Essays on Firms in the Globalized World

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Preface and Summary

Since the early 19th century, the world experienced an accelerating process of an integration of the national economies that is generally called globalization. Driven by the occurring technological and political progress, in particular the revolution of the transportation technology (railways, steamships and airplanes) and subsequently the information and communication technology (ICT; telephone and computer) as well as the liberalization, costs of transporting goods as well as barriers to trade, invest and travel declined and thus cross-border flows of goods (international trade), capital (international investment) as well as, to a smaller extent, people (international migration) surged. Recently, the world is economically (as well as politically and socially) by far more intertwined than ever before and any attempt of a deeper understanding of this process has to focus on its main economic drivers, inevitably leading to the following questions: How do the economic actors decide and behave in such an environment? Why do they decide and behave as they do? What are the (micro- and macro-)economic consequences of their actions?

This thesis contains five chapters that deal with firms in the globalized world. All of these self-contained essays are motivated and guided by the attempt to learn (more) about these smallest organizational units in the economy that aim at profitably producing and selling products to the market in which they are located in (domestic market), but especially to markets across their own national border (foreign markets) whose serving generates additional problems as well as options for the firms. The analyses that are conducted in these essays concentrate on the globally active firms’ decisions and behaviors, their corresponding determinants (factors) as well as implications. Chapter 1 theoretically studies the firm choice of how to serve foreign markets by choosing either exporting domestically manufactured goods or setting up a production facility abroad through foreign direct investment (FDI).

2For a comprehensive survey of the history of international trade and the non-monotone development of trade, i.e. the existence of phases of de-globalization due to political regresses (wars): See Findlay and O’Rourke (2009). Recently, Federico and Tena-Junguito (2017) revise the common temporal localization of the two waves of globalization.
3Based on the fundamental definition of economic organizations by Milgrom and Roberts (1992): “Economic organizations are created entities within and through which people interact to reach individual and collective economic goals. The economic system consists of a network of people and organizations, [...]” (p. 19).
The following essays all address a special type of firms, called *multi-product firms*, which are characterized by the fact that they produce and sell more than a single product. An encouragement, justification and requirement to (theoretically) concentrate on them when being interested in globally active firms and a detailed picture of their characteristics and behavior that binds the theory are provided by Chapter 2 that surveys a comprehensive body of empirical studies on multi-product firms in international trade. After a short introduction to the already existing theory of multi-product firms in international trade as a reference and starting point (Chapter 3), Chapter 4 and Chapter 5 theoretically study multi-product firms. The former takes a look into multi-product firms that both produce with a flexible manufacturing technology and are endowed with a resource, called organizational capital, in a limited amount and investigates those firms’ behavior in international trade. The latter analyzes the export behavior of multi-product firms in a world in which countries differ in their per capita income and preferences are non-homothetic.

**Chapter 1 - “Export versus FDI: Non-monotone Sorting under Quadratic Preferences”:** Monopolistic competition models with constant-elasticity-of-substitution (CES) consumer preferences do not replicate substantial empirical facts and in this way provide an incentive to look for an alternative theoretical assumption on consumption. By applying quadratic instead of CES preferences in a monopolistic competition model with heterogeneous firms, this chapter investigates the robustness of the conventional sorting of cross-border active firms that is derived by Helpman et al. (2004), and thereby detects a significant change at the upper end of the productivity range: While (sufficiently) productive firms export and even more productive ones engage in foreign direct investment (FDI), the most productive firms do not undertake FDI, but export again. This non-monotone sorting is consistent with the meta-analysis by Mrázová and Neary (forthcoming) and both direct and indirect empirical support for it can be found.

**Chapter 2 - “Multi-product Firms in International Trade - A Survey and Evaluation of Empirical Studies”:** Motivated by the intensive treatment of multi-product firms in the more recent empirical literature of international trade, which is based on the progressive availability of micro data sets, this chapter asks for the actual state of knowledge about this type of firms in international trade. It tries to generate a comprehensive answer by surveying and evaluating a large body of empirical studies that employ micro data sets both for developed and developing countries. Thereby, it turns out that several robust findings or stylized facts can be derived, even so definitions about firms and products vary across the studies: Multi-product firms (i) are national and especially international prominent, less prevalent but important, (ii) are singular economic units with premia in terms of their nature and performance, (iii) produce and export at different margins, (iv) are asymmetric within themselves (“superstar products”), (v) do not produce all their exports, (vi) are relatively diversified in production and export, (vii) compete on both a cost and quality
basis, (viii) are less prone to firm exit in the process of “creative destruction”, (ix) start exporting at a small scale and scope and increase them over time, (x) frequently and to a large extent change their product portfolio (“product switching”) and (xi) undertake adjustments in response to exogenous shocks. These derived empirical regularities bind both the existing and upcoming theoretical modeling of multi-product firms.

Chapter 3 - “Introduction to the Theory of Multi-product Firms in International Trade”: As a reference and starting point for the following chapters, this chapter introduces to the theory of multi-product firms in international trade, its evolution in the general theory of international trade and structurally presents its main contributions.

Chapter 4 - “Multi-product Firms with Flexible Manufacturing and Organizational Capital in International Trade”: Based on the traditional resource-based theory of firms, this chapter develops a theory of multi-product firms with both dis-economies of scope and asymmetric products in international trade. Firms with some flexible manufacturing technology in production are endowed with some production cost-reducing resource, called organizational capital, which is (in the short run) for each firm limited in volume and for which firms have to decide over its allocation across the product mix. In (sufficiently) heterogeneous industries, multi-product firms with more organizational capital as well as more productive multi-product firms have a larger product scope. Opening up to international trade, firms adjust their allocation of organizational capital, their intensive and extensive margin in response to changes in the cost parameters of trade, whereas a trade liberalization induces multi-product firms to increase their product scope. Under a (standard) functional specification for the production costs, large-scope firms respond less to trade shocks. Using firm-level data on production factors and performance, this chapter provides empirical evidence in support of some of the basic predictions of the theory and organizational capital in general is shown to be a relevant factor in the production process of firms.

Chapter 5 - “Multi-product Exporters in a World with Heterogeneous Countries and Non-homothetic Preferences”: With the increasing availability of disaggregate data, three facts about both countries and firms robustly appear in the empirical literature of international trade: Firstly, countries with a higher per capita income import more product varieties from other countries. Secondly, imports and exports are concentrated among a relatively small number of firms, so that firms in form of “export superstars” exist and represent the “granular” components of the set of internationally active players. And thirdly, multi-product firms are prevalent and dominant in international trade. In light of these observations, this chapter analyzes the export product scope of multi-product firms across countries that substantially differ in terms of their per capita income, which is so far largely ignored by the literature of international trade. In a non-standard model with non-homothetic pref-
erences and imperfect competition, multi-product firms export more varieties to countries with a higher per capita income, thereby dealing with both cannibalization and income effects.
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Chapter 1

Export versus FDI: Non-monotone Sorting under Quadratic Preferences

1.1 Introduction

As part of their business strategies, firms decide on the sales markets for their products. In times of globalization, they particularly face the question of whether to serve the market in a foreign country and, in the case of international engagement, how to serve the destination market abroad. Each firm has the choice between two alternative modes of operation: Either it exports its domestically manufactured goods, thereby becoming an exporter, or it undertakes foreign direct investment (FDI) to establish production facilities in the destination country, becoming a multinational enterprise (MNE). Internationally engaged enterprises, sometimes called internationalized firms (IFs), are different from those that limit their activity to the domestic market: Exporters and MNEs are unique in characteristics and superior in performance parameters - the group of these enterprises consists of larger, more productive and higher value added firms that pay higher wages, use more

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1 Throughout this chapter, the term FDI is used as a synonym for horizontal FDI, i.e. the replication of production and sales structures in a foreign country with the aim to serve its market (see for an illustrating model: Markusen (1984)), in contrast to vertical FDI, i.e. the fragmentation of production processes in the form of the relocation of stages abroad with the aim to generate benefits in serving the home market (see for an illustrating model: Helpman (1984)).

2 This chapter abstracts from a firm-level combination of these choices by assuming a cost structure for which firms do not have any incentive to serve a foreign market via both exports and affiliates. Beyond any bilateral country setting, a hybrid form of the two modes of international operation of firms is given by the export-platform FDI. For its theoretical treatment, see Motta and Norman (1996), Yeaple (2003), Grossman et al. (2006) and Ekholm et al. (2007) and its empirical treatment, see Hanson et al. (2005) and Baltagi et al. (2007) for firms simultaneously choosing both operation modes, each for a different market, see Oberhofer and Pfaffermayr (2012).

3 See Caves (2007), p. 1: A multinational enterprise is defined as “[a firm] that controls and manages production establishments - plants - located in at least two countries”.
capital per worker and employ more skilled workers relative to domestic firms. Among these outstanding IFs, each firm’s profit-maximizing decision about the operation mode, so either to become an exporter or a MNE, reflects its solution to the underlying proximity-concentration trade-off in a world with both trade barriers and economies of scale. While exporting allows the local concentration of production, and in this way enables the exploitation of economies of scale, FDI implies production close to the sales market and hence saves on transportation costs.

Analyzing the choices that are made by heterogeneous, i.e. productivity varying, firms regarding their preferred mode of operation in a country abroad, Helpman et al. (2004) establish a sorting of firm activities and operation modes that is denoted in the literature as conventional sorting: The least productive firms exit the domestic economy, firms with a low productivity restrict their activity to the domestic market, firms with an intermediate productivity in addition serve the foreign market via exports, and only the most productive firms sell domestically and engage in FDI. Empirical evidence for a productivity premium of the MNEs (relative to the exporters) and of the exporters (relative to the domestic firms) is found in Kimura and Kiyota (2006) and Tomiura (2007) for Japan, Girma et al. (2005) for the UK, Wagner (2006) and Arnold and Hussinger (2010) for Germany as well as Engel and Procher (2012) for France. In Helpman et al. (2004), the conventional sorting of firm activities and operation modes theoretically arises in a Melitz (2003)-type model of heterogeneous firms in monopolistic competition with constant-elasticity-of-substitution (CES) consumer preferences, iceberg transportation costs and fixed operating costs.

In general, however, monopolistic competition models with CES preferences, pioneered in international trade by Krugman (1980) and extended to heterogeneous firms by Melitz (2003), do not replicate substantial empirical facts, as recently addressed by Melitz (2018) himself. Firstly, by generating markups that are constant across heterogeneous firms in a market and across markets, they are not able to reproduce the empirically robust findings of variable markups, incomplete cost pass-through and markup responses to market-wide

---

4A comprehensive strand of the empirical literature of international trade documents the characteristics uniqueness and performance premia of IFs: See e.g. for exporters Bernard and Jensen (1995), Bernard and Wagner (1997), Bernard et al. (2007), for MNEs Doms and Jensen (1998) and Antrás and Yeaple (2014), and for both types of firms Mayer and Ottaviano (2008). An overview of the literature on firm heterogeneity and firm-level globalization strategies is provided by Greenaway and Kneller (2007).

5The proximity-concentration trade-off was established in the theoretical literature of horizontal FDI with the work by Brainard (1997) and is subsequently studied in several generalized setups: Markusen and Venables (2000) allow for factor endowment differences across countries and therefore incorporate country asymmetries. Helpman et al. (2004) extend the model to firm asymmetries (heterogeneity) and more recently Ramondo et al. (2013) set it into a stochastic environment.

6The model by Dixit and Stiglitz (1977), with which the idea of monopolistic competition as a market structure got formalized and customized to a broad professional audience, does not use the CES specification of preferences, but instead includes consumers with a direct utility that is symmetric and additive in the consumption of different goods, leading to a demand elasticity that depends on the consumption level.
changes. Possible ways to overcome the shortcoming and in this way to generate more flexible models with endogenous markups and incomplete cost pass-through are obtained by either changing the market structure, while maintaining the demand structure of CES preferences or changing the demand structure, while maintaining the market structure of monopolistic competition or changing both. Secondly, while in the empirical evidence, adjustments and reallocations are robustly reported to take place at the intensive margin, monopolistic competition models with CES preferences only allow for the extensive margin as their active channel.

Addressing and remedying the shortcomings of the monopolistic competition model of heterogeneous firms with CES preferences in its empirical consistency, Melitz (2018) generalizes the demand setup to separable preferences and requires that these preferences satisfy “Marshall’s Second Law of Demand”, i.e. the price elasticity of demand increases with the price along a demand curve, or equivalently - the derived demand curve is log-concave in log-price to generate empirically consistent model predictions. Any violation of this property would contradict the empirical evidence on (i) the markup variation across firms (more productive firms charge higher markups), (ii) the incomplete pass-through of cost changes (a change to marginal cost is passed-on less than one-for-one into prices, since the markups absorb the remaining variation), and (iii) the incomplete pass-through variation across firms (better performing firms absorb a greater proportion of a cost shock into their markups) as well as in particular (iv) the (pro-)competitive effects of trade, i.e. the response of markups and induced product reallocations to increases

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7 The case of constant markups by assuming CES preferences is not limited to the market structure of monopolistic competition. It also occurs in the case of Ricardian trade models with perfect competition, as in Eaton and Kortum (2002).

8 For an overview of the different approaches of estimating firm markups, see De Loecker and Goldberg (2014). And for variable markups across (heterogeneous) markets, see e.g. Campbell and Hopenhayn (2005), Hummels and Klenow (2005) and Barron et al. (2008).

9 In models of an oligopoly with firms of a non-zero measure, in which those firms internalize the effects of their price choices on market aggregates, variable markups occur even in the case of CES preferences, as for Bertrand competition in Bernard et al. (2003) and De Blas and Russ (2015) and for Cournot competition in Atkeson and Burstein (2008) and Edmond et al. (2015).


11 For the details on the empirical evidence: See the Section 1.3.

12 Equivalently, the price elasticity of demand falls with the quantity consumed.

13 Dixit and Stiglitz (1977) argue that this is the standard behavior of demand and propose a model specification with a variable elasticity of substitution form of demand. Krugman (1979) by himself also makes this assumption for the demand structure in his seminal work on monopolistic competition in international trade, before giving it up in favor of CES preferences in his subsequent contribution (Krugman 1980). For the property captured by Marshall’s Second Law of Demand, several other terms have been used in the literature: Zhelobodko et al. (2012) describe the preferences as to reveal an increasing “relative love for variety”, Bertoletti and Epifani (2014) as to represent a “decreasing elasticity of substitution” and Mrázová and Neary (2017 forthcoming) as to lead to a “subconvex” demand function.

14 In some cases, a slightly stronger assumption than Marshall’s Second Law of Demand is required: The marginal revenue (instead of the demand) becomes more inelastic as output increases.
in market size and competition in an exporter’s destination: Markups fall and market shares are reallocated towards the better performing products. As it is already evident, CES preferences do not satisfy this property and all the above mentioned facts therefore provide an incentive to look for an alternative theoretical assumption on consumption.

The present chapter takes the standard model of international trade, a monopolistic competition model with CES preferences and heterogeneous as well as internationally active firms à la Melitz (2003), and adds an operation mode for these firms, i.e. undertaking FDI. Moreover, it basically adjusts the model’s demand setup to perform the empirical facts by applying quadratic preferences, which leads to a linear demand with variable markups and this satisfies Marshall’s Second Law of Demand (as well as the slightly stronger assumption) as highlighted by Melitz (2018). By doing this, the chapter investigates the robustness of the conventional sorting that is derived by Helpman et al. (2004) and finds an alternative type of sorting, which is consistent with the meta-analysis of firm selection effects that is conducted by Mrázová and Neary (forthcoming): While (sufficiently) productive firms export and even more productive ones engage in FDI, the most productive firms do not undertake FDI, but export again, thereby establishing a switch of operation modes at the upper end of the productivity range.

The remainder of the chapter is organized as follows: After briefly outlining the essential elements of the monopolistic competition model with quadratic preferences that is developed by Melitz and Ottaviano (2008), the next section presents an appropriately modified and extended version of this model in which the export versus FDI decision of heterogeneous firms is both incorporated and analyzed. As the chapter’s main contribution and insight, the section’s analysis derives the non-monotone sorting of firm activities and operation modes. The section subsequently concludes with the presentation of the sorting’s consistency with the existing theoretical contribution about the firm selection effects. In the following section on the sorting’s empirical evidence, both direct support - in form of evidence for a more complex and less monotone sorting than the conventional one in case of some types of international firm activity - as well as indirect support - in form of evidence against the CES and for the quadratic preferences - are provided. Finally, the chapter ends with some conclusions in the last section.

1.2 Model

By incorporating the firms’ option to serve a foreign sales market through undertaking FDI, this chapter modifies the model by Melitz and Ottaviano (2008), as Helpman et al. (2004) do it for the model by Melitz (2003).
1.2. MODEL

1.2.1 Closed Economy

A country is populated by \( L \) individuals (consumers, workers), each endowed with one unit of labor. Consumers exhibit identical quadratic preferences, which are defined over a homogeneous good (chosen as the numéraire) and a continuum of horizontally differentiated product varieties indexed by \( i \in \Omega \), as represented by the utility function

\[
U = q_0^c + \alpha \int_{i \in \Omega} q_i^c di - \frac{1}{2} \gamma \int_{i \in \Omega} (q_i^c)^2 di - \frac{1}{2} \eta \left( \int_{i \in \Omega} q_i^c di \right)^2
\]

with \( q_0^c > 0 \) and \( q_i^c \) as the individual consumption of the numéraire and of each variety \( i \in \Omega \) respectively, and \( \alpha, \eta \) and \( \gamma > 0 \) as parameters, where \( \gamma \) describes the degree of product differentiation between the varieties. Maximizing utility and subsequently aggregating over all individuals, demand for variety \( i \) is given by

\[
q_i = \frac{\alpha L}{\gamma + \eta N} - \frac{L}{\gamma} p_i + \frac{\eta N}{\gamma + \eta N} \bar{p} \quad \forall i \in \Omega^*,
\]

where \( N \) denotes the measure of varieties in \( \Omega^* = \{ i : q_i^c > 0 \} \subseteq \Omega \), \( p_i \) the price of variety \( i \), and \( \bar{p} = \frac{1}{N} \int_{i \in \Omega^*} p_i di \) the average price. For all varieties within the consumption set then holds that

\[
p_i < \frac{\alpha \gamma}{\gamma + \eta N} + \frac{\eta N}{\gamma + \eta N} \bar{p} \equiv p_{\text{max}},
\]

and the price elasticity of demand for variety \( i \), \( \varepsilon_i \equiv -\left( \partial q_i / \partial p_i \right) (p_i/q_i) = \left( p_{\text{max}}/p_i - 1 \right)^{-1} \), depends both on the price of the variety and the price bound.

With a large number of small firms in the country, its market structure is characterized by monopolistic competition. Each firm produces a horizontally differentiated product variety \( i \in \Omega^* \), using labor as the single factor of production. The unit variable costs of a firm with the productivity level \( \varphi \) are given by \( c(\varphi) = \frac{w}{\varphi} \), where \( w \) represents the wage. Firms draw their productivity level from a common and known distribution \( G(\varphi) \) with support on \((0, \infty)\), after paying the sunk entry costs \( f_E > 0 \) (all fixed costs measures in labor units). Among entrants, only those firms that are able to cover their marginal costs produce; all other firms exit the economy. Maximizing its profits, a firm with a productivity level \( \varphi \) charges the price

\[
p(\varphi) = \frac{1}{2} (p_{\text{max}} + c(\varphi)).
\]

The marginal firm in the economy, i.e. the firm with the lowest possible productivity level to stay in the economy, satisfies the condition \( p(\varphi_E^*) = p_{\text{max}} = c(\varphi_E^*) = \frac{w}{\varphi_E^*} \), therefore just breaks even and so realizes zero profits. Finally, the economy-wide choke price \( p_{\text{max}} \) also co-determines firm-level variables like the markup \( \mu(\varphi) \), output \( q(\varphi) \), revenue \( r(\varphi) \) and operating profits \( \pi(\varphi) \):

\[
\mu(\varphi) = \frac{1}{2} (p_{\text{max}} - c(\varphi)) \quad \text{and} \quad q(\varphi) = \frac{L}{2\gamma} (p_{\text{max}} - c(\varphi)),
\]

\[
\pi(\varphi) = \frac{1}{2} (p_{\text{max}} - c(\varphi)) q(\varphi).
\]
\[ r(\varphi) = \frac{L}{4\gamma}(p_{\text{max}}^2 - c(\varphi)^2) \quad \text{and} \quad \pi(\varphi) = \frac{L}{4\gamma}(p_{\text{max}} - c(\varphi))^2. \]  

(1.6)

Facing quadratic preferences and linear demand with its decreasing price elasticity, firms with a higher productivity level charge lower prices and are able to realize higher markups, higher revenues and higher profits due to an incomplete pass-through of their cost and quantity of sales advantage.

1.2.2 Open Economy

The world consists of two countries \( j \) and \( k \). Their populations \( L^j \) and \( L^k \) are immobile across countries, markets are segmented, and each consumer in countries \( j \) and \( k \) exhibits preferences of type (1) and has a demand of type (2), with country-specific consumption variables and a country-specific degree of product differentiation as well as cross-country identical parameters \( \alpha \) and \( \eta \). Firms have an additional activity option in this setting: Besides serving the domestic market, they can extend their activity to the foreign country by either exporting domestically manufactured goods to its market or setting up a production plant and thereby investing in its market. Assuming country \( j \) as being a firm’s home country and thereby the location of both its headquarters and production plant, exports of products to country \( k \) entail iceberg transportation costs (including tariffs) \( \tau_{jk}^X \geq 1 \) and in addition some fixed costs \( f^X > 0 \), which are caused by the spatial separation of the headquarters and the sales location, requiring some extra entrepreneurial effort to arrange, coordinate and monitor sales services like marketing, distribution and logistics. Undertaking FDI in country \( k \) involves as well these fixed costs \( f^X > 0 \), but in addition is associated with some further fixed costs \( f^I > 0 \), which are caused by the duplication of production facilities and the spatial separation of the headquarters and the production location, requiring some extra entrepreneurial effort to communicate with the implementation unit and to ensure the intended realization of production plans.

A firm in country \( j \) makes the decision about its activities and modes of operation with the objective to maximize its overall profits, i.e. the sum of profits from domestic and foreign operations. Since these are independent of each other (separable and linearly additive), it is possible for the analysis to split up the decision-making of a firm with a productivity level \( \varphi \) into three separate profit maximization problems: Regarding domestic sales, denoted by \( q^D_j(\varphi) \), the firm solves the maximization problem

\[ \max_{p^j(\varphi)} \pi^j_D(\varphi) = (p^j(\varphi) - c^j(\varphi))q^j_D(\varphi) \]  

(1.7)

with \( p^j(\varphi) \) as the price of the variety produced by the firm in country \( j \) and \( c^j(\varphi) \) as its unit variable costs of production. The maximization problem regarding sales abroad, denoted by \( q^j_{M}(\varphi) \) with \( M \) as the mode of operation (\( X \) for export, \( F \) for FDI) from \( j \) as the source
country to \( k \) as the destination country, is given by
\[
\max_{p^k_M(\varphi)} \pi^{jk}_M(\varphi) = (p^k_M(\varphi) - c^*(\varphi))q^{jk}_M(\varphi) \quad \text{for } M \in \{X, F\},
\]
(1.8)
where \( p^k_M(\varphi) \) denotes the price of the variety supplied to country \( k \) by operation mode \( M \) and the unit variable costs of production are represented by
\[
c^*(\varphi) = \begin{cases} 
\tau^{jk}c^j(\varphi) & \text{for } M = X \\
c^k(\varphi) & \text{for } M = F
\end{cases},
\]
(1.9)
with \( c^k(\varphi) \) as the unit variable costs of production in country \( k \). The firm’s optimal prices then emerge as
\[
q^D_j(\varphi) = \frac{L_j}{\gamma_j}(p^j(\varphi) - c^j(\varphi)) \quad \text{and} \quad q^{jk}_M(\varphi) = \frac{L_k}{\gamma^k}(p^k_M(\varphi) - c^*(\varphi)) \quad \text{for } M \in \{X, F\},
\]
(1.10)
where the firm applies the aggregate demand in the domestic and in the foreign country respectively, and these prices are bound by the country-specific choke prices:
\[
p(\varphi^{m*}_E) = p^{m}_\text{max} = \frac{1}{\gamma^m + \eta N^m (\alpha \gamma^m + \eta N^m p^m)} \quad \text{for } m \in \{j, k\},
\]
pinning down the productivity level of the marginal firm in the respective market (\( \varphi^{j*}_E \) and \( \varphi^{k*}_E \)).

With the price and quantity determined, the optimal (operating) profits from each of the firm’s activities and operation modes are derived:

1. For the domestic market:
\[
p^j(\varphi) = \frac{1}{2}(p^{j}_\text{max} + c^j(\varphi)), \quad q^D_j(\varphi) = \frac{L_j}{2\gamma_j}(p^{j}_\text{max} - c^j(\varphi)) \quad \text{and} \quad \pi^D_j(\varphi) = \frac{L_j}{4\gamma_j}(p^{j}_\text{max} - c^j(\varphi))^2
\]
2. For the foreign market (for \( M \in \{X, F\} \))\footnote{See the Appendix A.1 for the details.}
\[
p^k_M(\varphi) = \frac{1}{2}(p^{k}_\text{max} + c^*(\varphi)), \quad q^{jk}_M(\varphi) = \frac{L_k}{2\gamma^k}(p^{k}_\text{max} - c^*(\varphi)) \quad \text{and} \quad \pi^{jk}_M(\varphi) = \frac{L_k}{4\gamma^k}(p^{k}_\text{max} - c^*(\varphi))^2.
\]

Taking the assignment of the productivity levels to the firms as given (i.e. the uncertainty of productivity draws is resolved) and thus the productivity-dependent firm-level variables as quantitatively determined, the firm’s decision about an activity and a mode of operation crucially depends on its productivity level. Heterogeneous firms self-select into those options that generate the highest profits among alternatives, conditional on the firm’s level of productivity. Given the outside option of refraining, a firm only takes an option of serving
a market into account when it is at least able to cover with its generated profits the fixed
costs associated with the activity. Levels of productivity at which the generated profits
and fixed costs equalize represent the minima required for firms to be profitably active and
are called zero-profit thresholds. In case of the domestic market with its fixed entry costs
$f_E > 0$, firms with a productivity level below the domestic sales threshold

$$\varphi_D^j = \frac{w^j}{p_{\text{max}}^j - 2\sqrt{f_E \gamma_j^j \ell_j}}$$

(1.11)
do not produce products for the domestic consumers and decide to exit the economy.
Foreign consumers in country $k$ however do not receive products through exports by firms
in country $j$ with a productivity level below the export sales threshold $\varphi_{X}^{jk}$ and they are
not served through FDI by firms in country $j$ with a productivity level below the FDI sales
threshold $\varphi_{F}^{jk}$.

$$\varphi_{X}^{jk} = \frac{\tau_{jk} w^j}{p_{\text{max}}^k - 2\sqrt{f_X \gamma_k^k \ell_k}} \quad \text{and} \quad \varphi_{F}^{jk} = \frac{w^k}{p_{\text{max}}^k - 2\sqrt{(f_X + f_I) \gamma_k^k \ell_k}}.$$

(1.12)

For a firm being sufficiently productive for a profitable activity abroad, productivity lev-
els at which the profits generated by exporting and undertaking FDI are equal indicate
an indifference in the choice of the operation mode and are called iso-profit thresholds.
Firms with a productivity level below and above those thresholds make different choices
and passing the threshold therefore would imply some change in the decision of firms.
The analysis works out that the activities and modes of operation can be sorted for the
heterogeneous firms by the productivity thresholds, between which some option yields the
highest (non-negative) profits and thus represents the preferred and chosen choice for firms
with corresponding levels of productivity. Restrictions on the model parameters need to
be imposed in order to fix the analysis. To arrange the zero-profit thresholds in a standard
way, this chapter assumes a lower bound on the transportation costs

$$\tau_{jk} > \frac{p_{\text{max}}^k \sqrt{L_k} - 2\sqrt{f_X \gamma_k^k \ell_k}}{p_{\text{max}}^j \sqrt{L_j} - 2\sqrt{f_E \gamma_j^j \ell_j}},$$

(A1)

ensuring that $\varphi_D^j < \varphi_{X}^{jk}$, which implies that all firms profitably exporting also serve the
domestic market, and an upper bound on the transportation costs

$$\tau_{jk} < \frac{w^k}{w^j} \frac{p_{\text{max}}^k \sqrt{L_k} - 2\sqrt{f_X \gamma_k^k \ell_k}}{p_{\text{max}}^j \sqrt{L_j} - 2\sqrt{(f_X + f_I) \gamma_k^k \ell_k}},$$

(A2)

replicating the order of the conventional sorting by ensuring that $\varphi_{X}^{jk} < \varphi_{F}^{jk}$. Assumption

\footnote{For (strictly) positive thresholds, this chapter assumes $2\sqrt{\gamma_j^j \sqrt{f_E} < p_{\text{max}}^j \sqrt{L_j}}$ and $2\sqrt{\gamma_k^k \sqrt{f_X + f_I} < p_{\text{max}}^k \sqrt{L_k}}$. See the Appendix A.2 for the details of derivation.}
1.2. MODEL

(A1) is more (less) restrictive for higher domestic (export) fixed costs, while assumption (A2) is less restrictive both the higher the FDI fixed costs and the higher the foreign wage premium.\footnote{See the Appendix A.3 for the details of derivation.}

\subsection*{1.2.3 Non-monotone Sorting}

Computing the iso-profit thresholds by setting equal the total profits from exporting and undertaking FDI, $\Pi_{M}^{jk}$ for $M \in \{X, F\}$, yields a quadratic equation and hence gives two possible solutions\footnote{See the Appendix A.4 for the details of derivation.}

\[ \varphi_{1,2}^{jk} = \frac{p_{max}^{k} \pm \sqrt{(p_{max}^{k})^2 - \frac{f_{I} \gamma^{k}_{j} \omega^{k}_{j}}{L^{k} \omega^{(-)}} \omega^{k}_{j}}}{\frac{f_{I} \gamma^{k}_{j} \omega^{k}_{j}}{L^{k} \omega^{(-)}}}, \]

with $0 < \omega^{k}_{j} \equiv \tau^{k} w^{j} - w^{k} < \omega^{k}_{j} \equiv \tau^{k} w^{j} + w^{k}$. Two distinct real solutions exist by assuming

\[ 2 \sqrt{\gamma^{k}} \sqrt{f_{I}} \sqrt{\frac{\omega^{k}_{j}}{\omega^{(-)}}} < p_{max}^{k} \sqrt{L^{k}}, \]

which requires sufficiently low FDI fixed costs, a sufficiently low wage in the destination country $k$ and a sufficiently high market potential in that country, all facts creating incentives to undertake horizontal foreign direct investment in the country abroad.

For the relation between the zero-profit and iso-profit thresholds and in detail the fact that the FDI sales threshold lies below the lower productivity level for which the profits equalize, i.e. $\varphi_{F}^{jk} < \varphi_{1}^{jk}$, this chapter consistently assumes\footnote{See the Appendix A.5 for the details of derivation.}

\[ 2p_{max}^{k} \Psi - \omega^{k}_{j} < \frac{4 \gamma^{k} f_{I} \Psi^{2}}{L^{k} \omega^{k}_{j}} \]

with $\Psi \equiv \frac{w^{k} \sqrt{f_{I}}}{p_{max}^{k} \sqrt{f_{I}} - 2 \sqrt{\gamma^{k} \sqrt{f_{X}} + f_{I}}}$, for which a sufficient condition is given by an upper bound on the transportation costs

\[ \tau^{k} \equiv \frac{w^{k}}{w^{j} (1 - z^{k}) \sqrt{f_{X}} + f_{I} p_{max}^{k} \sqrt{L^{k}} - 2 \sqrt{(f_{X} + f_{I}) \gamma^{k}}}, \]

with $z^{k} \equiv \frac{w^{k}}{\tau^{k} w^{j}} < 1$.

