \kappa_w)\pi^c + m
\]

\[
\hat{e}'(k_g) > 0, \beta'_{ew}(k_g) < 0, \beta'_{uw}(k_g) < 0
\]

\[
\hat{p} = \beta_p(u - \bar{u}) + \kappa_p(\hat{w} - m) + (1 - \kappa_p)\pi^c, \quad p(0) \text{ as initial condition, see } d_0
\]

\[
\hat{\pi}^c = \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p} - \pi^c) + (1 - \alpha_{\pi^c})(\pi^* - \pi^c))
\]

\[
\hat{p}_h = \beta_{hp}\left(\frac{C_w}{C_h} - \bar{u}_h\right) + \pi^c
\]
market, measured by the deviation of the rate of employment from the NAIRU rate of employment \( \bar{e} \), and on labor demand pressure within the firms, measured by the degree of over- or undertime work compared to the normal worktime of the employed. Cost pressure for wage earners is measured by two related expression. Firstly, and on the one hand, we assume – in order to show that myopic perfect foresight is not at all a problem for Keynesian macroeconomics – that workers have perfect knowledge of the short-term evolution of price inflation, but use in addition, and on the other hand, on the basis of this knowledge, an inflation rate expression, \( \pi^c \), representing the inflationary climate in which the current inflation regime is operating. The inflationary climate variable \( \pi^c \) is thus a magnitude that is related to the medium-run and is assumed to be updated in the adaptive fashion shown in the last equation in this module. Cost pressure for workers is then measured as a weighted average of these two expressions for price inflation \( \hat{p} \) and \( \pi^c \), implying that workers and their unions look beyond the short-run (for \( \kappa_w < 1 \)) and thus take into account also the climate in which current inflation is evolving. This guarantees that the dynamics of the model is not heavily dependent on whether short-term expectations are perfect or not. We here simply save, by the assumption of myopic perfect foresight, another dynamic law that would describe the evolution of short-term expectations, without much change in the implied dynamics if these expectations are revised sufficiently fast. We finally state that the assumption of Harrod neutral technical change of rate \( m \) requires that this term has to be added to the right-hand side of the money wage dynamics in order to allow for a steady state solution later on. Wage claims thus include the observed change in labor productivity in a one to one fashion, called a complete productivity pass-through in the literature (not fully confirmed by empirical estimates in this very strict way). The wage bargaining process is mitigated in an economy with a higher level of infrastructure \( k_g \), since the role of corporatism is increased thereby.

Turning next to price inflation we assume (analogously to wage inflation) that it is also based on demand pressure here measured by the rate of capacity utilization \( u \) in its deviation from what firms conceive as normal capacity utilization \( \bar{u} \). Regarding cost pressure we assume again myopic perfect foresight, now of firms with respect to wage inflation, and form again a weighted average with the inflationary climate \( \pi^c \) characterizing the medium-run expectations of firms. Note the presence of a negative coefficient of \( m \) in this condition.
Figure 6 shows in this context an example of adverse or destabilizing Rose-effects, the other adverse Rose (1967) effect being given by the situation where all arguments in this figure are reversed. Below we summarize all four possibilities by way of their partial feedback chains. Note that we do not yet consider asymmetric Phillips curves, which implies that Rose adjustment patterns can be explained in terms of inflation as well as deflation.

The case of a destabilizing Rose or real wage effect shown in Figure 6 is based on the situation that wages are less flexible than prices (the wage-price is then called labor-market led) and that investment is more responsive to real wage changes than consumption (the well-known profit led case). In deflationary periods, we therefore have that real wages will indeed rise, instead of falling as is generally assumed, which implies a decline in aggregate demand and thus further depressed situation with further increases in the real wage, further reduction in economic activity and so on, i.e., a deflationary spiral will be established in this way.

Two empirical observations are here needed in order to prevent such a spiral, the first being that the typical situation may rather be one where wages are more flexible with respect to demand pressure than prices which implies an "normal" real wage adjustment, see Flaschel and Krolzig (2003) for such empirical estimates of wage and price flexibilities for the US-economy. The second observation is that this may apply only to cases of high economic activity, while the hierarchy of price and wage flexibilities is reversed in depressions (though an asymmetry in the money wage Phillips curve), leading to stable real wage adjustments in such situations, see Hoogenveen and Kuipers (2000) for an empirical investigation of this type.

The four partial Rose wage adjustment mechanisms in sum are:

\[
\frac{w}{p} \uparrow \begin{cases}
C \uparrow & Y^d, Y^e, Y \uparrow \quad w \uparrow \quad \frac{w}{p} \uparrow \\
C \uparrow & Y^d, Y^e, Y \uparrow \quad w \uparrow \quad \frac{w}{p} \downarrow \\
I \downarrow & Y^d, Y^e, Y \downarrow \quad w \downarrow \quad \frac{w}{p} \downarrow \\
I \downarrow & Y^d, Y^e, Y \downarrow \quad w \downarrow \quad \frac{w}{p} \uparrow
\end{cases}
\]

Normal Rose Effects: Rose effect

1a) Real wage increases (decreases) will be reversed in the case where they reduce (increase) economic activity when nominal wages respond stronger than the price level to the decrease (increase) in economic activity.

1b) Real wage increases (decreases) will be reversed in the case where they increase (reduce) economic activity when the wage level responds weaker than the price level to the increase (decrease) in economic activity.

Adverse Rose Effect:

2a) Real wage increases (decreases) will be further increased in the case where they reduce (increase) economic activity when the wage level responds weaker than the price level to the decrease (increase) in economic activity.

2b) Real wage increases (decreases) will be further increased in the case where they increase (reduce) economic activity when the wage level responds stronger than the price level to the increase (decrease) in economic activity.

4.6 The dynamics of asset market prices and expectations

The sixth module lists the dynamic adjustment equations we assume to hold for the single flexible asset price of our model: long-term bonds, \( p_t \).

\[
\dot{p}_t = \beta_p (r_t + \pi_t - r), \quad dB_c + p_t dB_c^t = 0 \tag{4.54}
\]

\[
\dot{\pi}_{ls} = \beta_{\pi ls} (\dot{p}_l - \pi_{ls}) \tag{4.55}
\]

\[
\dot{\pi}_{lr} = \beta_{\pi lr} (\dot{p}_{lo} - \pi_{lr}) \tag{4.56}
\]

\[
\pi_t = \alpha_s \pi_{ls} + (1 - \alpha_s) \pi_{lr} \tag{4.57}
\]

Since we allow for only one risky financial asset, we simplify the dynamic portfolio approach on the basis of Walras' law of stocks to just one adjustment equation for the price of the considered perpetuities. Moreover we use the expectations formation approach of CFPS (2013)\(^2\) without the opinion dynamics investigated there in detail. We thus make use here of a fixed number of chartists and fundamentalist only, which are characterized by adaptive and regressive expectation formation, respectively. This approach is used here to indicate that the interest rate policy of the CB does not operated directly on the real part of the economy (as it is often assumed), but must channel itself through the financial markets first.

![Figure 5: Blanchard-type bond-market instability](image)

In the framework of an open economy, a Dornbusch-type Exchange Rate Dynamics can be formulated in a similar fashion by means of a delayed form of Uncovered Interest Rate adjustment process, leading from increasing expected depreciation of a currency to increasing expected returns of the foreign currency to increasing actual depreciation and from there again to further increases in expected depreciation of the domestic currency.

5 Collecting the Growth Laws of Motion

In order to study the dynamics of our disequilibrium growth model analytically and numerically it is necessary to reduce the equations of the model to intensive or per (value) unit of capital form. To simplify subsequent presentations of the dynamics of the model and also its steady state solution we assume in the remainder of this paper for the consumption of asset owners \(C_c(0) = 0\). Moreover we set \(\delta_f = \delta_h = \delta_y\). These two assumptions do not restrict the dynamical behavior of the system in an important way. We will also use the abbreviations \(\tilde{q}^r w = q^r (1 - \tau_w - \tau_{wp}) w, \quad \tilde{q}^u w = q^u (1 - \tau_w - \tau_{wp}) w\) in the following intensive form of the model.

Note also that the model is still based on a complete productivity pass-through into the wage share and the inflation rate, i.e., the rate of labor productivity growth does not yet appear as a parameter in the dynamics considered below.
5.1 The laws of motion

The Quantity Dynamics of Firms:
\begin{align*}
\dot{y}^e &= \beta_y(y^d/y^e - 1) + \gamma - (g_k - \delta) \quad [\text{Metzlerian Sales Expectations}] \\
\dot{v} &= y - y^d - (g_k - \delta)\nu \quad [\text{Metzlerian Inventory Adjustment}] \\
\dot{t}^{we} &= \beta_f(u_f^w - \bar{u}_f^w) + \gamma - (g_k - \delta), \quad t^{we} = L_f^u / l_y, \bar{u}_f^w = l^{de}_f / l^{we}_f \quad [\text{Okun’s Law}]
\end{align*}

The Growth Law of Industrial Wage Share \(v_f = \frac{w^{Ld}}{pY_f}\), Prices \(p, q_h = \frac{p_h}{p}\) and Inflation Climate \(\pi^c\):
\begin{align*}
\dot{v}_f &= \frac{(1 - \kappa_p)[\beta_{ew}(\frac{p^{we}}{\alpha^{we}T} - \bar{e}) + \beta_{uw}(u_f^w - \bar{u}_f^w) + \beta_{hw}(\frac{q_h}{q_o} - 1) - \beta_v(v - v_o)] - (1 - \kappa_w)[\beta_p(u - \bar{u})]}{1 - \kappa_p\kappa_w} \\
\dot{p} &= \frac{\beta_p(u - \bar{u}) + \kappa_p[\beta_{ew}(\frac{p^{we}}{\alpha^{we}T} - \bar{e}) + \beta_{uw}(u_f^w - \bar{u}_f^w) + \beta_{hw}(\frac{q_h}{q_o} - 1) - \beta_v(v - v_o)]}{1 - \kappa_p\kappa_w} + \pi^c \\
\dot{\pi}^c &= \beta_{\pi^c}(\alpha_{\pi^c}(\dot{p} - \pi^c) + (1 - \alpha_{\pi^c})(\pi^* - \pi^c)) \quad (5.5) \\
\dot{p}_h &= \beta_h(\frac{c_{wh}}{\alpha_hk_h} - \bar{u}_h) + \pi^c, \quad i.e. \\
\dot{q}_h &= \beta_h(\frac{c_{wh}}{\alpha_hk_h} - \bar{u}_h) + \pi^c - \dot{p} \quad (5.6)
\end{align*}