Under the assumptions (A1) – (A4), the following ordering of productivity thresholds pertains (Figure 1.1):

\[ 0 < \varphi_{D}^{j} < \varphi_{X}^{jk} < \varphi_{F}^{jk} < \varphi_{1}^{jk} < \varphi_{2}^{jk}. \]

This sequence implies the following sorting of firm activities and modes of operation, as
10  

CHAPTER 1. EXPORT VS. FDI: NON-MONOTONE SORTING

the productivity level successively increases \[\varphi^D\]. The least productive firms, i.e. firms whose productivity level is lower than \(\varphi^D\), exit the economy, while low- to medium-productive firms with a productivity level between \(\varphi^D\) and \(\varphi_X^F\) only serve the domestic market and operate as *domestic firms*. All firms with higher productivity levels engage additionally in the foreign market via one of the two available options, exporting or undertaking FDI. Those firms with intermediate productivity levels, i.e. with a \(\varphi\) between \(\varphi_X^F\) and \(\varphi_1^F\) (\(\equiv\varphi_X^{F,F}\)), export their products to the foreign market (*exporters*) and high-productive ones with productivity levels between \(\varphi_1^F\) and \(\varphi_2^F\) (\(\equiv\varphi_F^{F,X}\)) serve the foreign market by undertaking FDI, i.e. they establish production facilities in the foreign market and become *multinational enterprises*. Importantly, the highest productive firms, i.e. those with a productivity level above \(\varphi_2^F\), again decide to export their products to the foreign market (*exporters*). Hence, this chapter derives a non-monotone sorting that differs from the conventional one that is derived by Helpman et al. (2004), as firms at the upper end of the productivity range decide to export instead of undertaking FDI: Up to the threshold \(\varphi_2^F\), the conventional sorting is qualitatively replicated, but firms with a productivity level above this threshold (top performers) revert to a mode of operation (exporting) already chosen by firms with intermediate productivity levels, and not by the ones with adjacent levels of productivity.

In order to get an intuition for the result, one has to remember that firms charge lower prices, sell larger output volumes and are able to realize both higher markups and profits as the productivity level increases. While lower-productivity firms decide to serve the

\[\varphi^D\]\(\square\) For the sorting, the threshold \(\varphi^F\) is immaterial, since it does not indicate a switch in the decision of the firms, and therefore appears in Figure 1.1 as colored in gray only for completeness.

\(\square\) Deviations from the conventional sorting are theoretically also found in a general equilibrium Ricardian model with within-country and within-sector productivity homogeneity, see ?, and in a model with firm heterogeneity as well as cross-country factor-price and market-size heterogeneity, thereby allowing for vertical investment motives of firms, see Head and Ries (2003).
1.2. MODEL

Figure 1.2: Activity and Operation Mode Choices across Firm Productivity (Non-monotone Sorting)

foreign market via exports because of the associated low fixed costs and low total variable costs due to the limited sales volumes (concentration more relevant than proximity), higher-productivity firms would by this choice suffer from high total variable costs due to their large sales volumes. Therefore, these firms shift to undertaking FDI, being profitable enough to bear the associated high fixed costs while being able to save variable costs (proximity more relevant than concentration), thereby maximizing overall profits. So far, up to $\varphi_{2}^{jk}$, these solutions to the proximity-concentration trade-off give rise to the conventional sorting.

Importantly, under quadratic preferences, the highest-productivity firms export again instead of undertaking FDI (Figure 1.2) and thus the way, firms trade-off fixed and variable costs, becomes more complex. The firms with the highest productivity levels charge low prices and face a low-elasticity part of the demand curve, for which any further decrease in prices would lead only to a small increase in sales. So the incentive for those firms to cut variable costs through FDI, and thereby to reduce prices in exchange for high fixed costs, vanishes. And even though for the highest-productivity firms with large sales volumes total variable costs in case of exporting are high, the reduction in demand and operating profits due to exporting appears to be smaller than the savings on fixed costs, and this calculation induces those firms to choose exporting, leading to a non-monotone sorting.

Technically, the result arises because of the relative convexity of the export profit function (Figure 1.1). As pointed out by Cargo (1965) in its general form, a necessary and sufficient condition for this property is given by

$$\frac{\Pi''_{X}}{\Pi'_{X}} \geq \frac{\Pi''_{F}}{\Pi'_{F}},$$

with $\Pi''_{M}$ as the second derivative and $\Pi'_{M}$ as the first derivative of the total profit function for the operation mode $M \in \{X, F\}$. By inserting the respective expressions and simplifying the inequality, the condition reduces to $\tau^{jk}w^{j} \geq w^{k}$ or equivalently to $\omega_{(j)}^{jk} \geq 0$, which is fulfilled in the present analysis. For the special case of symmetric countries with identical wages, it is apparent that in both settings, the absence of transportation costs ($\tau^{jk} = 1$) and the existence of iceberg transportation costs ($\tau^{jk} > 1$), the export profit function is relative convex and thereby the non-monotone sorting of internationally active firms exists.

The conducted analysis and its result are consistent with Mrázová and Neary (forth-
who study in a general setting with different demand and supply assumptions
the firm’s choice between different modes of operation, whose result depends on the firm’s heterogeneity in productivity. In this way, firms (self-)select into a single option of a firm’s activity and operation mode and the aggregate firm behavior that is worked out by the meta-analysis is called selection effects. Importantly, the meta-analysis is thereby applicable to the choice between exporting and undertaking FDI, the relevant case for this chapter, and applies the concept of the modularity of the firm’s objective function, i.e. the (maximum operating) profit function \( \pi(t, c) \) with \( t \) as the transportation costs or tariffs and \( c \) as the marginal production costs. This concept turns out to be the distinguishing feature of the firms’ selection effects. A profit function reveals the property of super-modularity in its arguments if

\[
\pi(t_1, c_1) - \pi(t_2, c_1) \geq \pi(t_1, c_2) - \pi(t_2, c_2)
\]

for all \( t_1 \geq t_2, c_1 \geq c_2 \). In this case, more productive firms benefit more in terms of profits from a reduction in transportation costs, which seems to be a plausible relation, given that firms with larger amounts of goods internationally shipped take a larger advantage of cost reductions and those firms might probably be the more productive ones. Applying religious associations, the pattern of profit changes across firms in case of super-modularity reflect the “Matthew effect”: To those who have, more shall be given. The super-modularity of the profit function however ultimately implies as a sufficient condition the conventional sorting for the firms’ (self-)selection among the options of being exporters and multinational firms: More productive firms choose to serve the destination abroad through affiliates by undertaking FDI, while less productive firms export their domestically manufactured goods to the foreign market. Consequently, the intended determination of the selection effects reduces to the task of identifying the modularity of the firms’ profit function, whereas any violation of the super-modularity, which is called sub-modularity, does not by itself imply reversed selection effects.

The identification of the modularity is straightforward, if the profit function is continuous and differentiable: Given such a continuous and differentiable profit function, super-modularity is equivalent to a non-negative cross-derivative with respect to its arguments: \( \pi_{tc} \geq 0 \) for all \( (t, c) \). Any multiplicative specification of the cost parameters in the profit function allows for explicit demand conditions that can be derived for the super-modularity of the profit function. With both \( \varepsilon \equiv -(p/x)(\partial x/\partial p) = -p/xp' \) as the price elasticity of demand which represents a measure of its slope, and \( \rho \equiv -xp''/p' \) as a measure of its curvature, a demand function is called super-convex at a point if \( \log p \) is convex in \( \log x \) at that point and the super-convexity is equivalent to the properties that the demand function is more convex than a CES demand function with the same elasticity as well as its elasticity is increasing in sales, i.e. \( \varepsilon_x \) is positive. Any violation of the super-convexity is called sub-convexity, especially the case where \( \varepsilon \) is decreasing in \( x \), and is sometimes

\[
\frac{d^2 \log p}{d \log x} = \frac{1}{\varepsilon} (\rho - \frac{\varepsilon+1}{\varepsilon}) \geq 0,
\]

\[
\frac{d^2 \log x}{d \log p} = \varepsilon^2 (\rho - \frac{\varepsilon+1}{\varepsilon}) \geq 0.
\]
referred to as “Marshall’s Second Law of Demand”, as Marshall 1930 characterizes it as the benchmark for demand. Melitz 2018 points to its theoretical absoluteness to replicate substantial empirical facts. As pointed out by Mrážová and Neary forthcoming, the super-convexity of the demand function and the super-modularity of the profit function are then directly related by the following finding: A profit function $\pi(t, c)$ is super-modular in both its arguments $(t, c)$ at all levels of output if the demand function is weakly super-convex, i.e. if the elasticity of demand is non-decreasing in sales ($\varepsilon_x \geq 0$). Most notably for the analysis of the present chapter, Mrážová and Neary forthcoming derive modularity conditions: The super-modularity of the profit function $\pi(t, c)$ in both its arguments $(t, c)$ is equivalent to either of the following conditions that are equivalent among themselves: The elasticity of output with respect to marginal cost is greater than one in absolute value; the elasticity of marginal revenue with respect to output is less than one in absolute value; or the sum of the elasticity $\varepsilon$ and the convexity $\rho$ of the demand function is greater than three.

Let us now relate the insights of Mrážová and Neary forthcoming’s meta-analysis to the present chapter’s case: For a linear demand function, which is implied by (any specification of) quadratic preferences and which is as sub-convex, the meta-analysis reveals that the profit function switches with decreasing costs from super-modularity to sub-modularity, given that the curvature of the demand function is zero ($\rho = 0$) and its elasticity monotonically decreases with output/sales ($\varepsilon_x < 0$). Therefore, the conventional sorting gets established for the lower-performance firms, while it is reversed for the higher-performance ones, which contrasts the case of CES preferences with a weakly super-convex demand function, a super-modular profit function and hence the conventional sorting for all output/sales levels. Firms with low costs and high output levels however face a part of the demand function with a relatively low elasticity, i.e. the elasticity is less than three, and deal with a sub-modular profit function that indicates - if at all - a weaker Matthew effect. It provides insufficient incentives for the highest-performance firms to avoid additional variable costs by engaging in the activity with higher fixed costs, which leads them to choose exporting instead of undertaking FDI for serving the foreign market, and overall results in the derived non-monotone sorting.

1.3 Empirical Support

The non-monotone sorting that is derived in this chapter obtains empirical support both (i) directly from empirical evidence on a less monotone and more complex sorting compared to the conventional one, and (ii) indirectly from the empirical inconsistency of the assumptions under which the conventional sorting is derived, as well as the empirical consistency of its own assumptions.

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24For the details: See the Section 1.1
Let us first consider the direct empirical support for the non-monotone sorting. For some types of international firm activity, in particular investment and services trade, deviations from the structure and quantitative levels that are predicted by Helpman et al. (2004) occur, and a sorting that is less monotone and more complex compared to the conventional one can be directly observed in several cases. And this provides support for the present chapter’s non-monotone sorting. Analyzing firm-level data for U.S. multinational enterprises, Yeaple (2009) points out that the sales of a U.S. firm’s affiliates are indeed increasing in the parent firm’s productivity as measured by the parent firm’s U.S. sales, but this takes place at a rate which is lower than that predicted by the model. In addition, the standard model of the proximity-concentration trade-off with heterogeneous firms misses an element that is able to explain why the unit cost of serving foreign markets appears to rise in distance. And finally, the comparison of the actual and predicted engagement of firms yields that the large firms appear to invest in too few countries while the small firms invest in too many foreign locations. In all countries, more U.S. multinational activity is predicted than actually observed and in particular relatively less attractive countries experience much less multinational activity due to the underinvestment than would be predicted by the model.\footnote{Yeaple (2009) by himself offers a way to bring the theory more into line with the empirical facts: Allowing for variable markups, as it is the case with quadratic preferences.}

Spearot (2012) studies the investment choices between greenfield investment and acquisition (“brownfield investment”) of North-American industrial firms, primarily classified in the agricultural, commodity and manufacturing industries, between 1980 and 2004. He finds that those in a middle range of productivity engage in the largest amount of investment, both for greenfield investment and acquisition.

In addition, in case of services trade\footnote{For a literature overview of services trade and policy, see Francois and Hoekman (2010).} a robust finding in the literature suggests some sorting which is reversed compared to the conventional one: While more productive firms are engaged in exports, less productive suppliers of services undertake FDI. Bhattacharya et al. (2012) observe this fact in the Indian software industry between 2000 and 2008. It is also confirmed in a study by Foster-McGregor et al. (2014) for nineteen sub-Saharan African countries, based on a survey that is conducted in 2010-2011 and analyses 1,437 domestically owned services firms. Verifying the finding for a highly developed country, which likewise represents an important player in the service exports worldwide, Wagner (2014) provides the empirical evidence that those with FDI are less productive than firms that export for large German firms with at least 100 employees and an annual sum of turnover and other operating income equal or higher than €250,000 in business services trade. All these studies reveal for certain firm activities in selected countries deviations from the conventional sorting and point instead to a sorting that is more complex and less monotone, in line with this chapter’s result of the non-monotone sorting.

Let us now turn to the indirect empirical support for the non-monotone sorting. Contrary to the constant-markups implication of the assumption of monopolistic competition
1.3. **EMPIRICAL SUPPORT**

and CES preferences, an extensive empirical literature documents variable markups across firms and an incomplete pass-through that varies across firms, which represents an indication of markups that vary across firms. De Loecker and Warzynski (2012) detect significant differences in markups between exporting and non-exporting firms for Slovenian data, with robustly higher markups for exporters. Additionally, they provide empirical evidence at the dynamic level that firm markups increase when firms enter export markets. Examining firms’ markup adjustments to exchange rate shocks in Belgium, Amiti et al. (2014) document an incomplete pass-through of exchange rates on prices at the aggregate level and a variation in the pass-through across firms. Small exporters with zero import intensity and small market shares exhibit a nearly complete pass-through of exchange rates, while large exporters with a high import intensity and large market shares show an exchange rate pass-through of 55%, both the marginal cost and markup channels contributing roughly equally to the cross-sectional variation and overall leading to a low aggregate exchange rate pass-through of 62% due to an import intensity and export market shares distribution that is heavily skewed towards the largest exporters.

Based on direct evidence on markups in Pakistan, made possible by data collection in form of producer surveys, Atkin et al. (2015) find for soccer-ball manufacturers a greater markup dispersion than costs dispersion, at least in proportional terms, and a positive correlation of both costs and markups with firm size. Furthermore, the elasticity of markups with respect to firm size is significantly larger than the elasticity of costs and larger firms charge higher markups both because a larger share of higher-quality ball types, which carry higher markups, is produced and because those firms charge higher markups for a given type of ball. Marketing efforts thereby seem to contribute to the higher markups: The correlation between markups and measures of whether a firm attends an annual international sports goods trade fair and sells to richer countries is higher than the correlation between markups and measures of technical efficiency.

Bellone et al. (2016) investigate firm markups in the French manufacturing industry and show that these markups are higher with firm productivity and lower with the intensity of local competition as well as the degree of import penetration. Once again, markups are found to be positively related to the export participation of firms. In addition, the characteristics of the export destination seem to play a role: The exporters’ markups are higher for countries with a higher average wealth and for countries on average further away from the exporters’ location. Explicitly accounting for multi-product firms in times of a comprehensive trade liberalization in India, De Loecker et al. (2016) observe not only variable markups across firms, with higher markups for firms with larger outputs and higher accounting profits, but even within firms: For products further away from the firms’

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27 A vast strand of literature exists on the exchange rate pass-through, thereby documenting a heterogeneity in the degree of pass-through incompleteness across firms and across products within firms: See Berman et al. (2012) in terms of firm size and productivity for France, Chatterjee et al. (2013) for Brazil and Li et al. (2015) in terms of firm productivity and other dimensions (i.e., intensity of imported inputs) for China.
core competency variety, they have higher marginal costs and realize lower markups. In
the course of the trade liberalization with both reductions in input and output tariffs,
associated changes in marginal costs are not perfectly accompanied by changes in prices,
indicating variable markups and incomplete pass-through of costs to prices at the firm
level: While the average output and input tariffs decline by a substantial amount of 62
and 24 percentage points, respectively, marginal costs experience a decline on average by
31 percent due primarily to input tariff liberalization while prices fall on average by 18
percent. The difference in the response of marginal costs and prices reflects adjustments
in the markups: On average, markups increase by 13 percent, even so the output tariff
reductions have some pro-competitive effects by putting downward pressure on markups,
with substantially more pronounced effects on products with initially high markups. Firms
with larger increases in average markups apparently face a less restrictive surrounding and
are found to be more likely to introduce new products, being able to cover the costs
associated with their development. Especially for those firms with high initial markups,
the trade reform’s pro-competitive effect caused by output tariff reductions also restricts
the firms’ ability to raise the markups even further.

With US data on household purchases, Hottman et al. (2016) structurally estimate a
model of heterogeneous multi-product firms and thereby analyze the firm composition and
heterogeneity of sectors, finding substantial differences in both size and markups across
firms. A typical sector consists of a few large firms that occupy substantial market shares
and simultaneously a competitive fringe of firms with only trivial market shares. Their price
behavior can be approximated by the monopolistic competition benchmark of constant
markups, while the very largest firms realize substantially higher markups. Moreover, the
variation in markups turns out to be greater under quantity than under price competition.
Quantitatively, in a typical sector, the largest firm with a market share of above 20%
is able to implement a markup that is 24% higher than that of the median firm under
price competition and almost twice as large as that of the median firm under quantity
competition. However, the markup differentials quite rapidly cut back for the second- and
third-largest firms, indicating that only a few firms can make use of their market power.

Amiti et al. (forthcoming) study firms’ price setting reactions in response to cost shocks
for Belgian data and find strong evidence for an incomplete cost pass-through and strategic
complementarities: While the typical firm adjusts its price with an elasticity of 0.65 to 0.70
in response to its own cost shocks, its response to the price changes of its competitors takes
place with an elasticity of 0.35 to 0.40. These average reactions however conceal a lot of
heterogeneity across firms, in the way that small firms exhibit no strategic complementarities
and have a complete pass-through of own cost shocks, whereas large firms react

\[A\text{ large empirical literature on the relationship between trade liberalization and firm markups exists.}
The depressing effects of import (output) tariff cuts on markups (pro-competitive effects) are thereby
considered in a number of empirical case studies, see e.g. Levinsohn (1993) for Turkey, Harrison (1994)
for the Ivory Coast and Brandt et al. (2017) for China (with the enhancing effects of input tariff cuts on
markups).\]
strongly to competitors in their price setting and charge variable markups, accounting for large shares of sales and thereby shaping the average patterns.

While the assumptions of the present chapter’s analysis are consistent with the above empirical evidence (in contrast to those of the model that yields the conventional sorting), whether the empirically documented variability of markups across firms has a relevant normative implication in case of trade is the subject of an active discussion in the theoretical literature of international trade. Theoretically, monopolistic competition models with heterogeneous firms in international trade provide in general three types of gains from trade: Besides the gains from the access to new import varieties of differentiated products (variety expansion effect), they propose the self-selection of more efficient firms into exporting and the market exit of less efficient firms, overall leading to gains due to the rise of the average productivity (firm selection effect), as well as the reduction in markups charged by firms due to the import competition and gains due to the lower consumer prices (pro-competitive effect). By the application of CES preferences as the demand structure of the models, the third source of gains is by construction closed down and thereby a priori excluded from any welfare evaluation. However, Arkolakis et al. (2019) show that even in the case of non-CES preferences and so the case of allowing for variable markups, the pro-competitive effect does not lead to gains from trade, questioning the normative relevance of variable markups. Replying to the study, Feenstra (2018) points to the sensitivity of the result with respect to the assumed unbounded Pareto distribution of productivity and demonstrates that the pro-competitive effect regains a role in the normative considerations in the case of a bounded, i.e. truncated from above, Pareto productivity distribution. In a structural estimation with a monopolistic competition and translog preferences model specification, Feenstra and Weinstein (2017) ascertain that the pro-competitive effect, together with the variety expansion effect, which gets also restored by the distribution adjustment, leads to cumulative welfare gains to the United States of 0.85% of total GDP over 1992-2005 and Feenstra (2018) then draws the conclusion that both effects thereby account for about 75% of the increase in U.S. welfare over this period resulting from the expansion in trade, while the residual firm selection effect contributes in an upper bound the remaining 25%.

An additional shortcoming in the empirical consistency of monopolistic competition models with CES preferences is that they do not capture reallocations of firms at the intensive margin due to variations across sales markets and changes within a sales market.

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29 When preferences are non-homothetic, total gains are reduced by the pro-competitive effect.
30 In his study, Feenstra (2018) applies a quadratic mean of order r (QMOR) expenditure function.
31 Translog preferences represent a special case of the QMOR expenditure function with a parameter restriction. Feenstra (2018) obtains a similar result (0.93%) for the welfare gains of the two effects for the same country and period, an explicit quantitative assessment of an upper-bound of the firm selection effect and a small adjustment/extension in the estimation approach, thereby being able to draw qualitatively and quantitatively equivalent conclusions for both studies.
32 For a normative analysis of a monopolistic competition model with heterogeneous firms and either quadratic preferences or a general system of demand: See Nocco et al. (2014, 2017) and Dhingra and Morrow (2017, forthcoming).
Mayer et al. (2014) document for French multi-product exporters reallocations of their products’ market shares (product mix) across destinations: They skew their export sales towards their better performing products in markets where they face tougher competition, i.e. in bigger markets (measured by destination’s GDP in a given year) and in markets with more competing firms (measured by the geography of the destination in a given year). In a follow-up study, Mayer et al. (2016) take a look at the within-exporter adjustments of the mix of products sold in a particular destination over time and find that firms skew their sales towards their better performing products in response to a positive demand shock in the export destination. As repercussions of the induced reallocations, multi-product firm productivity increases following the demand shock, with an elasticity of the labor productivity between 5% and 11%, and the productivity in the French manufacturing sector between 1995-2005 increases on average by 1.2% per year due to the growth in world trade.

To summarize the empirical support for the non-monotone sorting, one can find both direct evidence from case studies indicating a less monotone and more complex sorting compared to the conventional one, and indirect evidence from the empirical inconsistency of the assumptions of the conventional sorting and the empirical consistency of the assumptions of the non-monotone sorting. Especially, the robust empirical fact of variable markups across firms can not be captured by the demand assumptions of the conventional sorting, but the demand assumptions of the non-monotone sorting.

1.4 Conclusion

Monopolistic competition models with CES consumer preferences do not replicate substantial empirical facts and thereby provide an incentive to look for an alternative theoretical assumption on consumption. This chapter now applies quadratic instead of CES preferences in a monopolistic competition model of international trade with heterogeneous firms, being consistent with “Marshall’s Second Law of Demand” and the empirical evidence, and thereby investigates in a setting of a firms’ choice between exporting and undertaking FDI the robustness of the conventional sorting of cross-border active firms that is derived by Helpman et al. (2004). At the upper end of the productivity range, this chapter detects a significant change: While (sufficiently) productive firms export and even more productive ones engage in FDI, the most productive firms do not undertake FDI, but export again. The non-monotone sorting that this chapter derives is consistent with the meta-analysis of firms’ selection effects by Mrázová and Neary (forthcoming). In addition, both direct and indirect empirical support in its favor are found in form of the empirical evidence on a less monotone and more complex sorting compared to the conventional one in the investment and services trade as well as the empirical inconsistency of the assumptions under which the conventional sorting is derived and the empirical consistency of the non-monotone sorting’s own assumptions.
Appendix A

A.1 Profit Functions

The (operating) profits that are generated by export sales are given by

\[ \pi_{jk}^X (\varphi) = \frac{L_k}{4\gamma_k} \left( p_{max}^k - \frac{\tau_{jk} w_j}{\varphi} \right)^2. \]

A positive derivative of the export profit function with respect to \( \varphi \) requires

\[ \frac{\partial \pi_{jk}^X (\varphi)}{\partial \varphi} = 2 \frac{L_k}{4\gamma_k} \left( p_{max}^k - \frac{\tau_{jk} w_j}{\varphi} \right) \left( \frac{\tau_{jk} w_j}{\varphi^2} \right) > 0 \]

\[ \iff \quad \frac{p_{max}^k - \tau_{jk} w_j}{\varphi} > 0 \]

\[ \iff \quad \varphi > \frac{\tau_{jk} w_j}{p_{max}^k}. \]

A positive second derivative of the export profit function with respect to \( \varphi \) requires

\[ \frac{\partial^2 \pi_{jk}^X (\varphi)}{\partial \varphi^2} = 2 \frac{L_k}{4\gamma_k} \left( \frac{\tau_{jk} w_j}{\varphi^2} \left( \frac{\tau_{jk} w_j}{\varphi^2} \right) + \left( \frac{p_{max}^k - \tau_{jk} w_j}{\varphi} \right) \left( -2\frac{\tau_{jk} w_j}{\varphi^3} \right) \right) > 0 \]

\[ \iff \quad \left( \frac{\tau_{jk} w_j}{\varphi^2} \right)^2 > 2 \frac{\tau_{jk} w_j}{\varphi^3} \left( \frac{p_{max}^k - \tau_{jk} w_j}{\varphi} \right) \]

\[ \iff \quad \frac{\tau_{jk} w_j}{\varphi} > 2 \left( \frac{p_{max}^k - \tau_{jk} w_j}{\varphi} \right) \]

\[ \iff \quad 1.5 \frac{\tau_{jk} w_j}{p_{max}^k} > \varphi. \]

Therefore, the export profit function is (strictly) increasing and

\[ \begin{cases} 
(\text{strictly) convex} & \text{for } \varphi \in \left( \frac{\tau_{jk} w_j}{p_{max}^k}, 1.5 \frac{\tau_{jk} w_j}{p_{max}^k} \right) = \frac{\tau_{jk} w_j}{p_{max}^k} (1, 1.5), \\
(\text{strictly) concave} & \text{for } \varphi \in \left( 1.5 \frac{\tau_{jk} w_j}{p_{max}^k}, \infty \right). 
\end{cases} \]
The (operating) profits that are generated by FDI sales are given by
\[
\pi_{jk}^F (\varphi) = \frac{L^k}{4 \gamma^k} \left( p_{max}^k - \frac{w^k}{\varphi} \right)^2.
\]
A positive derivative of the FDI profit function with respect to \( \varphi \) requires
\[
\frac{\partial \pi_{jk}^F (\varphi)}{\partial \varphi} = 2 \frac{L^k}{4 \gamma^k} \left( p_{max}^k - \frac{w^k}{\varphi} \right) \left( \frac{w^k}{\varphi^2} \right) > 0
\]
\[\iff p_{max}^k > \frac{w^k}{\varphi} \]
\[\iff \varphi > \frac{w^k}{p_{max}^k}.\]
A positive second derivative of the FDI profit function with respect to \( \varphi \) requires
\[
\frac{\partial^2 \pi_{jk}^F (\varphi)}{\partial \varphi^2} = 2 \frac{L^k}{4 \gamma^k} \left( \frac{w^k}{\varphi^2} \right) + \left( p_{max}^k - \frac{w^k}{\varphi} \right) \left( -2 \frac{w^k}{\varphi^3} \right) > 0
\]
\[\iff \left( \frac{w^k}{\varphi^2} \right)^2 > 2 \frac{w^k}{\varphi^3} \left( p_{max}^k - \frac{w^k}{\varphi} \right)
\]
\[\iff \frac{w^k}{\varphi} > 2 \left( p_{max}^k - \frac{w^k}{\varphi} \right)
\]
\[\iff 1.5 \frac{w^k}{p_{max}^k} > \varphi.\]
Therefore, the FDI profit function is (strictly) increasing and
\[
\begin{cases} 
\text{(strictly) convex} & \text{for } \varphi \in \left( \frac{w^k}{p_{max}^k}, 1.5 \frac{w^k}{p_{max}^k} \right) = \frac{w^k}{p_{max}^k} (1, 1.5), \\
\text{(strictly) concave} & \text{for } \varphi \in \left( 1.5 \frac{w^k}{p_{max}^k}, \infty \right).
\end{cases}
\]
With \( \tau_{jk} w^j > w^k \) and \( 1.5 w^k > \tau_{jk} w^j > w^k \), the following cases of the curvature of the profit functions are given:

(i) for \( \varphi \in \left( \frac{\tau_{jk} w^j}{p_{max}^k}, 1.5 \frac{w^k}{p_{max}^k} \right) \): the export profit function is convex, the FDI profit function is convex,

(ii) for \( \varphi \in \left( 1.5 \frac{w^k}{p_{max}^k}, 1.5 \frac{\tau_{jk} w^j}{p_{max}^k} \right) \): the export profit function is convex, the FDI profit function is concave,

(iii) for \( \varphi \in \left( 1.5 \frac{\tau_{jk} w^j}{p_{max}^k}, \infty \right) \): the export profit function is concave, the FDI profit function is concave.
A.2 Zero-profit Thresholds

The domestic sales threshold derives as

$$\frac{L^j}{4\gamma^j} \left( p_{max}^j - \frac{w^j}{\varphi} \right)^2 = f_E,$$

and

$$p_{max}^j - \frac{w^j}{\varphi} = \sqrt{\frac{4\gamma^j L^j}{L^j}} f_E$$

(A.1)

The export sales threshold derives as

$$\frac{L^k}{4\gamma^k} \left( p_{max}^k - \frac{\tau^j w^j}{\varphi} \right)^2 = f_X,$$

$$p_{max}^k - \frac{\tau^j w^j}{\varphi} = \sqrt{\frac{4\gamma^k L^k}{L^k}} f_X$$

and

$$\varphi^k_X = \frac{\tau^j w^j}{p_{max}^k - 2\sqrt{\frac{L^k \gamma^j}{L^j}}}.$$  (A.2)

The FDI sales threshold derives as

$$\frac{L^k}{4\gamma^k} \left( p_{max}^k - \frac{w^k}{\varphi} \right)^2 = f_X + f_I,$$

$$p_{max}^k - \frac{w^k}{\varphi} = \sqrt{\frac{4\gamma^k L^k}{L^k}} (f_X + f_I)$$

and

$$\varphi^k_F = \frac{w^k}{p_{max}^k - 2\sqrt{\frac{f_X + f_I \gamma^k}{L^k}}}.$$  (A.3)

A.3 Ordering - Part I

In order to ensure that

$$\varphi^j_D < \varphi^k_X$$

holds, the following assumption on the transport costs has to be established:

$$\frac{w^j}{p_{max}^j - 2\sqrt{\frac{L^j \gamma^j}{L^j}}} < \frac{\tau^j w^j}{p_{max}^k - 2\sqrt{\frac{L^k \gamma^k}{L^k}}}.$$
\[ \Leftrightarrow p_{\text{max}}^k - 2\sqrt{\frac{E \gamma^k}{L_k}} < \tau^j \left( p_{\text{max}}^j - 2\sqrt{\frac{E \gamma^j}{L_j}} \right) \]

\[ \Leftrightarrow p_{\text{max}}^k \sqrt{L_k} - 2\sqrt{E \gamma^k} < \tau^j \sqrt{L_k} \left( p_{\text{max}}^j - 2\sqrt{E \gamma^j} \right) \]

\[ \Leftrightarrow \frac{p_{\text{max}}^k \sqrt{L_k} - 2\sqrt{E \gamma^k}}{p_{\text{max}}^j \sqrt{L_j} - 2\sqrt{E \gamma^j}} < \tau^j. \quad (A.4) \]

In order to ensure that

\[ \varphi_{\mathcal{X}}^j < \varphi_F^j \]

holds, the following assumption on the transport costs has to be established:

\[ \frac{\tau^j \varphi^j}{w^j} \left( p_{\text{max}}^j - 2\sqrt{\frac{(f_X + f_I) \gamma^j}{L_j}} \right) < \frac{w^j}{w^k} \left( p_{\text{max}}^k - 2\sqrt{\frac{f_X \gamma^k}{L_k}} \right) \]

\[ \Leftrightarrow \tau^j \varphi^j \left( p_{\text{max}}^k \sqrt{L_k} - 2\sqrt{(f_X + f_I) \gamma^k} \right) < \frac{w^j}{w^k} \left( p_{\text{max}}^k \sqrt{L_k} - 2\sqrt{f_X \gamma^k} \right) \]

\[ \Leftrightarrow \tau^j < \frac{w^j}{w^k} \frac{p_{\text{max}}^k \sqrt{L_k} - 2\sqrt{f_X \gamma^k}}{p_{\text{max}}^k \sqrt{L_k} - 2\sqrt{(f_X + f_I) \gamma^k}}. \quad (A.5) \]

### A.4 Iso-profit Thresholds

The iso-profit thresholds derive as

\[ \frac{L_k}{4 \gamma^k} \left( p_{\text{max}}^k - \tau^j \varphi^j \frac{w^j}{\varphi} \right)^2 - f_X = \frac{L_k}{4 \gamma^k} \left( p_{\text{max}}^k - \frac{w^k}{\varphi} \right)^2 - (f_X + f_I), \]

\[ \frac{L_k}{4 \gamma^k} \left( p_{\text{max}}^k - \tau^j \varphi^j \frac{w^j}{\varphi} \right)^2 = \frac{L_k}{4 \gamma^k} \left( p_{\text{max}}^k - \frac{w^k}{\varphi} \right)^2 - f_I, \]

\[ \left( p_{\text{max}}^k \right)^2 - 2p_{\text{max}}^k \tau^j \varphi^j \frac{w^j}{\varphi} + \left( \tau^j \varphi^j \frac{w^j}{\varphi} \right)^2 = \left( p_{\text{max}}^k \right)^2 - 2p_{\text{max}}^k \frac{w^k}{\varphi} + \left( \frac{w^k}{\varphi} \right)^2 - \frac{4 \gamma^k}{L_k} f_I, \]

\[ \left( \tau^j \varphi^j \frac{w^j}{\varphi} \right)^2 - \left( \frac{w^k}{\varphi} \right)^2 - 2p_{\text{max}}^k \tau^j \varphi^j \frac{w^j}{\varphi} + 2p_{\text{max}}^k \frac{w^k}{\varphi} + \frac{4 \gamma^k}{L_k} f_I = 0, \]

\[ \left( \tau^j \varphi^j \frac{w^j}{\varphi} - w^k \right) \left( \tau^j \varphi^j \frac{w^j}{\varphi} + w^k \right) \varphi^{-2} - 2p_{\text{max}}^k \left( \tau^j \varphi^j \frac{w^j}{\varphi} - w^k \right) \varphi^{-1} + \frac{4 \gamma^k}{L_k} f_I = 0, \]
with \( \omega_{(-)}^j \equiv \tau^j w^j - w^k \) and \( \omega_{(+)}^j \equiv \tau^j w^j + w^k \), and

\[
\varphi^2 - 2p_{\text{max}}^k L^k f_l A_\gamma^k \omega_{(-)}^j \varphi + \frac{L^k}{f_l A_\gamma^k \omega_{(-)}^j} \omega_{(+)}^j \omega_{(-)}^j = 0.
\]