Asset Prices and Medium-run Expectations:
\begin{align*}
\dot{p}_t &= \beta_p[r_l + \pi_t - r] = -\dot{r}_t \quad [\text{Interest-spread driven Bond-price Dynamics}] \\
\dot{\pi}_{ls} &= \beta_{\pi_t}(\dot{p}_t - \pi_{ls}) \quad [\text{”Naive” Expectations}] \\
\dot{\pi}_{lr} &= \beta_{\pi_t}(\dot{p}_o - \pi_{lr}) \quad [\text{”Rational” Expectations}] \\
\pi_t &= \alpha_s\pi_{ls} + (1 - \alpha_s)\pi_{lr} \quad [\text{Average Expectations}] 
\end{align*}

Growth Dynamics:
\begin{align*}
\dot{e} &= \bar{e} - (g_k - \delta), \quad \bar{e} = \frac{L/l_y}{K} = \frac{L/L^d}{K/Y} \quad [\text{Labor Intensity}] \\
\dot{k}_h &= g_h - \delta - (g_k - \delta) \quad [\text{Housing Stock}] \\
\dot{k}_g &= \alpha_g g_y \bar{u}^p - (g_h - \delta)k_g \quad [\text{Public Capital Stock}] 
\end{align*}

Monetary and Fiscal Policy Rules:
\begin{align*}
\dot{r} &= \beta_{rfr}(r_l - \pi^*) + \beta_{qr}(\frac{q_h}{q_o} - 1) + \beta_{pr}(\dot{p} - \pi^*) + \beta_{ur}(u - \bar{u}) \quad [\text{Interest Rate Policy}] \quad (5.13) \\
\dot{\tau}_{fp} &= \beta_{d:\tau fp}(\frac{b + p_i b_i}{y^e} - d_o) - \beta_{d:\tau fp}(\tau_{fp} - \tau_{fp0}) \quad [\text{Payroll Tax Policy}] \\
s_g^n &= t_g - g - (rb + b), \quad s_{g0}^n = t_0 - g_0 - (r_o b_o + b'_0) \\
t &= (1 - q^n)v_f y + \tau_f v_f y + \tau_c(v^d c_c + c + g) + \tau_c[p_f g + \rho_h k_h] \\
g &= g_y \bar{u}^p - g_p(y - y_o), \quad g_y = g_{y0} - g_{y1}(d - d_o)
\end{align*}

Government Debt Accumulation: \(d = (b + p_i b_i)/y^e, b = \frac{B}{pK}, b_l = \frac{B}{pK}\)
The above laws of motion do not yet form an autonomous system of differential equations, but must be augmented by certain identities and algebraic equations about macro-economic behavior in order to become a complete Keynesian model of the working of the macro-economy.

Supplementing Static Relationships for the Laws of Motion of the Baseline Model:

\[
\begin{align*}
    y^d &= c_{wc} + c_e(0) + g_k + g_h k_h + g_y \bar{y} p - g_p (y^e - y^e_o) \\
    y_{ww}^D &= q^n v_f l^{we} + q^n v_f (\alpha w l^e - l^{we}) + q^n v_f \bar{\alpha}^e l^e \\
    y &= y^e + \beta_n (\alpha_n s y^e - \nu) + \bar{\gamma} \alpha_n s y^e \\
    c_{wc}^g &= c_{wc}(q_h) y_{ww}^D \\
    c_{wh}^g &= c_{wh}(q_h) y_{ww}^D, \text{ in terms of goods, just as supply } \alpha_h K_h \\
    \rho^{eg} &= y^e / (1 + \tau_v) - (1 + \tau_f) y_f - \delta \\
    \rho_h &= q_h c_{wh}^g / k_h - \delta \\
    g_k &= \alpha_p (\rho^{eg} - (r_l - \pi^e)) - \alpha_r (r_l^l - r^e) + \alpha_u (u - \bar{u}) + \bar{\gamma} + \delta \\
    g_h &= \alpha_p h (\rho_h - (r_l - \pi^e)) - \alpha_r h (r_l^l - r^e) + \alpha_u h (\tilde{c}_h / \alpha_h k_h - \bar{u}_h) + \bar{\gamma} + \delta \\
    d &= \frac{b + p_t b_t}{y^e}, \quad d_0(t) = d(t) / p(0)!
\end{align*}
\]

The logic of Keynes’ approach to macro-statics (and macro-dynamics in the chapter “Notes on the Trade Cycle” of his “General Theory”) is summarized in the following diagram which shows the assumed market hierarchy of his theory, the repercussions this theory allows for and the impact of fiscal and monetary policy, nowadays often formulated as fiscal and monetary policy rules.

The diagram in particular shows that the argument that nominal wage decreases immediately imply price level decreases must be considered as rather naive from his perspective, to say the least. Note in this respect that the contributions by Goodwin (1967) and Rose (1967) have significantly enlarged the perspective of Keynesian macro-dynamics due to the complex working of the wage-price spiral their approaches have led us to.

Note also that the contribution by Metzler, his completion of the dynamic multiplier story, is a compelling one, since the dynamic multiplier simply ignores what happens to inventories when the goods market is not in equilibrium. By contrast, assuming that it is always in IS-equilibrium in our view represents an assumption which is hard to swallow.
Traditional Keynesian Theory: Summary

Market Hierarchies and Supply Side Features

Feedback Mechanisms

FEEDBACK POLICY RULES

Asset Markets

$ r, r_1, ...$

Investment

Dornbusch exchange rate dynamics

Blanchard equity and bond dynamics

Money Supply Rule

Taylor Interest Rate Rule

FISCAL POLICY RULES

Goods Markets

Saving, Investment propensities

Fisher and Pigou effect

Debt

Metzlerian expected sales inventory adjustments

Capacity effect on I

Expected medium-run inflation

Mundell effect

Fiscal Policy Rules

Capacity effect of I

Labor Markets

Wage price spiral

Real wage dynamics

Wage inflation

Production function

How dominant is the downward influence? How strong are the repercussions?

How dominant are the supply-side dynamics?

Figure 6: Keynes’ causal nexus and his repercussion analysis
6 The ”Balanced Growth Reference Path” of the model

In this section of the paper we show that there is a uniquely determined, economically meaningful balanced growth path or steady state solution of our model which provides us with a useful reference path for the dynamical evolutions over time the model implies, which may or may not converge to this steady state reference solution.

The first set of steady state conditions presented below concerns the growth rates of our economy:

\[ g_{k_o} = g_{d_{k_o}} = \bar{\gamma} + \delta \]
\[ g_{h_o} = g_{d_{h_o}} = \bar{\gamma} + \delta \]

These equations state that capital (and also output) will grow with the external rate \( \bar{\gamma} \), to which also the natural rate of growth of the working population will adjust (in the present model through instantaneously fast ”migration” processes, giving rise to their growth rate \( \bar{\gamma}(k_{go}) - m \)).

The next set of steady state conditions concerns inflation and expected inflation – for all prices and the capital gains that exist in our model which are all equalized, except for wage rates. Note that only the state variables of the model are numbered in the following set of steady state equations:

\[ \pi^* = \pi^c_o = \hat{p}_o = \hat{p}_{go} = \hat{p}_{ho} \] (6.1)
\[ r^* = r_{lo} = 1/p_{lo} = r_o \] (6.2)
\[ \pi_{ls} = 0 \] (6.3)
\[ \pi_{lp} = 0 \] (6.4)
\[ r^* - \pi^* = \rho_{o}^e = \rho_{ho} \]

The next block concerns the steady state determination of various quantities of the model. If one wants also \( y^e_o \) to depend on the relative size of the infrastructure \( k_{go} \) by way of a function \( f(\cdot) \), one has to solve the first of the following equations in a different way, by inverting the function \( f(k_{go})/k_{go} \) and applying it to the argument \( \frac{1}{\alpha g y_o} \) thereafter.

\[ k_{go} = \text{from } \alpha g y_o \bar{u} y^p_o = \bar{\gamma} k_{go}, \text{ i.e.: } \]
\[ k_{go} = \frac{\alpha g y_o \bar{u} y^p_o}{\bar{\gamma}} \] (6.5)
\[ y_o = \bar{u} y^p_o \]
\[ l^d_{fo} = l^w_{fo} = y_o \] (6.6)
\[ l^w_{go} = g_{yo} y_o \]
\[ l^w_o = l^w_{fo} + l^w_{go} = (1 + g_{yo}) y_o \]
\[ l^w_o = l^w_{fo} / (\alpha^w e_o) \]
\[ [e_o = \bar{e}(k_{go})] \] (6.7)

We clearly see how government can influence the size of its infrastructure relative to the capital stock of firms. Moreover it can create in this way a more tranquil scenario for wage negotiations, in particular concerning the level of the so-called NAIRU.

Further steady state relationships on the side of quantities are:

\[ y^e_o = \frac{y_o}{1 + \bar{\gamma} \alpha_{n,d}} \] (6.8)
\[ \nu_o = \alpha_{n,d} y^e_o \] (6.9)
\[ y^d_o = c^q_{wco} y^D_{wwo} + \bar{\gamma} + \delta + (\bar{\gamma} + \delta) \frac{c^q_{who} y^D_{wwo}}{\bar{u}_h \alpha_h} + g_{yo} y_o \]
\[ = (c^q_{wco} + (\bar{\gamma} + \delta) \frac{c^q_{who}}{\bar{u}_h \alpha_h}) y^D_{wwo} + \bar{\gamma} + \delta + g_{yo} y_o = y^e_o \rightarrow \]
We see that a variety of parameters influence the relative level of the disposable wage-related income of worker households which is determined in the market for goods and there in fact from the side of uses, not resources. We in particular see that increasing long-run government expenditure does not have a positive influence on this income. There is thus from this angle no ”free lunch” for the creation of public infrastructure, which however may increase $y^e$ via $y^p$, if the linear function assumed in module 3 is suitably modified.

Similar effects hold for the wage share in GDP of workers in the private sector of the economy, which is dependent on all the sources of worker households’s incomes. The minus sign in front of $\tilde{q}^u$ is dominated by the first appearance of $\tilde{q}^u$ in the denominator below and thus not giving rise to a positive impact of this parameter on $v_{fo}$. The negative impact of unemployment benefits on the share of wages may also be small.

Increasing the tax rates on workers’ income influences the gross wage share of the workforce of firms in a positive way, while it has a negative effect on their net wage income $q^n v_{fo}$. And a higher level of the ratio $k_g$ can improve their wage share if it really has the effect of making the bargaining process more moderate.

We see that an increase in the value added tax rate and the long-run interest rate decreases the payroll tax burden for firms. The determination of the relative size of the housing stock is as expected.

Note again that the fraction $y^{D\text{wwo}}_{y}$ is the share of work-related incomes in gross output, while $v_{fo}$ is the share of wages paid by firms per output unit and is thus also measuring the unit wage costs of firms. One has to take note of the fact that the logic of steady states is not mirroring the logic of the Keynesian business cycles of the model, since decreases in the consumption coefficients imply increases of the above income fraction. Note however that the goods consumption of workers plus gross investment in housing per unit of capital remain unchanged in such cases.

An isolate decrease of $c^g_{\text{wco}}$ must therefore result in a decrease of $c^g_{\text{wco}} y^{D\text{wwo}}_{y}$ and an increase of $c^g_{\text{wco}} y^{D\text{wwo}}_{y}$ and thus a shift from goods consumption to ”flat let for rent” consumption in the long-run, with no definite conclusion concerning the total consumption effect. An increasing portion of pensioners $\alpha^r$ works on the balanced growth path via $v_{fo}$ and $t_o$, see below. It affects $v_{fo}$ negatively and increases public debt without much consequences as long as the economy remains stable.
And for the aggregate of government bonds we finally get (per unit of \( pK \)):

\[
(\bar{\gamma} + \pi^*)(b_o + b'_o/r^*) = -s_{go} = g_o - t_o + r^*(b_o + b'_o/r^*), \quad \text{i.e.}
\]

\[
b_o + b'_o/r^* = \frac{g_o - t_o}{\bar{\gamma} + \pi^* - r^*}, \quad \text{and thus have as result of debt financing:}
\]

\[
d_o = \frac{g_o - t_o}{(\bar{\gamma} + \pi^* - r^*)y_o}, \quad \text{and in the "actual world":}
\]

\[
d_0 = \frac{g_o - t_o}{(\bar{\gamma} + \pi^* - r^*)p(0)y_o}
\]

\[
t_o = (1 - q^n)\nu_f(k_g)y_o + \tau_{fp} v_f(k_g)y_o + \tau_v(c_{wc}(q_{ho})y^D_{wwo} + g_o) + \tau_c[(r^* - \pi^*)(1 + k_{ho})]
\]

\[- [\tilde{q}^n u_f(k_g)(\alpha w l^w_e - l^w e) + \tilde{q}^r v_f(k_g)\tilde{\alpha} r l^e_o + q^n v_f(k_g)g_{ho}y_o]
\]

\[
= [(1 - q^n)y_o + \tau_{fp}y_o - [\tilde{q}^n((1 + g_{ho}y_o)\frac{1 - e_o}{e_o}) + \tilde{q}^r \tilde{\alpha} r (1 + g_{ho})y_o + q^n g_{ho}y_o]v_f(k_g)
\]

\[
+ \tau_v(c_{wc}(q_{ho})y^D_{wwo} + g_o) + \tau_c[(r^* - \pi^*)(1 + k_{ho})]
\]

\[
g_o = g_{ho}\tilde{\alpha} y^D_{wwo}
\]

from which the individual distribution of bonds (between workers and pure asset holders) can be derived if this is desired.

The tax to capital-stock ratio \( t_o \) is a fairly complicated expression, due to the encompassing tax and transfer system that characterizes the considered economy, where \( \tau_w, \tau_{wp}, \tau_c, \tau_v \) are exogenously given and \( \tau_{fp} \) endogenously (as well as export and import taxation).

Summing up – and this conclusion holds due to the admitted neoclassical or Friedmanian supply side influences on the long-run output-capital ratio \( y_o \) and the long-run employment rate \( e_o \) – we have that the most effective way to increase the wage-related share of incomes \( y^D_{wwo} \) is to increase the steady state level of the stock ratio \( k_g \), the size of the public capital stock relative to the industrial capital stock, because of its assumed impact on the potential output of firms (and also on taxes per unit of capital).

Such an assumption is a very natural one and also often assumed as a positive externality in the environment of neoclassical production functions when issues of for example endogenous growth are investigated from the perspective of policy making.

We conclude that an adequate distribution of income between capital and labor in capitalist economy that can be considered as an advanced one demands an advanced government sector with an advanced system of public investments in all sorts of things in order to create the frame within which capitalism can develop his innovative potential without endangering a social structure as we have experienced it in the property phase after World War II.
7 Numerical investigations

7.1 "Barebone" Capitalism: A core case, and also a case of "Government-funded" People’s Capitalism

We here collect the equations\(^3\) of the model which characterize a type of "barebone capitalism," lacking any supply of public goods and services, a concept comparable to a "barebone computer". The positive influence of the building-up of an infrastructure by the government remains totally untapped here. We here consider inflation-free steady states for simplicity only.

\[
\begin{align*}
\dot{y}^c &= \beta y^c(y^d/y^c - 1) + \gamma - (g_k - \delta) \\
\dot{v} &= y - y^d - (g_k - \delta)v \\
\dot{\nu}^w &= \beta f(u^w_f - \bar{u}^w_f) + \gamma - (g_k - \delta), \quad u^w_f = \nu^w_f/\nu \\
\dot{v}_f &= \frac{1}{1 - \kappa_p\kappa_w}[(1 - \kappa_p)\beta_{ew}(k_g)(\frac{\nu^w_e}{\nu^e e} - \bar{e}(k_g)) + \beta_{uw}(k_g)(u^w_f - \bar{u}^w_f) + \beta_{hw}(q_h/q_o - 1) - \beta_v(v - v_o)] \\
&\quad - (1 - \kappa_w)\beta_p(u - \bar{u}), \quad v_f = \frac{wL^d_f}{pY} \\
\dot{\hat{p}} &= \frac{\beta_p(u - \bar{u}) + \kappa_p[\beta_{ew}(k_g)(\frac{\nu^w_e}{\nu^e e} - \bar{e}(k_g)) + \beta_{uw}(k_g)(u^w_f - \bar{u}^w_f) + \beta_{hw}(q_h/q_o - 1) - \beta_v(v - v_o)]}{1 - \kappa_p\kappa_w} + \pi^c \\
\hat{\pi}_c &= \beta_{\pi c}(\alpha_{\pi c}(\hat{p} - \pi^c) + (1 - \alpha_{\pi c})(0 - \pi^c)) \\
\hat{q}_h &= \beta h(\frac{c^g_{wh}}{\alpha_{kh} k_h} - \bar{u}_h) + \pi^c - \hat{p} \\
\hat{p}_t &= \beta p_t [r_t + \pi_t - r^*] = -\bar{r}_t, \quad \pi_t = \alpha_s \pi_{ts} + (1 - \alpha_s)\pi_{t'} \\
\hat{\pi}_{ts} &= \beta_{\pi ts} (\hat{p}_t - \pi_{ts}) \\
\hat{\pi}_{t'} &= \beta_{\pi t'} (\hat{p}_o - \pi_{t'}) \\
\hat{i}^c &= \gamma - (g_k - \delta) \\
\hat{k}_h &= g_h - \delta - (g_k - \delta) \\
s^h_g &= \tau_{\beta b}(s^h_g - (g_k - \delta)b) \\
\hat{b} &= -\alpha_{\beta b} s^h_g - (g_k - \delta)b \\
\hat{b}' &= -\frac{(1 - \alpha_{\beta b})s^h_g}{p_t} - (g_k - \delta)b', \quad d_0 = (b_o + p_{i\beta}b')/p(0) \\
\hat{r} &= \beta_{\pi r}(r_t - r^*) + \beta_{qr}(q/q_o - 1) + \beta_{\pi r} \hat{p} + \beta_{ar}(u - \bar{u})
\end{align*}
\]
Supplementing Static Relationships:

\[ y = y^e + \beta_n (\alpha_n y^e - \nu) + \gamma_n y^e, \quad u = y^p (k_g) \]

\[ y^d = \beta_n y^e + \beta_p (\alpha_n y^e - \nu) + \gamma_n y^e, \quad u = y^p (k_g) \]

\[ y_{ww}^D = y_{fye} = y_{fy} \]

\[ c_{we}^g = c_{we} (q_h) y_{ww}^D \]

\[ c_{wh}^g = c_{wh} (q_h) y_{ww}^D \]

\[ \rho^g = y^e - (1 + \tau fp) v_f y - \delta \]

\[ \rho_h = q_h c_{h}^g / k_h - \delta \]

\[ g_k = \alpha_h (\rho^g - r^*) - \alpha_r (r_l - r^*) + \alpha_u (u - \bar{u}) + \gamma + \delta \]

\[ g_h = \alpha_h (\rho_h - r^*) - \alpha_r (r_l - r^*) + \alpha_u (c_{wh}^g / k_h - \bar{u_h}) + \gamma + \delta \]

\[ \tau fp = \frac{y^e - v_f o y_o - \delta - \rho^g}{v_f o y_o} \]

PARAMETERS (yeshock=1.02, TimeHorizon=50)

betqw = 1; betp = 1; betpic = 0.1;
bepl = 0.5; betpils = 0.2; betpilr = 0.2;
betye = 5; betn = 1; betlf = 0.5;
beth = 0.5;
alpnd = 0.1;
alppic = 0.5; alppils = 0.5;
alpw = 1; alpeh = 0.1;
alpuh = 0.3; alprhoh = 0.2; alprh = 0.2;
alpu = 0.5; alprho = 0.5;
kapp = 0.5; kapw = 0.5;
baru = 1; baruw = 1.0; baruh = 1; alph = 0.15;
bargam = 0.06; del = 0.1; yp0 = 1; yp1 = 0;
betuw0 = 0.4; betuw1 = 0;
betew0 = 0.4; betew1 = 0;
barew0 = 0.96; barew1 = 0;
cw0 = 0.6; cw1 = 0;
ch0 = 0.4; ch1 = 0;
betur = 0; betpr = 0; betrlr = 0; betqr = 0;
betd = 0; betdd = 0; gy0 = 0; gy1 = 0; alpg = 0; gp = 0;
tauv = 0; tauwp = 0; tauw = 0; tauc = 0;
rf = 0.05; tilqu = 0; tilqr = 0; tilalpr = 0; alpfp = 1;
We can see from the set of selected steady state values (all per unit of the capital stock) that the government runs a prime surplus (solely through collecting taxes from firms, at a fairly low rate) and buys and accumulates on this basis short-term bonds (like "gold", issued by the asset holders (workers do not save, due to the high level of residential prices, equal to the goods prices here). The government expenditure quota and the public stock are both zero. The simulation run, where sales expectations $y^e$ are shocked by a factor 1.02 and show that this type of economy exhibits stable adjustment processes. Note that the wage share is high due to the assumed low level of the interest rate (and the lack of any risk premium here).