The solution to the quadratic equation is given by

\[
\varphi_{1,2}^j = \frac{p_{\text{max}}^k L^k}{f_l A_\gamma^k \omega_{(-)}^j} \pm \sqrt{\left(\frac{p_{\text{max}}^k L^k}{f_l A_\gamma^k \omega_{(-)}^j}\right)^2 - \frac{L^k}{f_l A_\gamma^k \omega_{(-)}^j} \omega_{(+)}^j}.
\]

and

\[
\varphi_{1,2}^j = \frac{p_{\text{max}}^k \pm \sqrt{(p_{\text{max}}^k)^2 - \frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j}}{\frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j}}.
\]

(A.6)

For the existence of two distinct real solutions, the following assumption has to be established:

\[
p_{\text{max}}^k \sqrt{ \frac{L_k}{x} } > 2 \sqrt{ \gamma_x f_l A_\gamma^k \omega_{(-)}^j } \left[ \frac{\omega_{+}^j}{\omega_{-}^j} \right].
\]

A.5 Ordering - Part II

In order to ensure that

\[
\varphi_{F}^j < \varphi_{1}^j
\]

holds, the following assumption on the transport costs has to be established:

\[
\frac{w^k}{p_{\text{max}}^k - 2 \sqrt{ \frac{(x + f_l)^k}{L^k} } } < \frac{p_{\text{max}}^k - \sqrt{(p_{\text{max}}^k)^2 - \frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j}}{\frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j}}
\]

\[
\Leftrightarrow \frac{w^k}{p_{\text{max}}^k - 2 \sqrt{ \frac{(x + f_l)^k}{L^k} } } < \frac{\frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j}{p_{\text{max}}^k - \sqrt{(p_{\text{max}}^k)^2 - \frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j}}
\]

\[
\Leftrightarrow \sqrt{(p_{\text{max}}^k)^2 - \frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j} < \frac{w^k}{p_{\text{max}}^k - 2 \sqrt{ \frac{(x + f_l)^k}{L^k} } } < \frac{\frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j}{p_{\text{max}}^k - \sqrt{(p_{\text{max}}^k)^2 - \frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j}}
\]

\[
\Leftrightarrow (p_{\text{max}}^k)^2 - \frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j < \left( \frac{p_{\text{max}}^k - \frac{w^k}{p_{\text{max}}^k - 2 \sqrt{ \frac{(x + f_l)^k}{L^k} } } }{p_{\text{max}}^k - \sqrt{(p_{\text{max}}^k)^2 - \frac{f_l A_\gamma^k}{L^k \omega_{(-)}^j} \omega_{(+)}^j}} \right)^2
\]
\[ \Leftrightarrow \left( \frac{p_{\text{max}}}{k} \right)^2 - \frac{f_1 A^k_\gamma}{L k \omega_{(\text{+})}} < \left( \frac{p_{\text{max}}}{k} \right)^2 - \frac{w^k \sqrt{L^k}}{p_{\text{max}} \sqrt{L^k}} - \frac{f_1 A^k_\gamma}{L k \omega_{(\text{+})}} \]
\[ \Leftrightarrow -\omega_{(\text{+})} < -2p_{\text{max}} \frac{w^k \sqrt{L^k}}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}} \frac{f_1 A^k_\gamma}{L k \omega_{(\text{+})}} \]

with \( \Psi \equiv \frac{w^k \sqrt{L^k}}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}} \), and

\[ 2p_{\text{max}} \Psi - \omega_{(\text{+})} < \frac{4 \gamma f_1 L k \omega_{(\text{+})}}{} \Psi^2. \tag{A.7} \]

A sufficient condition for the assumption (A.7) exists:

\[ \Leftrightarrow 2p_{\text{max}} \frac{w^k \sqrt{L^k}}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}} - \omega_{(\text{+})} < \frac{4 \gamma f_1 L k \omega_{(\text{+})}}{} \left( \frac{w^k \sqrt{L^k}}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}} \right)^2 \]
\[ \Leftrightarrow \frac{2p_{\text{max}} w^k \sqrt{L^k} - (\tau_{(\text{+})} w^j + w^j) \left( p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k} \right) \frac{w^k \sqrt{L^k}}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}}}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k} \omega_{(\text{+})}} < \frac{4 \gamma f_1 L k \omega_{(\text{+})}}{} \left( \frac{w^k \sqrt{L^k}}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}} \right)^2 \]
\[ \Leftrightarrow -\omega_{(\text{+})} < -2p_{\text{max}} \frac{w^k \sqrt{L^k} + \omega_{(\text{+})} \sqrt{L^k} + \omega_{(\text{+})} \sqrt{L^k}}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}} \left( p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k} \right) \]
\[ \Leftrightarrow \frac{w^k \left( p_{\text{max}} \sqrt{L^k} + 2 \sqrt{(J_X + f_I) \gamma^k} \right) - \tau_{(\text{+})} w^j \left( p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k} \right)}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}} < \frac{w^k \left( p_{\text{max}} \sqrt{L^k} + 2 \sqrt{(J_X + f_I) \gamma^k} \right) - w^k \left( p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k} \right)}{p_{\text{max}} \sqrt{L^k} - 2 \sqrt{(J_X + f_I) \gamma^k}} \]
A.6 Relative Convexity

The concept of relative convexity is defined by Cargo (1965) (Theorem 2): Let \( I(a, b) \) denote the set of all continuous, strictly increasing functions on the interval \([a, b]\). Suppose that \( \zeta, \chi \in I(a, b) \) and that \( \zeta'' / \zeta' \) and \( \chi'' / \chi' \) are defined throughout \((a, b)\). Then \( \chi \) is convex with respect to \( \zeta \) (relative convexity) if and only if

\[
\frac{\chi''(t)}{\chi'(t)} \geq \frac{\zeta''(t)}{\zeta'(t)},
\]
whenever \( a < t < b \).

While the convexity of a (continuous and differentiable) function on an interval describes the relationship between the function and a linear model of the function (reference) based on the function value and its first derivative, the relative convexity of a (continuous and differentiable) function on an interval characterizes the relationship between the function and a model of a (reference) function based on the function values and their first derivatives.

The ratio of the first and second derivative of the export profit function is given by

\[
\frac{\pi''_X}{\pi'_X} = \left( \frac{\tau_{jk} w_j}{\varphi} \right)^2 + \left( p_{\text{max}}^k - \frac{\tau_{jk} w_j}{\varphi} \right) \left( \frac{-2 \tau_{jk} w_j}{\varphi^2} \right),
\]

\[
\frac{\pi''_X}{\pi'_X} = \frac{\tau_{jk} w_j}{\varphi^2} + \left( p_{\text{max}}^k - \frac{\tau_{jk} w_j}{\varphi} \right) \left( \frac{2}{\varphi} \right),
\]

\[
\frac{\pi''_X}{\pi'_X} = \tau_{jk} w_j \left( \frac{\varphi}{p_{\text{max}}^k \varphi - \tau_{jk} w_j} \right) - \frac{2}{\varphi},
\]

and

\[
\frac{\pi''_X}{\pi'_X} = \frac{\tau_{jk} w_j}{\varphi} \left( p_{\text{max}}^k - \frac{\tau_{jk} w_j}{\varphi} \right) - \frac{2}{\varphi}.
\]
APPENDIX A.

The ratio of the first and second derivative of the FDI profit function is given by:

\[
\frac{\tilde{\pi}_F''}{\tilde{\pi}_F'} = \left(\frac{w_k}{\varphi}\right)^2 + \left(\frac{p_{\text{max}}^k - w_k}{\varphi}\right) \frac{-2w_k}{\varphi^2},
\]

\[
\frac{\tilde{\pi}_F''}{\tilde{\pi}_F'} = \frac{w_k}{\varphi^2} + \left(\frac{p_{\text{max}}^k - w_k}{\varphi}\right) \frac{-2}{\varphi},
\]

\[
\frac{\tilde{\pi}_F''}{\tilde{\pi}_F'} = \frac{w_k}{\varphi^2} \left(\frac{\varphi}{p_{\text{max}}^k \varphi - w_k}\right) - \frac{2}{\varphi}
\]

and

\[
\frac{\tilde{\pi}_F''}{\tilde{\pi}_F'} = \frac{w_k}{\varphi} \left(\frac{\varphi}{p_{\text{max}}^k \varphi - w_k}\right) - \frac{2}{\varphi}.
\]

The condition for the relative convexity of the export profit function derives as

\[
\frac{\tilde{\pi}_X''}{\tilde{\pi}_X'} \geq \frac{\tilde{\pi}_F''}{\tilde{\pi}_F'} \iff \frac{\tau_{jk} w^j}{\varphi (p_{\text{max}}^k \varphi - \tau_{jk} w^j)} \geq \frac{w_k}{\varphi} \left(\frac{p_{\text{max}}^k \varphi - w_k}{\varphi}\right) - \frac{2}{\varphi}
\]

\[
\iff \tau_{jk} w^j \varphi (p_{\text{max}}^k \varphi - \tau_{jk} w^j) \geq w_k \varphi (p_{\text{max}}^k \varphi - w_k) \iff \tau_{jk} w^j \varphi^2 p_{\text{max}}^k \geq w_k \varphi^2 p_{\text{max}}^k \iff \tau_{jk} w^j \geq w_k.
\]

Figure A.1: Wage Allocations and Relative Convexity: Sets of Wage Allocations for which the Relative Convexity is (not) Fulfilled (Given that \(\tau_{jk} > 1\), i.e. Existence of Transportation Costs)
Chapter 2

Multi-product Firms in International Trade - A Survey and Evaluation of Empirical Studies

2.1 Introduction

With the progressive availability of (disaggregated) micro, i.e. product- and firm-product-level, data sets, products and in particular multi-product firms attracted more and more attention in the empirical literature during the last decade, leading to a meanwhile intensive treatment and a large body of research output. In an early as well as seminal study on the products that are purchased by a representative sample of households in 23 cities in the United States, Broda and Weinstein (2010) examine the turnover of products mainly in the grocery, drugstore and mass-merchandise sectors for the year 1994 and the period 1999-2003. They document high dynamics in the product market: In a four-year period, there is four times more entry and exit in the product market data than that found in the establishment and labor market data, since most of the product turnover takes place within existing firms: At four-year frequencies, 82% of product creation and 87% of product destruction happens within existing manufacturers. The firms consequently have a multi-product nature: A typical firm sells eight different products in two different brand modules. However, the distribution of products per firm is highly skewed, with a large number of firms having only a small number of products and a relatively small number of large and highly diversified firms that sell over 700 different products in over 35 different brands.

1For an overview about a likewise large body of research output in the theory of multi-product firms in international trade: See the Chapter 3.
2Product creation and destruction represent the weighted analogues of the entry and exit rates. The value in consumption is thereby used as the weight. Hence, creation (destruction) measures the share in total expenditure in period t (s) on those goods that were consumed in period t (s) but were not available in period s (t).
3Products are identified by the Universal Product Codes (UPCs), which are also denoted as "bar codes" and represent the lowest level of aggregation. They are aggregated to brand modules, which in turn are ordered into product groups.
CHAPTER 2. MULTI-PRODUCT FIRMS - SURVEY OF EMPIRICAL STUDIES

and 18 different product groups, thereby being responsible for over 60% of the sales (in the forth quarter of 2003).\footnote{A highly skewed sales (exports/imports) distribution across firms is frequently documented in the literature: See [Bernard et al. (2007)] for the U.S., [Mayer and Ottaviano (2008)] for European countries, [Freund and Pierola (2015)] for a set of developing countries and [Hottman et al. (2016)] for the U.S. The phenomenon of the existence of a few large firms (“superstars”) that decisively influence aggregate patterns and outcomes is also denoted as “granularity”, with these firms as the “granular elements” (grains) of the economy. For the macroeconomic implications of the granularity: See [Gabaix (2011)] and the application to international trade: See [Di Giovanni and Levchenko (2012)]. For the comparative advantage implications of the granularity: See [Gaubert and Itskhoki (2016)].} In this way, the average firm sells about 40 different products under about four different brands in about three modules, which are contained in about two product groups. Product creation and destruction by firms do not happen independently from the overall demand conditions, but it turns out that the net creation, i.e. creation less destruction, is strongly pro-cyclical with the total sales growth and primarily driven by creation rather than destruction.

The studies on multi-product firms in international trade by themselves mainly investigate the type-specific characteristics as well as the behavior of firms that simultaneously produce multiple products and are active in trade across national borders: In addition to the (standard) decision on the production (output) and sales amount of a product (scale: how much to produce and sell in a market (abroad)?), those firms have to make a choice on their supplied product portfolio, i.e. the number of products produced and sold (scope: how many to produce and sell in a market (abroad)?) as well as its composition (space: what to produce and sell in a market (abroad)?)\footnote{In this way, the average firm sells about 40 different products under about four different brands in about three modules, which are contained in about two product groups. Product creation and destruction by firms do not happen independently from the overall demand conditions, but it turns out that the net creation, i.e. creation less destruction, is strongly pro-cyclical with the total sales growth and primarily driven by creation rather than destruction.}. In particular, with this type of firms, a new and additional margin of adjustment to exogenous shocks in international economics, in particular trade liberalization and exchange rate fluctuations, is introduced and can be examined in empirical studies. Given the meanwhile extensive work already done, one may tend to raise the question: Summarizing, what do we empirically know about multi-product firms that binds the existing and upcoming theoretical modeling of this firm type being progressively treated?

The contribution of this chapter now consists of surveying and evaluating a large number of the empirical studies by taking up a novel cross-study perspective to get to know the actual state of knowledge about multi-product firms in international trade. Despite the methodological problem of a non-uniform definition of a product across the studies, several robust findings or stylized facts (empirical regularities) can be derived: Multi-product firms in international trade are not an exception, but represent and account for large shares of economic agents and activity: They (i) are national and especially international prominent, less prevalent but important (Section 2.2.1); a fact that justifies, encourages and also requires the attempt to collect the knowledge about this firm type. As it turns out by doing this, multi-product firms in international trade feature the following characteristics: They (ii) are singular economic units and reveal premia in terms of their nature and performance (Section 2.2.2), they (iii) produce and export at different margins (Section 2.2.3), they (iv) are asymmetric within themselves (“superstar products”, Section 2.2.4),
2.2. EMPIRICAL REGULARITIES

they (v) do not produce all their exports (“carry-along trade”, Section 2.2.5), they (vi) are relatively diversified in production and export (Section 2.2.6), they (vii) compete on a cost and quality basis (Section 2.2.7) and they (viii) are less prone to firm exit in the process of “creative destruction” (Section 2.2.8). Given the dynamically varying economic environment in which they operate, multi-product firms in international trade reveal the following behavior: They (ix) start exporting at a small scale and scope and increase them over time (Section 2.2.9), they (x) frequently and to a large extent change their product portfolio (“product switching”, Section 2.2.10) and they (xi) undertake adjustments in response to exogenous shocks (Section 2.2.11).

The remainder of the chapter is then organized as follows: The next section presents the empirical regularities on multi-product firms that are derived by surveying and evaluating a large number of empirical studies. The last section summarizes the main findings.

2.2 Empirical Regularities

The attempt of deriving empirical regularities about multi-product firms is undertaken by surveying and evaluating 50 empirical studies dealing with those firms in international trade. These studies use data sets gathered both in developed and developing countries, in American, Asian and European countries and mostly over a decade between the mid-1990s and mid-2000s. Because of the large geographical, sectoral/industrial and temporal coverage of the volume of the empirical studies, results out of a comparison of their findings can claim some validity for multi-product firms. However, as hinted above, an obvious limitation of this procedure must be mentioned and stressed beforehand: Products are typically defined according to categories of international classifications, which are hierarchical and top-down structured with an increasing level of disaggregation going further down the classification. Empirical studies in general and likewise in this survey however apply different classifications and levels of disaggregation to fix their definition of the term *product*5, which restricts a reasonable comparison of the findings across the studies, given that the findings, their robustness and their significance in each study are sensitive to the chosen product definition and those definitions describe significantly varying ideas of what a product represents. Therefore, any results in this survey must be treated with some caution and one should be aware of those methodological difficulties of a comparative evaluation of empirical studies of multi-product firms.

An overview about the evaluated empirical studies, their samples, unit of observation, industrial coverage and product definition used is given by Table B.1 Table B.2 Table B.3 Table B.4 and Table B.5 in the Appendix B.2.

---

5An overview about the commonly used product classifications and their levels of disaggregation is provided by the Appendix B.1.
## Empirical Study | Share of Firms$^a$ | Share of Output$^b$ | Average Number of Products$^c$ |
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Aw and Lee (2009) (Taiwanese data (1992-1999))</td>
<td>9%</td>
<td>40%</td>
<td>2.09†</td>
</tr>
<tr>
<td>Elliott and Virakul (2010) (Thai data (2001-2004))</td>
<td>43%</td>
<td>57%</td>
<td>-</td>
</tr>
<tr>
<td>Bernard et al. (2010) (U.S. data (1997))</td>
<td>39%</td>
<td>87%</td>
<td>3.5</td>
</tr>
<tr>
<td>Goldberg et al. (2010b) (Indian data (2003))</td>
<td>47%</td>
<td>80%</td>
<td>3.06</td>
</tr>
<tr>
<td>Navarro (2012) (Chilean data (1996-2003))</td>
<td>52%</td>
<td>56%</td>
<td>3.9</td>
</tr>
<tr>
<td>Söderbom and Weng (2012) (Chinese data (2004-2008))</td>
<td>47%</td>
<td>50%</td>
<td>2.76</td>
</tr>
<tr>
<td>Bernard and Okubo (2015) (Japanese data (2006))</td>
<td>39%</td>
<td>77%</td>
<td>2.8</td>
</tr>
<tr>
<td>Lo Turco and Maggioni (2016) (Turkish data (2009))</td>
<td>44%</td>
<td>68%</td>
<td>3.9</td>
</tr>
<tr>
<td>Boehm et al. (2016) (Indian data (2000-2008))</td>
<td>39%</td>
<td>71%</td>
<td>2.81</td>
</tr>
<tr>
<td>Caselli et al. (2017) (Mexican data (1994-2007))</td>
<td>58%</td>
<td>67%</td>
<td>2.86</td>
</tr>
<tr>
<td>Lopresti and Shiferaw (2017) (Ethiopian data (1996-2007))</td>
<td>34%</td>
<td>42%</td>
<td>-</td>
</tr>
<tr>
<td>Dhyne et al. (2017) (Belgian data (1997-2007))</td>
<td>-</td>
<td>74%††</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes: $^a$: share of multi-product firms in all country-wide active firms, $^b$: share of multi-product firms’ output in country-wide output, $^c$: average number of products produced by multi-product firms, †: weighted average across the four main industries in the Taiwanese electronics sector and ††: at the 2-digit PRODCOM level: 22% (firms producing 3+ products: 5% → firms specialize by typically producing goods within the same 2-digit category.

Table 2.1: Prevalence and Importance of Multi-product Firms in (National) Economies

### 2.2.1 Prevalence and Importance

#### Regularity 1

Multi-product firms are national and especially international prominent, less numerically, i.e. prevalent, but quantitatively, i.e. important.

Multi-product firms are national and especially international prominent, less numerically, i.e. prevalent, but quantitatively, i.e. important (Table 2.1 and Table 2.2). While these firms represent only a minority or slight majority (Navarro (2012), 52%; Caselli et al. (2017), 58%) of firms in a country, their output dominates national accounts. With the exception of the studies by Aw and Lee (2009), Liu (2010) and Lopresti and Shiferaw (2017) using relatively aggregated firm-product-level data (comparable to 4- or 5-digit SIC categories as products) and reporting a lower prevalence (Liu (2010)) as well as importance (Aw and Lee (2009) and Lopresti and Shiferaw (2017)) of multi-product firms, the studies
document that between 39% and 47% of the firms in a country produce more than one product; in detail, they produce on average between three and four products (2.76-3.9).\footnote{One exception is Aw and Lee (2009): They report a lower average number of products (2.09).}

Regarding output, one can split the country-level studies into three groups: The first group reports a national output share of multi-product firms of 56% or 57% (with Söderbom and Weng (2012) representing an outlier at the bottom limit), the second one demonstrates a greater relevance of multi-product firms by documenting an output share of about 69%, while the third group gives evidence of an again greater quantitative dominance of multi-product firms, as the studies of this group reveal that multi-product firms are responsible for about 79% of the national output (with Bernard et al. (2010) representing an outlier at the upper limit and Dhyne et al. (2017) representing a study between the second and third group).

Looking at international transactions, the prominence of multi-product firms, numerically and quantitatively, is even enhanced: They build the majority of exporting firms in a country (only the studies by Iacovone and Javorcik (2008), Elliott and Virakul (2010) and Chatterjee et al. (2013) with shares slightly below 50%) and account for almost all of the national export values (only the studies by Iacovone and Javorcik (2008), Elliott and Virakul (2010), Navarro (2012) and Chatterjee et al. (2013) with shares below 90%).

As a consequence, these facts about the national and international prevalence and importance of multi-product firms justify, encourage and likewise require by themselves an intensive theoretical analysis as well as an empirical examination of multi-product firms in international trade.

### 2.2.2 Singular Economic Units - Premia of Multi-product Firms

**Regularity 2** Multi-product firms are different in terms of premia of characteristics and performance, in particular size, productivity and export propensity, compared to single-product firms.

Empirical Study | Share of Exporting Firms$^d$ | Share of Export Value$^e$
---|---|---
Bernard et al. (2007) (U.S. data (2000)) | 58% | 99%
Mayer and Ottaviano (2008) (French data (2003)) | 65% | 98%
Iacovone and Javorcik (2008) (Mexican data (2003)) | 43% | 63%
Elliott and Virakul (2010) (Thai data (2001-2004)) | 47% | 52%
Navarro (2012) (Chilean data (1996-2003)) | 57% | 58%
Görg et al. (2012) (Hungarian data (2003)) | 96% | -
Chatterjee et al. (2013)† (Brazilian data (1997-2006)) | 49% | 78%
Amador and Opromolla (2013) (Portuguese data (1996-2005)) | 61% | 92%
Bernard et al. (2014) (Belgian data (2005)) | 66% | 98%
Damijan et al. (2014) (Slovenian data (2008)) | 83% | 99%

Notes: $^d$: share of multi-product exporters in all exporting firms in a country, $^e$: share of multi-product exporters’ value in total export value in a country and †: share of multi-product exporters’ employment in total employment in the export sector: 65% and 86% of unit-value observations from multi-product firms, overall average number of products exported by a firm to a given country: 5.2 (median number of products: 2) - heterogeneity across industries.

Table 2.2: Prevalence and Importance of Exporting Multi-product Firms in Intern. Trade


the firms’ diversification (diversification discount, Section 2.2.6), as well as Lopresti and Shiferaw (2017), who find that multi-product firms are not more productive in terms of labor productivity.

$^8$A restriction is given by Elliott and Virakul (2010) for Thailand, who find the ownership structure of firms to be an important determinant of firm behavior, and, in detail, they derive a negative correlation between the multiple product production status and foreign ownership, as well as report that only foreign-owned multi-product firms are more likely to export. Furthermore, an exception is given by Lopresti and Shiferaw (2017) for Ethiopia, who find that multi-product firms are not more likely to export.
rean data and Lopresti and Shiferaw (2017) for Ethiopian data, and more capital-intensive than single-product firms. Accordingly, Lopresti and Shiferaw (2017) detect that multi-product firms are more likely to invest in physical capital and are more likely to undertake large investments than single-product firms.

Inherently to (productive) firms producing more than a single product, they further empirically derive the result that those firms more often undertake research and development (R&D). The positive correlation between the multiple product production status and R&D is confirmed by several following studies, namely the one of Elliott and Virakul (2010) for Thai data, in which multi-product firms are more active in product and process innovation, latter especially in case of domestic firms, and also by Liu (2010) and Lopresti (2016), both reporting higher R&D expenditures of multi-product firms for U.S. data, as well as Choi and Hahn (2017), who find for Korean data a positive correlation between the multiple product production and innovation status.

The general discussion in the international trade literature about the productivity premium of internationally active firms and its reasons finds its extension to the empirical literature of multi-product firms, revealing both characteristics: (disproportional) international participation and a productivity premium. While Bernard et al. (2010) document that single-product firms that subsequently become multi-product firms are more productive than other single-product firms and interpret this finding as a contribution to some evidence for the selection approach, Navarro (2012) rejects that approach (in the sense that plants that add products to their portfolio are ex-ante more productive) in favor of the alternative one of learning.

2.2.3 Margins of Production and Export

Regularity 3 Multi-product firms produce and export at different margins.

The analysis of the margins of production and export, which becomes relevant in the case of multi-product firms and possible with the availability of disaggregated data, can be conducted schematically by deducing margins through aggregate data and data on other inverse margins or analytically by employing firm- and firm-product-level data. A firm’s output (sales) can be schematically decomposed into two margins, the product-extensive margin, i.e. the number of products produced (sold) by the firm, and the product-intensive margin, i.e. the product output produced (sold) by the firm. Denoting firm $f$’s extensive margin with $n_{if}$ and product $p$’s intensive margin with $x_{ifp}$, firm output (sales) $x_{if}$ is then given by: $x_{if} = n_{if} \times x_{ifp}$ and describes the firm-intensive margin. To get the aggregate, i.e. country-, sector- or industry-level output (sales) $X_{i}$, firm output (sales) $x_{if}$ has to be multiplied with the firm-extensive margin, i.e. the number of firms $N_{i}$ in the respective unit $i$: $X_{i} = N_{i} \times x_{if}$.

In their seminal paper, Hummels and Klenow (2005) analyze a country’s exports by decomposing it into an extensive (number of goods) and intensive (volume of each good) margin. They show that larger
CHAPTER 2. MULTI-PRODUCT FIRMS - SURVEY OF EMPIRICAL STUDIES

Figure 2.1: Decomposition of Aggregate Output (Sales) in the (Firm- and Product-level) Margins

product-level) data, the decomposition becomes possible to be conducted by empirical studies in international trade, thereby investigating firm behavior in more detail.¹⁰

The correlation between the product-extensive and the product-intensive margin is examined by several studies that derive in part opposite results. Iacovone and Javorcik (2008) (in exporting), Adalet (2009) (in exporting), Goldberg et al. (2010b) (in output), Elliott and Virakul (2010) (in exporting), Liu (2010) (in sales), Bernard et al. (2011)¹¹ (in exporting, per destination) find a positive correlation¹² in the sense that firms producing (exporting) a larger set of products also produce (export) more of each product in their portfolio. In contrast, Elliott and Virakul (2010) (in production) and Navarro (2012) (in sales and exporting) report for Thai and Chilean data respectively a negative correlation¹³ between the product-intensive and product-extensive margin, i.e. firms producing (exporting) a larger set of products produce (export) less of each product in their portfolio.

Another correlation between the product-extensive margin and the firm-intensive margin exists. As documented by Iacovone and Javorcik (2008), Adalet (2009), Bernard et al. (2011), Berthou and Fontagné (2013) (per destination) and Bernard et al. (2014), both margins are positively related in the way that larger exporters export more products. In addition, Berthou and Fontagné (2013) (per destination) find a positive correlation between the product-intensive margin and the firm-intensive margin for exporting, i.e. exporting economies export more in absolute terms than smaller economies and the extensive margin accounts for 62% of these larger exports.¹⁴

¹⁰ In the following, the analysis do not explicitly distinguish between schematically deduced values and analytically estimated values for the margins in the estimation results, since such a distinction do not provide exceeding insights (exception: the estimations by Bernard et al. by construction).
¹¹ In their baseline econometric exercise, Bernard et al. (2011) use the exports of the firm’s (5th-)largest product as a measure for the product-intensive margin; different to what is used in other studies. In addition, they state that the exports of this top product increase more sharply with the number of products exported than the exports of an average product; even so, they still confirm a positive correlation between the latter and the number of exported products.
¹² In line with the theoretical model of multi-product firms by Bernard et al. (2011).
¹³ In line with the theoretical models of multi-product firms by Eckel and Neary (2010) and Nocke and Yeaple (2014), incorporating crucial within-firm cannibalization effects and dis-economies of scope.
firms with larger exports per product are also larger in their aggregate exports.

Bernard et al. (2011) take a look at the margins of trade across countries and thereby analyze their responsiveness to variable trade costs (proxied by distance) and the market size of the destination country. As it is well established in the literature, exports decline with the distance of the source country to the destination country and increase with the destination country’s market size (Bernard et al. (2014)). The decomposition however yields that this negative relationship between the export value and distance is caused entirely by the extensive margin, i.e. the number of firm-product observations in the destination country, while the intensive margin, i.e. the average firm-product exports (product-intensive margin), seems not to be significantly influenced by distance (Bernard et al. (2014)). Even so the number of firms (firm-extensive margin; Bernard et al. (2014)) and the number of products (Bernard et al. (2014)) decline with distance (combined effect of the extensive margins almost completely explains cross-country variation in export value: Bernard et al. (2014)), the opposite is true for the density as a measure of the extent to which each firm supplies each product (Bernard et al. (2014)): The number of firm-product observations increases less than proportionately than the number of firms times the number of products as market size increases, illustrating the restricted set of products that firms are active in. In the analysis, Bernard et al. (2011) show that the number of products per firm (product-extensive margin) and the number of firms per product also decline with distance. Overall, the exports of a given product by a given firm and distance are negatively related.

Arkolakis and Muennder (2013) confirm the finding that the product-extensive margin is negatively related to distance, while they find that it is not significantly related to the destination country’s market size; so whether the country is large or small in terms of its market does not play a significant role for the number of products that a firm exports to it. In contrast, Berthou and Fontagne (2013) derive a positive correlation between the product-extensive margin and the destination country’s market size. This factor also positively influences the export decision, the average value exported per product by each firm (product-intensive margin) and the total export value of each firm (firm-intensive margin). Arkolakis and Muennder (2013) similarly identify that the product-intensive margin is significantly positively related to the destination’s market size, but unrelated to distance. Furthermore, the firm-extensive margin is driven by the destination’s market size; so finally, they conclude that the market size drives firm entry but not meaningfully influence subsequent product entry decisions of firms.

In addition to looking at the market potential (market size) and a barrier for goods delivery (trade costs) as potentially important factors of the margins of trade, Berthou and Fontagne (2013) take into account the competition from the rest of the world and figure out that both the product-intensive margin and the firm-intensive margin are negatively correlated with it. A 10% increase in the competition from other Euro-area countries leads to a reduction in the product-intensive margin by 5.8%.
So far, firm-external factors of the margins of trade were investigated, but it seems plausible to extend the consideration also to firm-internal factors. Firm characteristics that indicate the firm’s overall performance, especially its productivity, thereby represent natural candidates for potentially important factors of the margins of trade. With a higher productivity, firms are more likely to export products (Berthou and Fontagné (2013) and Masso and Vahter (2015)). More productive firms typically export larger volumes of goods (firm-intensive margin), documented by Berthou and Fontagné (2013) (per destination) and Bernard et al. (2014). According to the latter, a 10% increase in total factor productivity (labor productivity) is associated with a 0.7% (7.6%) increase in firm exports. More productive exporters are also found to be those firms that export a larger number of products (product-extensive margin), which is reported by Bernard et al. (2011), Berthou and Fontagné (2013) (per destination) and Bernard et al. (2014) (aggregate and per destination), and those firms that export to a larger number of destinations, according to Bernard et al. (2011) and Bernard et al. (2014). As Berthou and Fontagné (2013) (per destination) and Bernard et al. (2014) (per destination) show in addition, the product-intensive margin positively correlates with the firm’s productivity, which implies that more productive firms export more of each of their supplied products. In Bernard et al. (2014), the density measure falls with productivity: More productive firms export more products and reach more destinations but do not ship every product to every country. The aggregate increase in firm exports associated with a higher firm productivity can be attributed to more than a half to the extensive margin. Overall, the higher productivity at the firm level is associated with greater exports of a given product to a given country.