The economy is profit-led if there holds:

$$ (1 + g_0)[c_w(q_h)(q^n - \tilde{q}^u) + (\tilde{q}^u \alpha^w + \tilde{q}^r \tilde{\alpha}^r)/(\alpha w e_o)] < \alpha \rho (1 + \tau_{fpo}). $$

It is wage led if the opposite inequality holds. In the presently considered barebone case, this boils down to the simple condition:

$$ 0.6 = c w_0 > \alpha \rho (1 + \tau_{fpo}) = 0.5 * 1.038976290 \approx 0.519, \quad i.e. $$

the economy is wage-led in this case. This corresponds to the anti-clockwise orientation of the goods-demand augmented distributive cycle, see the following figure top-left.

The phase diagrams plot (top-left to bottom-right) are: Wage share vs. the employment rate, and vs. the government’s stock of private bonds, the Metzlerian inventory adjustment process and finally the relative price $q_h$ against the stock variable $k_h$.

All partial cycle mechanisms are counter-clockwise in their orientation, the first is therefore not an example for the Goodwin/Marx model of the distributive cycle. Overshooting is relatively weak in the first three cycles, while the stock of dwellings keeps on rising for a longer while, even after the rental price has passed its maximum value already.

We next consider an example of this special choice of our general model which at first sight appeared to be a miscalculation:
Figure 7: Simple barebone capitalism with a prime surplus of the government
PARAMETERS

\[
\begin{align*}
\text{betqw} &= 1; \\
\text{bethe} &= 5; \\
\text{beth} &= 0.5; \\
\text{alpw} &= 1; \\
\text{alpeh} &= 0.1; \\
\text{alpu} &= 0.5; \\
\text{kapp} &= 0.5; \\
\text{kapw} &= 0.5; \\
\text{baru} &= 1; \\
\text{baruh} &= 1; \\
\text{bargam} &= 0.06; \\
\text{del} &= 0.1; \\
\text{yp0} &= 1; \\
\text{betu0} &= 0.4; \\
\text{betew0} &= 0.4; \\
\text{barew0} &= 0.96; \\
\text{cw0} &= 0.6; \\
\text{ch0} &= 0.3; \\
\text{taufpo} &= \text{yeo-vfo*baru*yp0-del-rhoego/(vfo*baru*yp0)};
\end{align*}
\]

The steady value of the wage share is now approximately 128 percent of the output of firms. We notice however that firms are subsidized by the government by a high negative “payroll” tax rate. And the government is now running a prime deficit and has accumulated a high level of debt. Otherwise it is doing nothing for the society. The balanced growth path is again a stable one, i.e., it is surrounded by centripetal forces as the next figure shows.

Notice that the stock of houses is supplying more residential area now (alph=1) which lets the steady state value of rental prices drop dramatically to \(q_{ho} = 0.15\). Workers now only spend \(c_w = cgwc + 0.15 * cgwh << ywwo\) and therefore can save a lot by buying short-term government debt which is used to finance firms. This is a situation comparable to Pasinetti’s people capitalism of the capital debate of the 1960’s. A crude form of capitalism may therefore look quite comfortable from a simple work-only perspective, but lacks public goods and services and thus any public infrastructure.
Figure 8: Credit-funded barebone capitalism: Convergence to an exceptional steady state
7.2 Socially-protected capitalism: Numerical investigation of an opposite "limit" case of the model

We now consider a case where the government intervenes heavily into the private sector, also by way of an anti-cyclical fiscal policy rule. Payroll taxes still remain fixed at their steady state value and the interest rate is pegged by the central bank to a low level.

PARAMETERS

\[
\begin{align*}
\text{betqw} &= 1; & \text{btep} &= 1; & \text{betpic} &= 0.1; \\
\text{betpl} &= 0.5; & \text{betpils} &= 0.2; & \text{betpi} &= 0.2; \\
\text{betye} &= 5; & \text{betn} &= 1; & \text{betlf} &= 0.5; \\
\text{beth} &= 0.5; \\
\text{alpnd} &= 0.1; & \text{alppic} &= 0.5; & \text{alppils} &= 0.5; \\
\text{alpw} &= 1; & \text{alpeh} &= 0.1; \\
\text{alpuh} &= 0.3; & \text{alprhoh} &= 0.2; & \text{alprh} &= 0.2; & \text{alph} &= 1; \\
\text{alpu} &= 0.5; & \text{alprho} &= 0.3; & \text{alprg} &= 0; \\
\text{kapp} &= 0.5; & \text{kapw} &= 0.5; & \text{baru} &= 1; & \text{baruw} &= 1; & \text{baru} &= 1; \\
\text{bargam} &= 0.06; & \text{del} &= 0.1; & \text{yp0} &= 1; & \text{yp1} &= 0; \\
\text{betuw0} &= 0.8; & \text{betuw1} &= 0; \\
\text{betew0} &= 0.8; & \text{betew1} &= 0; \\
\text{barew0} &= 0.96; & \text{barew1} &= 0; \\
\text{cw0} &= 0.6; & \text{cw1} &= 0; \\
\text{ch0} &= 0.3; & \text{ch1} &= 0; \\
\text{betur} &= 0; & \text{betpr} &= 0; & \text{betlr} &= 0; & \text{betq} &= 0; & \text{betd} &= 0; \\
\text{gy0} &= 0.4; & \text{gy1} &= 0.05; & \text{gp} &= 0.1; \\
\text{tauw} &= 0.2; & \text{tauwp} &= 0.1; & \text{tauw} &= 0.2; & \text{tauc} &= 0.5; \\
\text{rf} &= 0.03; & \text{tilqu} &= 0.8; & \text{tilqr} &= 0.65; & \text{tilalpr} &= 0.3; & \text{alp} &= 0.1; \\
\text{t} & \text{dtg} & \text{d} & \text{vf} & \text{taufp} & \text{gov/y} & \text{kg} \\
50 & -0.19277630 & 0.32123871 & 0.51094112 & 0.36693316 & 0.40292269 & 3.3384221
\end{align*}
\]

The government runs a prime deficit and accumulates public debt, still in short-term bonds solely. The wage share is at the lower end and taxed with 30 percent, but firms and asset holders are also strongly taxed. Moreover, the value added tax rate is high. Yet, the government expenditure quota is also high and the supply of public infrastructure more than three times as high compared to the private capital stock. Residential prices are very low \(q_{ho} = 0.13\) and workers consumption is \(c_w = (0.6 + 0.13 * 0.3) * ywwo \approx 0.64 * ywwo\) that is they save at a very high rate (or change their consumption pattern significantly).

The dynamics shown in the next figure exhibits strongly damped oscillation (stochastic shocks as always neglected) with some monotonic adjustment to balanced growth at a later stage. The anti-clockwise orientation is as in the case of barebone capitalism. We have added now at the bottom of the figure the interaction of the prime deficit with government debt \(d\) and also the one
The positive shock in the sales expectations of firms lowers the deficit instantaneously, but it starts rising immediately after the shock, with government debt passing its steady state level, before strongly stabilizing forces take over and drive the economy towards a level that is lower than steady state debt and accompanied by a somewhat higher prime deficit. Thereafter, a monotonic adjustment back to the steady state values of these two state variables takes place. Something opposite happens in the partial phase plane shown bottom-right, due to the anti-cyclical policy of the government.

We consider next a case where business cycles become nearly undamped, associated with a higher share of wages however.
PARAMETERS

betqw = 1; betp = 1; betpic = 0.1;
betpl = 0.5; betpils= 0.2; betpilr=0.2;
betye = 5; betn = 1; betlf = 0.5;
beth = 0.5;

alpnd = 0.1;
alppic = 0.5; alppils =0.5;
alpw= 1; alpeh = 0.1;
alpuh= 0.3; alprhoh = 0.2; alprh= 0.2; alph= 1;
alpu = 0.5; alprho = 0.5;

kapp = 0.5;
kapw = 0.5;
baru= 1; baruw= 1.0;
baruh= 1;

bargam = 0.06;
del = 0.1;
yp0 = 1; yp1= 0;

betuw0= 0.4; betuw1= 0;
betew0= 0.4; betew1= 0;
barew0= 0.96; barew1= 0;
cw0= 0.6; cw1= 0;
ch0= 0.3; ch1= 0;

betur=0; betpr= 0; betrlr = 0; betqr= 0; betd=0;

gy0=0.4; gy1=.062; gp=0;
tauv = 0.2; tauwp= 0.1; tauw= 0.2; tauc = 0.5; alpg=0.5;
rf = 0.03; tilqu = 0.8; tilqr=0.65; tilalpr=0; alpfp = 1;

t    dtg    d    vf    taufp    gov/y    kg
50  -0.19305402  0.32175670  0.65241069  0.070434870  0.40240000  3.3333333
Figure 10: Socially-protected capitalism: Instability through debt-dependent government expenditures and inactive fiscal policy: $g_{y1} = 0.062, g_{p} = 0$. 
### PARAMETERS

\[
\begin{align*}
\text{betqw} &= 1; & \text{btep} &= 1; & \text{betpic} &= 0.1; \\
\text{betpl} &= 0.5; & \text{betpils} &= 0.2; & \text{betpilr} &= 0.2; \\
\text{betye} &= 5; & \text{betn} &= 1; & \text{betlf} &= 0.5; \\
\text{beth} &= 0.5; \\
\text{alpnd} &= 0.1; & \\
\text{alppic} &= 0.5; & \text{alppils} &= 0.5; \\
\text{alpw} &= 1; & \text{alpeh} &= 0.1; \\
\text{alpuh} &= 0.3; & \text{alprhoh} &= 0.2; & \text{alprh} &= 0.2; & \text{alph} &= 1; \\
\text{alpu} &= 0.5; & \text{alprho} &= 0.5; \\
\text{kapp} &= 0.5; & \\
\text{kapw} &= 0.5; & \\
\text{baru} &= 1; & \text{baruw} &= 1.0; \\
\text{baru} &= 1; \\
\text{bargam} &= 0.06; & \\
\text{del} &= 0.1; & \\
\text{yp0} &= 1; & \text{yp1} &= 0; \\
\text{betuw0} &= 0.4; & \text{betuw1} &= 0; \\
\text{betew0} &= 0.4; & \text{betew1} &= 0; \\
\text{barew0} &= 0.96; & \text{barew1} &= 0; \\
\text{cw0} &= 0.6; & \text{cw1} &= 0; \\
\text{ch0} &= 0.3; & \text{ch1} &= 0; \\
\text{betur} &= 0; & \text{betpr} &= 0; & \text{betrlr} &= 0; & \text{betqr} &= 0; & \text{betd} &= 0.001; \\
\text{gy0} &= 0.4; & \text{gy1} &= 0.05; & \text{gp} &= 0; \\
\text{tau} &= 0.2; & \text{tauwp} &= 0.1; & \text{tauw} &= 0.2; & \text{tauc} &= 0.5; \\
\text{rf} &= 0.03; & \text{tilqu} &= 0.8; & \text{tilqr} &= 0.65; & \text{tilalpr} &= 0; \\
\text{alpfp} &= 1; & \text{alpg} &= 0.5; \\
\text{t} & \quad \text{dtg} & \quad \text{d} & \quad \text{vf} & \quad \text{taufp} & \quad \text{gov/y} & \quad \text{kg} \\
300 & -0.19305402 & 0.32175670 & 0.65241069 & 0.070434870 & 0.40240000 & 3.3333333
\end{align*}
\]
Figure 11: Convergence and after year "100" partial cumulative instability through a very weak adjusting, debt dependent payroll tax rule for firms: $\beta_d = 0.001$. 
PARAMETERS

betqw = 1; betp = 1; betpic = 0.1;
betpl = 0.5; betpils = 0.2; betpilr = 0.2;
betye = 5; betn = 1; betlf = 0.5;
beth = 0.5;
alpnd = 0.1;
alppic = 0.5; alppils = 0.5;
alpw = 1; alpeh = 0.1;
alpuh = 0.3; alprhoh = 0.2; alprh = 0.2; alph = 1;
alpu = 0.5; alprho = 0.5;
kapp = 0.5;
kapw = 0.5;
baru = 1; baruw = 1.0;
baruh = 1;
bargam = 0.06;
del = 0.1;
yp0 = 1; yp1 = 0;

betuw0 = 0.4; betuw1 = 0;
betew0 = 0.4; betew1 = 0;
barew0 = 0.96; barew1 = 0;
cw0 = 0.6; cw1 = 0;
ch0 = 0.3; ch1 = 0;

betur = 0; betpr = 0; betrlr = 0; betqr = 0; betd = -0.01;
gy0 = 0.4; gy1 = 0.05; gp = 0;
tauv = 0.2; tauwp = 0.1; tauw = 0.2; tauc = 0.5;
rf = 0.03; tilqu = 0.8; tilqr = 0.65; tilalpr = 0;
alpfp = 1; alpg = 0.5;

t dtg d vf taufp gov/y kg
3000 -0.19305402 0.32175670 0.65241069 0.070434870 0.40240000 3.3333333
Figure 12: Convergence, but still partial cumulative instability through a weak, now negatively debt-dependent payroll tax of firms: $betd = -0.01$. Time Horizon 3000 years!
We finally consider the impact of financial markets and the conduct of monetary policy on the real part of the economy.

**PARAMETERS**

\[
\begin{align*}
\text{betqw} &= 1; & \text{betp} &= 1; & \text{betpic} &= 0.1; \\
\text{betpl} &= 0.5; & \text{betpils} &= 0.2; & \text{betpilr} &= 0.2; \\
\text{betye} &= 5; & \text{betr} &= 1; & \text{betlf} &= 0.5; \\
\text{beth} &= 0.5; &
\end{align*}
\]

\[
\begin{align*}
\text{alpnd} &= 0.1; & \text{alppic} &= 0.5; & \text{alppils} &= 0.5; \\
\text{alpw} &= 1; & \text{alpeh} &= 0.1; &
\end{align*}
\]

\[
\begin{align*}
\text{alpuh} &= 0.3; & \text{alprhoh} &= 0.2; & \text{alprh} &= 0.2; & \text{alph} &= 1; \\
\text{alpu} &= 0.5; & \text{alprho} &= 0.5; & \text{alprg} &= 0.0001; \\
\text{kapp} &= 0.5; & &
\end{align*}
\]

\[
\begin{align*}
\text{kapw} &= 0.5; &
\end{align*}
\]

\[
\begin{align*}
\text{baru} &= 1; & \text{baruw} &= 1.0; & \text{baruh} &= 1; &
\end{align*}
\]

\[
\begin{align*}
\text{bargam} &= 0.06; & \text{del} &= 0.1; & \text{yp0} &= 1; & \text{yp1} &= 0; &
\end{align*}
\]

\[
\begin{align*}
\text{betuw0} &= 0.4; & \text{betuw1} &= 0; & \text{betew0} &= 0.4; & \text{betew1} &= 0; & \text{barew0} &= 0.96; & \text{barew1} &= 0; & \text{cw0} &= 0.6; & \text{cw1} &= 0; & \text{ch0} &= 0.3; & \text{ch1} &= 0; &
\end{align*}
\]

\[
\begin{align*}
\text{betur} &= 0.0001; & \text{betpr} &= 0; & \text{betrlr} &= 0.0001; & \text{betqr} &= 0; & \text{betd} &= -0.01; & \text{betdd} &= 0.5; & \text{gy0} &= 0.4; & \text{gy1} &= 0.05; & \text{gp} &= 0; & \text{tauw} &= 0.2; & \text{tauw} &= 0.1; & \text{tauw} &= 0.2; & \text{tauc} &= 0.5; & \text{rf} &= 0.03; & \text{tilqu} &= 0.8; & \text{tilqr} &= 0.65; & \text{tilalpr} &= 0; & \text{alpfp} &= 1; & \text{alpg} &= 0.5; &
\end{align*}
\]

The outcome of various simulation runs of this extension is not very convincing yet. Monetary policy is – if really operate, see the parameters here chosen – destabilizing. We conclude that we should follow Keynes (1936) suggestion which recommended that monetary policy should operate directly on the long end of the financial markets (as the ECB is doing it now).
Figure 13: Switching on the market for long-term bonds, but not the monetary policy rule
8 Conclusions

This paper has been explorative in nature as it was intended to explore in a first attempt the properties of the here proposed Keynesian model of monetary growth from the mathematical perspective primarily, testing so to speak the limits of the approach we are proposing. Nevertheless it, of course, exhibits and displays the essential partial feedback structures of Keynesian macroeconomic theory as they were stressed by James Tobin primarily, while the traditional Keynes effect is covered behind the working of the so-called Taylor interest rate policy rule. Its traditional counterpart, the stabilizing Pigou effect, and its opponent, the destabilizing Mundell effect, are however not yet present in the here considered model structure, since we do not consider wealth or real rate of interest effects in this model type (which thus remain to be integrated in future research). But even without these effects the task of creating a viable economic behavior through public policy intervention was not an easy one, due to the high dimensional nature of the considered laws of motion of this macro-dynamic approach.

In a companion paper we shall then reconsider the modules of the present hierarchically structured continuous-time model of Keynesian growth extended to the case of a small open economy reformulating and modifying the type of approach introduced Charpe, Chiarella, Flaschel and Semmler (2010) such that ”social protection” of worker households become the focus of interest. Their model was already sufficiently rich with respect to markets, sectors and agents in order to allow to capture the important details of actual macro-economies and was therefore well-suited to serve as the basis for modelling issues of choosing a social protection program for workers by the government which is effective and preserves the macro-efficiency of the economy.

Of primary interest was then the question how the many tax-, transfer- (unemployment benefits and pensions payments) and government-expenditure-parameters of this model could be used to improve the social protection of worker households, without loosing the efficiency of a well-performing labor market (with its partial modelling of Friedmanian supply side forces), and also without neglecting the creation of a sufficient ”infrastructure” for an modern educational system, well-equipped medicare and thoughtful care for the elderly, i.e., the corner-stones for the young, the labor market participants and the retired.

Concerning the topics just enumerated we have provided a wide range of numerical answers showing the macro-advantages of the development of an modern type of capitalist economy for the ”social protection” of workers, where high output-capital ratios and high productivity as well as work-related income growth was based on public investments into the ”infrastructure” of the country, various types of income transfers, and counter-cyclical fiscal and monetary policies. We have however also seen some obstacles in the promotion of such a development, preventing the creation of ideal situation of what is called a ”free lunch” by mainstream economics.

These aspects will be illustrated through numerous simulations of the laws of motion of the formulated macro-dynamical system in the sections that follow the detailed determination of the reference balanced growth path, used in this paper to start the dynamics with a situation, where the intensive form state variables are constant, but then disturbed at time t=1 by a unit-wage cost shock for example, with convergence to a new or the old balanced growth path thereafter. The reference balance growth path could therefore in particular be shown to be non-uniquely determined from the global perspective, due to nonlinearities though positive externalities as they were created by the public investment into the public capital stock run by the government.


Curriculum Vitae

Date/Place of Birth: September 30, 1943; Zittau/Saxony

Citizenship: German.

Marital Status: Married, no children.

Education:

1968 Diploma in Mathematics (Theme of the Thesis: The Calculus of Variations in the Large), Department of Mathematics, University of Bonn.


Awards and Grants:

1969 Felix Hausdorff Memorial Price (Theme of the Essay: Geometry of the Space of Closed Curves. Closed Geodesics.), Department of Mathematics, University of Bonn.

2006 Theodor Heuss Professor at the New School for Social Research, New York

2007/8 Opus Magnum Research Grant from the Fritz Thyssen / Volkswagen Foundations
Academic positions and visits:

1969-75  C1-Assistant, Department of Mathematics, University of Bonn.

1975-81  C1-Assistant, Department of Economics, Free University of Berlin.

1981-85  C2-Professor of Economics, Department of Economics, Free University of Berlin.

1985-2006  C3-Professor of Economics, Faculty of Economics, University of Bielefeld.

1996/97  Dean of the Faculty of Economics, University of Bielefeld.

Since 2000  Board Member of the ‘Center for Empirical Macroeconomics’, Bielefeld University

Since 2001/2  Member of the ‘Bielefeld Graduate School for Economics and Management’

1984  Visiting Professor, New School for Social Research, New York, Spring term

1985  Visiting Professor, New School for Social Research, New York, spring term

1985  Visiting Professor, Bonn University

1987  Visiting Professor, University of Vienna

1993  Research Project: Oscillations in Macroeconomic Dynamics.

1994  Fall Term, UTS Sydney. Financially supported by Deutschen Forschungsgemeinschaft and the UTS, invitation by Professor Carl Chiarella.