2.2.4 Heterogeneity within Firms - Product Asymmetry

Regularity 4 Multi-product firms reveal a within-firm product heterogeneity and hierarchy in the way that they sell more of some products and less of others, i.e. asymmetric products.

The sales distribution across products within firms is non-uniform but skewed and points to the existence of core or “superstar” varieties of firms and a product hierarchy within firms (Aw and Lee (2009), Adalet (2009), Goldberg et al. (2010b), Bernard et al. (2010), Liu (2010), Bernard et al. (2011), Navarro (2012), Söderbom and Weng (2012), Görg et al. (2012), Amador and Oprromolla (2013), Chatterjee et al. (2013), Masso and Vahter (2014), Aronson (2016), Lo Turco and Maggioni (2016), Lopresti (2016), Boehm et al. (2016), Caselli et al. (2017), Dhyne et al. (2017), Lopresti and Shiferaw (2017)). Regarding the theoretical foundation of selection effects in models with heterogeneous firms, see the seminal work by Melitz (2003). Miranda et al. (2012) show that the firm-extensive margin (firm exit) is mainly determined by firm characteristics and Masso and Vahter (2015) derive a productivity premium of multi-product or multi-market new exporters. For the “superstar” phenomenon (“granularity”) at the firm margin, i.e. a few large exporters account for large shares of a country’s exports: See Bernard et al. (2007) for the U.S., Mayer and Ottaviano (2008) for European countries, Freund and Pierola (2015) for a set of developing countries and Hottman et al. (2016) for the U.S.
2.2. EMPIRICAL REGULARITIES

magnitude of the distribution skewness, [Adalet (2009), Liu (2010), Bernard et al. (2011) and Navarro (2012)] report that the tails of this distribution are thinner than that of the Pareto distribution. [Liu (2010)] further documents that the average sales share of the largest product is about two times larger than the average sales share of the second largest product, while [Chatterjee et al. (2013)] find that on average the export value generated by the top product is approximately three times larger than the export value generated by the second-largest product and roughly twice as large as the export value generated by the sum of all the products exported by the firm. For firms with ten export products, a single product accounts on average for about 70% of export sales and the top-5 export products are responsible for about 98% of export sales ([Masso and Vahter (2014)]). According to [Arnarson (2016)], the top (second top) ranked export product accounts for 44% (18.3%) of the aggregate firm trade value, while the first three products in sum are responsible for over 73% of the aggregate firm trade value. Similarly, [Lopresti (2016)] reports a sales share of the firm’s core product of at least 45%.

The contribution of the top product to firm sales decreases with the scope of the firm [Navarro (2012), Arnarson (2016), Lo Turco and Maggioni (2016), Dhyne et al. (2017), Lopresti and Shiferaw (2017)]. [Dhyne et al. (2017)] (Lopresti and Shiferaw (2017)) document that the core product accounts for 77.5% (80%) of firm sales for firms that produce only two products, whereas its share decreases to 69.5% (66%) for firms that produce three products and to 49.4% (45%) for firms that produce six or more (six) products. For firms producing nine or more products, the sales share of the core product falls down below 50% and the shares of the second and third largest products amount then to 20% and 10%, respectively ([Lo Turco and Maggioni (2016)]).

For internationally active firms, the sales distribution across destinations within firms is also skewed towards a top element, even so this skewness is less pronounced compared to the product level ([Amador and Oprimolla (2013)]). Differentiating between products, the sales of the top product are much more concentrated in the top destination, while less important products are more homogeneously sold across markets; so here the degree of skewness is positively related to the product rank. [Arnarson (2016)] looks at the number of destination per product and finds that over 65% of the products are only exported to a single destination and a large majority of products (81%) with a total firm trade value of 9% are exported to three or less destinations. The skewness within firms is further expressed by the fact that products exported to three or less destinations and ranked second or lower account for 71.4% of the products but only for 5% of the firm export value. Contrary, a few products (5.2%) that are exported to four or more destinations and ranked among the three highest in the firm sales distribution are responsible for 66.9% of the firm export value.

In a study on the firm-product-level efficiency, [Garcia-Marin and Voigtländer (2017)] find that productive plants tend to be relatively efficient across their whole product portfolio and not just for their core product, which is consistent with the concept of flexible
CHAPTER 2. MULTI-PRODUCT FIRMS - SURVEY OF EMPIRICAL STUDIES

Moreover, the variable on which the product rankings (hierarchies) within firms are established turns out to be relevant: The standard concept of establishing product ladders according to the sales of the products leads to a seemingly contradictory pattern: Products that are sold at the largest scale and are therefore top-ranked exhibit higher physical efficiency but also higher marginal costs. The discrepancy is likely to be driven by product quality, which exerts an upward pressure on marginal costs, due to higher input prices, but leaves physical efficiency largely unaffected.

2.2.5 Divergence of Production and Exports - Carry-along Trade

Multi-product firms in general have three extensive margins that do not necessarily coincide: (i) the production scope, i.e. the number of products that the firm produces by itself, (ii) the domestic scope, i.e. the number of products that the firm sells in the domestic market and (iii) the export scope, i.e. the number of products that the firm sells abroad by exporting. A divergence between the firms’ production and sales can also exist at their intensive margin: Firms sell as many products to the domestic market or the market abroad as they produce, but the sales amount of a product is larger than its production amount.

By examining jointly the production and exporting of multi-product firms in Belgium, Bernard et al. (forthcoming) detect that the overwhelming majority of manufacturing exporters export more products or more of a product than they produce and thus undertake some (complementary) export activities of non-produced goods, which is labeled by them as “carry-along trade” (CAT): Except for the category of single-product exporters, firms in all other categories report greater numbers of products exported than products produced and 89% of the firms are CAT exporters, i.e. exporting firms with at least one CAT product. A CAT firm-product is thereby defined as a product of a firm whose value of exports by the firm is greater than the value of production by the firm.

Given that about 16% of the firms are single-product exporters, CAT is widespread across multi-product exporters. CAT firm-products can be differentiated between two non-overlapping, exhaustive categories: Pure-CAT firm-products, for which the firm export value is positive but no production in that product is recorded, and mixed-CAT firm-products, for which production and export values are positive but the value of exports is greater than that of production.

For the importance of the quality (vs. productivity) dimension of firm heterogeneity: See the Section

Besides the CAT firm-products, two further mutually-exclusive categories at the firm-product level exist: Non-exported firm-products, i.e. products that are reported as produced by the firm but not recorded as exports, and regular export firm-products, i.e. products that are reported both as produced and exported by the firm and whose value of exports is less than or equal to their value of production.
Product exports without any production of the product by the firm is common across the firms that are active in CAT: 98.3% of the CAT exporters are firms with at least one pure-CAT product. While the number of exported products is given by the sum of the number of regular products and the number of CAT products, the number of produced products is given by the sum of the number of non-exported products, the number of regular products and the number of mixed-CAT products.

Along the categories, the firm export value can be divided into a produced export value, which is given by the sum of exports of regular firm-products and the exports of mixed-CAT firm-products that are produced by the firm, and a sourced export value, which is given by the sum of exports of pure-CAT firm-products and the exports of mixed-CAT firm-products that are sourced by the firm.

Multi-product exporters do not produce the majority of the products that they export: Even if multi-product exporters are also multi-product domestic producers, the number of exported products increases much more rapidly than the number of produced products, as one follows the categorization of firms according to their export scope from small to large. Sourcing is done in almost all product categories, in particular 98.7% of the products are reported as CAT by at least one firm, and for a large number of firm-product pairs, in particular 87% of the firm-products are reported as CAT. Within firms, the probability that a product is produced rather than sourced is increasing in the number of its export destinations.

CAT is both widespread and quantitatively important: CAT exports account for at least 5% of total firm exports for more than two-thirds (67.3%) of all firms and more than three-fourths (75.5%) of CAT firms, and it even accounts for at least 10% of total firms exports for more than 60% (61.2%) of all firms and more than two-thirds (68.7%) of CAT firms. Focusing solely on pure-CAT exports, the numbers are quite large: Pure-CAT exports account for at least 5% of total firm exports for more than fifth-ninths (58.5%) of all firms and more than two-thirds (66.9%) of pure-CAT firms, and it even accounts for at least 10% of total firms exports for more than half (51.1%) of all firms and more than fifth-ninths (58.4%) of pure-CAT firms. Even though the share of CAT exports in the total value of exports is quite sizeable, i.e. 29.9% of total exports, and pure-CAT products in turn are relevant as they account for 96% of the number of CAT products and 74% of the value of sourced exports, the majority of the export value is made in-house by the firms.

In general, the margins of firm exports and a distinctive feature of the firm, namely its total factor productivity, are related as follows: The total firm exports are strong positively correlated with firm productivity, which is revealed by an elasticity significantly greater than one. Both margins, the number of distinct products exported and the average

\[ X_f = P_f \times \bar{X}_f, \]

whereas \( \bar{X}_f = 1/P_f \times \sum_p X_{pf}. \)

\[ X_f = P_f \times \bar{X}_f, \]

whereas \( \bar{X}_f = 1/P_f \times \sum_p X_{pf}. \)
exports per product, are also strong positively correlated with firm productivity. The more productive firms export more products and more of each product. Differentiating the total firm exports into produced and sourced product exports reveals notable differences in the relation with productivity between them: While the value of both produced and pure-CAT exports increases with firm productivity, both total factor productivity and value-added per worker and pure-CAT exports at a slightly lower rate, the rise in produced exports is mostly caused by a rise in the average exports per product and much less by an increase in the number of produced products that are exported. Exports of pure-CAT products are significantly lower than produced exports within the firm and their increase with productivity is roughly evenly caused by rising numbers of pure-CAT products and an increase in their average export value. While the extensive margin of pure-CAT exports is larger than that of produced products and increases much faster in firm productivity, the intensive margin of pure-CAT exports is lower than that for produced products and increases at a much lower rate as firm productivity increases.

The increase in the number of exported products for more productive firms is mostly caused by an increase in the number of pure-CAT products, as a strong (weak) positive relationship between firm productivity and the number of sourced (produced) products that are exported leads to a rising share of pure-CAT products in total exported products for more productive firms. The most productive manufacturing exporters sell more products to the market, whereas they both produce more products and source more products.

CAT appears to be some feature of firms’ cross-border activities which is both pervasive across firms and products (prevalence) and responsible for a substantial share of exports (importance). As ascertained by company interviews, firms are most likely active in CAT in order to extend their range of products that are produced in-house and thereby to meet and enhance customer demand, as the bundling of manufactured goods with complementary sourced products turns out to be vital for this firm objective.

### 2.2.6 Production and Export Diversification

**Regularity 6** Multi-product firms are remarkably diversified in the way that they jointly produce and/or export products belonging to different industries or even sectors.

Firms producing multiple products have to decide about their product portfolio, in particular composition, and therefore the degree of diversification. A remarkable share of firms produce products belonging to different industries (multi-industry firms): Most studies report that about 30% of firms in an economy are diversified in this form\[19\] (Goldberg et al. (2010b) [33%], Bernard et al. (2010) [28%], Navarro (2012) [22%], Söderbom and Weng (2012) [34%], Bernard and Okubo (2015) [28%], Lo Turco and Maggioni (2016) [30%] and Boehm et al. (2016) [32%]). On average, those firms are then active in two to three industries (Goldberg et al. (2010b) [2.01], Bernard et al. (2010) [2.8], Navarro (2012) [2.6],

\[19\] An exception is given by Adalet (2009): She reports a share of 61%.
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Söderbom and Weng (2012) [2.25], Lo Turco and Maggioni (2016) [2.7] and Boehm et al. (2016) [2.55]). For almost all studies, a considerably smaller share of firms is even more diversified in the sense of producing products that belong to different sectors (multi-sector firms) \(^{20}\) Goldberg et al. (2010b) [24%], Bernard et al. (2010) [10%], Navarro (2012) [9%], Söderbom and Weng (2012) [9%], Bernard and Okubo (2015) [14%], Lo Turco and Maggioni (2016) [16%] and Boehm et al. (2016) [19%]. On average, those firms are active in two sectors (Goldberg et al. (2010b) [1.68], Bernard et al. (2010) [2.3], Navarro (2012) [2.2], Söderbom and Weng (2012) [2.08], Lo Turco and Maggioni (2016) [2.2] and Boehm et al. (2016) [2.34]).

Multi-industry and multi-sector firms represent large firms being responsible for large shares of country-wide output: For India, they account for 62% and 54% of sales respectively (Goldberg et al. (2010b)), for Japan, 70% and 52% respectively (Bernard and Okubo (2015)), for Turkey, 44% and 27% respectively (Lo Turco and Maggioni (2016)) and for India again, 62% and 49% of sales (Boehm et al. (2016)). Comparing sizes of firm types, Bernard and Okubo (2015) further report that multi-sector firms are more than five times larger than multi-product firms being active in only one industry.

Lo Turco and Maggioni (2016) undertake an analysis of diversified Turkish firms: Internationalized, i.e. exporting, large firms and firms being active in a more competitive market (for their main (core) product) are more diversified in their activity across products. Over the period of observation (2005-2009), all domestic firms as well as exporters and importers experienced a slight increase in the diversification measure (entropy index), while foreign owned firms became less diversified.\(^ {21}\) A foreign competition threat regarding the core product has no impact on the diversification (strategy) of firms and a tariff cut (firm-weighted tariffs) is associated with a higher concentration of firms’ sales.

Looking at the extent of diversification, Amador and Opromolla (2013) report that exporters in Portugal are relatively diversified: About half of the exporters that ship two products are active in two different sectors, almost two-thirds of three-product exporters ship products that belong to more than one sector and more than 80% of firms exporting between eleven and 50 products are active in five and more sectors. Based on an alternative indicator of diversification, which takes into account the firm’s product scope, the distribution of sales across the products as well as the input relatedness of the products and therefore represents a generalized index of diversification, Lopresti and Shiferaw (2017) detect for Ethiopia that all single-product firms and 18% of the multi-product firms are non-diversified, whereas a firm for which the diversification index is larger than the industry average is considered to be diversified.

In a study on the (dynamic) firm structure in a developing (African) country, in particular Ethiopia, Lopresti and Shiferaw (2017) analyses the firms’ transition process from a single-product to a multi-product firm: The vast majority of start-ups (70%) enters the

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\(^{20}\) An exception is given by Adalet (2009): She reports a share of 56%.

manufacturing sector as single-product firms and only few of them extend their product portfolio to multiple products: 25% of single-product firms observed at a given point in time change into multi-product firms in the subsequent six years and the rate of transition further decreases after that interval, letting transition from a single-product to a multi-product firm status be extremely rare (negative duration dependence). Most of the single-product firms are likely to remain so and do not expand. Nevertheless, among them, differences in the likelihood of becoming a multi-product firm exist: Single-product firms with lumpy investments\textsuperscript{22} are nearly twice as likely as their counterparts without lumpy investments to become multi-product firms in a given year. A further factor that facilitates the transition process of firms is labor productivity: Single-product firms with higher initial levels of labor productivity are more likely to become multi-product firms, as a one standard deviation increase in the logarithm of initial labor productivity increases the likelihood of a transition to a multi-product firm by 11% to 14%.

In addition, firm-level investments and especially lumpy investments have a significant positive effect on diversification: Firms with lumpy investments are nearly 1.9 times more likely to change to becoming a diversified multi-product firm than firms that do not undertake such investments. Taken both together, conditional on the initial levels of firm size, labor productivity and firm age, firms that undertake substantial investments in a given year are more likely to either expand or diversify product offerings than firms without such investments, which indicates the existence of substantial investment costs to product scope expansion and diversification.

Investigating the firms’ motives for product scope expansion and diversification, \textcite{loprestishiferaw2017} find that more diversified firms have a lower variance of sales, where the sales volatility of multi-product firms with a median generalized index of diversification is 11% lower than that for a single-product firm. Furthermore, an increase in the index of diversification from 0 to 0.1 lowers the firm’s coefficient of variation by 0.063 or 9% of the mean. Across heterogeneous industries, single-product firms in industries with a high sales variance\textsuperscript{23} are 46% more likely to become multi-product firms than their counterparts in industries with a low sales variance and firms in high-variance industries are 51% more likely to introduce markedly distinct products compared to firms in industries with a low sales variance. As a result, firms expand their product scope or diversify their product portfolio in order to mitigate risk which is accompanied with their operations.

2.2.7 Firm and Firm-product Competences

\textbf{Regularity 7} Multi-product firms operate and compete on the basis of their overall and/or product-specific productivity or quality and especially the latter plays an important role for

\textsuperscript{22}Lumpy investments are defined as investment spikes for which the investment rates exceed 20% of the capital stock.

\textsuperscript{23}High(low)-variance industries are defined as industries with a coefficient of variation above (below) the global median.
this firm type. A product market characteristic, namely the market’s degree of product differentiation, decides about the relative contribution of the two dimensions: Product quality is more relevant in more differentiated markets.

The heterogeneity of firms and products within firms rests on either their productivity respectively cost of production or their quality of output and firms thus compete with their products on the basis of their superiority in either of the two dimensions, providing the sets of possible fundamentals for their activity and performance differentials.

Manova and Yu (2017) examine the role of product quality in the operations of Chinese multi-product firms during the period 2002-2006 and find that the differentiation of product quality across firms and across products within firms represent a key feature for understanding the differentials in the firms’ export performance and the effects of trade reforms.

Firms that export products at higher average prices and firms that vary prices more across their product range export more and exhibit a faster export growth. Similar patterns also hold when output quality instead of the price of the output as its quality indicator is employed. Furthermore, superior firms, i.e. firms with a higher productivity, employment and intensity of skill, capital, advertising as well as research and development, sell their products at higher average prices and quality as well as reveal a greater price and quality dispersion across their products. Analyzing the variation within a product across firms, the study finds that the export price and the export revenue are positively correlated. A one-standard-deviation increase in the export revenues is accompanied by 20% higher export prices. The output quality used likewise positively correlates with the export revenue, overall providing an indication for a differentiation of quality across firms.

In addition, differentials are also in place across the products within firms: The export prices and revenues are systematically positively correlated across a firm’s product range: Products that are the firms’ top sellers tend to be their most expensive ones. In 45% and 19% of the time, the firm’s best-performing product in terms of the export revenues is also its most or second-most expensive product. Conversely, the firms’ most expensive products tend to be their top-selling ones. In 45% and 18% of the time, the firm’s most expensive product is its best- or second-best-performing one in terms of the export revenues. Due to the obtained finding that, across all the products in a firm’s portfolio, the price rank of a given product most likely coincides with its sales rank, indicating a differentiation of

\[\text{Eckel et al. (2015)}\] develop a model of multi-product firms, in which the firms’ competence is based either on cost or quality, being identifiable by a negative or respectively positive correlation of the firms’ price and sales profiles across their products. They empirically test the model’s key prediction that firms in sectors with differentiated products exhibit a quality-based competence and find its confirmation in a Mexican data set. In addition, the empirical analysis yields the compatible as well as reinforcing result that the export sales of firms in non-differentiated-product sectors exhibit the opposite pattern.\[\text{Output quality is inferred from data on export prices and quantities or proxied with the price or the inferred quality of imported inputs.}\]

\[\text{As in Eckel et al. (2015), the theory on which Manova and Yu (2017)’s study is based states that a negative correlation between prices and revenues indicates an efficiency sorting, while a positive correlation a quality sorting.}\]
quality across the products within firms, one can infer that the firms’ core products are the expensive and high-quality ones that generate the highest revenues, while the cheap and low-quality products that are sold at low volumes represent the firms’ peripheral ones.

Analyzing the variation across products within a firm, the study reveals that firms allocate their activities across products in accordance with their differentiation of quality and therefore both the quality differentiation and the resulting product hierarchies govern the operations of multi-product firms. The export price and the export revenue are once again positive correlated. On the one hand, more expensive goods generate systematically higher global sales. An increase by a one-standard-deviation in export revenues is associated with 10.6% higher export prices. As to bilateral sales on the other hand, exporters earn higher revenues from their more expensive products also within each destination. A one-standard-deviation increase in bilateral exports is related to 9.5% higher bilateral units values. As before, in addition to the export price as the product’s quality indicator, the study more directly employs output quality. And for the two specifications, qualitatively similar results are obtained, even so quantitative differences exist: The global export revenue positively correlates with the output quality across the products within a firm and also a strong positive correlation between the bilateral export revenue and the output quality across the products within a firm-destination exists. By contrast, the point estimates in the latter specification are substantially higher. A one-standard-deviation increase in the bilateral export revenues is associated with a 280% higher product quality. The firms’ core products reveal both high quality and high production efficiency as manifested in low quality-adjusted prices, thereby overall indicating a quality differentiation across a firm’s products. The positive correlations between prices and revenues, both within a product across firms and across products within a firm, are invariant to a firm’s market power both in the output market and in the input market as well as invariant to processing imports and matching procedures. Moreover, the firms’ export prices rise substantially more quickly.

Multi-product firms differentiate the output quality across their products by making use of inputs of varying quality levels. Input prices positively correlate with the output price across the products within a firm, with an elasticity ranging between 0.11 and 0.13, depending on the matching procedures between inputs and outputs. With input quality instead of the price as its quality indicator, a strong positive correlation between input and output quality across the products within a firm is found. The relationship between the input prices and the output price is thereby invariant to the firms’ market power across its products, possibly influencing its pricing strategy, and the associations are stronger for products with arguably greater scope for quality differentiation, such as differentiated goods and sectors intensive in research and development (R&D) as well as combined R&D and advertising.

Investigating the correlation based not only on the levels of the export price and revenue but on the within-firm ranks, a strong positive correlation between the products’ global and bilateral rank by price and by revenue across the products within exporters can be found and reinforces the study’s findings.

Input quality is inferred from data on import quantities and prices.
with their input prices when the output product is differentiated.

For the variation across destinations within a firm, Manova and Yu (2017) detect that the export product scope and the export revenues are positively correlated. Exporters thus generate systematically higher revenues in destinations where they sell more products. The typical firm earns 85% higher bilateral revenues when it exports 50% more products to a given country. Thereby, the distribution of the sales across its products is not independent from its export scope. Instead of, firms concentrate more on their core competence products in markets being smaller for them, in the way that they skew their exports more towards their top-selling, most expensive and highest-quality products in markets with a smaller export scope. Reducing the product range by 50% comes along with a 21% (8.5%, 15.5%) rise in the revenues from the best-selling (most expensive, highest-quality) product relative to the second-best (second-most, second-highest). Qualitatively similar results are obtained when separately considering firms’ homogeneous and differentiated products. Quantitatively however, the concentration of sales towards the expensive and towards the high-quality products falls faster with the export product scope for differentiated products than for homogeneous products, while the opposite is true for the concentration of sales towards the best-selling product. Therefore, firms reallocate their activities towards their core, high-quality products in destinations with a smaller export scope both along their extensive margin, by dropping lower-quality peripheral products, and their intensive margin, by concentrating sales towards their high-quality products.

In addition, the export product scope is also related to the average price at which firms sell their products: Both are negatively correlated. The relationship is not driven by cross-country differences in a firm’s market power and existent in the sample of differentiated products with some potential for quality upgrading, while being absent among homogeneous products. The typical firm lowers its average bilateral price by 1% when it exports 50% more products to a given destination. For differentiated products, the correlation is 20% higher. As firms adjust their export scope, they operate at the lower end of their product hierarchies: Exporters expand (restrict) their product range across markets by consistently exporting their core, expensive products of high quality and adding (dropping) their peripheral, cheaper products of lower quality. So, a negative correlation between the export product scope and the average output quality across a firm’s products exists. Even though, the weighted average quality is significantly positively correlated with the export product scope across destinations within a firm-year.

Aw and Lee (2017) investigate the relative importance of the two dimensions of firm heterogeneity, productivity and quality, for the export performance of Taiwanese multi-product firms in the period 2000-2004. Firms with high demand and productivity measures

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29 Similar results are received in case of an alternative sales concentration measure: The Herfindahl index.
30 According to the theory, the findings are inconsistent with constant markups, but variable markups importantly affect the sales decisions of multi-product firms.
31 However, the relationship between export product scope and the revenue-weighted average price is markedly less negative and not statistically different from zero.
have a higher likelihood to export and to export a large number of products. As the firms decide about their export participation and export product scope, quality plays a larger role for export decisions of firms in more differentiated product markets and for export decisions of products with lower cost elasticities of quality improvements, while productivity has a larger influence on export decisions of firms in markets with lower degrees of product differentiation and on export decisions of products with higher cost elasticities of quality improvements. Regarding product-level characteristics, the lowest-selling products of large-scope exporters reveal firm-product demand and productivity measures that are progressively lower than that of the lowest-selling products of small-scope exporters, indicating that large-scope exporters face lower thresholds in productivity and demand for their products to be existent in the export market.

2.2.8 Production Innovation and Firm Survival

A strand of the literature looks at the effect of a product innovation on the survival of the innovating firm, which is ex ante ambiguous and therefore worth to be investigated empirically. In a competitive and rapidly changing economic environment, a product innovation and some degree of diversification by adjusting or extending the firm’s product portfolio commonly prove to be necessary for firms to ensure their sustainable performance and existence in the economy by matching the consumers’ demand, capturing a market position and balancing the segments’ volatility. A firm’s product innovation however arrives only at the discount that it generates some entrepreneurial risks due to uncertainties inherent in the realization of its commercialization. At the time of the market launch of the new product, it is eminently unclear for the firm to what extent the innovation fits the consumers’ demand, whether it is not quickly imitated or replaced by the new products developed by competitors and therefore to what extent the innovation can be commercialized, which is essential to at least amortize the typically high fixed costs associated with any innovation process and to pay off the investments undertaken by the innovating firm. Any medium- or long-term absence of a commercial success of the innovation in the market therefore jeopardizes to some degree the survival of the innovator. Firms producing multiple products are by nature more or less engaged in product innovation as well as diversification and the insights of the literature on the innovation-survival relationship thus apply to a comprehensive consideration of this type of firms.

In a recent contribution to the literature, Fernandes and Paunov (2015) basically observe a negative relationship between a product innovation and the exit probability of the conducting plant, i.e. product innovating plants reveal a lower hazard of exit, and the relationship holds for a qualitative as well as a quantitative measure of innovation: A plant’s

32In the study, a product innovation describes a product that is new to the plant and not supplied by it in previous periods, but the product is not necessarily new to the country or the rest of the world. Therefore, research and development (R&D) expenditures do not necessarily correlate with the product innovation measure that is applied in the study.
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engagement in product innovation and its introduction of a larger number of new products lead to a reduction of its exit probabilities. In particular, the higher is the number of new products that are introduced by a plant, the lower is the plant’s hazard of exit. In terms of the magnitudes, the choice of a plant to engage in product innovation decreases its death probability by 22% and the introduction of an additional new product reduces a plant’s death probability by 11%. Applying plant and industry controls to the analysis, survival odds turn out to be varying both across heterogeneous plants and industries: Plants with a higher sales growth and firms consisting of multiple plants exhibit lower exit probabilities.

2.2.9 Export Dynamics

The export strategies of multi-product firms in entering and defending foreign sales markets are investigated by studies by Iacovone and Javorcik (2008) and Iacovone and Javorcik (2010). They find that new exporters33 start small in terms of both values and number of exported products. Firms begin their endeavor abroad with one or two export varieties (on average: 1.7 (Iacovone and Javorcik (2008)); 1.5 (Iacovone and Javorcik (2010))), and already two years after entering the average number of exported products raises to two. The products with which the firms start their export activity are typically those for which the firms have gained some experience in the domestic market: In about 68% to 90% (on average: about 85%) of the cases, new exporters enter foreign markets with a variety that is previously sold domestically (Iacovone and Javorcik (2010): in at least one of the previous three years: 79% of the cases, within the past two years: 75% of the cases, in the year before: over two-thirds of cases (68%)). Although, the total number of varieties sold by the firm increases slightly in the year of entering foreign markets, but is stable afterwards. The volume of exports with which the firm starts is very small relative to the total sales of the variety and the first-year exports represent only a small fraction of the firm’s total sales (about 12% (Iacovone and Javorcik (2010))). This share increases subsequently, but less than firm’s total sales (close to 20% in the fifth year of exporting (Iacovone and Javorcik (2010))). The whole distribution of export values of varieties in their first year of being exported, normalized by the total sales of the variety by a given firm, is skewed to the left and therefore indicates that most trading relationships start with small volumes.

With increasing familiarity with the export market, firms are willing to launch products that are new to them: In nearly one-third (on average: about 31%) of the cases, experienced exporters34 introduce to export markets a variety that has not been previously sold domestically by them. And those products start to be exported at a larger scale, i.e. with a larger volume in absolute terms and relative to domestic sales: 43% of the production is exported relative to 13% for the other group of products. The introduction of a new export

33 According to Iacovone and Javorcik (2008, 2010), new exporters are firms that were not exporting in the previous year, but are exporting in the current year under observation.
34 According to Iacovone and Javorcik (2008, 2010), experienced exporters are firms that are in their second or later year of exporting activity.
product by foreign firms\textsuperscript{35} is characterized by the fact that this product is more likely to be of higher quality, i.e. higher unit value. About a quarter (28\%) of newly introduced export varieties are export discoveries, which are newly introduced export varieties that were not exported before by any firm, and those are more frequent in the immediate period after a devaluation of the domestic (Mexican) currency, the Peso. In detail, all types of firms (experienced and new exporters, domestic and foreign exporters) are accountable for the export discoveries, since they represent comparable shares of newly introduced export products across firm types: Out of the new export products introduced by experienced exporters and new exporters, export discoveries take a share of 30\% and 27\%, respectively, while for domestic (foreign) firms, they account for 29\% (29\%).

Export products that are exported for the first time at the country level experience a process of fast diffusion. These export discoveries are typically introduced by more than one firm - between one and two firms - and for approximately half of them, an additional firm starts exporting in the following year. Not before three years after the discovery, the rate of diffusion slows down but continues until at least the eighth year. Before and after starting an export activity, within-firm adjustments are undertaken. A surge in investment in physical assets and technology acquisition precedes the introduction of new export varieties, while at the same time, there is no evidence for increased research and development activity in preparation for exporting. Instead of, research and development increases after the introduction of a new export variety. And this pattern of investment is not only restricted to new exporters, but also experienced exporters show higher volumes of investment in the surrounding of the introduction of new export varieties, suggesting the existence of fixed costs associated with the export variety introduction and not with the export activity by itself. In line with this, the export status of firms play an important role for investments undertaken by them. While only 51\% of non-exporters invest, more than 70\% of the exporters are active in some forms of investment, indicating a substantial difference in the frequency of investments between exporters and non-exporters. The difference however is much smaller between exporters introducing a new export product and those not introducing a new export product (77\% versus 71\%), nonetheless some difference exists.

Iacovone and Javorcik (2010) further report that a vast percentage of the export varieties does not survive for more than a year in the foreign market. In particular, only about two-thirds of the new export varieties are exported for more than one year, between 46\% and 60\% of the varieties are able to stay in the foreign markets for at least three years and only a bit more than one-third (37\%) of the export varieties introduced to foreign markets in 1995 are still exported eight years later. In addition, the export variety’s survival rate increases with its tenure in the export market. Specifically, the survival rates are lowest in the varieties initial period of being supplied abroad, only between half and two-thirds of

\textsuperscript{35}According to Iacovone and Javorcik (2008, 2010), foreign firms are firms with some foreign (vis-a-vis Mexico) participation in 1994.
the varieties survive between the entry year and the subsequent period and between 76% and 88% survive between the second and third year after entry.

In addition to the empirical evidence above, which suggests among others that firms feel their way into export markets by starting small and with a domestically approved portfolio, Álvarez et al. (2013) explicitly analyze the question of whether some previous export experience can be an important factor for prospective exports and find that the firm’s own previous export experience with a product or market plays an important role. An increase in the cumulative export value of a particular product in the previous period by 1% increases the probability that the firm will export the same product to a new market by 2.3% and an increase in the previous experience in a given market by 1% increases the probability of exporting different products to that market by 2.7%. Not only the firm’s own experience with export markets and products has an influence on its exports, but also other firms’ export experience seems to have an impact on the firm’s export behavior. The probability that a firm exports a new product-market pair increases by 0.69% when the cumulative export amount of the same product by other exporters to the same market increases by 1%. The increase in probability is even higher (0.74%) when other firms export the product to other markets at a larger scale, i.e. when the cumulative export amount of the same product by other exporters to other markets increases by 1%. Furthermore, an increase in the cumulative export value of different products to the same market by other firms by 1% increases the probability of introducing a new product to that market by 1.16%.