2001  Visiting Professor, University of Antwerp (UFSIA)

2005  Visiting Professor, Ca’ Foscari University, Venice (Sokrates-Erasmus Program)

2006  Theodor Heuss Professor at the New School for Social Research, New York

2012  Visiting Professor, University of Vienna (Sokrates-Erasmus Program)
Books (publications or forthcoming):

1. Early work:


2. Reconstructing Macroeconomics:


3. Real-Financial Market Interactions:


4. **Applied Macrodynamics:**


5. **Social Capital Accumulation and Social Evolution: Towards the modelling of a unified Goodwinian MKS System**


• Unbalanced Growth from a Balance Perspectice (with R. Araujo M.Charpe, C. Chiarella, A. Szczutkowski). *Forthcoming: Edward Elgar Publishing: Cheltenham Glos, UK. Forthcoming: 2020 (Opus Magnum Research Grant Funding by the Fritz Thyssen / Volkswagen Foundations is gratefully acknowledged).*


Papers (journal publications):


Papers in Edited Volumes:


Just published as joint work with Roberto, Jonathan, Reiner and Nils, 200 years after Marx was born (see Edward Elgar, 2018, in the bibliography in this open access publication):

As Heuss Professor at the New School for Social Research, NY, I used in the discussion of my Heuss lecture the sentence: "I am not interested in what Marx really meant, but only interested in what we can use him for today" (with very limited approval, if I remember correctly). And in the discussions in the lobby of the lecture hall after my talk I was made aware of the quotation (not being a philosopher): "Theory without empirics is empty. Empirics without theory is blind" from Immanuel Kant (1724-1804), which immediately convinced me, due to the many discussions with Reiner I had on the role of empirical research in the past.

I came into economics via joint seminars at our Mathematical Department of Bonn University starting 1973 with Werner Hildenbrand and others with a comparison of Debreu's *Theory of Value* with the Value Theory of Andras Bródy: *Proportions, Prices and Planning: A mathematical restatement of the labor theory of value*, 1970, a strictly empirically oriented input-output approach -- distinguishing capital stocks and sectoral flows from the empirical point of view -- in the spirit of Wassily Leontief, Nobel Laureate in Economics of 1973.


We show in the book the empirical validity of the Theory of Surplus Value of Marx's Capital Vol. I (if a very secondary -- for “Capital” not visible, since unsystematic -- "Marxian" uncertainty relationship between his general profit rate and the general actual price rate of profit is taken note of. An abstract mathematical proof of their secondary actual discrepancy should be an easy mathematical task, for any reasonable deviation between labor values and actual prices (normalized just as LVs).

We prove among others -- and demonstrate empirically in the case of Germany for an economy with 7 sectors including services (certainly not an econometrically over-parameterized point of departure, as compared to the barely applicable, since technology-based neoricardian theory) -- as basis for Marx's three sources of exploitation in production, the "Law of Falling Labor Content" under profitable capital-using, labor-saving technical change, and this in the general framework once attacked in Steedman's...

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1 See also https://www.marx-capital.eu/
book: "Marx after Sraffa", by using Stone’s SNA for open economies – due to our general view that sectoral disaggregation cannot go beyond a certain meso-level.2

"Steedman after Stone", the implicit message of our book, therefore shows that his critique of Marx was completely beside the point. The so-called "transformation problem" and its implied artificial identities, in particular between the average value and (for us) actual price rate of profit, resembles the "squaring of the circle" (there are no "invariance principles" in the social sciences!).

More than 100 years of discussion with their defenders were therefore by and large just a waste of time (with the so-called Marxists -- Marx denied in a letter to Engels to be one -- as well as with many Marxian Economists, John Roemer being one of the rare exceptions, due to his significant contributions to "Analytical Marxian Economics", though he considered these topics as outdated the last time I had the pleasure to meet him in NY in 2006 -- but can proved theorems become outdated if not meanwhile replaced by superior ones?).

A System of National Accounts is a theoretical construction of “real” concepts (definitions), behind the nominal magnitudes, we actually observe -- which leaves Marx’s Abstract Labor motivation of the measure of total labor costs to the philosophers -- concepts which we teach in any undergraduate course, though often with severe mistakes, when the SNA of Stone is not taken into account properly. A transformation of such real magnitudes back into the sphere of actual prices is therefore a meaningless task. The only alternative to the need for such a SNA is the statement: There is nothing "real" behind the nominal, certainly a view not shared by too many empirically working economists!

Moreover, the discussion of prices of production in Ricardo (the best general price theory existing at Marx’s time) is from today's perspective of a completely academic nature, where the empirics of our book indicates its blindness and earlier theoretical work of mine its emptiness -- compared to the many market forms we observe in reality, with in our book several stages in the manufacturing part of the economy, various types of service sectors, and now also the digital sectors. There is no general price theory whatsoever – in particular not one based on technological data, but only a chaotic mixture of partial market forms, with no overall tendency for equalizing profit rates -- between for example Windows X and the production of bananas.

We observe finally that Stone's derived IO tables, based on his alternative "commodity technology assumption", provide a simple way for the general construction of Sraffa's Standard Commodity, a construction which solely shifts an unavoidable non-linearity in the wage-profit-curve into another area (where it is hidden from the eyes of the uninformed

2 Or should we distinguish in a single enterprise in its capital stock each different chair and table they are using for example etc., in place of just its capital stock, maybe subdivided into machinery, work equipment and more – as if Economics can compete with microphysics. All data we can get are deflate or so-called real data, and not technological ones, and are thus price dependent in one way or another. This does not hurt the definition of total costs, if it allows for meaningful proposition concerning the dynamics of capitalism, since it is indeed just a definition in Stone (as well as in Marx’s Capital). But a general price theory -- following a unique principle -- runs into problems if its assumed “technological data” are market-price dependent.
There is thus also no solution for Ricardo’s search for an invariable measure of value.

In a forthcoming book on “Unbalanced Growth”, also published by EE, we add Keynes’ General Theory of Effective Demand and Financial Markets from various points of view to the Goodwin-type Marxian macroeconomic theory of the reserve army mechanism, augmented also by Keynes-Schumpeter aspects of animal spirits in a first attempt. We also reconsider and improve here Schumpeter’s foundation of modern democracy theory. We do this in view of the fifth long wave we are now living in (which increasingly is turning into an Orwellian nightmare). In a further future work published by EE: “Capitalism, Inclusive Growth, and Social Protection: Inherent Contradiction or Achievable Vision?” we, i.e. in fact my younger coauthors, finally consider the long-run tendencies and long waves of capitalism in more detail – the latter now better understood using David Gordon’s concept of “Social Structures of Capital Accumulation” in order to reconsider anew the vision of the EE-book of Flaschel and Luchtenberg (2012) for the construction of the pillars of a viable type of capitalism: Social Capitalism, where we argued that the social relations of capitalism should regulate its productive forces and not vice versa (as it is again the case in the fifth wave so far). This vision attempts to counteract the rise of façade democracies (“Big Brother” is watching, “faking” and using you). A return to nationalism as it existed before and caused WWII – on a more sophisticated level now of course is already on its way. Now fakenews have become the basic means to win elections – by dismantling thoroughly investigating media – and has meanwhile led us indeed into a phase of rapidly evolving variants of façade democracies, often just bringing crazy people into power who can be selected from a fairly limited class of candidates solely (“all men are equal, but some are more equal”).

Goodwin’s MKS-System – the basis of my contribution to the above works – if critically investigated further as point of departure by -- in the minimum -- appropriately educated researchers of the cosmopolitan society may be the right starting point for really understanding the dynamics of capitalism – dismantling the Ptolemaic approach of the ruling orthodoxy -- and may show us a way out of the current crisis wave of ruthless real and financial globalization, quality-destroying international wage-price competition coupled with the development of a very questionable type of nationalism at one and the same time – plus a variety of further disaster scenarios not to be touched upon here.

Summing up, I hope for the future generation that the current digital SSCA and the many worldwide problems ruthless capitalism has already started to accumulate in the preceding democratic wave, and extended thereafter, will not become the next catastrophic wave after the third, the martial wave, which Schumpeter did not accomplish to fully analyze in his famous reconsideration of Kondratieff waves.

Peter Flaschel

See the bibliography

A term coined by Habermas

not necessarily of course the opinion of my very valued and truly independently thinking rigorous co-researchers in this Marx@200 book -- which therefore is the result of various reflections on the usefulness of the LTV
Joint Work with Christian R. Proaño Acosta


Quito, capital of Ecuador, García Moreno street in the historic centre of the city. El Panecillo is seen in the background.

https://commons.wikimedia.org/wiki/
File:Quito_calle_Garc%C3%ADa_Moreno.jpg
‘You have to regulate capitalism, otherwise the criminals will dominate it’

Interview with Peter Flaschel

Peter Flaschel is Professor Emeritus at Bielefeld University, Germany. He holds a PhD degree in Mathematics and a Habilitation degree in Economics. He has extensively published on Classical Economics and Heterodox Macrodynamical Model Building. He was on numerous occasions Visiting Professor at the University of Technology, Sydney, and was invited in 2006 as Theodor Heuss Professor to the New School for Social Research, New York. He received an Opus Magnum Grant from the Fritz Thyssen/Volkswagen Foundations in 2007–2008.

Your first major work was on ‘Riemannian Hilbert manifolds’. Riemann and Hilbert are not economists but mathematicians. So therefore our first question would be: How did you become an economist?

Well, the – may I say so – leading guy in the residential community I was living in proposed to read Marx’ *Capital*, Volume I. We started reading Marx in 1972, when I still was assistant at the University of Bonn in the department of mathematics. But Marx was so fascinating that I decided on the next holiday trip to Corsica to read also Volumes II and III of *Capital*. And Volume III was of course a bit of a surprise to me, because everybody was so focused on the labour theory of value as presented in Volume I. In Volume III I had to realize that labour values are not immediately regulating the exchange ratios of commodities.

To a certain degree I lost interest in mathematics at that time, though in particular Riemannian Geometry and its application to Hilbert manifolds was a very beautiful theory for me. It also happened then that I told Werner Hildenbrand, Professor of Economics at the economics department in Bonn, about my interests in Marx. He responded, ‘Well, we have a much better theory of value’, and so we decided to have a joint seminar, reading Bródy on Marxian labour theory of value and Debreu’s theory of value. Carl Christian von Weizsäcker participated in this seminar, as well as important members of the department of economics at Bonn University. It was a very interesting seminar, but at the end Egbert Dierker concluded that its aim was not really reached, because many of the participating mathematicians became interested in Marx and not so much in Debreu.

How did you move on in your professional career? Did you get involved more and more in applying your mathematical tools to economics?

First I had to learn more matrix algebra, which I had of course dealt with in my undergraduate studies, but not followed up. And I started writing on Marx, because the problem of joint production within the labour theory of value was becoming interesting to me. And funnily enough I formulated a solution where I disentangled joint production into equal value proportions. But Carl Christian von Weizsäcker in a private conversation on a trip from Bielefeld to Bonn suggested to me that I should not take equal
proportions, but use relative sales values. I realized that this was a very good proposal and started writing a longer paper on the transformation problem in joint production economies.