The effect of past export activity on the firm’s exports, which is contextualized by getting or making some experience in exporting and thereby reducing uncertainty associated with the export process, differs across heterogeneous firms and products. While larger firms are in general more likely to introduce a new product, smaller firms benefit more from having exported a product in the past. The effect of having experience in exporting a product is also stronger for relatively more heterogeneous products and gets particularly important in those sectors that are more dependent on external finance. Furthermore, firms benefit more from previously exporting relatively simple products and thereby from experience in those goods’ exporting. Overall, one can observe diminishing returns to experience at some point in the firm’s learning process and a congestion effect among firms exporting the same product. However, firms whose export product-market pairs are more concentrated show a lower probability of introducing new products in new markets.

Lo Turco and Maggioni (2015) analyze the impact of the export (and import) activity of firms on their (product) innovation activity and therefore on their scope. They find that firms which start to export their own produced products increase both their product scope and their product innovation intensity. In the entry year, the number of products increases by 18% and a year later by 14%. The probability of expanding the product scope

\footnote{In case of a broader treatment of exported goods, i.e. exports also include non-produced goods, the positive effect of exports is relevantly downsized.}
shifts upwards by about 6\% in the entry year. With regard to new products, their number increases by 20\% in the entry year and the probability that the firm introduces a new good is higher by 9.1\% in the entry year. In contrast, starting to import has no significant effect on scope and innovation. This observation alters if one takes a look at firms that enter the export and import market at the same time: Those firms are more likely to both expand scope and innovate and this effect is higher than for new exporters, which implies that there is some complementarity between export and import activities. However, starting to export and starting to import have not symmetric effects on the innovation activity of firms that consequently switch to two-way trading: While the innovation activity of importing firms that start exporting increases, adding an import activity does not substantially improve the innovation activity of the exporting firms. Giving up exporting by contrast implies a contraction of both scope and innovation: Switching from export starters to import starters, a substantial and significant reduction in the firms’ innovation propensity and product scope is observable. Therefore, product innovations seem to be attributable to the firm’s relationship to foreign customers and the effect is enhanced when the firm sources a part of its inputs from abroad. A learning process in firms takes particularly place in case of a two-way trading of the firms\textsuperscript{37} Using foreign intermediates alone however does not contribute to an intensified innovation activity of firms. Regarding the timing of the firms’ activity in their sales endeavor abroad, Lo Turco and Maggioni (2015) report that firms are most active in the same year they enter the foreign market, i.e. right at the beginning of their activity. The possibility of exporting turns out to be especially important in enhancing the weight of newly introduced products in the firm’s production and product portfolio and starting to import simply reinforces the positive effects of the export entry.

In addition to the insights above, two-way traders reveal a higher likelihood for the introduction of new goods of higher quality and starting to export exerts a mild effect on the probability to start introducing new goods of higher quality the year after the entry.

\subsection{Product Switching}

\textbf{Regularity 8} Multi-product firms frequently and to a large extent change their product portfolio by adding and/or dropping products. Nevertheless, notable differences in the product switching behavior exist both across firms and products within firms being heterogeneous in terms of productivity and size.

A strand of the studies takes up a dynamic perspective when looking at multi-product firms and analyses their product portfolio changes, their characteristics, relevance(s) and
implications. Due to the availability of micro (panel) data sets the studies can track firms and products over time and thereby identify firms (products) that add products (that are added), firms (products) that drop products (that are dropped) and firms (products) that both add and drop products simultaneously (that are added and dropped simultaneously). The latter course of action should be called “product churning” and the overall sum of actions “product switching”.

In their seminal paper to the phenomenon, Bernard et al. (2010) take a look at (frequent) product portfolio changes of multi-product firms in the U.S. manufacturing and document their prevalence. Overall, more than half of all surviving firms (54%) are active in some product mix changes over a 5-year period with about 46% of them simultaneously adding and dropping products, indicating some more fundamental changes of the product supply. Restricting the sample to multi-product firms, product mix changes are an even more widespread firm strategy: 80% of multi-product firms change their product mix during a 5-year period, and 45% of them add and drop products at the same time. Therefore, multi-product firms are more likely to change their product supply than single-product firms.

Liu (2010) confirms with another study to the firm behavior in the U.S. the widespread product churning and switching across multi-product firms: According to his analysis, 73% of multi-product firms alter their product mix during each of 5-year periods, with a minority of them (10%) only adding products, about 42% of them dropping products only and a relative majority (48%) being active in churning products.

Analyzing manufacturing firms’ behavior in Japan, Bernard and Okubo (2015) show that 20% of all firms change their product mix each year, whereby multi-product firms are more likely to undertake product portfolio adjustments (33%). These changes take primarily place through product churning (11% of all firms and 16% of multi-product firms active in), whereby multi-product firms are twice as likely to churn their product mix compared to single-product firms.

For a developing country like India, Goldberg et al. (2010b) report a less prevalent product switching behavior of firms: Only 28% of continuing firms are active in some product mix changes over a 5-year period with about 79% of them only adding products and a less smaller share of firms (7%) churning products. During a one-year period, 10% of continuing firms switch their supplied products, whereas 70% of them only add products and none are simultaneously active in adding and dropping products. They also find multi-

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38 For the quantitative dimension of product portfolio changes: See Broda and Weinstein (2010). They find that most of the sectoral product turnover takes place within existing firms, i.e. 92% (82%) of product creation happens within existing manufacturers and 97% (87%) of product destruction happens within existing manufacturers at a 1(4)-year frequency. Depending on the time frame used and looking at consumption values, product entry and exit appear to be 6-30 times as important as manufacturer entry and exit, illustrating the differences of the margins.

39 This relation holds also true for several other dimensions: Exporters (compared to non-exporters), large firms -firm’s output above the 75th percentile- (compared to small firms) and multi-plant firms (compared to single-plant firms), even so in each category a smaller fraction of firms is active in product switching compared to multi-product firms.
product firms to be more likely to change their supplied products than single-product firms, but less evidence for within-firm Schumpeterian creative destruction compared to firms’ behavior in U.S. manufacturing.

Contrary results for India are found in a study by [Boehm et al. (2016)]: Documenting product switching as a prevalent firm behavior, they find that 65% of all firms are active in product mix changes over a 5-year period and even higher across multi-product firms, 85% of them change their product mix over this horizon. A decomposition reveals that a minority of them (7%) only add products and another quarter only drop products, but the large majority (68%) of multi-product firms add and drop products simultaneously.

The study by [Navarro (2012)] for Chilean plants documents that 32% of continuing plants are active in product switching between two consecutive years, while the share for multi-product plants (48%, compared to 15% for single-product plants) is somewhat higher; in a 7-year period (1996-2003) however, 75% of continuing plants and the overall majority of multi-product plants (93%, compared to 47% of single-product plants) change their supplied product mix and thereby show relatively high shares in comparison to the above mentioned evidence. Confirming the figures, [Alvarez et al. (2016)] find that 68% of Chilean plants are not active in product switching between two consecutive years, and vice versa 32% of the plants either add, drop or simultaneously add and drop products over this horizon. Dropping products is thereby the most prevalent change (17% of plants), followed by adding products (15% of plants) and churning products (9% of plants).

[Lo Turco and Maggioni (2016)] find a share of about 24% of Turkish manufacturing firms to be active in adding, dropping or churning products during a 1-year interval, almost doubling it (45%) for 3-year periods. A similar pattern is found by [Lopresti and Shiferaw (2017)] for Ethiopian firms. Over a 1-year horizon, 22% of the firms add at least one product and 17% of the firms add exactly one product, while the share of firms that add at least one product over a 3-year horizon almost doubles (41%). Product dropping is thereby less frequent than product adding: Only 13% of the firms drop at least one product over a 1-year horizon. Across industries, the frequency of product adding varies between a high of 31% (metal and light machinery sector) and a low of 6% (beverage sector). According to [Söderbom and Weng (2012)], 61% of surviving Chinese firms and 78% of those firms producing multiple products in China realize some product mix changes over the sample period of four years (2004-2008). Most of these changes consist of actions in which products are added or churned rather than just dropped and multi-product firms are found to be more likely to undertake those product mix changes than single-product firms.

Exceptionally high shares of firms being active in product mix changes are reported by a recent study by [Timoshenko (2015)] for Brazilian firms: Nearly three quarters (72%) of surviving exporters alter their product mix each year.

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40 Across multi-product exporters, 94% switch their products during a 7-year period (1996-2003) and 90% switch products they are exporting, slightly exceeding the share of product-switching multi-product plants.
Iacovone and Javorcik (2010) report substantial numbers of exporting Mexican firms being active in some form of product switching, even so an average number hides such a behavior: They find a remarkable stability in the average number of exported varieties per exporter over time. In 1994, an exporter sells on average 1.95 varieties abroad and this number rises up to 2.2 in 1999, afterwards remaining closely around this level up to 2003. The variation in the average number of exported varieties per firm over time is even smaller for existing exporters\footnote{Existing exporters are defined as those in their second or later year of exporting.} who send between 2.18 and 2.26 varieties abroad. In contrast and besides the difference in the level, new exporters reveal a much larger difference and more volatile pattern in the number over time, as it varies more discontinuously between 1.2 and 1.58. Over the period of the study (1995-2003), in each year, about 250 to 700 exporters introduce new export varieties, with some declining pattern over time except for 2003, while between about 250 and 400 firms drop existing export varieties, with some peak in 1999.\footnote{Exiting firms are included among those dropped export products.} A much smaller number of firms, about 30 to 90, simultaneously introduce new export varieties and drop existing ones, with some peak in 1997. In relative terms at the variety level, the export variety creation\footnote{The export variety creation is defined as the number of new export varieties introduced at time $t$ divided by the total number of varieties exported at $t$.} amounts to 18.8%, while the export variety destruction\footnote{The export variety destruction is defined as the number of varieties ceasing to be exported at time $t$ divided by the total number of varieties exported at $t$.} accounts for 11.4% (for the whole economy and over all years); that is, about every fifth export variety represents a new variety and about every tenth export variety is a variety that is dropped out. Net switching, as the difference between export variety creation and destruction, with a figure of 7.4% is much lower than gross switching, defined as the sum of export variety creation and destruction, with a figure of 30.2%. Across the sectors, high gross switching rates are observed ranging from a low of 19.8% in mineral-based industries to a high of 38% in textiles, garments and leather as well as of 42.8% in wood products. Export variety creation varies between 13.2% in mineral-based industries and 27.7% in wood products, while export variety destruction runs from 6.6% in mineral-based industries to 16.1% in textiles, garments and leather.\footnote{Excluding exiting firms from the calculation of the export variety destruction does not change much: It ranges from 6% in mineral-based industries to 16% in textiles, garments and leather, and economy-wide reaching 11.5%.

Varying the degree of aggregation of the unit of treatment of the firm switching behavior (product, industry, sector) and thereby capturing different levels of the extent of changes, a robust finding across some studies is a (weakly) decreasing prevalence of the switching behavior with an increasing degree of aggregation, i.e. product switching is more prevalent than industry switching and the latter more prevalent than sector switching, which can be made plausible with the more radical change of the firm nature and scope and so a higher associated risk being inherent in a switching behavior of an increasing degree of aggregation. \cite{Bernard2010} report for 5-year intervals that while more than half
(54%) of the firms in the U.S. are active in product switching, only 41% switch industries they are active in and less than every fifth firm (16%) switch (even more aggregated) sectors. Furthermore, product adding implies for 27% of the firms to enter at least one new industry and for 9% of the firms to get active in at least one new sector every five years.

In the study by Liu (2010) for U.S. firms, industry switching is relatively prevalent across firms (73%), reaching the same firm share as product switching but it decreases for sector switching (61%), even so this number of firms is high compared to the other studies. Boehm et al. (2016) add evidence for a developing country, in particular India: Over a 5(1)-year horizon, product switching represents a preferred product strategy for 65% of the firms and 49% (38%) undertake some industry switching. An even smaller share of firms (34% (26%)) is also willing to switch sectors they are active in. Across multi-product firms, switching behavior is even more prevalent: While 85% of those firms switch their products during a 5(1)-year interval, 78% (64%) are active in industry switching and only a slightly smaller fraction of firms (74% (59%)) engage also in sector switching.

Overall, the studies reveal a remarkable prevalence of firm switching behavior, especially across multi-product firms and irrespectively of the varying definitions of the observation units. Given this fact, further aspects of this phenomenon are analyzed in the mentioned studies: Along the two dimensions of the applied data sets, the questions of what type of firms is active in product switching and of what type of products is switched arise. Looking at the characteristics of product-switching firms, Bernard et al. (2010) find a positive correlation (for both, single- and multi-product firms) between subsequent product adding and initial firm productivity (TFP and labor productivity, given the firm’s existing product mix), i.e. (ex ante) more productive firms are more likely to add products, whereas this correlation is robust to controls such as firm size (employment) and age. It yields some evidence for selection, representing an explanation approach for the productivity premium of multi-product firms (apart from the alternative approach of learning). For Japanese firms, Bernard and Okubo (2015) also report that firm productivity is positively related to the probability of adding a product. By focusing in contrast on the survival of products in the firms’ export mix, Görg et al. (2012) document a decreasing hazard of dropping the product with increasing firm productivity. So, more productive firms export products with on average higher survival probabilities in export markets. Iacovone and Javorcik (2010) detects that older producers are less likely to discontinue the production of a variety.

Besides productivity and age, firm size represents another potential characteristic being correlated with the product-switching behaviour. The state of knowledge generated by the studies is however ambiguous: While some studies suggest a higher probability of product

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46 One has to take into account the unit definitions: In Bernard et al. (2010), products are defined as 5-digit SIC categories, industries as 4-digit SIC categories and sectors as 2-digit SIC categories. In Liu (2010) however, 4-digit SIC categories represent products or industries alike and 2-digit SIC categories sectors.

47 See the Section 2.2.2.
2.2. EMPIRICAL REGULARITIES

switching among smaller firms (Söderbom and Weng 2012) (smaller firms (output) more likely to switch product lines), Görg et al. (2012) (larger firms (employment) less likely to drop products, growing firms less likely to drop export products) and Bernard and Okubo (2015) (firm size (employment) negatively related to probability of adding products)), others indicate the opposite (Miranda et al. (2012) and Lo Turco and Maggioni (2016) (larger firms (employment) more likely to add new products, larger firms (employment) more likely to drop products)) and Navarro (2012) (even distribution of product switching across plant sizes, i.e. plant sales) cannot find any relation between size and switching. As given by Bernard et al. (2010) for the U.S., 89% of output is attributed to firms that are active in product switching in a 5-year interval. And firms that churn products produce 68% of output. In the sample of multi-product firms, the shares further increase to 94% and 75%, respectively. In India, product-switching firms account for smaller shares of aggregate output, as documented by Goldberg et al. (2010b): Only 43% of total output is produced by firms that switch products over a 5-year interval. Product-churning firms are responsible for 12% of output. The shares slightly increases for the multi-product firms: Among those firms, 47% of total output goes back to product-switching firms, while firms being active in product churning account for 15% of output. Equivalent relevance of product-switching is reported for the Chinese as well as Chilean economy: Due to Söderbom and Weng (2012), product-switching firms (over a 4-year interval) are responsible for 36% of output, while only 13% of output is attributed to product-churning firms. Once again, shares are higher in the sample of multi-product firms (45% and 25% respectively), but much smaller compared to the U.S. economy. Similarly, Álvarez et al. (2016) document that two-thirds (66%) of the output are attributed to products that are not added, dropped or churned, while adding (dropping, churning) products accounts for 20% (16%, 9%) of the output.

With an alternative dataset, Boehm et al. (2016) however show a significantly higher relevance of product-switching for the Indian economy compared to other studies and developing countries: Firms that switch products over a 5-year horizon produce 68% of total output and product-churning firms are responsible for 47% of the output in the economy. For multi-product firm, those shares further increase to 85% and 59%, respectively, and suggest similar structures among those firms as in the U.S. economy.

By analyzing two subsets of firms in the countries, i.e. foreign-owned firms and exporters, the studies derive further insights into the firm switching behavior. With respect to the firms’ ownership structure, foreign firms are less active in product switching activities and show a more stable product mix according to Görg et al. (2012). The survival probability of a product is about 18% higher when being part of the product mix of foreign firms compared to domestic firms. In addition and with respect to the firms’ export status, firms with a higher export-sales ratio export products for a longer period and some previous exporting experience seems to be a positive factor for success in export markets. In

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48 See foreign ownership definition in Görg et al. (2012): More than 10% of firm shares owned by foreigners.
Chapter 2. Multi-Product Firms - Survey of Empirical Studies

The study by Lo Turco and Maggioni (2016), non-exporters are firms that are more likely to add new products.

The cross-study analysis of the characteristics of products being switched leads to the results that firms drop products with a lower probability that have a high firm-level sales amount and/or a large time of being part in the firm’s product portfolio, representing products that are relevant for the revenue of the firm and turn out to be successful in the sales market. According to Bernard et al. (2010), firms are less likely to drop a product if its firm-level shipments and tenure are large relative to the average values across firms producing the same product. Given the positive correlation between relative firm-product size and tenure on the one hand and firm-product productivity on the other hand, the firm behavior reveals some systematic within-firm reallocation of resources towards activities generating more revenue per unit of factor input. In their study, Iacovone and Javorcik (2010) find that products with a lower share in the firm’s total sales (relevance), a lower value of the firm’s sales (sales volume) or a lower firm’s share in the national sales of the product (market share) are more likely to be dropped. Navarro (2012) gets similar results by deriving a decreasing probability of dropping a product with increasing (relative to average across plants) product size (sales) and (relative to average across plants) tenure. Not calculating a relative measure of product size across firms as the relevant covariate, but instead within firms, i.e. the share of product sales in the firm’s export revenue, Görg et al. (2012) show a negative correlation between the importance of the product in the firm’s total export revenue and the probability of dropping the product. Core products, i.e. those with the highest shares in the firm’s export revenue, are exported longer than other products and the hazard for the top three products of the firm is approximately 40% smaller. As before, their study finds in addition that the hazard of dropping a product decreases with its tenure. Setting the time since introduction for a product in relation to the duration of the other products of the firm, thereby following the idea that a firm starts to build up its (export) portfolio with those product it has an expertise in, it is demonstrated that the later the firm started to export the product, the higher the hazard of dropping the product. In line with previous results, Lo Turco and Maggioni (2016) report a higher probability of product dropping for fringe products, i.e. products with a smaller share in firm output.

Görg et al. (2012) consider further product characteristics that are potentially influential for product dropping. They calculate a variable for the share of wholesalers and retailers in the export of each product and derive the result that a higher export involvement of intermediaries is associated with a higher hazard of dropping the product. Furthermore, the differentiation of products is taken into account by calculating for each product the price dispersion across firms and indicating products with an above-median level of price dispersion as high-price-dispersion products. Products more differentiated in this sense feature a higher hazard of product dropping than less differentiated products. Interacting

49 See the Section 2.2.11.
the product differentiation measure with a proxy for product prices (relative unit value) yields however that higher prices for more differentiated products are related to improved survival, possibly reflecting their higher quality. Görg et al. (2012) additionally differentiate products according to their subsequent use (consumer, capital and intermediate goods) and find differential export durations for them. Consumer goods have the highest probability of survival, whereas intermediate goods are most likely to be dropped. As before, interacting those categories with the relative unit value as a proxy for product prices, the effect of a higher product price is stronger on survival for consumer goods.

With the exception of the last paragraph, the empirical studies suggest that the interaction of firm and product characteristics, forming the so-called firm-product characteristics, is the influential determinant of firms’ product switching. Product characteristics and some product-specific superiority alone seem not be able to sufficiently explain the phenomenon. Bernard et al. (2010) and Bernard and Okubo (2015) support this insight by deriving a positive cross-product correlation between the rates at which products are added and dropped. So, the products that are added by many firms are the same products that are dropped by many other firms. On net, however, some products are added by firms, while others are dropped and some net transfer of output across products takes place in the U.S. and Japanese economies.

Product-switching behavior is not only numerically widespread across firms, but products being switched are also quantitatively important within firms. According to Bernard et al. (2010), products which (surviving) firms added within the previous five years accounted for 26% and 31% of firm output in the years 1992 and 1997, respectively. With a similar order of magnitude, leading to the fact that gross changes in firm output being substantially larger than net changes, 29% and 26% of firm output in the years 1987 and 1992, respectively, are associated with products which (surviving) firms drop in the following five years. For the case of Brazilian exporters, Timoshenko (2015) finds higher figures, given the smaller (one-year) horizon of changes: 29% of firm-level export sales are derived by products which surviving exporters adding at least one product in a given year add to their export portfolio. The same share of firm-level export sales, 29%, is given by products which surviving exporters dropping in the subsequent year at least one product discontinue selling abroad. At the core of the study, showing the conditional export experience (age) dependence of product switching, it derives that these shares gradually decline with the age of the exporter: While for firms with two years of exporting, more than half (51%) of total sales are attributed to added products, firms with five years of exporting only derive 28% of their total export sales from added products. Similar pattern emerges for product

50This fact is already indicated by the characteristics of products being switched, which are described before in this section. For the country-wide quantitative importance of product-switching firms and its variance across countries, see description some paragraphs before.

51In the study by Timoshenko (2015), figures for added (to-be-dropped) products are calculated across exporters which added (will drop) at least one product between two consecutive years (in the subsequent year). Those exporters represent large shares of surviving exporters (56% and 54%, respectively) and account for large shares of Brazilian exports (74% and 70%, respectively).
dropping, suggesting less engagement in product adding and dropping for firms exporting for longer periods of time.

Several studies examine the contribution of the product-switching behavior of firms to the growth of aggregate (country-wide) output or exports and thereby detect a significant role of this type of firm activity. Navarro (2012) finds that about 85% of the aggregate sales growth of continuing plants can be attributed to some product switching and in the same way Söderbom and Weng (2012) are able to identify a significant contribution of changes in firms’ product mix to aggregate output growth. For Brazilian exporters, Timoshenko (2015) reports that nearly 40% of the annual export growth is due to the product adding and dropping margin among surviving exporters, much larger than the contribution of the firm entry and exit margin (exporter-turnover margin accounts for less than 15% of annual export growth, gross contributions about five to six times larger than the net), while gross contributions of product adding and dropping are three to four times larger than the net, illustrating the enormous intensity of firm activities in both directions at the product-level.

Similarly documented by Lopresti and Shiferaw (2017) for Ethiopian firms, the manufacturing sales growth among continuing products at surviving firms amounts to 7%, while the respective values for added and dropped products are 8% and -5%, respectively. Given a yearly total manufacturing sales growth of 11%, the net effect of product churning among incumbent firms (3%) thus accounts for about 30% of the total sales growth, which is much more than the share for which the net effect of firm churning (1%, with growth due to firm entry (exit): 5%(-4%)) is responsible.

The product-switching behavior of firms exhibits some correlations with changes in firm characteristics, besides the immediate effect on the firm-product scope. Bernard et al. (2010) document that, among surviving firms, net product adding is associated with an increase in firm size (both output and employment), as intuitively suggested, and firm productivity (both labor and total factor productivity), which is more surprising following the theoretical concept of flexible manufacturing. Net product dropping however is associated with a decrease in firm size (both output and employment) and firm productivity (total factor productivity). In another study for firms in the U.S., Liu (2010) examines the relationship between firms that (only) add and drop products and changes in their characteristics. For firms dropping products, he finds a decrease in their product scope and increases in the Herfindal index of sales, the sales share of core products as well as the firm-level supply, demand, production and sector relatedness measures.

Álvarez et al. (2016) study the (immediate and lagged) impact of product mix changes on productivity and other plant-specific outcomes. The product dynamics at the firm level

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52 For the (individual) contributions of the extensive and intensive margins to aggregate and firm-level growth: See the Section 2.2.3.

have positive effects on total factor productivity and labor productivity two years after the product mix changes were introduced (relative to plants with no changes in the product mix: about 17% (18%, 30%) on total factor productivity for adding (dropping, churning) products; similar effects for labor productivity), whereas these effects are mainly driven by the firms that are active in product churning. Product creation and destruction in isolation do not significantly affect the productivity and thus positive effects only appear when product adds and drops are accompanied by another. Furthermore, positive effects of product mix changes on employment and sales, but not on wages are observed (relative to plants with no changes in the product mix: sales: plants adding (dropping, churning) products: about 19% (12%, 24%); employment: plants adding (dropping, churning) products: about 5% (5%, 9%) only immediately (1 year, 1 year) after the change). Once again, no effects on outcomes exist when the analysis is restricted to firms that only add or drop products. Across the active and in-active plants, characteristics differ: Plants with product mix changes are larger in terms of output and employment than plants that do not change their product mix. However, there are no significant differences in the plants’ total factor productivity growth before and after the changes in the product mix and plants with product mix changes reveal a better performance along several indicators before and after the treatment. Both facts reject the hypothesis that these changes would be the plants’ response to a poor past performance.

Based on propensity scores, Álvarez et al. (2016) find that total factor productivity growth in the previous year does not have a significant effect on the probability of changing products, once again rejecting the hypothesis of product mix changes as a (defensive) firm measure of rehabilitation, while the number of products and the plant’s market share positively affect the probability of introducing changes in the product mix. In contrast, larger firms are less likely to introduce product mix changes and the firms’ export status does not play any role for changes in the product mix.

### 2.2.11 Firm-product Adjustments to Exogenous Shocks

With the appearance of (disaggregated) micro, i.e. firm-level, data sets in the 1990s, the empirical literature of international trade documents and analyses the heterogeneity of firms within narrowly defined industries and within-industry reallocations across firms following exogenous shocks, in particular trade liberalizations.

Besides the documentation of a within-firm heterogeneity of products the appearance of (even further disaggregated) micro, i.e. firm-product-level, data sets then allows the analysis of a new and additional margin of adjustment following exogenous shocks in international trade, i.e. within-firm reallocations at the product-extensive and the product-
intensive margins. Their nature, relevance and implications at the micro and aggregate level are at the core of some recent empirical studies, investigating in detail trade liberalizations\(^{57}\) and exchange rate fluctuations as examples of those exogenous shocks.

**Trade Liberalization**

**Regularity 9** Multi-product firms in general adjust their product portfolio in the way that they drop peripheral products (product-extensive margin) and skew sales towards their core products (product-intensive margin), thereby refocusing in their product portfolio on their core competencies. Nevertheless, notable differences in the responses to both trade liberalization and import competition exist across firms being heterogeneous along several dimensions, especially the size as well as status of domestic and international engagement (innovator and exporter status).

With the bilateral reduction in trade costs between countries, i.e. trade liberalization, (multi-product) firms in general may face three and in part adversely working effects: 

(i) They compete more with foreign firms offering output goods that are more or less substitutable to their own goods (output trade liberalization - import competition for the domestic market), exporting firms get an improved access to the foreign sales market (output trade liberalization - improved access for the foreign market) and (ii) they get an improved access to foreign-made input goods (input trade liberalization)\(^{58}\).

Let us start with the multi-product firms’ responses to one of the unilateral components of the output trade liberalization \([(i)]\), in particular the import competition. Liu (2010)\(^{57}\)


Beyond the effects on firms and industries (investment responses: Pierce and Schott (2018a)), a recently emerging strand of the empirical literature investigates the effects of trade liberalization and import competition on an array of socio-economic outcomes: Health of workers and mortality (McManus and Schaur (2016), Lang et al. (forthcoming), Pierce and Schott (2018b)), marriage and fertility (Autor et al. (forthcoming), Keller and Utar (2018)), public goods (Feler and Senses (2017)), crime (Che et al. (2018)), Dix-Carneiro et al. (2018), political elections (Autor et al. (2017), Che et al. (2017)), media coverage (Lu et al. (2018)).

By analyzing bilateral trade shocks along the two dimensions of their impacts, i.e. the range (output - input) and the direction (increased competition in the domestic market - improved access to the foreign market), four effects and therefore one additional effect can be simply identified: With the output trade liberalization, a firm faces both an increased import competition in the domestic market and improved export opportunities to the foreign market. When the input trade gets liberalized between two countries, a firm not only gets an improved access to foreign-made inputs, but also faces increased competition in the domestic market on the home-made inputs (input competition). However, the empirical evidence in the literature on the last effect is very limited and it is therefore not further considered here.
studies the effect of the import competition on the product portfolio of multi-product firms in the United States. Being confronted with an increase in import competition, multi-product firms refocus in their product portfolio on their core competencies by dropping peripheral products and skewing sales towards their core products. As the study reveals, a product exit is in general more likely as imports rise, but firms are relatively less (more) likely to drop core (peripheral) products and products being closer to the firm’s core product. In this way, a 10% increase in the imports across all a firm’s product lines raises the probability that the firm exits a peripheral product by 5.2%, while at the same time, the probability of exiting the firm’s core product decreases by 6.7%. However, the probability of a peripheral product being dropped varies across those products and is determined by the strength of the linkages it shares with the firm’s core product. So, the stronger the linkages in terms of a larger extent of joint sales to buyers, joint purchases of inputs, joint production and joint sectorship, the less likely the peripheral product is to be divested in the face of import competition. When a firm is confronted with a 10% increase in the imports, the probabilities of exiting are 4% (3.7%) lower for core products and 5.7% (8.2%) higher for peripheral products that share no supply (demand) relationship with the core. In general, the initial product size, its initial profitability, the firm growth and the industry growth are all negatively related to the probability of product exiting. Furthermore, the reallocation within a multi-product firm due to the import competition, i.e. the firm’s refocusing on the core production, does not only take place at the extensive margin by dropping peripheral products, but also at the intensive margin by skewing sales towards the firm’s core products. A rising import competition generally leads to declining sales shares of all the firm’s products, but is also associated with an increasing sales share of core products and a decreasing share of peripheral products; the latter effect is accentuated for products being further away from the core product and sharing fewer linkages with it in terms of supply, demand, production and sector membership.

The study by Lacovone et al. (2013) conducts an analysis of the Mexican economy when being challenged by a surge of Chinese exports and the competition for Mexican manufacturing (multi-product) producers associated with it. The increased competition from China leads to plant and product exits as well as to sales contraction. These effects however are highly asymmetric at the plant and product level and suggest some reallocation.

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50 Measure for the import competition: The industry/product-level imports.
51 A core product is defined as the firm’s product with the largest sales share (peripheral products as the residual products) and product closeness to the core is measured by the within-firm sales rank of the corresponding product.
52 Using the product sales rank within a firm as the measure for the product-level differentiation within firms, the values change to 32% and 3.8%, respectively.
53 Within-firm linkages between products produced at a lower scale and the firm’s core product are quantified by relatedness measures regarding sales, procurement, production and sector membership.
54 Measure for the import competition: At the product level, the share of Chinese imports in total imports; at the plant level, the sales-weighted average product-level Chinese import share.
55 In the study by Lacovone et al. (2013), the terms firm and plant are used interchangeably due to their approximate equivalence in the data.
both between products within plants and between plants within sectors. Comparing firms facing low (below median) and high (median and above median) competition, respectively, firm survival and evolution seem to be more difficult in the latter case, i.e. higher plant exit, lower probability of one-product firms evolving into two or more product firms, higher probability of five-plus product firms cutting back below five products or of exiting altogether and higher probability of a change in product numbers, with slightly higher chances of cutting back than expanding. However, the exit probability of plants decreases with their size. In industries with higher competition, products reveal a higher dynamic in the sense that they are more likely to enter and exit the market, and less likely to continue. And at the product-intensive margin, products facing higher competition are found to grow by a lower extent. As at the plant level, the competition effect is mitigated for products produced at a larger scale (core products). These results regarding plant and product entry or exit as well as product contraction remain the same for export products facing competition in the U.S.

Mayer et al. (2014, 2016) analyze French multi-product manufacturers’ exports across and within export market destinations. In the study by Mayer et al. (2014), it is shown that the skewness of a firm’s exported product mix \(^{65}\) consistently varies with destination characteristics such as size (gross domestic product (GDP)) and geography (proximity to other big countries)\(^{66}\). Firms sell relatively more of their better performing products in bigger and more centrally located markets and therefore skew their export sales towards their core competencies in destination markets with a higher competition. Subsequently, Mayer et al. (2016) examine the response of a firm’s export sales skewness \(^{67}\) to demand shocks \(^{68}\) within a destination over time. Thereby, they observe that positive demand shocks in a destination market induce multi-product exporters to increase their export sales to that destination along both margins and to skew their export sales to that destination towards their better performing products, in this way reacting to the increased competition in that destination market. Furthermore, these positive destination-level demand shocks, aggregated across a firm’s export destinations to get a firm-level demand shock, and their induced reallocations within a firm have a substantial positive effect on a multi-product firm’s productivity \(^{69}\) whereas the elasticity of labor productivity to trade shocks lies between 5% and 11%. The firm productivity response is concentrated within the quartile of exporters with the highest export intensity. For the aggregate sector, the trade shocks

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\(^{65}\) Measures for the product mix skewness: The firm’s export sales ratio of the two or three top selling products, the standard deviation of log export sales, a Herfindahl index and a Theil index.

\(^{66}\) Measure for the geography following the supply potential concept: The destination’s foreign supply potential as the aggregate predicted exports to that destination based on a bilateral trade gravity equation with exporter fixed effects and standard bilateral measures of trade barriers/enhancers.

\(^{67}\) Measure for the export sales skewness: A Theil index.

\(^{68}\) Underlying measure for the demand shock: The gross domestic product (GDP) of the destination, the total imports into the destination (excluding French exports), the imports into the destination in product categories a specific French firm also exports to the destination.

\(^{69}\) Measure for the (firm-level) productivity: The labor productivity as the deflated value added per worker.
account for a 1.2% average annual increase in the French manufacturing productivity over the ten year sample.\footnote{Garcia-Marin and Voigtländer (2017) stress that the result of export skewness in response to competition crucially depends on the efficiency measure that is used to rank the products within firms: With sales- and physical-efficiency-based product rankings, the result still holds, while there is no export skewness in case of marginal-costs- and revenue-efficiency-based product rankings.}

In a study on Korean manufacturing plants, Choi and Hahn (2017) consider their responses to import tariff reductions. Those are found to be associated with a decrease in plant shipments, product scope and production scale\footnote{For a correspondence to the notation in the Section 2.2.3, plant shipments or plant size denote the firm-intensive margin, product scope the product-extensive margin and production scale the product-intensive margin.}, whereas this effect is only significant in the domestic market, in which firms face a higher competition, and not the export market. Taking thereby into account possible differential responses across heterogeneous firms, it turns out that plant total factor productivity (TFP) and a plant’s exporter status do not matter in the plants’ responses to tariff reductions, but a plant’s innovation status\footnote{Measure for the innovator status: An engagement in research and development (R&D) in the beginning year of the three-year period.} indeed does. In this way, the study detects that innovators react quite differently in comparison to non-innovators, particularly in the domestic market, in which all these firms are confronted with an increased competition. Following the import tariff reductions, innovators reduce their product scope but increase their production scale in total and in the domestic market, while non-innovators show no significant change of the product scope but a decrease in the production scale in the domestic market, overall leading to a significant reduction in domestic shipments. A one-standard-deviation reduction in import tariffs or a 3.9 percentage points reduction in the tariff rate in a three-year period causes innovators to reduce their domestic product scope in the three-year period by about 2.6% more than non-innovators. However, a one-standard-deviation reduction in import tariffs causes innovators to increase the scale of their domestic production in the three-year period by about 4.4% more than non-innovators. Further differentiating across the plant size\footnote{Measure for the plant size: The number of employees (large plants: more than 100 employees, small plants: less than 100 employees).} reveals that large innovators reduce their product scope but increase their production scale in total and in the domestic market, as it was the case for the innovators when not differentiated. While small innovators undertake a reduction in their production scale in the export market, small non-innovating plants reduce both their production scale and plant shipments in the domestic market. Finally, when distinguishing between exporting and non-exporting innovators and non-innovators, the study reaches the result that exporting innovators reduce their product scope and increase their production scale in total and in the domestic market, as it is done by large innovators. Exporting and non-innovating plants instead increase their product scope and plant shipments in total. Plants that neither export nor innovate undertake a reduction in their plant shipments in total and in the domestic market and reduce their product scope in the domestic market.
Liu and Rosell (2013) study the relationship between the import competition that firms in the United States are confronted with and the nature of the innovations that they undertake. An innovation is called basic if it is tended to expand the general knowledge about the physical world, while it is called applied if it intends to give a specific understanding of a particular problem or application. The study finds that the firm innovation becomes less basic when firms face a higher import penetration. A 10 percentage point increase in the average import penetration decreases the research and development (R&D) expenditures of firms by 0.014% and decreases the basic research expenditures of firms by about 0.4% to 1%. The indirect channel through which this works is given by the product diversity responses of multi-product firms: Import competition induces multi-product firms to narrow their firm activities and to become more focused by dropping peripheral products and concentrating on those in which the firms are most competent in. But the narrowing raises for the firms the basic innovation’s uncertainty to get its outcomes incorporated by them and thus lowers the incentive to undertake this type of innovation. This view is supported by the study’s findings that firms with more product lines generate more basic innovations and a higher import competition lowers the number of product lines a firm operates. In addition, it is found that more effective patents, which better allow firms to sell or licence its innovations and generate benefits from basic innovations without needing to use them itself, mitigate the impact of the import competition on the innovation basicness by diminishing the importance of the firm product diversity.

Based on the estimation of multi-product production functions, which confirms the theoretical proposition that their input coefficients are positive and the output coefficients negative, Dhyne et al. (2017) investigate the effect of import competition on the technical efficiency with which Belgian multi-product firms produce their products. They find that increases in the import share are positively correlated with the technical efficiency, which is characterized by a high persistence on the firm-product level over time, indicating that changes in the technical efficiency are long-lived. However, the magnitude of the effect crucially depends on the method with which it is measured: Applying ordinary least squares estimators, an increase of 10% in the import share is only associated with a 1.0% increase in firm-product technical efficiency, while the effect increases to 8.7% when applying instrumental variables (IV) estimators. Given an average change in import shares of

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74 Measures for the innovation basicness: The Herfindahl basicness measure based on patent citations and technology classifications of different levels of aggregation (as well as an industry classification), the share of citations received from university patents and the count of non-patent citations received.

75 Measure for the import competition: The firm’s import penetration ratio in a year, i.e. the revenue-weighted average industry-level import penetration measure.

76 Note that the main purpose of the study is to look at the nature of the innovation, not its quantity. However, Liu and Rosell (2013) detect a positive relationship between the import competition and the level of the firm inventive activity. This finding is consistent with other studies, as those suggest that an increase in competition may spur firm innovation (e.g. Aghion et al. (2005)) and a trade liberalization in form of an increased import competition increases firm innovation (e.g. Bloom et al. (2016)).

77 Measure for the import competition: The import share as the product-level import penetration on net (adjustment for re-exporting) imports.

78 Instruments for the import share, which ought to be correlated with the shares but uncorrelated with
4.7%, OLS estimators state that import competition plays only a relatively minor role in promoting economic growth.

Provided that an asymmetry within firms in form of a product heterogeneity and hierarchy exists, import competition affects the products across the portfolio differently: The within-firm ranking of products that is based on their revenue on average coincides with the ranking that is based on the technical efficiency with which they are produced (Garcia-Marin and Voigtländer (2017)). Including interactions between the import share and the rank of the product within the firms in the regressions to account for differences in the competitive effect across the products within firms increases the estimate to 1.2% for the OLS estimators and 10.5% for the IV estimators. In the former case, the interactions are all negative and a 1% increase in the import share is associated with a 1.05% increase in the technical efficiency of the first and second ranked products, and a 0.65% increase in the technical efficiency of all other products produced by the firm.

Another set of empirical studies looks at the output trade liberalization and its effects on multi-product firms in a more comprehensive manner, by taking into account its bilateral character between participating countries. Baldwin and Gu (2009) study all the scope, size and diversification responses of Canadian manufacturing plants to the trade liberalization with the United States between 1984 and 1997, given the free trade agreement between the two countries (Canada-United States Free Trade Agreement (CUSFTA)) of 1989. They find that tariff cuts lead to a reduction in the number of products produced by non-exporters; in particular, a 1 percentage point decline in tariffs is associated with a 0.6% decline in the number of products. In contrast, the effect of tariff cuts on the number of products produced by exporters is only limited in the way that it is not statistically significant, as it is the case for large plants. For plants that are one standard deviation smaller than the average plant however, a one percentage point tariff cut is associated with a 5% decline in the number of products. Overall, tariff cuts are linked to a larger rate of decline in the number of products at smaller non-exporters than at larger non-exporters. While exporters reduce their product ranges relative to non-exporters, the declines in the number of products produced are not related to tariff cuts. The effect of being an exporter exists for smaller plants but is not relevant for large plants. In general, large plants add new products to their portfolio relative to small plants.

Looking at the plants’ product diversification, a similar pattern appears as for the number of products: Tariff cuts are associated with a decline in the product diversification index of non-exporting plants, while their effect on the product diversification of exporting plants is only limited in the way that it is not statistically significant. The product diversification of smaller plants is to a larger extent affected by tariff reductions than that of larger plants. In this way, a one percentage point decrease in tariff rates is associated with

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79See the Section 2.2.3
a 0.2% decline in the product diversification index for plants that are a one standard deviation smaller than the average plant, where the effect on plants that are a one standard deviation larger than the average plant is statistically insignificant. Putting both plant characteristics then together, tariff cuts decline the product diversification index of smaller non-exporters more than that of larger non-exporters. In general, exporters reduce their product diversification relative to non-exporters and the impact primarily exists for small exporters. Large plants increase their product diversification relative to small plants. For entrants to the export market, tariff reductions do not significantly affect their product diversification.

Tariff cuts are associated with a decline in the size of non-exporters, while, once again, the effect on exporters turns out to be not statistically significant. The same is true for new exporters. The contraction effect of tariff cuts however is more pronounced for larger plants than for smaller plants. A one percentage point decline in the tariff rates is associated with a 0.6% decline in the size of plants that are a one standard deviation larger than the average plant, with only an insignificant effect for plants that are a one standard deviation smaller than the average plant. For non-exporters, the negative effect of tariff reductions on their size increases with the plant size. The rate of the decline in the plant size following the tariff cuts is larger for larger non-exporters than for smaller non-exporters. Even without a significant effect on the size of the average exporters, tariff reductions reduce the size of larger exporters. In general, exporters and new exporters increase their plants’ size relative to their reference groups.

Considering the production-run length of Canadian manufacturing plants, tariff cuts do not have a significant effect on it for non-exporters, exporters and new exporters. In general however, exporters increase their production-run length compared to non-exporters. Iacovone and Javorcik (2010) look at Mexican firms’ response at the product-extensive margin to trade liberalization. In case of an industry-level consideration, it is found that industries that face a larger decline in U.S. tariffs on Mexican imports experience a larger increase in the number of exported varieties. In addition, industries with a larger decline in Mexican tariffs on U.S. imports have a larger increase in the number of varieties sold domestically. Notably, at the firm level, a positive relationship between Mexican tariffs on U.S. imports and the probability of a variety’s survival in the domestic market is found. This effect of tariff protection turns out to be stronger for core varieties.

Using the introduction of the Euro in 1999 as a natural experiment, Berthou and Fontagné (2013) analyze the responses of French multi-product exporters to a change in the trade costs. They find that the introduction of the Euro has only a negligible positive aggregate effect on the firm-level exports to destinations in the Euro area, thereby combining such adverse effects as the trade-creation and competition effect due to the lower trade costs. In detail, the export propensity and the product-extensive margin are not affected, while there is a weakly significant positive effect on the product-intensive margin. Overall and also being only weakly significant, the introduction of the Euro increased the firm-level
exports by about 5%. This finding however hides substantial differences across exporters and destination markets. Taking into account the heterogeneity among exporters, it turns out that while the Euro introduction has no effect on the exporters’ selection in Euro-area markets, it fosters the exports only for the most productive firms. Those firms in the quartile with the highest productivity increase their exports by 8.4%, whereas about a quarter of the effect comes from the product-extensive margin and the rest from the product-intensive margin. Furthermore, a significant difference exists across the Euro-area destinations, considering their heterogeneity with respect to the intensity of the monetary policy coordination before 1999. For destinations with a less close coordination before 1999, the introduction of the Euro has a positive effect at all: In destinations of the Euro area outside the D-mark zone, firm-level exports increased by 12.8%. More than three quarters of this effect go back to the product-intensive margin, and the product-extensive margin accounts for the rest of the effect. By contrast, the introduction of the Euro negatively affects the firm-level exports, with a weakly significant decline of 5.5%, and the export propensity in Euro-area destinations within the D-mark zone. The time dimension reveals a steady increase in the effects over time, which suggests that firms adjust to the changed trade conditions over time.

Decomposing the Euro introduction’s aggregate effect for French exporters into a trade-expansion effect and a competition effect by explicitly accounting for the competition from other Euro-area countries reveals that the competition within the Euro area plays a significant role: A 10% increase in the competition from other Euro-area countries reduces the product-intensive margin by 5.8%. For the trade-expansion effect, the reduction in trade costs due to the Euro introduction has strong positive effects on the firm-level exports, as they increase by about 7%. The increase in the product-extensive margin contributes 18% to this effect, and the remaining 82% comes from the product-intensive margin. However, the export propensity is not affected by the introduction of the Euro. The competition effects are concentrated among the intermediate productivity firms. Finally, the Euro introduction increases industry-level French exports to the destinations in the Euro area, whereas the effect turns out to be larger when controlling for the within-area competition, and even larger effects for the Euro area excluding the D-mark zone. For the Euro area within the D-mark zone, industry-level French exports decrease with falling trade costs, though reflecting a weakly significant effect, while the effect gets insignificant in case of a competition control.

Another study about the firm-level responses to changes in trade costs is provided by Lopresti (2016). Taking the Canada-US Free Trade Agreement (CUSFTA) of 1989 as the policy event being responsible for a decrease in trade costs between Canada and the United States, the study finds a differential product portfolio response among U.S. firms that are heterogeneous in terms of their involvement in foreign markets, thereby explicitly trying

\[80\] By contrast, the differential pattern can not be found with respect to relative firm sales, as a second form of firm heterogeneity.
CHAPTER 2. MULTI-PRODUCT FIRMS - SURVEY OF EMPIRICAL STUDIES

to reconcile contradictory conclusions drawn in the models of multi-product firms. Firms that are mainly concentrated on the domestic market and thus are engaged in foreign markets only to a limited extent, i.e. with less than 10-20% of total sales accounted for by foreign markets, reduce their product diversification as trade costs fall, while more internationally oriented firms increase their product diversification or show no response.

A 10 percentage points reduction in bilateral tariffs is associated with a reduction in the diversification index of 0.11 (0.173 with an alternative definition of involvement) for the first type of firms, while it is associated with an increase of 0.065 (0.075) for the second firm type. The change point between those two groups of firms lies around 18%. In general, it is found that firms with higher levels of sales experience a rise in the diversification between the years of the analysis relative to firms with lower sales and a negative correlation exists between the foreign market orientation in levels and the changes in diversification. Overall documented, trade increases the costs of production for approximately 13% of the firms.

Let us now turn to the effect of the input trade liberalization on multi-product firms (ii), which is also considered in the study by Iacovone et al. (2013). Besides increased competition, the availability of inputs for producers in Mexico increases with increased Chinese exports to the country and larger plants and core products benefit disproportionately from the expanded access to inputs. Firms are able to make better use of the availability of Chinese inputs for their core products and the expanded access to cheaper Chinese inputs helps them to improve the competitiveness of their core products. As a result of increased penetration of Chinese input imports, the output of core products expands more, whereas the output of marginal products do less so or not at all. Similarly, larger plants are better able to make use of Chinese inputs.

Goldberg et al. (2010a) report for India that a decline in input tariffs is associated with an increase in firm scope; in detail, a 10 percentage points fall in tariffs leads to a 3.2% expansion of a firm’s product scope. Altogether, changes in the input tariffs account for a substantial share of firm scope adjustments and contribute to aggregate output increases: 31% of the observed expansion in the firm’s product scope goes back to the declines in input tariffs; thereby being responsible for 7.8% of the manufacturing output growth. Furthermore, firms adjust more to changes in the extensive margin than to changes in the price margin on the input side: The elasticity of the firm scope with respect to the input extensive margin is higher than the elasticity with respect to the input price margin.

Further insights into the product-level adjustments of multi-product firms to input trade policy changes are provided by Vandenbussche and Viegelahn (2018). By investigating all Indian anti-dumping measures or cases on the imports of raw material inputs that were initiated between 1992 and 2007, they study within-firm input and output reallocations of importing firms in response to the trade shocks on raw material inputs that are used in their firm-level production. Each case involves one or several products, leading up to 1300 different firm-inputs that were subject to anti-dumping measures. Firms in the treatment

81 Measure for the product diversification: Herfindahl-Hirschman-style index of firm diversification.
lower the use of treated relative to non-treated inputs on average by 25-40% and thus more than control firms do and thereby skew their input use away from protected inputs towards unprotected inputs when they are confronted with trade protection: The induced reallocations however do not take place in form of contractions at the extensive margin of the firms’ input usage, as trade protection does not have a significant impact on input dropping, but firms that use protected inputs evade to alternative ones, as they tend to increasingly start using other inputs than the protected ones and tend to add protected inputs less frequently, when protection is imposed.

In terms of the firm-level impact of input trade policy changes, differences exist across the anti-dumping measures and firms: The input reallocation effect is larger or stronger, the longer the protection is in force, and the results on input reallocation are mostly driven by the large firms as the effect is only significant for them. Moreover, the intensity with which firms employ the protected input in their production plays some role: For the firms that are insufficiently involved in making use of the protected input, i.e. with a share of the protected input in the total value of inputs below 20%, the input reallocation effect is weaker, even so still observable in input quantities, but no longer for input values.

The import protection on inputs has also effects on the firm behaviour outside the period of its validity, both before and after the trade shock takes place: Firms anticipate the import protection in the way that they already reduce their corresponding input use in the year before trade protection is in force (investigation effect) and firms do not, or at least not fully, return to their pre-protection input allocation, once protection has expired (post-treatment effect), which illustrates that input trade policy changes exert a permanent impact on the firms’ input usage, even so the measures by themselves are only temporary.

In general, the point estimates of the effects are similar for input values and quantities, which indicates that the input purchase prices of protected inputs are unchanged on average, while however only the low intensive users of the protected inputs observe an increase in the average input prices of the protected inputs.

Exchange Rate Fluctuations

Regularities 10 Multi-product firms increase their producer prices in response to a real exchange rate depreciation, whereas an incomplete pass-through of the exchange rate shocks

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82 The treatment group consists of importers of raw material inputs that use an input in production on which anti-dumping protection is imposed in the year the anti-dumping case is initiated (treated inputs in treated firms). All other inputs within the treated firms are referred to as non-treated inputs in treated firms. A year is considered as a treatment year if an anti-dumping measure has been in force for at least six months in that year.

83 Firms that employ an input on which anti-dumping protection is imposed, but are not importers of raw material inputs belong to the non-importer control group. The treated inputs in control firms are the inputs that are employed by these firms and are under import protection by the anti-dumping measure. The non-treated inputs in control firms are given by all the other inputs that are employed by these firms.

84 Firms are assigned to the categories of small or large firms based on the criterion of whether their size is below or above the median size, whereas the firm size is measured in terms of net fixed assets in the year the anti-dumping case is initiated.
exists. The responsiveness however differs across the firms as well as across the products within the firms.

In a study of Brazilian data for 1997-2006, Chatterjee et al. (2013) investigate the effects of exchange rate shocks on the export behavior of multi-product firms. They basically find that increases in the real exchange rate, i.e. real depreciations, are associated with increases in producer prices and the quantity exported, while the producer price responsiveness turns out to be highly heterogeneous across different industries. On average, the estimated producer price elasticity amounts to approximately 0.23, which implies that the exchange rate pass-through to import prices abroad (in the destination’s currency) appears to be around 0.77, implying some incomplete pass-through of exchange rate shocks. For multi-product firms, the producer price responsiveness varies across products and the relative position of a product within a firm represents its statistically and economically significant determinant. In particular, products further away from the firm’s core expertise have a significantly lower producer price response; for non-core products, it is 6% lower than for core products and the producer price responsiveness for products with below median sales is 8 percentage points lower than for products with above median sales.

In addition, the productivity of firms, which can be measured by different proxies, plays a key role in determining the exchange rate pass-through. In the wake of a depreciation, more productive firms increase their mark-ups to a greater extent than less productive firms. But in case of multi-product firms, the heterogeneity in price responses can be certainly attributed to both the productivity dispersion across firms and the within-firm productivity dispersion across products. Further firm-level determinants of price responsiveness and therefore factors for heterogeneity across firms seem to be the firm size, the wage bill and the number of products exported (product-extensive margin). To be more concrete, it emerges that the higher the ratio between imports of inputs and the wage bill of the firm, the higher the response of prices to a depreciation. Furthermore, an increase in bilateral distance and hence transportation costs reduce the producer price responsiveness, while an increase in distribution costs increases the producer price responsiveness. In an environment with a higher exchange rate volatility and a smaller market potential, firms show a smaller response in producer prices to real exchange rates.

Besides the adjustments in producer prices due to exchange rate shocks, firms react through their product-extensive margin in the way that real exchange rate depreciations are found to be associated with an increase in the number of products exported. As the product-level response to one-standard-deviation shocks in exchange rates is larger than the response to one-standard-deviation changes in firm-level characteristics (employment, wage bill, skill composition, imports and exports), one can draw the conclusion that fluctuations in exchange rates are at least as important as firm-level fluctuations in explaining the product-extensive margin of trade. However, the product-level response is found to be heterogeneous across firms, based on the general result that more productive firms (productivity measured in terms of firm size, fraction of skilled workers and export per-
formance) export a larger number of products: More productive firms adjust their scope to a lesser extent in response to exchange rate fluctuations. An increase in the firm size by a one-standard-deviation of the cross-sectional distribution leads to an exchange rate response that is lower in 0.02, an increase in average wages by a one-standard-deviation of the cross-sectional distribution results in a response that is lower in 0.04 and a simultaneous increase in each of the characteristics (employment, skill composition, wage bill, imports, exports) by a one-standard deviation of the respective cross-sectional distribution yields a product-level response that is lower in 0.09, which represents almost half of the average response across firms.

A third margin of adjustment due to exchange rate shocks is given by the product-intensive margin. Following a real exchange rate depreciation, the within-firm sales distribution across products becomes less skewed, since firms reallocate resources towards their less efficient usage. [Bernard et al. 2014] disaggregate the changes in bilateral trade flows due to exchange rate fluctuations into the two margins, the extensive and intensive margin, and document in a study for Belgium firms for the period 1998-2005 that both margins matter: A 1% depreciation of the Euro is correlated with a 0.35% increase in exports and this increase is driven by both changes in the extensive margins (increases in the number of firms and the number of products at the country level) and the product-intensive margin (increase in the average exports per firm-product). Thereby, the response is roughly evenly split in magnitude between the extensive and intensive margins. It is also shown that the within firm-product response is strongly increasing as the currency depreciates.

As reported before, significant variations in the effects exist across both firms with different productivity and the products within firms: In terms of the export price, exchange rate changes exhibit a higher influence for more productive firms, and adjustments due to exchange rate shocks are less pronounced for products further away from the core competency of the firm. Stressed by [Chatterjee et al. 2013], exchange rate shocks absorption into mark-ups varies across products, being larger for core products, which account on average for two thirds of total firm-level exports. For the export quantity however, it is exactly the other way round as compared to prices: More productive firms react to exchange rate movements to a lesser extent, while adjustments are more marked for products being further away from the core product of the firm. Focusing on the different margins of adjustment, a real appreciation is associated with a reduction in the export scope (product-extensive margin) and the export sales get more skewed towards the best performing products. Firms concentrate on the products in which they are most efficient in by reallocating resources towards the more productive exporting products. In line with this result, the export duration of core products broadens, while it decreases for non-core products following a real exchange rate appreciation. Furthermore at the firm level, more productive, larger, more profitable and less finance-constrained firms have a lower hazard rate of exiting an export market.

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85For the skewness in the within-firm sales distribution: See the [Section 2.2.4]
Generally consistent (similar) results are drawn in two recent studies by Xu et al. (2016) for China in the period 2000-2007 and Caselli et al. (2017) for Mexico in the period 1994-2007. Looking at a real currency appreciation, Xu et al. (2016) document decreasing effects on both export prices and export quantities. Quantitatively, an appreciation of 10% leads to a drop of the export price by around 0.81%, indicating an incomplete pass-through effect of the real effective exchange rate on firms’ export price.\footnote{In addition, Xu et al. (2016) summarize that exporters reveal a pricing-to-market behavior; for the meanwhile large literature on pricing-to-market, see e.g. Hsieh and Klenow (2007), Alessandria and Kaboski (2011) and Simonovska (2015).} For the export quantity, an appreciation of the same size implies a drop by around 4.5%.

Caselli et al. (2017) study the effects of exchange rate fluctuations on the markups across the products within firms. They detect that a real depreciation induces an increase in markups and producer prices of exports, whereas the former’s elasticity with respect to the real exchange rate amounts to 0.06 and the latter’s elasticity with respect to the real exchange rate is 0.29, which implies that the exchange rate pass-through to import prices in foreign currency appears to be 0.71. Once again, the relative position of a product within a firm determines the markup and producer-price responsiveness to real exchange rate fluctuations: The markup and producer price of products with a higher productivity, and thus products closer to the core competency variety, increase by a larger amount. Given a value of -0.021 in the main specification for producer prices, the producer price of the third-highest ranked product increases by 3% less than that of the second-highest ranked product in response to a real depreciation of the exchange rate. Thereby, the response in producer prices is accompanied by a qualitatively equivalent responsiveness of markups and producer prices respond heterogeneously across the products within firms due to the heterogeneous response of markups: Given a value of -0.013 in the main specification for markups, the markup of the third-highest ranked product increases by about 7% less than that of the second-highest ranked product in case of a real exchange rate depreciation. In addition, differences in the producer price and markup responsiveness exist across industries and firms: The increase in producer prices and markups as well as their heterogeneous response within firms is particularly strong in industries with higher local distribution margins and the increase in marginal costs due to a real exchange rate depreciation is larger for firms with a higher share of imported inputs.

All in all, a robust finding in this strand of literature is an incomplete pass-through effect of exchange rate changes on firms’ variables, suggesting some absorption of aggregate fluctuations by exporters. As those firms are the ones that are large and productive, and larger exporters are even more productive, they are able and willing to partially absorb exchange rate fluctuations in their mark-ups, leading to the robust empirical finding of relatively muted responses of aggregate exports to exchange rates.
2.3 Conclusion

Motivated by the intensive treatment of multi-product firms in the more recent empirical literature of international trade, which is based on the progressive availability of micro data sets, this chapter asks for the actual state of knowledge about this type of firms in international trade. It tries to generate a comprehensive answer by surveying and evaluating a large body of empirical studies that employ micro data sets both for developed and developing countries. Thereby, it turns out that several robust findings or stylized facts can be derived, even so definitions about firms and products vary across the studies: Multi-product firms (i) are national and especially international prominent, less prevalent but important, (ii) are singular economic units with premia in terms of their nature and performance, (iii) produce and export at different margins, (iv) are asymmetric within themselves (“superstar products”), (v) do not produce all their exports, (vi) are relatively diversified in production and export, (vii) compete on both a cost and quality basis, (viii) are less prone to firm exit in the process of “creative destruction”, (ix) start exporting at a small scale and scope and increase them over time, (x) frequently and to a large extent change their product portfolio (“product switching”) and (xi) undertake adjustments in response to exogenous shocks. These empirical regularities bind both the existing and upcoming theoretical modeling of multi-product firms in international trade.
Appendix B

B.1 Product Classifications

Typical classifications that are used to define products in empirical studies are the following:

- Harmonized System (HS) Classification
- Standard Industrial Classification (SIC)
- International Standard Industrial Classification (ISIC)
- Combined Nomenclature (CN) Classification
- Statistical Classification of Economic Activities in the European Community (NACE)
- Classification for the Production of Manufactured Goods (PRODCOM)

B.2 Empirical Studies
Table B.1: Empirical Studies on Multi-product Firms in International Trade

<table>
<thead>
<tr>
<th>Study</th>
<th>Country/Region</th>
<th>Sample</th>
<th>Unit of Observation</th>
<th>Industrial Coverage</th>
<th>Product Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernard et al. (2007)</td>
<td>USA, 1992-2000</td>
<td>Firms</td>
<td>All international trade transactions</td>
<td>10-digit HS</td>
<td></td>
</tr>
<tr>
<td>Iacovone and Javorcik (2008, 2010)</td>
<td>Mexico, 1994-2003</td>
<td>Plants</td>
<td>Largest firms (plants), with the exception of assembly plants (&quot;maquiladoras&quot;) in classes of activity with more than 20 plants; all entities, about 85% of industrial activity, over 50% of total exports</td>
<td>4-digit ISIC</td>
<td></td>
</tr>
<tr>
<td>Aw and Lee (2009)</td>
<td>Taiwan, 1992-2000</td>
<td>Plants</td>
<td>Plants or firms (SIC 314-317) in major industries (within the electronics sector)</td>
<td>5-digit SIC</td>
<td></td>
</tr>
<tr>
<td>Baldwin and Cua (2009)</td>
<td>Canada, 1996-2001</td>
<td>Firms</td>
<td>Plants inside the manufacturing sector, (9661-9669, 9676-9679, 9681, and 9699) and (9660, 9664-9666, 9670-9675)</td>
<td>4-digit ISIC</td>
<td></td>
</tr>
<tr>
<td>Elliott and Virakul (2010)</td>
<td>Thailand, 2001-2004</td>
<td>Firms</td>
<td>All exporting firms</td>
<td>10-digit HS</td>
<td></td>
</tr>
<tr>
<td>Liu (2010)</td>
<td>USA, 1984-1996</td>
<td>Firms</td>
<td>7,098 public firms, 72 industry sectors (20 manufacturing sectors)</td>
<td>4-digit SIC</td>
<td></td>
</tr>
<tr>
<td>Goldberg et al. (2010a)</td>
<td>India, 1989-1997</td>
<td>Firms</td>
<td>2,927 manufacturing firms, about 60-70% of the organized industrial sector</td>
<td>National, 6-digit and 8-digit HS</td>
<td></td>
</tr>
<tr>
<td>Goldberg et al. (2010b)</td>
<td>India, 1989-2003</td>
<td>Firms</td>
<td>About 9,500 publicly listed firms, all entities, about 90% of national industrial activity, all firms</td>
<td>National</td>
<td></td>
</tr>
<tr>
<td>Elliott and Virakul (2010)</td>
<td>Thailand, 2001-2004</td>
<td>Firms</td>
<td>All exporting firms</td>
<td>10-digit HS</td>
<td></td>
</tr>
<tr>
<td>Bernard et al. (2009)</td>
<td>USA, 1992-2000</td>
<td>Firms</td>
<td>All international trade transactions</td>
<td>10-digit HS</td>
<td></td>
</tr>
<tr>
<td>Baldwin and Cua (2009)</td>
<td>Canada, 1996-2001</td>
<td>Firms</td>
<td>Plants inside the manufacturing sector, (9661-9669, 9676-9679, 9681, and 9699) and (9660, 9664-9666, 9670-9675)</td>
<td>4-digit ISIC</td>
<td></td>
</tr>
<tr>
<td>Elliott and Virakul (2010)</td>
<td>Thailand, 2001-2004</td>
<td>Firms</td>
<td>All exporting firms</td>
<td>10-digit HS</td>
<td></td>
</tr>
<tr>
<td>Liu (2010)</td>
<td>USA, 1984-1996</td>
<td>Firms</td>
<td>7,098 public firms, 72 industry sectors (20 manufacturing sectors)</td>
<td>4-digit SIC</td>
<td></td>
</tr>
<tr>
<td>Goldberg et al. (2010b)</td>
<td>India, 1989-2003</td>
<td>Firms</td>
<td>About 9,500 publicly listed firms, all entities, about 90% of national industrial activity, all firms</td>
<td>National</td>
<td></td>
</tr>
<tr>
<td>Elliott and Virakul (2010)</td>
<td>Thailand, 2001-2004</td>
<td>Firms</td>
<td>All exporting firms</td>
<td>10-digit HS</td>
<td></td>
</tr>
<tr>
<td>Bernard et al. (2009)</td>
<td>USA, 1992-2000</td>
<td>Firms</td>
<td>All international trade transactions</td>
<td>10-digit HS</td>
<td></td>
</tr>
</tbody>
</table>
### Empirical Studies