**What was next in your professional career?**

First of all I tried to contact people in economics – I was still in Riemannian Geometry – people who could offer me an assistant position in economic theory. I went to Elmar Wolfstetter and later on to Malte Faber, and Faber brought me into contact with Klaus Jaeger, who was just moving to the Free University of Berlin. He was very open-minded and helped me to get the position of an assistant researcher at his institute in Berlin in 1975. My mathematical doctoral thesis was written in little more than a year, but the Habilitation thesis needed 5 years in Berlin to be completed. So the adjustment to economics was not an easy one.

**And who were the economists who impressed you most, making this transition?** You talked about Marx already and about the three volumes of Capital, but when we read your books or the titles of your books, it seems it was not only Marx you became interested in, but also Keynes and Schumpeter. Was this already in this period?

I think Klaus Jaeger basically hoped that I would skip Marx and do other more Keynesian type of research. But in fact my Habilitation thesis was on Marx, Sraffa and Leontief, and he was nevertheless very supportive in this respect. In addition, I had to teach macroeconomics in Berlin. Since I had not learned macro at all, this was a very hard time for me. I had a colleague, Michael Ambrosi, who was a definite Keynesian and we did many things together, and so Keynes became more and more of interest to me.

**So then you got interested in synthesizing or bringing together Marxian and Keynesian elements to macroeconomics?**

There were further colleagues in Berlin like Michael Krüger and Jörg Glombowski – and I think Michael Krüger brought the work of Goodwin to my attention, and therefore Marx and Macro became the next objective on my agenda. The marriage between Keynes and Goodwin (Marx), in the form of demand-driven distributive cycles, was done much later on – but first of all I started from the supply side and the Goodwinian conflict about income distribution.

**Then you finished your Berlin period with the Habilitation?**

Yes, but I had the luck to get a 4-year temporary position in 1980 as professor there, where Klaus Jaeger was again very supportive. During that time I of course applied at other universities. Willi Semmler invited me for two terms, 1984 and 1985, to the New School for Social Research, New York, but then came a tenure offer from Bielefeld, which I accepted and where I have stayed until now.

**Maybe you could explain your idea of a Marx–Keynes–Schumpeter (MKS) synthesis?**

The concept of a MKS-system may sound very unattractive at first. So I would prefer to put it simply under the post-Keynesian umbrella and use only in brackets the term MKS as a proposal where post-Keynesian economics could go to. Very briefly, Marx is about labour productivity and the distribution of the product of labour. Keynes is about effective demand, not potential output, and about financial markets and interest. And Schumpeter is about product innovation, potential output–capital ratios and...
banking, because banking is the big push behind this type of innovation. So bringing the three researchers together just gives you six variables, so to speak, as a basis that you can make a theory of.

*Only for clarification: Does labour theory still play a role in that? There has been a huge discussion on the value of the labour theory of value.*

Yes, and it can in principle be very easily solved, but I think the last 30 years were lost years in this respect. The generation of Duncan Foley, John Roemer and others really contributed to Marxian economics but what happened thereafter was very disappointing from a scientific point of view. There is another prominent contributor however. I already mentioned the von Weizsäcker proposal on the sales value method of firms, and when I read Richard Stone’s 1968 *System of National Accounts* I discovered that he had labour values involved, when he measured labour productivity within this system. He even solved the problem of joint production in the same way as von Weizsäcker proposed it to me, calling it the industry technology assumption. So labour values are part of conventional input–output accounting – they are nothing fancy. We have prices, we have quantities, what we want to have from a Marxian perspective is a scientific language behind prices and quantities. And total labour costs or, in reciprocal form, labour productivity as in Stone, provides this language. You thereby look at the behaviour of the agents of the economy in terms of labour time embodied in the various commodities. The Law of Falling Labour Content is the most basic proven assertion of this approach.

*What would you say that you – as an educated mathematician – have contributed to a MKS synthesis in terms of methods?*

The macroeconomy is a high frequency economy. It changes every day, for example the price level changes every day, since each day a few prices will change. So basically you have to use continuous time dynamics in macro, and not the oversynchronized period models that are the fashion today. And if you use a continuous time approach, or a high frequency discrete time model, you also have much more powerful tools from mathematics at your disposal. So my claim first of all would be that applicable macroeconomics is continuous time macro, and then we should slowly expand this modelling framework into the area of delayed differential equation systems, as for example Kalecki did it.

To have the full picture it is also helpful to use the feedback structures that have been introduced by Keynes, Tobin, Fisher and others, and get from their perspective, so to speak, an ensemble of feedback channels, where we can work on partial aspects as well as integrated ones. This is the way I would proceed in macrodynamics.

*Some heterodox economists hold a more or less antiformalist, antimathematical view of economics. So would you say that, if you use the right models, the right formalization, the right tools, mathematics is very valuable for economics and should be used, or what is the relationship between mathematics and economics or empirical facts?*

Economics is about quantities and interdependence, and I think our brain is fairly limited in studying such issues in purely verbal ways. For example, if you have three types of financial assets and the labour market, and the goods market and maybe something else, you cannot think about this in detail without mathematics. Economics is a quantitative science, so it is already about numbers. Using numbers, but not mathematics, is not very plausible. Also, I would point out that already Marx studied mathematics. He wrote mathematical manuscripts and there you can see that he should have

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studied linear algebra first. But he studied analysis, and the miracles of the division of zero by zero, where you can see that he took such things very seriously.

Would you see any conflict between the use of the models you apply and the term ‘uncertainty’, which is a main topic among many post-Keynesians? As you mentioned in the beginning, the model you are using is from your point of view a post-Keynesian model, so is uncertainty covered in this model?

No. Not even stochastic elements are really covered yet. But stochastic elements are of course easy to introduce; you just add a stochastic process to the given deterministic set-up. Currently I would say, from the formal point of view, what can be done in post-Keynesian model building is ‘dynamic stochastic general disequilibrium’ (DSGD) modelling, because in continuous time, you cannot assume market clearing. No market clears every second. So you have everywhere gradual adjustment, it may be very fast, but it is always gradual. So DSGD would be the thing from the formal perspective I would pursue next, but uncertainty is still a very big issue that is left aside, as is product- but not so much process-innovation. Formalizing animal spirits as the human response to uncertainty has been done by Reiner Franke for example.

A couple of years ago you started with the idea of flexicurity as an attempt at taming capitalism, in the sense that we can get rid of unemployment as a disciplining device regarding wage demands or wage claims. Could you please explain this concept a little bit and how it has developed over the last couple of years?

Flexicurity as a concept is the combination of flexibility and security. In fact I read Schumpeter’s *Capitalism, Socialism and Democracy*, and there he is saying in 1942 that socialism will not come from the East, in fact it will fail in the East, but it will come from the West and will be built on what the Vanderbilts, the Carnegies and the Rockefellers have established. This was very plausible to me. Well, I read it again – and it was no longer so plausible in the (basically Walrasian) setup he had chosen to model his competitive type of socialism.

But from thereon I thought, one has to use Western methods of production also under socialism, because they are the most advanced ones, to control the production of an enterprise and to lead an enterprise into the future. But one has of course to avoid the Marxian consequences for the labour market. So flexibility in the way enterprises are conducted is very important for evolution, but security for households and safe life course perspectives, in particular normal working days, no segmented labour markets and so on, are equally important.

I think, as the situation is today, that we indeed should, on the one hand, say yes to the productive forces of capitalism. But, on the other hand, the relations of production which govern them can be formed and can be given a shape so that the forces of production can remain capitalistic in nature, while we nevertheless arrive at much more than just the welfare state, which means safe life course perspectives, a very well-balanced education system, citizenship-education, and finally, the conduct of elites that is democratically-oriented and not, as in Bourdieu, habitus-based. I would call such a democracy-based social structure of capital accumulation ‘Social Capitalism’, which in fact only adds a few more letters to the word ‘Socialism’, but changes its essence significantly.

Would this change the structure of the firm, or is this only a new institutional framework for the firm?

The objective of the firm would still be profit-seeking, because I think the iPad would not have been invented otherwise. But I also think capitalism, to a certain degree, has
always been criminal in nature and indeed has become criminal again after the prosperity phase after World War II. So you have to regulate capitalism, otherwise the criminals will dominate it.

You can produce spoiled meat and sell it on the market and securities in the financial markets are just spoiled meat. If you sell spoiled meat in the real markets, you go to prison. However, if you sell spoiled meat in the financial markets, you do not really face imprisonment, but indeed you should be sentenced to go to prison for that. Because these agents know that you are providing toxic assets to customers.

Consequently, capitalism always needs to be regulated in strong and well-reflected ways. I indeed believe that the profit-seeking motive can be regulated successfully in fairly strict ways, so that you can not only avoid criminal activities, but also the exploitation of labour power within the enterprise.

A final question: You are now retired as a professor in Germany. What is your view after most of your academic life in economics in Germany on the development of non-neoclassical economics, what is your recommendation to the younger generation, those who are still going for their PhD, and are still looking for jobs?

First of all, I would always state that my work is embedded in a relatively large group of old, intermediate and young economists. I never had assistants, because my position has not been one with such an endowment, but it happened in the last decade that I came into contact with a lot of young people.

I could name probably ten young researchers who are not just doing what we did, but who are going their own way by applying similar methods. So I see there is indeed a group of young researchers that will do very good work in the future in a post-Keynesian way. And I think there are also many others, whom I do not know yet, who will go into the same direction. Therefore, I expect very much from this generation which moreover is well ‘adapted’ to the ridiculous publish-or-perish mainstream straitjacket we are living in.

These people are not committed to mainstream economics – quite the opposite – but they are subdivided into doing just things that can be published in a mainstream journal and doing the work they find important. This combination of being forced to ‘howl with the wolves’ and doing post-Keynesian or Marxian work in addition is what I see is developing, and I find this very promising. And that is why I also think that Keynes in my view is a bit superior over Kalecki. Marx knew the political economy of his time in great detail and he wrote a critique of it. Keynes knew the mainstream of his time very well and his General Theory was an important step forward. Kalecki wrote a lot of insightful essays, but they are not really a critique of the mainstream, because he was not too much acquainted with it. Schumpeter, finally, started from Walras to formulate his theory of economic development. I therefore think that it is important to know the mainstream in detail, because then you may see clearer what you can do against it.

The interview was conducted by Eckhard Hein and Torsten Niechoj in October 2011.