<table>
<thead>
<tr>
<th>Empirical Study</th>
<th>Sample</th>
<th>Unit of Observation</th>
<th>Industrial Coverage</th>
<th>Product Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernard et al. (2011)</td>
<td>USA, 1987-2004</td>
<td>firm</td>
<td>455 industries</td>
<td>10-digit HS and 5-digit SIC</td>
</tr>
<tr>
<td>Görg et al. (2012)</td>
<td>Hungary, 1992-2003</td>
<td>firm</td>
<td>1,587 large manufacturing exporters, 89% of all manufacturing exports and 75% of total manufacturing turnover</td>
<td>6-digit HS</td>
</tr>
<tr>
<td>Söderbom and Weng (2012)</td>
<td>China, 2004 and 2008</td>
<td>firm</td>
<td>68% of total timber volume</td>
<td>national</td>
</tr>
<tr>
<td>Álvarez et al. (2013)</td>
<td>Chile, 1991-2001</td>
<td>firm</td>
<td>all exporters</td>
<td>8-digit HS</td>
</tr>
<tr>
<td>Amador and Opromolla (2013)</td>
<td>Portugal, 1995-2005</td>
<td>firm</td>
<td>about 80% of total exports and about one quarter of all exporters</td>
<td>4-digit HS</td>
</tr>
<tr>
<td>Arkolakis and Muendler (2013)</td>
<td>Brazil, Chile, Denmark and Norway, 2000</td>
<td>firm</td>
<td>n/s</td>
<td>6-digit HS</td>
</tr>
<tr>
<td>Berthou and Fontagné (2013)</td>
<td>France, 1995-2003</td>
<td>firm</td>
<td>15,088 exporters with more than 20 employees</td>
<td>8-digit CN and aggregation to 6-digit HS</td>
</tr>
<tr>
<td>Chatterjee et al. (2013)</td>
<td>Brazil, 1997-2006</td>
<td>firm</td>
<td>manufacturing firms</td>
<td>national (~ 6-digit HS)</td>
</tr>
<tr>
<td>Iacovone et al. (2013)</td>
<td>Mexico, 1994-2004</td>
<td>plant</td>
<td>about 85% of industrial output, with the exception of “maquiladoras”</td>
<td>national and 6-digit HS</td>
</tr>
<tr>
<td>Liu and Rosell (2013)</td>
<td>USA, 1978-1998</td>
<td>firm</td>
<td>majority of patents that have been granted to corporations</td>
<td>n/s</td>
</tr>
<tr>
<td>Lu et al. (2013)</td>
<td>China, 2000-2006</td>
<td>firm</td>
<td>all exporters</td>
<td>6-digit HS</td>
</tr>
<tr>
<td>Bernard et al. (2014)</td>
<td>Belgium, 1998-2005</td>
<td>firm</td>
<td>85% of total exports</td>
<td>8-digit CN</td>
</tr>
<tr>
<td>Colantone and Crinò (2014)</td>
<td>Europe, 1995-2007</td>
<td>firm</td>
<td>entire manufacturing sector with at least 90% of the annual production of each industry in each country</td>
<td>8-digit PRODCOM, 8-digit CN and 6-digit HS</td>
</tr>
</tbody>
</table>

Table B.2: Empirical Studies on Multi-product Firms in International Trade
Table B.3: Empirical Studies on Multi-product Firms in International Trade

<table>
<thead>
<tr>
<th>Study</th>
<th>Country</th>
<th>Period</th>
<th>Unit of Observation</th>
<th>Sample Description</th>
<th>Product Definition</th>
<th>Industrial Coverage</th>
<th>Unit of Operation</th>
<th>Sample</th>
<th>Sample Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Io Turco and Aligronotov</td>
<td>USA</td>
<td>2005-2009</td>
<td>Plant</td>
<td>Around 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2016</td>
</tr>
<tr>
<td>Liopesilo</td>
<td>USA</td>
<td>1996-2009</td>
<td>Plant</td>
<td>Around 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2016</td>
</tr>
<tr>
<td>Holviren and al.</td>
<td>USA</td>
<td>2004-2011</td>
<td>Plant</td>
<td>Around 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2016</td>
</tr>
<tr>
<td>Arneson</td>
<td>Sweden</td>
<td>1977-2015</td>
<td>Plant</td>
<td>Approximately 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2016</td>
</tr>
<tr>
<td>Alvarez et al.</td>
<td>Chile</td>
<td>1996-2006</td>
<td>Plant</td>
<td>Around 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2016</td>
</tr>
<tr>
<td>Timoshenko</td>
<td>Brazil</td>
<td>1995-2000</td>
<td>Plant</td>
<td>More than 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2015</td>
</tr>
<tr>
<td>Hazum and Walker</td>
<td>Chile</td>
<td>1996-2003</td>
<td>Plant</td>
<td>More than 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2015</td>
</tr>
<tr>
<td>Renames and Sanuvo</td>
<td>Japan</td>
<td>1992-2006</td>
<td>Plant</td>
<td>More than 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2015</td>
</tr>
<tr>
<td>Breuer and Oehiro</td>
<td>Japan</td>
<td>1992-2006</td>
<td>Plant</td>
<td>More than 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2015</td>
</tr>
<tr>
<td>Hazum and Walker</td>
<td>Chile</td>
<td>1997-2003</td>
<td>Plant</td>
<td>More than 20% of all manufacturing products with more than 10 employees, manufacturing firms.</td>
<td>6-digit HS</td>
<td>89% of manufacturing</td>
<td>Plant</td>
<td>Yes</td>
<td>2015</td>
</tr>
</tbody>
</table>

APPENDIX B.
### Table B.4: Empirical Studies on Multi-product Firms in International Trade

<table>
<thead>
<tr>
<th>Empirical Study</th>
<th>Sample</th>
<th>Unit of Observation</th>
<th>Industrial Coverage</th>
<th>Product Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xu et al. (2016)</td>
<td>China, 2000-2007</td>
<td>firm</td>
<td>manufacturing firms with more than 8 employees and positive trade value, industrial value added, intermediate input, fixed assets, sales, average wage, age, excluding intermediary firms</td>
<td>8-digit HS</td>
</tr>
<tr>
<td>Aw and Lee (2017)</td>
<td>Taiwan, 2000-2004</td>
<td>plant</td>
<td>about 55% of all manufacturing plants, 95% of total revenue of plants</td>
<td>7-digit SIC</td>
</tr>
<tr>
<td>Caselli et al. (2017)</td>
<td>Mexico, 1994-2007</td>
<td>plant</td>
<td>3,923 plants, all manufacturing plants with more than 100 employees and selection of smaller plants, roughly 85% of all manufacturing output value, excluding “maquiladoras”</td>
<td>national</td>
</tr>
<tr>
<td>Choi and Hahn (2017)</td>
<td>Korea, 1991-2002</td>
<td>plant</td>
<td>70,000-100,000 plants for each year, about 70-80% of the plants with 5 or more employees in the manufacturing sector, more than 84% of shipments and almost all (99.9%) of exports</td>
<td>national and 10-digit HS</td>
</tr>
<tr>
<td>Manova and Yu (2017)</td>
<td>China, 2002-2006</td>
<td>firm</td>
<td>176,116 manufacturers, universe of firms that participated in international trade, excluding wholesalers</td>
<td>6-digit and 8-digit HS</td>
</tr>
<tr>
<td>Dhyne et al. (2017)</td>
<td>Belgium, 1997-2007</td>
<td>firm</td>
<td>manufacturing firms, at least 90% of production value in each of the 12 largest industries by including all firms with a minimum of 10 employees or total revenue above €2.5 million (exclusion of outliers)</td>
<td>8-digit PRODCOM</td>
</tr>
</tbody>
</table>
### Table B.3: Empirical Studies on Multi-product Firms in International Trade

<table>
<thead>
<tr>
<th>Study</th>
<th>Country</th>
<th>Time Period</th>
<th>Sample Details</th>
<th>Industrial Coverage</th>
<th>Observation Unit of</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garcia-Marin and Voigtländer (2017)</td>
<td>Chile</td>
<td>1996-2007</td>
<td>∼5,000 plants for each year, universe of manufacturing plants with 10 or more employees, 67% small (less than 50 employees), 20% medium-sized (50-150 employees), 12% large (more than 150 employees), national (7-digit ISIC), 8-digit HS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lopresti and Shiferaw (2017)</td>
<td>Ethiopia</td>
<td>1996-2007</td>
<td>1,008 firms, 41 industries, all manufacturing establishments that employ at least 10 employees and use power-driven machinery, national</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vandenbussche and Viegelahn (2018)</td>
<td>India</td>
<td>1992-2007</td>
<td>Around 60-70% organized industrial activity, primarily medium- and large-sized firms, 75% of all corporate taxes and more than 95% of excise duties collected by the government, national (6-digit HS)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bernard et al. (forthcoming)</td>
<td>Belgium</td>
<td>2005</td>
<td>Manufacturing exporters, (i) firms with a primary activity in manufacturing employing at least 10 employees and (ii) firms with a primary activity outside manufacturing (but with manufacturing production) employing more than 20 employees, 17% of exporters and 43% of total exports in the products of each four-digit HS group</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
- HS6: 6-digit HS
- HS8: 8-digit HS
Chapter 3

Introduction to the Theory of Multi-product Firms in International Trade

In its historical evolution, the theory of international trade experienced a trend towards a disaggregation and generalization of its models’ components, thereby driven by the mutually dependent phases of theorizing, empirical investigating, and updating existing models or developing novel models: While Ricardo (1817) as well as Heckscher and Ohlin (1919 and 1924) (old trade theory) considered countries and trade between them based on labor productivity and endowment differences (inter-industry trade), Krugman (1979) turned towards firms and introduced increasing returns to scale to the production technology of (homogeneous) firms that produce differentiated product varieties and are not strategically active (monopolistic competition). In this way, he allowed for the empirically observable but so far not theoretically captured trade within industries (intra-industry trade; new trade theory) and the effect of the opening up to trade on firms got part of the modern trade analysis. Induced by the progressive availability of micro, i.e. firm-level, data sets in the 1990s and the arising gap between the empirical evidence - within a typical industry, firms vary greatly in size, productivity, composition of inputs, wages and participation in foreign trade - and the theoretical modeling, Melitz (2003) broke up the assumption of ho-

1 The numbers in the parentheses represent the dates of publication of the most influential contributions for the development of the respective theory. For the references (reprints and translations): See Sraffa (ed., 1951) [On the Principles of Political Economy and Taxation (1817)] and Flam and Flanders (eds., 1991) [The Effect of Foreign Trade on the Distribution of Income (1919, pp. 43-69) and The Theory of Trade (1924, pp. 75-214)].

2 In the theoretical literature of international trade, two main concepts exist for modeling the opening up to trade as switching from the autarky to the open economy: In the sense of an integration of once separated country-level markets and thus an increase in the sales market as well as in the competition for each country’s firms, the opening up to trade is modeled by one part of the literature as an increase in the population size of a country (as in Krugman (1979)) and, where appropriate explicitly, an increase in the number of competitors. In contrast, the other part emphasizes the liberalization of trade between once separated countries by the reduction of tariffs and therefore models the trade opening as a decrease in the costs of transportation of goods between countries.
mogeneous firms by developing a way to model the heterogeneity of (single-product) firms in a monopolistic competition (new new trade theory) and intra-industry reallocations across firms due to an exposure to trade.

In times of the collection and publication of (even further disaggregated) micro, i.e. product- and firm-product-level, data sets a further part of the empirical evidence that was so far not covered by the theoretical modeling - simultaneous production of several products by a firm - became apparent and models of multi-product firms in international trade emerged during the last decade and therewith broke up the assumption of single-product firms, by obviously allowing firms to produce and sell more than only one product. In this way, a novel and likewise the most recent stage in the evolution of the theory of international trade has been entered. Given its meanwhile establishment in terms of the extent of the research output, the existing models of firms producing multiple products vary in several dimensions, in particular the market structure or the across-firms competition (monopolistic competition versus oligopoly), the within-firm heterogeneity (symmetric versus asymmetric products) and the within-firm cannibalization (existence versus absence).

Along the dimension of within-firm heterogeneity, its establishment in the theory essentially proceeds with the seminal contribution by Eckel and Neary (2010). With their model, in which firms of a non-zero measure compete in a (Cournot) oligopoly and a negative demand linkage (spillover) between a firm’s varieties (cannibalization) exists, they introduce the concept of flexible manufacturing (supply (cost) linkage (spillover) between a firm’s varieties) to the theoretical literature of multi-product firms in international trade, and thereby concentrate for the within-firm heterogeneity on the firms’ cost side: Each firm has a core competency variety with minimal unit production costs and the produc-

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3 Alternative, even if less often employed, approaches of modeling firm heterogeneity in international trade are given by Bernard et al. (2003) and Yeaple (2005).

4 For the empirical motivation of the development of a theoretical literature on firm heterogeneity in international trade and (i) a review of the empirical literature on firm heterogeneity in trade: See Bernard et al. (2012), or (ii) an exposition and review of the theoretical literature on firm heterogeneity in trade: See Redding (2011) and Melitz and Redding (2014).

5 For the normative implications of trade (gains from trade) in the presence of heterogeneous firms in a monopolistic competition and in particular the gains’ sources: (i) Both an exposition and a review: See Melitz and Trefler (2012), (ii) an exposition: See Feenstra (2016) and (iii) a review: See Feenstra (2018).

6 For the role of oligopoly in the theory of international trade (“two and a half theories of trade”: perfect competition, monopolistic competition and oligopoly), its empirical justification and possible ways of development: See Neary (2010).

7 For their utilization in (a meanwhile comprehensive body of) empirical studies on multi-product firms in international trade: See the Chapter 2.

8 In the industrial organization (IO) literature, multi-product firms are units of observation and analysis since the 1980s: See Brander and Eaton (1984), Klemperer (1992), Baldwin and Ottaviano (2001), Johnson and Myatt (2003), Allanson and Montagna (2005) and Ottaviano and Thisse (2011).

9 For other directions in the recent evolution of the international trade theory: (i) Endogenous firm productivity through firm decisions in different fields, in particular technology adoption (Costantini and Melitz (2008)), innovation (Sampson (2016)) and workforce composition (Helpman et al. (2010) and Helpman et al. (2017)), (ii) (international) organization of production respectively firms: (a) A review: See Antrás and Rossi-Hansberg (2009) and (b) both an exposition and review: See Antrás (2016).

10 For an overview of the models of multi-product firms in international trade, arranged along these three dimensions: See the Figure C.1 in the Appendix C.1.
tion of additional varieties being increasingly further away from the core is associated with increasingly higher unit production costs.\textsuperscript{11}

Several subsequent contributions adopt the concept of flexible manufacturing and thus also deal with products being asymmetric within firms, where the products of a firm are sold in different amounts and at different prices, but the contributions vary either on one or both of the other two dimensions: Eckel and Irlacher (2017) as well as Flach and Irlacher (2018) consider a multi-product firm in a monopoly, which can be easily extended to an oligopoly, whereas Qiu and Zhou (2013), Mayer et al. (2014) and Arkolakis et al. (2015) analyze multi-product firms in a monopolistic competition and in case of the absence of a within-firm cannibalization\textsuperscript{12} among themselves differing in the assumptions on the demand side, i.e. the preferences of consumers: While Arkolakis et al. (2015) apply constant-elasticity-of-substitution (CES) preferences with constant firm markups induced, Qiu and Zhou (2013) and Mayer et al. (2014) assume quadratic preferences, leading to linear demand.\textsuperscript{13} With Bernard et al. (2011) however, the source for within-firm asymmetries across products varies, while maintaining the monopolistic competition and the absence of a within-firm cannibalization: They focus on the demand side and incorporate random product attributes that determine together with the firm ability (productivity) the profitability and survival in the economy.

Feenstra and Ma (2008), Baldwin and Gu (2009), Dhingra (2013) and Nocke and Yeaple (2014) follow the contrary approach of modeling symmetric products within multi-product firms, where the products of a firm are sold in the same amount and at the same price. While Baldwin and Gu (2009) take into account a cannibalization within firms that are of a non-zero measure, have some market power and compete in an oligopoly, the other three contributions assume a monopolistic competition for their models’ market structure. With free entry of firms and no strategic interaction between firms of a limited measure, Nocke and Yeaple (2014) do not consider a within-firm cannibalization, whereas Feenstra and Ma (2008) as well as Dhingra (2013) incorporate it into their models: The former contribution relaxes the constant aggregate price index assumption and thus enables the existence of

\textsuperscript{11}In a way, two extensions to Eckel and Neary (2010)’s baseline model exist: Besides the model’s configuration, in which the price and sales profiles of firms across their products are negatively correlated, interpreted as the cost-based competency of firms, Eckel et al. (2015) develop a second configuration, in which firms invest in their brand and products quality, leading to a positive correlation between the price and sales profiles of firms across their products and interpreted as the quality-based competence of firms. Eckel et al. (2016) however incorporate the investment of firms in their export market penetration and are thereby able to reconcile the model with the empirical evidence that most firms sell more of all their products at home than they export and only the largest firms exhibit a dominance of exports over home sales.

\textsuperscript{12}In their baseline model, Arkolakis et al. (2015) do not consider a within-firm cannibalization. Nonetheless, they incorporate in a generalization of their model a within-firm cannibalization by employing nested constant-elasticity-of-substitution (CES) preferences, with different within-nest substitution elasticities between the nests. In particular, in the case that the products of a firm in the inner nest are closer substitutes to each other than the product lines are substitutable across the firms in the outer nest, a within-firm cannibalization exists.

\textsuperscript{13}Qiu and Zhou (2013) additionally extend their model to CES preferences and stress the robustness of their results with respect to this alternative demand configuration.
negative demand linkages between a firm’s products, while with the identification of firms as brands with multiple products within each, the latter contribution allows for a within-brand cannibalization by introducing a parameter for the differentiation across brands into the consumer’s utility function, which leads to negative demand linkages between products of the same brand.

**Product Scope Responses to Changes in International Trade**

In general, multi-product firms do not only decide about the production (output) and sales amount of a product (*scale*), as single-product firms do, but they also have to make a choice on their supplied product portfolio, i.e. the number of products produced and sold (*scope*) as well as its composition (*space*). Thereby, the multi-product firms’ choices on product scope are constrained on the demand side through (within-firm) cannibalization and/or on the supply side through (within-firm) dis-economies of scope. A firm does not expand its scope any further if the additional profits of the last added product do not exceed (or are equal to) the additional costs of the last added product. Within-firm cannibalization occurs either as a result of the firms’ optimization (Feenstra and Ma (2008), Baldwin and Gu (2009), Eckel and Neary (2010), Eckel et al. (2015, 2016), Eckel and Irlacher (2017) and Flach and Irlacher (2018)) or the modification of the demand function in form of the introduction of a product substitution parameter (Dhingra (2013)).

The product scope choice of multi-product firms in international trade evidently depends not only on the firms’ characteristics, but also on a number of factors of the trade setting in which the firms operate and hence a new and additional margin of adjustment to changes in the setting of international trade (changes in trade costs: (de-)liberalization and integration of formerly segregated economies: globalization\(^{14}\)) is established in the international trade theory: The number of products that a firm produces and/or exports (product scope) can be changed by it to re-optimize the firm’s objective variable, in particular profits. As the firm type’s unique feature, the study of the extensive margin and its adjustment (pattern) is central to all models in theory\(^{15}\).

The predictions of the models of multi-product firms in international trade on the firms’ scope adjustments can be categorized into two groups, according to qualitative differences throughout the firm distribution\(^{16}\): (i) Homogeneous response and (ii) heterogeneous response. For the first group [(i)], Feenstra and Ma (2008), Eckel et al. (2016), Eckel and Irlacher (2017) and Flach and Irlacher (2018) find that all firms increase their product scope in response to liberalization or globalization, while Baldwin and Gu (2009), Eckel and Neary (2010), Bernard et al. (2011), Mayer et al. (2014) and Arkolakis et al. (2015) detect that all firms reduce their product scope in response to liberalization or globalization.

\(^{14}\)The modeling concept of globalization goes back to Krugman (1979).

\(^{15}\)For the study of the extensive margin and its adjustment (pattern) in the empirical contributions on multi-product firms: See the Chapter 2.

\(^{16}\)For an overview of the models’ predictions on the multi-product firms’ scope adjustment to changes in international trade: See the Table C.1 and Table C.2 in the Appendix C.2.
In detail, while Eckel and Irlacher (2017) identify an increase in scope at the expense of core products as each firm’s response to globalization, Eckel and Neary (2010) derive that each firm’s decrease in scope takes place to the benefit of its core products, which implies that multi-product firms get “leaner and meaner” in response to globalization. Furthermore, Eckel et al. (2015) find that globalization has an ambiguous effect on the scope of all firms and trade de-liberalization leads all multi-product firms to decrease their scope.

For the second group [(ii)], the contributions find that the firms that are ex-ante superior positioned, in particular exporters and/or more productive firms (exporters), are more able to make use of and to cope with the changed environment and that these firms expand their scope, whereas the other firms reduce it: Dhingra (2013) detects that non-exporters and less productive exporters reduce their scope, while more productive exporters increase their scope in response to globalization. Additionally, Qiu and Zhou (2013) predict a decrease in scope for less productive firms, while more productive firms may increase their scope in response to globalization. In the contribution by Nocke and Yeaple (2014), it is found that initially domestic firms decrease their scope and continuing exporters increase their scope in response to trade liberalization.

The relevance of the step towards multi-product firms in the historical evolution of the international trade theory gets illustrated by Breinlich and Cuñat (2016): They evaluate different versions and extensions of Melitz (2003)’s heterogeneous firms model by checking their ability to quantitatively replicate the changes in trade flows and productivity in Canada following the trade liberalization by the Canada-US Free Trade Agreement (CUSFTA) of 1989. While the baseline model is inherently incapable of matching both trade and productivity increases, the only extension that substantially improves the model’s performance in providing a good fit to the data and passing the over-identification tests is one in which multi-product firms are considered. Thus, multi-product firms, their firm type’s unique feature, in particular product scope, and their (more extensive set of) margins of adjustment need to be included in the international trade models to capture the quantitative features of trade liberalization.
Appendix C

C.1 Models of Multi-product Firms in International Trade

Figure C.1: Models of Multi-product Firms in International Trade: Representation along Three Dimensions: Market Structure or Across-firms Competition (Monopolistic Competition versus Oligopoly), Within-firm Heterogeneity (Symmetric versus Asymmetric Products) and Within-firm Cannibalization (Existence versus Absence) [*: Within-firm cannibalization absent in the baseline model, but existent in a generalization of the model in the online supplement, **: Monopoly in the baseline model, but an extension to the oligopoly either done in the appendix or possible with the respective reference]

C.2 Product Scope Responses to Changes in International Trade
<table>
<thead>
<tr>
<th>Model</th>
<th>Comp. Heterog. Cannibal. Change in Trade Scope Adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feenstra and Ma (2008)</td>
<td>MC globalization† homogeneous response: increase in scope</td>
</tr>
<tr>
<td>Baldwin and Gu (2009)</td>
<td>O trade liberalization‡ homogeneous response: decrease in scope</td>
</tr>
<tr>
<td>Eckel and Neary (2010)</td>
<td>O O trade liberalization‡ homogeneous response: decrease in scope</td>
</tr>
<tr>
<td>Dhingra (2013)</td>
<td>MC globalization† heterogeneous response: decrease in scope for non-exporters, increase in scope for more productive exporters</td>
</tr>
<tr>
<td>Qiu and Zhou (2013)</td>
<td>MC globalization† heterogeneous response: decrease in scope for less productive firms, increase in scope for more productive firms</td>
</tr>
<tr>
<td>Mayer, Melitz and Ottaviano (2014)</td>
<td>MC globalization△ homogeneous response: decrease in scope, skewness in exported product mix</td>
</tr>
</tbody>
</table>

Notes: M: monopoly, MC: monopolistic competition, O: oligopoly; †: existent, ‡: absent; ††: increase in the number of countries participating in the global economy, †††: reduction in the offshoring costs, △: increase in the toughness of competition (market size and geography); and ⋆: qualitative differences throughout the firm distribution.
<table>
<thead>
<tr>
<th>Model</th>
<th>Comp.</th>
<th>Heterog.</th>
<th>Cannibal.</th>
<th>Change in Trade</th>
<th>Scope Adjustment*</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nocke and Yeaple (2014)</td>
<td>MC</td>
<td>✗</td>
<td>✗</td>
<td>trade liberalization†</td>
<td>heterogeneous response: decrease in scope for initially domestic firms, increase in scope for continuing exporters</td>
<td>organizational capital</td>
</tr>
<tr>
<td>Arkolakis, Ganapati and Muendler (2015)</td>
<td>MC</td>
<td>✓</td>
<td>✗</td>
<td>globalization†</td>
<td>homogeneous response: decrease in scope</td>
<td>market-specific product entry cost</td>
</tr>
<tr>
<td>Eckel, Iacovone, Javorcik and Neary (2015)</td>
<td>M</td>
<td>✓</td>
<td>✓</td>
<td>globalization† and trade de-liberalization††</td>
<td>ambiguous effect on scope and decrease in scope</td>
<td>cost- and quality-based competency</td>
</tr>
<tr>
<td>Eckel, Iacovone, Javorcik and Neary (2016)</td>
<td>any</td>
<td>✓</td>
<td>✓</td>
<td>trade liberalization†</td>
<td>homogeneous response: increase in scope</td>
<td>market-size puzzle</td>
</tr>
<tr>
<td>Eckel and Irlacher (2017)</td>
<td>M</td>
<td>✓</td>
<td>✓</td>
<td>globalization††</td>
<td>homogeneous response: increase in scope at the expense of core products</td>
<td>relocation of product lines (multi-product offshoring)</td>
</tr>
<tr>
<td>Flach and Irlacher (2018)</td>
<td>M</td>
<td>✓</td>
<td>✓</td>
<td>globalization†</td>
<td>homogeneous response: increase in scope</td>
<td>product and process innovation</td>
</tr>
</tbody>
</table>

Notes: M: monopoly, MC: monopolistic competition, O: oligopoly; ✓: existent, ✗: absent; †: increase in the (market) size of an (autarkic) economy, ††: increase in the number of countries participating in the global economy, †††: reduction in the offshoring costs, †: reduction in the transportation costs (a: iceberg trade costs, b: uniform trade costs), ††: increase in the (uniform) transportation costs, △: increase in the toughness of competition (market size and geography); and *: qualitative differences throughout the firm distribution.

Table C.2: Theoretical Studies on Multi-product Firms in International Trade - Product Scope Responses to Trade Changes
Chapter 4

Multi-product Firms with Flexible Manufacturing and Organizational Capital in International Trade

4.1 Introduction

Multi-product firms quantitatively dominate manufacturing production both within national borders and in international trade. A rich body of empirical studies documents this fact for several developed and developing countries and varying definitions of products. Bernard et al. (2007) and Bernard et al. (2010) show for the U.S. that multi-product firms account for 87% of the national output and are responsible for almost all (99%) of the export value. For Chile, Navarro (2012) detects some of the lowest figures across the studies, but still reveals a significant importance of multi-product firms for domestic production and activity across borders: 56% of the national output is produced by multi-product firms and those account for 58% of the export value.

A similar robust fact that is documented in the empirical studies is the size and productivity premium of multi-product firms. They are on average larger in terms of output and employment and more productive than their single-product counterparts (Bernard et al. (2010) for the U.S., Goldberg et al. (2010b) for India, Navarro (2012) for Chile, Bernard et al. (2014) for Japan). Goldberg et al. (2010b) and Navarro (2012) report that Indian and Chilean multi-product firms respectively produce on average an about 25% higher output, while Navarro (2012) finds Chilean multi-product firms to be on average 3% more productive than single-product firms.

Given these facts of superiority of multi-product firms in several dimensions of economic activity⁴ the question of how these firms are able to realize them arises and requires a

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⁴I would like to thank my supervisors, Hale Utar and Gerald Willmann, as well as Wen Cheng, Carsten Eckel, Giuseppe Vittucci Marzetti, Claudia Tronconi and the participants of the 20th Workshop “Internationale Wirtschaftsbeziehungen” in Göttingen for helpful comments, suggestions and support.

⁵In a recent work, Eckel and Irlacher (2017) offer an explanation approach to multi-product firms’
detailed analysis of those firms and especially of their internal organization. In economics and management, the firm as the central vehicle of economic transaction is naturally at the core of some theoretical approaches, trying to obtain a deeper understanding of its nature, existence, boundaries and organization. An attempt regarding the nature of the firm, developed to explain the varying degree of competitive advantage of firms, goes back to early contributions by Coase (1937) and Penrose (1959) and finds its fundamental conceptualization in the studies by Rumelt (1984) and Wernerfelt (1984). It focuses on the characteristic endowment of firms with some resources and therefore finds its way in the literature as the resource-based theory of the firm.

Starting with the assumption of a resource endowment of firms, thereby picking up the traditional resource-based theory of firms and combining it with the most relevant form of firms in international trade, this chapter tries to shed light on the internal organization of resource-endowed and -constrained multi-product firms and their adjustment patterns to trade liberalization. Thereby, it builds on the work by Nocke and Yeaple (2014) and as they do, the resource of firms is identified by the concept of organizational capital. Its allocation to a firm’s product reduces the product’s cost of production and any increase of scope reduces the amount of organizational capital allocated to each product, thereby increasing its production costs (dis-economies of scope). In the model by Nocke and Yeaple (2014), products within firms are symmetric in output and price, i.e. they contribute the same amount to the firm’s output and they are supplied all with one price. The main contribution of the chapter is now to reconcile dis-economies of scope, caused by the limited endowment of firms with organizational capital and in turn building on the traditional resource-based theory of firms, with the robust empirical finding of within-firm asymmetry. In particular, Adalet (2009), Aw and Lee (2009), Bernard et al. (2010), Goldberg et al. (2010b), Liu (2010), Bernard et al. (2011), Görg et al. (2012), Navarro (2012), Söderbom and Weng (2012), Amador and Opromolla (2013), Chatterjee et al. (2013), Masso and Vahter (2014), Arnarson (2016), Boehm et al. (2016), Lo Turco and Maggioni (2016), Lopresti (2016), Caselli et al. (2017), Dhyne et al. (2017) and Lopresti and Shiferaw (2017) document for multi-product firms located in several different countries substantial product differences within these firms.

The remainder of the chapter is organized as follows: The next section describes the resource-based theory of the firm and the concept of organizational capital, as well as draws the existing connections to multi-product firms. In the following section, a model of multi-product firms with organizational capital and flexible manufacturing is developed, starting with a closed economy and then opening it up to trade. The subsequent section provides superiority by introducing labour market imperfections/frictions into a model of multi-product firms.

For an overview about and a critical appraisal of (the development of) the resource-based theory of the firm (in management): See Lockett et al. (2009).

In the literature, there is no uniform term for the employed concept. Organizational capital, organization capital and organizational resource are largely taken as synonyms and therefore interchangeably used. However, in this chapter, preference is given to the term organizational capital.
empirical evidence in support of (i) the general relevance of the organizational capital in the firms’ production process and (ii) some of the main predictions of the developed theory. The last section summarizes the main results.

4.2 Resource-endowed Multi-product Firms

4.2.1 Resource-based Theory of the Firm

In each economy, firms represent the central vehicle of transaction of factors and goods. Naturally, they are at the core of some theoretical approaches in economics and management, which try to gain a deeper understanding of the firms’ existence, nature, boundaries and organization. One of those approaches, that is fundamentally conceptualized in 1984, analyses the nature and thereby aims to develop an explanation for the varying degree of competitive advantage of firms. The resource-based theory characterizes a firm as a historically determined (path-dependent) collection or bundle of assets or resources, which are tied semi-permanently to the firm and its management, and states that the heterogeneous distribution of resource endowments among firms translates into firm performance differences. Subsequent contributions in the literature, in particular Amit and Schoemaker (1993) and Makadok (2001), divide the encompassing term of resources into resources and capabilities: While resources describe stocks of (externally) available and transferable factors that are owned or controlled by the firm, for example financial or physical assets (e.g. property, plant), know-how (e.g. patents, licenses) and human capital, the firm’s capacities to deploy those resources are named capabilities. By being largely responsible for the sustainable competitive advantage of a single firm, these capabilities fulfill some unique characteristics, beyond which and given that they feature a high degree of idiosyncracy in the way that they cannot be bought “off the shelf”: They are valuable in the sense that they enable the firm’s outperformance or the reduction of the firm’s weaknesses, in short, its success in competition (Barney (1991)), they are rare as they are limited in supply (Barney (1991)), they are inimitable as their replication is difficult for competitors (Barney (1991), Peteraf (1993)) and they are non-substitutable in the way that a simple replacement by the firm is not possible (Barney (1991), Peteraf (1993)). According to Barney (1991), these four characteristics form the so-called VRIN characteristics. In addition, the capabilities are also appropriate in the sense that firms can take control of the generated rents (Grant (1991)), durable as

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4 Early contributions to the approach are provided by Coase (1937) and Penrose (1959). For the advancement of the approach, given its essential features as well as the alternative approaches and their shortcomings: See Montresor (2004). The alternative approaches are, for example, the transaction-cost theory, pioneered by Coase (1937) and decisively developed by Williamson (1971, 1975, 1985), and the property-rights theory, pioneered by Grossman and Hart (1986).

5 Large parts of the development of the resource-based theory take place in (the literature of) management. For the impediments of its application to economics and future potentials: See Lockett and Thompson (2001).
they can repeatedly employed in the firm (Grant (1991), Prahalad and Hamel (1990)) and non-transferable as their purchase and sale at the market is not possible (Grant (1991), Peteraf (1993)). According to Amit and Schoemaker (1993), the capabilities with the above characteristics do not only enable firms to make use of their resources, but provide the efficient employment of factors within the firms. In the same way, Makadok (2001) describes capabilities as some firm-specific and implicit productivity-enhancing means of firms.

Another strand of literature examines the relative importance of firm-specific and industry effects for long-run, outstanding firm performance and thereby makes possible to empirically verify the explanatory power and relevance of the resource-based theory of the firm, which attributes a firm’s sustainable comparative advantage to firm-specific capabilities. Setting the standards for the analysis in the subsequent literature, McGahan and Porter (1999) employ some data on a sample of U.S. firms across industries to study the persistence of incremental industry, corporate-parent and business-specific effects on profitability. They find that the incremental industry effects persist longer than the other two effects, indicating that changes in the industry structure exert a more persistent impact on firm profitability than changes in the firm structure. By differentiating across firms, the business-specific effects however are found to erode faster for low performers than high performers.