SELECTED PUBLICATIONS OF PETER FLASCHEL


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Urheber: Jörg Blobelt (1949–) Blue pencil.svg wikidata:Q28598952

Doch dann kam 1950 der Weg durchs Brandenburger Tor nach Koblenz (Späteinschüler), zum Studiun nach Bonn (Mathematik, gegen den Rat meines Mathe-Klasssenlehrers, Diss Bestnote), die damals übliche Lektüre von "Das Kapital I" (wo ich mich auf einer Ferienreise anschliessend durch Band 2 und 3 quälte), mich das Marx-Engels'sche Transformationsproblem (Böhm-Bawerk) der Werte in die Preise irritierte und ich -- durch die Frage der Behandlung von Wertbildung bei Kuppelproduktion -- mich zum Lösen dieser Probleme entschloss. Insbesondere Egbert Dierker hat mich in sehr liebenswerter Weise in 5 Seminaren danach mit den Inhalten der mathematischen Ökonomie dieser Zeit vertraut gemacht (ein sehr anstrengender Prozess, der mich aber bewog in die Ökonomie und zu Klaus Jaeger als Assistent zu wechseln, der mich sehr gefördert hat (trotz meines fokusierten mathematischen Interesse an der Marxschen Werttheorie) und auch gleich eine Makro-Vorlesung halten liess, obwohl ich noch nie ein Makro-Buch in der Hand gehalten hatte. Es folgte die Habilitation und durch seinen und anderen Support schliesslich die Berufung nach Bielefeld. Man kann also meinen akademischen Lebensweg kurz mit BBB bezeichnen (C4 und der Osten waren keine Option für mich), wobei ich zwischen Bonn und Westberlin meine Lebensphilosophie gefunden habe -- treu dem Kölner Grundgesetz folgend, in der einzigen Sprache der Welt, die man auch trinken kann, im Vergleich zu dem Zeuch in dem Dorf da jejenüwer.

Der Düsseldorfer Heinrich Heine war beiden Dialekten gegenüber kritisch: In Köln klängele „Köbes“ mit „Marizzebill“ in einer Mundart, „die wie faule Eier klingt, fast riecht“, die Sprache seiner Heimatstadt disqualifizierte er, man könne dieser schon „das Froschgequake der holländischen Sümpfe“ anmerken.
Et kölsche Jrundjesetz

§1: Et es wie et es!

§2: Et kütt wie et kütt!

§3: Et hät noch immer jot jejange!

§4: Wat fott es, es fott!

§5: Et bliev nix, wie et wor!

§6: Kenne mer nit, bruche mer nit, fott domet!

§7: Wat wellste maache!

§8: Maach et jot, ävver nit ze of!

§9: Wat sull dä Quatsch?

§10: Dringste eine met?

§11: Do laachs dich kapott!

Dann jitt et do noch die volljende Zusatzartikelsche:

§12: Wat däm ein sing Ül eß däm andere sing Naachtijall.
§12a: Mir sin ävver nit aaberläubisch!
§13: Wer fiere kann, dä kann och arbeide!
§14: Mer weiss et nie, mer stich nit drinn.
§15: Jede Jeck es anders.
§16: Dröm Jeck loß Jeck elahns!
§17: Och dä raderdollste Aasch hät sing Visaasch!
§18: Jeddem Dierche sing Pläsierche.
§19: Dat jitt et nur, nur, nur in Kölle!
§20: Dat sull ech jesaat hann?
§21: Du bes Kölle!
§22: Der wiess nix
§23: Hammer nit, künstemer nit.
Book review


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More than 75 years ago, John Maynard Keynes wrote in his magnum opus: ‘The outstanding faults of the economic society in which we live are its failure to provide for full employment and its arbitrary and inequitable distribution of wealth and incomes’ (Keynes 1936 [1978]: 372). This statement still applies today and has gained even more significance in recent times. In contrast to all the grandiose promises which accompanied the march back to Manchester liberalism during the past decades, present-day capitalism performs rather poorly. Most people did not benefit from the laissez-faire programme propagated by orthodox economists and conservative politicians but many are worse off. Such sad circumstances have escalated in a number of countries as a consequence of the latest financial crisis. In order to overcome the apparent deficiencies in living conditions for the great majority of the population, a conclusive alternative to the ruling doctrine is urgently needed. The contribution to this demanding task by Peter Flaschel and Sigrid Luchtenberg offers remarkable insights into the workings of capitalism and advances innovative ideas to shape the system’s outcome.

The authors depart from their basic insight that ‘the future of capitalism will depend very much on its capability to integrate its “dynamic forces of production” with a truly “social mode of production”’ (p. 11). In their view, ‘social capitalism’ is neither a contradiction in itself nor does this ideal coincide with a ‘social market economy’. The latter concept of a ‘third way’ between the extremes exerted a considerable influence on the economic order in (Western) Germany after World War II. Flaschel and Luchtenberg separate themselves from this approach (p. 21) as well as from other notions of ‘social capitalism’ (p. 307 et seq.).

The framework of the enquiry is provided by the theories of Marx, Keynes and Schumpeter ‘on ruthless capitalism, regulated capitalism and in the case of Schumpeter also competitive socialism’ (p. xi). This so-called MKS system had been proposed by Richard M. Goodwin (1986). The relevant paper was reprinted in a publication by Flaschel (2009: 376–382) which dealt with issues similar to those addressed in the book under review. Anyway, the authors are in a position to commence several chapters with reference to previous research work by one or both of them.

Though Flaschel and Luchtenberg on occasion invoke Marx, Keynes and Schumpeter, Goodwin’s class struggle model (1967) serves as the main analytical foundation of their reasoning. Doubts about whether the approach leaves any room at all for the strategic behaviour of capitalists (see Wörgötter 1986) are not discussed. Furthermore, it seems that the contemporary reality is not very well depicted by Goodwin’s equations, at least in the more recent past. In Germany, for example, the volume of work (not the number of employed persons) has been fairly stable since 1991. Yet labour’s share of national income has fallen over the years without any indication of an approaching significant rise. In the long
run, however, we may be taught otherwise. Entrepreneurs, it’s worth noting, appear not always to be unanimously interested in lowering wages (Helmedag 2012).

The volume’s ‘General introduction’ characterizes the most important varieties of modern capitalism: the United States of America, the European Union and the People’s Republic of China. The following three parts are entitled ‘Failing capitalism: baseline scenarios and social reforms’, ‘The forces to cope with: effective demand, finance and innovation’ and ‘Systemic crises, policy responses and the road to social capitalism’.

Part I rests upon Marx’s reserve army mechanism. The ensuing distribution cycle is studied under the assumption of homogenous labour as well as in case of segmented labour markets. In the concluding chapter, an ‘employer of first resort’ ensures that full employment prevails. The resulting ‘flexicurity model’ is compared with the debate in the European Union and particularly its realization in Denmark.

In part II, the MKS system is elaborated in more detail. The first chapter covers the interaction between the distribution conflict and a Keynesian demand analysis. The authors then introduce a narrow commercial banking system with a Fisherian 100% reserve ratio. The third chapter, which is written by Reiner Franke, investigates Schumpeterian innovation waves.

Part III starts with the dynamics of capital accumulation and the decay of the infrastructure and the environment. Based on a flexicurity system, Flaschel and Luchtenberg present a normative system designed for a sustainable type of growth. The next chapter focuses on some governments’ ‘rampant’ fiscal policies and the ensuing danger for the survival of the eurozone. In this context, the policy of the IMF is also considered. In conclusion, the authors present the basic principles constituting their approach to ‘socialize’ capitalism.

The vision of Flaschel and Luchtenberg encompasses three pillars. First, flexible and socially oriented labour market institutions guarantee full employment. In addition, extensive health services and care for the elderly have to be provided by ministries with their own budget. The second pillar consists of an educational system granting equal opportunities for all people, skill formation during lifelong learning and a political education for the citizens. The last pillar, constituting social capitalism, concerns the election of executive persons and the creation of elites. Since the authors, following Schumpeter, do not recognize the basic aim of democracy as representing the will of the constituency but as a means of changing the government, they prefer a qualified type of majority voting (p. 321).

Interestingly enough, Flaschel and Luchtenberg even venture as far as applying their programme to a country in serious trouble: they ‘formulate a strategy for the evolution of the Greek society in the longer run where the three pillars … are the compass for the intended radical socio-economic evolution’ (p. 324). The hope remains that the rich there, who will have to carry the main burden on the road to a better future, do indeed understand their contribution as an act of solidarity.

At any rate, Flaschel and Luchtenberg present profound and thorough considerations about how to improve the capitalism under which we currently live. They do not restrict themselves to a mere adumbration of the societal system we should be striving for. What is more, the authors propose concrete ways and means to get there. Their monograph abounds with useful information, thought-provoking suggestions and substantial instructions, written in a spirit of humanity and democracy. These characteristic features alone were already reason enough to praise the book as an outstanding example of economic literature.

REFERENCES


Aber was ist denn heute los?: "The party’s over, they burst your pretty balloon,..."

Can capitalism survive? No, I do not think it can. The thesis I shall endeavor to establish is that the actual and prospective performance of the capitalist system is such as to negative the idea of its breaking down under the weight of economic failure, but that its very success undermines the social institutions which protect it, and inevitably creates conditions in which it will not be able to live and which strongly point to socialism as the heir apparent. J. Schumpeter: *Capitalism, Socialism and Democracy*, 1942 (first edition).

But "Competitive Socialism" rested in Schumpeter on an unproven general price theory, and definitely has become much more perverted in today’s literature on socialism. And in the case of Germany, slave of US orthodoxy, flexicurity and the "Road to Social Capitalism" degenerated to the "Black Zero" ideology of the currently advancing facade (fakebook) democracies. ... Addios MKS, addios RMG, the road we have travelled has come to an end. Addios compadre, what must be must be -- Remember to name one muchacho for me.
Die Marxsche Reservearmee auf Basis seiner Theorie segmentierter Arbeitsmärkte

Goodwin's (1967) Growth Cycle model for the FRG after WW!!: Marx's General Law of Capital Accumulation

Stattdessen: Hydraulischer "Keynessianismus", Marx Phobie und die Folgen

Und dann erneut:

Sozialdemokratischer Tango: Ein Schritt vorwärts, (nur?) zwei Schritte zurück???

Der putinesque Erfinder des modernen Lumpenproletariats (Hartz IV) und der "besoffene" Anfang des Niedergangs der alternden SDP

Details in dem SPD Buch der Bibliographie.

"Aufstehen" ja, aber das Schlachtfeld ist die Produktion, nicht die individuelle Verteilung der "Beute", siehe Marx@200 in der Bibliographie