With a sample of firms in different sectors of the U.S. food economy, Schumacher and Boland (2005) replicate the analysis on profitability persistence of industry, corporate-parent and business-specific effects and derive the result that profits are more persistent within an industry than within a specific corporation. Lin et al. (2014) apply the analysis on a developing country by making use of data on firms in China: They show that the incremental effects of industry on profitability persist longer than the incremental effects of the firm. The competitive advantage of firms seems to be more predictable and sustainable

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6See Amit and Schoemaker (1993): “They [capabilities] can abstractly be thought of as “intermediate goods” generated by the firm to provide enhanced productivity of its resources, as well as strategic flexibility and protection for its final product [...]” (p. 35).

7See Makadok (2001): “[... a] special type of resource - specifically, an organizationally embedded non-transferable firm-specific resource whose purpose is to improve the productivity of the other resources possessed by the firm.” (p. 389).

8In a dynamic and Schumpeterian perspective of firms, Teece et al. (1997) and Teece (2009, 2014, 2016) identify capabilities that ensure the long-run and sustainable corporate success, called dynamic capabilities, and distinguish them from ordinary ones: While, in the tradition of the literature, the latter’s purpose is to provide the technical efficiency of production, ensuring the firm’s technical fitness, by concentrating on the appropriate implementation of chosen opportunities (“doing things right”), dynamic capabilities represent the firm’s ability to integrate, build and reconfigure internal and external competences to address rapidly changing environments, thereby aiming to create a congruence of the firm’s agenda with customer needs and business opportunities. Firms with these dynamic capabilities are characterised by evolutionary fitness, realized through the appropriate choice of given opportunities (“doing the right things”). Dealing with the same facts, Fujimoto (1998) distinguishes three levels of a firm’s capability: (1) Static capability, which affects the level of competitive performance, (2) improvement capability (first-order dynamic capability), which affects the pace of performance improvements and (3) evolutionary capability (second-order dynamic capability), which is related to the accumulation of the above capabilities themselves. The constructed trichotomy is then applied to the Japanese automobile industry.
based on industry factors than firm factors. Results in the literature however are not unambiguous in favor of a major role of industry effects in profit persistence, as several other studies draw the opposite conclusion and thereby support the resource-based view of the firm: For the largest manufacturing firms in Turkey, [Yurtoglu (2004)] detects in a dynamic analysis with persistent profitability differences across firms a moderately quick erosion of rents, except for the most highly profitable firms. Firm characteristics rather than industry effects account for the differences in permanent profits. [Bou and Satorra (2007)] apply structural equation models to data on Spanish firms. While they observe significant and permanent differences between profit rates both at industry and firm levels, the variation of abnormal returns at the firm level is greater than at the industry level. [Chen and Lin (2010)] look at the IT industry in Taiwan and find that the incremental effects of firms on profitability persist longer than the incremental effects of industry. In a recent study on Chinese machinery manufacturing firms, [Guan et al. (2015)] get the result that both industry and firm effects contribute to the profit persistence, but firm effects persist longer than industry effects.

4.2.2 Organizational Capital

Parts of the economics literature refer to a specific firm capability as organizational capital. In line with the resource-based theory of the firm, organizational capital generally represents an intangible input into the production process of each product produced and managed by a firm, whereas it cannot be completely codified and hence transferred to other firms, either upon the firm’s approval or against its approval through the imitation by competitors. More precisely, [Evenson and Westphal (1995)] characterize organizational capital as the firm’s knowledge of how to combine the inputs of the production process, in particular physical and human capital, into systems for producing and delivering want-satisfying products. By its nature, much of the knowledge is tacit, thus not feasibly physically embodied and neither codifiable nor readily transferable. In the same way, [Atkeson and Kehoe (2005)] describe organizational capital as the firms’ stored and accumulated knowledge that affects their technology of production. However, the within-firm components in which the capability materializes are identified differently in the literature, thereby establishing two strands of a perspective on firms and their organizational capital, whereas the second one becomes prevalent over time:

(a) With connections to the concepts or models by [Becker (1975) and Jovanovic (1979)], [Prescott and Visscher (1980)] stress in a dynamic study on firm characteristics the employees of firms as their central agents, about which the firm has some information with respect to their suitability for particular tasks and information on their ability to work as a team with particular fellow employees, which represent together with the firm-specific human

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[Evenson and Westphal (1995)] further distinguish technological capabilities into firms’ production capabilities, investment capabilities and invention capabilities.
capital vested in individual employees an asset to the firm, which is called organization capital. The information by itself is distinguished between some information about the abilities of the firm’s personnel and the match between the workers and tasks (personnel information), some information about the match between employees working in teams (team information) and some information about the skills of workers (human capital information). Consequently, the firm is described as a storehouse of such information, in which incentives for both the efficient accumulation and use of that information are generated, overall providing and ensuring the efficiency in the production of the firms.

(b) With connections to the concepts or models by Arrow (1962) and Rosen (1972), Tomer (1981) stresses the organization of firms in terms of their systems, procedures, structures, and interpersonal and inter-groups relationships, which have been developed in the firms, or more precisely, which have been jointly produced with the output (learning by doing), to ensure their effectiveness and efficiency in the production and which all together form an asset that is called organizational capital. In a subsequent study, Tomer (1987) classifies as before organizational capital as some type or subcategory of human capital, whereas the former is vested only in organizational relationships, while the latter is vested only in individuals, but now differentiates more between human and organizational capital by setting up a human-organizational capital spectrum, in which pure human capital and pure organizational capital form the extremes and some human-organizational capital hybrids lie in between them. According to Tomer (1987), Prescott and Visscher (1980)’s concept of organizational capital captures simply one of those hybrids.

As organizational capital is defined as an intangible production input, the theory of intangible assets of firms and its identification and classification approaches provide characterizations of organizational capital which mainly follow the dominating approach (b) and evidently have an overlap with the resource-based theory of firms: Intangible assets are in general characterized as those assets of the firm that lack a physical existence or embodiment, but contribute to the value of the firm or represent sources of future corporate benefits (Edvinsson and Malone (1997), Lev (2005), Webster and Jensen (2006)). According to Lev (2005), the unique features of intangible assets are their partial excludability in the sense that the property rights over intangible assets are not as tightly defined and secured as they are for physical and financial assets as well as their non-marketability in the sense that intangibles are by and large not traded in active and transparent markets. In particular, knowledge or information that exists in a firm to generate comparative advantage by satisfying customer needs is also referred to as an intangible asset (Fernández 2004).
Edvinsson and Malone (1997) divide intellectual capital into human capital, which is the combined knowledge and skills of the firm’s individual employees at work as well as the firm’s value and culture, and structural capital such as hardware and software, databases, organizational structure, trade and brand names and everything else of organizational capability that supports those employees’ productivity. In short, the latter is “everything left at the office when the employees go home” (p. 11). Structural capital in turn is further classified into organizational, process and innovation capital, whereas organizational capital contains the firm’s systems, tools, and operating philosophy that accelerates the flow of knowledge through the organization and out to the supply as well as distribution channels.

Several further approaches of a classification of the intangibles and thereby of a characterization of organizational capital exist as alternatives: Fernández et al. (2000) differentiate intangible assets into human capital, organizational capital, technological capital and relational capital. Organizational capital includes the firm’s norms and guidelines, databases, organizational routines and corporate culture as well as its strategic alliances. Kaplan and Norton (2004) classify intangible assets into human capital, information capital and organization capital, which is composed of the firm’s culture, leadership, alignment of its goals and incentives with the strategy and teamwork. Lev (2005) sorts intangible assets according to their categories of products and services, customer relations, human resources and organizational capital. The latter includes unique corporate organizational designs and business processes that allow companies to outperform competitors in generating revenues or by economizing on production costs. Webster and Jensen (2006) differentiate intangibles into human capital, organizational capital, marketing (or relational) capital and production capital, whereas organizational capital includes the organizational architecture and the systems for monitoring activity and communicating within the firm.

Focusing on organizational capital and thereby likewise following its dominating approach (b), Lev and Radhakrishnan (2005) address the systematic out-performance of single firms in almost all industries and sectors and identify its source precisely not in some monopoly power or competition-restricting regulation but in the firms’ organization which gets manifested by their systems, practices, processes and designs. All that together builds a collective capability as the firm’s major and unique factor of production, which is called organization capital and which is predominantly tacit, therefore not or not completely codifiable, transferable to and imitable by the competitors, and capable of providing the competitive advantage and so yielding superior returns for the firm. By contrast, almost all of the other production factors, in particular capital and labour, merely represent commodities or resources to which all the firms have equal access and which yield, at best, the cost of capital. Taken all together, organizational capital is described as some agglomeration of technologies, including business practices, processes, designs and systems, that together enable the firms to consistently and efficiently extract from a given level of

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12For the VRIN characteristics of the capabilities: See the Section 4.2.1
physical and human resources a higher value of products than other firms would find possible to attain. Typical examples of specific business processes and designs as components of organization capital are Wal-Mart’s supply chain, which features the direct transmission of barcodes of purchased products, registered at the checkout, to suppliers who are largely responsible for the inventory management, Cisco’s Internet-based product installation and maintenance system with a huge cost savings potential and Dell’s path-breaking built-to-order distribution system, allowing unique levels of customization.

Black and Lynch (2005) single out three firm practices as paradigmatic components of organizational capital that have been shown to be associated with a higher productivity for the firms and/or higher wages for the workers: workforce training, employee voice and workplace organization. While the decisions about worker education (level of human capital) are assumed to be individual-based and taken independently from the specific employment relationship, workforce training is the result of a decision of investment in skill upgrading that is jointly made by the worker and the firm at which he is employed (investment in human capital). The component that is called employee voice subsumes all organization structures contributing to and promoting the participation of (non-managerial) workers in the decision making associated with the design of the production process and greater autonomy and discretion in the structure of the employees’ work. Finally, workplace organization\^{13} includes as examples of applied practices restructuring activities that lead to adjustments in the occupational structure of the workplace, the number of workers per supervisor, the number of levels of management within the firm, the existence and diffusion of job rotation and job share arrangements. In addition, methods of monitoring the applied practices and connected comparisons to competitors (benchmarking) also belong to the firm practices. Brynjolfsson and Saunders (2010) merely take the last component mentioned and refer to these workplace practices in form of an asset as organizational capital. The practices include the allocation of decision rights, the design of incentive systems, cumulative investments in training and skill developments and even supplier and customer networks\^{14}.

Stressing the role and importance of capabilities in the competition between firms in a globalized world, Sutton (2012) follows as well the organization approach (b) and so locates organization capital in the organizational structure of the firm, which is the architecture of a firm’s organization, originating in some framework of rules, routines and tacit understandings that have been put in place or have evolved over time, combined with the assembling of a team of people who work effectively together.

Lev and Radhakrishnan (2005) exceed by far with their study a theoretical conceptual-

\(^{13}\)For the (firm-level) complementarity between the changes in workplace organization and the investments in information technology (IT): See Bresnahan et al. (2002). For the relation between computerization and organizational capital: See Brynjolfsson et al. (2002) and the explanations below.

\(^{14}\)For an overview about the literature on human resource management practices (employee training, hiring criteria, teamwork, job design and employee hierarchies), whose empirical part is generated through an “insider econometrics” approach: See Ichniowski and Shaw (2003).
ization and practical illustration of organization capital, as they also undertake therewith the first attempt of its estimation, quantification and valuation. Thanks to its nature as a capability, organizational capital is difficult to operationalize and document and so there exits no evident and widely accepted operational measure of it, neither at the input (investments or costs) nor output (e.g. designs and processes) level. While parts of the investment in organizational capital are not fully tracked and recorded by firms, especially by smaller firms, its output is given in form of an intermediate product without an observable market price. Consequently, firms’ existing accounting systems do not explicitly capture organizational capital and do not distinguish its contribution to the final output of the firm from the contribution of the other inputs in the production process, therefore failing to be adequately equipped to constitute a direct reference of any estimation, quantification and valuation task of organizational capital.

Based on a standard Cobb-Douglas production function, which puts a firm’s output (sales) into a unique relation to its inputs, in particular physical capital, labour as well as research and development (R&D) capital, and which exhibits constant returns to scale, organizational capital can be estimated in two different ways: First, the residual of the function can be taken as an estimate of organizational capital, which can be achieved either by extracting the systematic component of the error term or by inserting a firm-specific dummy variable. Such a procedure follows the pioneering attempt by Solow (1957) to estimate total factor productivity (TFP). The drawback of the residual estimate however is that it essentially represents a black box, reflecting all told random shocks and various omitted variables along with TFP and organizational capital, which can not be separately identified. Second and as an estimation alternative, the income statement item “selling, general, and administrative (SGA) expenses” can be used as a proxy for organizational capital, since SGA expenses capture the non-production related costs of the business operation that are interpreted as investments in the capability of the firm. While the selling expenses capture all the spending related to the process of putting produced products to customers (e.g. salaries of sales people, advertising costs, brand enhancement costs, costs of Internet-based supply and distribution channels), general and administrative expenses arise from the management of the business (e.g. salaries of executives, payments to systems and strategy consultants, employees training costs, IT outlays, insurance costs, office rents). Even so this income statement item indeed includes most of the expenditures that generate and enhance organizational capital, it goes in part beyond what can be viewed as an expenditure that is relevant for the organizational capital of the firm, e.g. compensation of executives.

In their estimation approach, Lev and Radhakrishnan (2005) consider two types of organizational capital: An economy-wide, common organizational capital, which is available to all firms, and a firm-specific organizational capital, which is developed and maintained by each firm. The volume of a firm’s SGA expenses is in turn determined by the level of its activity as a short-term scaling factor, measured by its output, and the committed
portion of expenditures as a long-term scaling factor, measured by the lagged value of its SGA expenses. To account for the endogeneity of the SGA expenses, they are modelled as a function of both elements. After taking logarithms of annual changes, the production function, which is enriched by the SGA expenses as a proxy for organizational capital, and the relation for the SGA expenses are estimated by using a two-stage least squares procedure cross-sectionally on the COMPUSTAT database with listed firms that operate in twelve major industry categories. The sample for the period 1978-2002 is restricted to firms with both annual sales and total assets greater than $5 million, which leads to a total of 90,237 firm-year observations.

The research and development (R&D) capital of a firm as a stock is computed by capitalizing and amortizing its annual R&D expenditures as a flow variable over five years, whereas an annual amortization rate of 20% is employed, and the sample is then divided into two groups of firms: Those with R&D expenditures, in short R&D firms, and those without R&D expenditures, in short non-R&D firms. At the mean, R&D firms are larger in terms of output, physical capital and employment than non-R&D firms, while at the median, the reverse applies, indicating the existence of some very large R&D firms in the sample. For both R&D and non-R&D firms, the logarithm of growth in the economy-wide, common organizational capital is 0.03, which represents approximately 3% of the average output growth. The marginal productivity of physical capital in case of R&D (non-R&D) firms is 0.08 (0.10), while the one of labour is 0.33 (0.15), the one of R&D is 0.09 and the one of organizational capital is 0.41 (0.58). Deducing from the statistical (non-)significance of the difference between the marginal productivities in the two groups, R&D turns out not to essentially influence the efficiency of physical capital, while it enhances the efficiency of employees in form of some process R&D and non-R&D firms have some competitive edge through organizational processes and designs to compensate for the absence of R&D. Overall, organizational capital is a major input in the production process of firms.

Lev and Radhakrishnan (2005) furthermore provide firm-specific estimates of the annual contribution of organizational capital to output (sales) growth. Specifically, given two defined expectations of a firm’s output, in particular the expected output of a given firm in a given year with and without the common and firm-specific organizational capital, the firm-specific measure of organizational capital is obtained by taking the difference between these two sales expectations. The mean of organizational capital amounts to $96 million, which represents about 4% of average sales, while the mean annual change in sales is $90 million, implying that the estimated average contribution of organizational capital to sales growth is almost 100% and illustrating the relevance of organizational capital in generating

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15The industry categories are the following: Consumer non-durables: food, tobacco, textiles, apparel, leather, toys; consumer durables: cars, TVs, furniture, household appliances; manufacturing: machinery, trucks, planes, office furniture, paper, commercial printing; energy, oil, gas, and coal extraction and products; chemicals and allied products; business equipment: computers, software, and electronic equipment; telecom, telephone, and television transmission; utilities; wholesale, retail, and some services (laundries, repair shops); health care, medical equipment, and drugs; money, and finance; other: mines, construction, building materials, transportation, hotels, business services, and entertainment.
output growth.

Bresnahan (2005) however is critical of Lev and Radhakrishnan (2005)'s approach by questioning the appropriateness of the measure of organization capital and the estimation methodology. Given the chosen measure, firm-level inefficiencies, which lead to higher SGA expenses, would imply higher levels of organization capital for the firm, which is not intended by the measure’s choice. In addition, the measure includes marketing as well as managerial expenditures, but does not capture potentially relevant aspects that are not directly correlated with the observable expenditures, e.g. managerial talent, and it does not consider the organization of firms (hierarchy, centralization, dynamic incentive contracts, divisional structure, workplace organization in productive units, corporate culture, role of management).

Furthermore, the estimation of the organizational capital basically operates with production function residuals, even in the second proposed way, with all their drawbacks in the identification: In the second proposed way, a productivity residual is projected onto SGA expenses and the interaction of SGA expenses with other inputs and then called organizational capital. Consequently, organizational capital is measured either by a production function residual at the firm level or by the portion of a production function residual that is explained by the variation in SGA expenses. In this case, the difficulty both to make sure and convince that production function residuals at the firm level in fact measure organizational capital, and not any other feature of differences in firms or operating circumstances, turns out to be especially serious for the paper’s statement of the relevance of organizational capital. In addition to it, any estimation at the firm level is connected to some problems due to the heterogeneity of firms, making it difficult to distinguish between selection, the endogeneity in the inputs, and the true advantages of the firm.

Further estimation attempts exist that employ the production function framework as in Lev and Radhakrishnan (2005), but differ in the choice of the organizational capital measure and its application in the estimation: De and Dutta (2007) pick the item administrative expenses, which is a sub-class of the SGA expenses, out of the income statement of firms and compute the organization capital by capitalizing (a fraction of, i.e. a capitalization rate of (i) 20% and (ii) 10%) the expenses with the perpetual inventory method while assuming a depreciation rate of (i) 10% and (ii) 20%, respectively, yielding two stock measures. Based on the estimation of a more augmented Cobb-Douglas production function, which includes physical, brand, human and organization capital as factors of production, on a sample of 165 Indian IT firms over the period 1997-2005, they find that organization capital has the highest output elasticity among the inputs.

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16 For an overview about firm organization: See Milgrom and Roberts (1992).
17 In an early estimation attempt, Brynjolfsson et al. (2002) document a substantial output elasticity of organizational capital for firms that have invested in IT.
18 The perpetual inventory method is a widely used concept in the R&D literature to construct a stock of innovation (knowledge) capital: See e.g. Hall et al. (2010) for an overview about the literature on measuring R&D returns.
and Vittucci Marzetti (2011) replicate the task for a sample of 828 European firms over the period 2005-2006. Thereby, they follow the approach of choosing a capitalized income statement item as a proxy for organization capital, but different from De and Dutta (2007), they do not apply the administrative expenses but rather the more general SGA expenses, and capitalize them by using the perpetual inventory method with a capitalization rate of 20% as well as a depreciation rate of 10%. For a production function that includes four inputs, in particular physical capital, labor, R&D stock and organization capital, and specialized in two functional forms (Cobb-Douglas and translog\(^{19}\)), it is found that labor and organization capital have the highest output elasticities, once more confirming the significance of organization capital for production.

Lev and Radhakrishnan (2005) undertake an approach of valuation of organizational capital by employing an equity valuation model, which relates the firm’s stock price, i.e. the firm’s market value, to its existing assets, i.e. the firm’s book value, and its growth potential, which is represented by the firm’s future abnormal earnings that describe the earnings in excess of the required rate of return on the assets, i.e. the cost of capital. The difference or gap between the firm’s assessment in the market and its assessment on the balance sheet reflects the value of the firm’s intangible assets, given that these are included in the investors’ evaluation of the assets of the firm but excluded in its own evaluation because of limiting accounting rules. This forms a basic valuation principle for intangible assets\(^{20}\) and provided that organizational capital captures important elements of the firm’s future abnormal earnings potential, the value of the firm’s intangible assets measures to a large degree the value of organizational capital.

With a regression of the firm’s contribution of organizational capital to sales on the difference between the firm’s market value, that is computed as the stock price multiplied by the number of common shares outstanding and based on the Center for Research on Stock Prices (CRSP) database, and book value, that is based on the COMPUSTAT database, all elements scaled by sales, Lev and Radhakrishnan (2005) obtain an estimate for the degree by which the value of organizational capital captures the value of intangibles, based on a sample of 44,073 firm-year observations for the period 1978-2002. An adjusted \(R^2\) value of the regression of about 24% indicates that organizational capital by itself explains about a quarter of the cross-sectional variation in the difference between market and book values. Consequently, organizational capital captures a major component of the firm’s intangible assets and is indeed closely connected to the firm’s capacity to generate future abnormal earnings or growth, as stated by the theory. Furthermore, the contribution of organizational capital in a given year to the future abnormal earnings lasts for about three years, given a coefficient on organizational capital of 2.26 and a discount rate of 12%. By differentiating the sample into groups of firms with yearly high, medium and low market-

\(^{19}\)The transcendental logarithmic (translog) functional form of a production function goes back to Berndt and Christensen (1973) and finds several applications in the empirical literature (e.g. Kim (1992)).

\(^{20}\)For alternative measures based on the same principle (market-to-book ratio and Tobin’s q): See the

Footnote 19
to-book value\textsuperscript{21}, whereas firms with a high market-to-book value can be assumed to be relatively more intensive with intangibles or organizational capital and for those firms organizational capital ought to contribute more to future abnormal earnings, it turn out that the coefficient on organizational capital is 4.67 for the high market-to-book firms, which implies that the contribution of organizational capital to future abnormal earnings lasts for about five year, while the coefficient on organizational capital is 1.65 (0.35) for the medium (low) market-to-book firms, which implies that the contribution of organizational capital to future abnormal earnings lasts for about two years (less than a year).

Introducing financial analysts’ forecasts of earnings as a proxy for future earnings into the regression and conducting it both with and without organizational capital, \textsuperscript{22}\textsuperscript{21} \textsuperscript{22}Lev and Radhakrishnan (2005) detect that the addition of organizational capital to the valuation substantially improves the model’s explanatory power from 24% to 32% and that the coefficient on organizational capital is 2.02 and highly statistically significant, which indicates that financial analysts fail to capture with their evaluations a major contributor to future benefits. In a sample that is partitioned into groups of firms with different market-to-book-values, organizational capital improves the model’s explanatory power for the gap between the market and book values for all groups, while the increase is largest for the medium and high market-to-book firms: The adjusted $R^2$ raises for the medium (high) market-to-book firms from 22% (32%) to 52% (55%). Thus, financial analysts are not able to fully comprehend the value of organizational capital and their failure is especially pronounced for the high-growth firms.

Lev and Radhakrishnan (2005) do not only estimate the organizational capital in the production process of firms and value it in their market capitalization, but also consider the relation between organizational capital and information technology (IT). Thereby, it is possible for them to add once more to the literature that evaluates the contribution of organizational capital to the market valuation of firms, in this way valuing organizational capital itself: First, as several business practices and process, such as Internet-based supply and distribution channels as well as production design and control systems, depend heavily on the IT infrastructure of a firm, IT theoretically represents a major promoter of the generation and enhancement of organizational capital. Based on annual data on IT expenditures of firms from the Information Week reports that are published in its IT 500 survey (1991-1997), \textsuperscript{23}\textsuperscript{22} \textsuperscript{23}Lev and Radhakrishnan (2005) detect that a correlation between IT investments and organizational capital exists and IT investments explain to some degree organizational capital in a regression\textsuperscript{22} They investigate the Spearman rank correla-

\textsuperscript{21}The market-to-book ratio is defined as the ratio of the firm’s value in the capital market to its value on the balance sheet. It represents the gap between the investors’ assessment of the firm’s assets and their assessment in the firm’s accounting system, that exists mainly due to the omission of the intangible assets by the latter, therefore serving as a measure of the intangibility of the firm’s assets (cf. Tobin’s q as an alternative measure).

\textsuperscript{22}For the effects of the information and communication technology (ICT) on the firm organization: See e.g. Bloom et al. (2014). For the effects of the information technology (IT) on the firm organization and the firm performance: See Brynjolfsson and Hitt (2000) as a survey.
ations among organizational capital, IT investments, lagged IT investments and the market share. Organizational capital is found to be positively correlated with both the current IT spending as well as the lagged IT spending, which provides some simple evidence that IT spending is associated with building organizational capital. \cite{brynjolfsson2002s} as well compute the Spearman rank correlations and get the same result in the sense that firms that invest in IT are more likely to build up organizational capital. In addition to the correlation analysis, \cite{lev2005organizational} conduct a regression analysis with organizational capital as the dependent variable and specifications of IT as the independent variables. The coefficients on both the prior year’s IT expenditures and the current year’s change in IT expenditures are found to be positive and highly significant, which indicates that IT is a major contributor to organizational capital. By differentiating the sample into groups of firms with yearly high, medium and low market-to-book values, whereas firms with a high market-to-book value can be assumed to be relatively more intensive with intangibles or organizational capital, an asymmetric effect of IT investments across heterogeneous firms turns out to exist in the way that IT expenditures contribute much more strongly to organizational capital for the group with medium and high market-to-book ratios. \cite{brynjolfsson2002s} also perform a regression in which organizational capital is the independent variable and either computer assets, capital or other assets serve as the dependent variables. The coefficient on organizational capital is only positive and significant in the computer assets specification. Taken all together, not only IT investments seem to explain to some degree organizational capital, but the inverse relation seems also to be true.

Second, \cite{brynjolfsson2002s} as well as \cite{brynjolfsson2016computers} find that financial markets put a higher value on firms with more installed computer assets and obtain a large estimate of the contribution of IT to the market value of firms: $1 of IT investment raises the firm value by about $10. The apparent excess valuation of IT by the market may lead to the presumption that substantial intangible assets, adjustment costs or other components of the market value that are correlated with computer assets are present but omitted in the analysis. As organizational capital is included into the regression as it is done in \cite{brynjolfsson2002s}, it appears that firms with organizational capital have higher market valuations, so an increasing effect of the intangible asset on the firms’ valuation exists, and the coefficient on computer assets drops, which indicates that at least some part of the high observed value of IT is the result of computers serving as a proxy for organizational capital. Third, computer assets and organizational capital behave as complementarities for the market valuation of firms: As it gets revealed by employing an interaction term in the regression, firms with both installed computer assets and organizational capital have disproportionately higher market valuations than firms that invest only

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23 Organizational capital is formed by (a cluster of) human resource as well as workplace practices, which consists in turn of structural and individual decentralization, team incentives and skill acquisition, and so gets quantified by their measures.
in one of these dimensions and so much of the apparent excess value is located in firms that have high levels both of IT and of organizational capital. The potential benefits of computerization can be disproportionally realized by firms that simultaneously build up organizational capital and thus IT is most valuable when it is coupled with organizational capital.

### 4.2.3 Multi-product Firms

The resource-based theory of firms and in particular organizational capital are to a special degree exposed and in fact applied to firms that produce multiple products. Compared to single-product ones, those firms add to their internal space of action a second dimension: Besides *scale*, i.e. the decision of how much to produce of a supplied product, multi-product firms have to decide about *scope*, i.e. the number of supplied products. The additional dimension becomes part of a large literature, trying to obtain a deeper understanding of its existence, nature and organization (cf. the analogy to the theory of firms). In the light of the significant presence of diversified firms in the U.S. economy, Montgomery (1994) gives a summary of the theoretical approaches explaining corporate diversification. While the *market power view* takes up the position that firms diversify to wield conglomerate power across markets and thereby identifies the reason for diversification in the firm’s interaction with its competitive environment, the *agency view* and the *resource view* as alternatives focus on the firm’s inside. The agency view argues that managers pursuing their own interests at the expense of the firm’s owners push on with the diversification. With reference to Penrose (1959), the resource view explains diversification as a way to utilize excess capacity in productive factors. Montgomery (1994) finally reports that empirical research finds only little support for the market power view, but is to a large extent consistent with the within-firm explanation approaches, the agency view and the resource view.

Boehm et al. (2016) analyze the nature of corporate diversification by looking at the firms’ product space and its determinants. Based on the theory that firms can be thought of as a comprehensive bundle of productive capabilities and the approach of uncovering them by the firm’s applied inputs and produced outputs, which are intended to illustrate the performance-relevant realizations of the firm’s underlying capabilities, they empirically test for the influence of firm-level capabilities on the direction of product adoption. Thereby, horizontal and vertical firm-industry input-output (IO) measures indicate the linkages between the firm and potential industrial areas of diversification and therefore

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24 Based on the resource view, Chatterjee and Wernerfelt (1988, 1991) investigate the relation between the type of the underlying resources of a firm and its type of diversification.

25 Additional work on (similarity or “distance”) factors (input similarity, physical distance to existing locations of production, upstream-downstream connectedness) shaping the firm’s product mix is provided by Flagge and Chaurey (2014).

26 According to Teece (1982), the fact that the firm’s productive capability “lies upstream from the end product” (p. 45) allows firms to diversify into its varying applications and to become multi-product firms.
incentives to expand scope in a specific direction: Horizontally, economies of scope due to inputs can motivate a firm to expand to an industry with on average similar input shares and strategic complementarities in outputs represent an incentive for a firm to diversify to an industry with on average similar output shares. Vertically however, a firm can move to an industry that is upstream to its outputs or downstream to its inputs, both realizing synergies with the existing product mix. In the empirical analysis, [Boehm et al. 2016] use the measures of input and output similarity as well as expected upstream and downstream shares to predict the pattern of product adoption. They find that firms are more likely to move into products that have horizontal and vertical linkages to the existing product line, whereas idiosyncratic firm-level input linkages explain more product adoption, suggesting that input (upstream) linkages form a relevant factor of firm diversification and firm capabilities are more embodied in applied inputs.

The view of diversification as a firm’s strategy to leverage idiosyncratic resources often includes the proposition of the existence of (management) costs that arise when firms reallocate scarce internal resources from existing to added products. These internal resources are for a long time subject of the literature and take different terms in different settings, called span of control by [Lucas Jr. 1978], organizational ability by [Maksimovic and Phillips 2001], knowledge capital by [Klette and Kortum 2004] and organizational capital by [Santalo 2002] in the industrial organization literature and [Nocke and Yeaple 2014] in the international trade literature. [Roberts and McEvily 2005] moreover argue that numerous complementary resources are needed to make use of the firm’s intangible resources. In case that these resources are limited in supply and acquiring and assimilating additional resource capacity takes some time, any expansion of scope imposes costs on already existing products by cannibalizing firm resources (resource cannibalization). Along these lines, the costs or inefficiencies that emerge in the case of scope expansion due to the inextricably linked reallocation of resources across the products within the firms are taken in this chapter as dis-economies of scope. The empirical analysis of product introductions in the pharmaceutical industry conducted by [Roberts and McEvily 2005] supports the result of resource cannibalization and suggests that product market experience helps to offset some of the costs.

A large literature investigates the impact of diversification on the aggregate value of firms, trying to determine whether a premium or discount exists. [Lang and Stulz 1994] and [Berger and Ofek 1995] show that diversified firms exhibit a discount in relation to a portfolio of comparable non-diversified firms, suggesting that diversification annihilates to some extent firm value and more diversified firms are less efficient. More recent contributions to the literature however draw a far more nuanced picture by showing heterogeneity across firms with similar diversification as well as industry settings and economic environments (e.g. [Santalo and Becerra 2008], [Hovakimian 2011] and [Kuppuswamy and Villalonga 2016]).
4.3 Model

The following section analyses multi-product firms that are endowed with a (fixed) amount of organizational capital and use a flexible manufacturing technology in their production process. Firstly, a single autarkic economy with its demand, supply and resulting equilibrium is considered, and secondly, the economy is opened up to international trade. Due to methodological reasons and without any contradictions to the literature, the model thinks of organizational capital more as a resource than a capability in its physical embodiment.

4.3.1 Closed Economy

Preferences and Demand

The economy consists of \( L \) identical individuals (consumers, workers) with CES preferences and a per capita income \( y \). The (upper-tier) utility (function) is given by

\[
U = \sum_{k=0}^{1} U_k, \tag{4.1}
\]

where \( k \in \{0, \ldots, 1\} \) represents an industry and \( U_k \) denotes the utility index of industry \( k \), which takes the form

\[
U_k = \left[ \int_{\Omega_k} q_k(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \tag{4.2}
\]

where \( q_k(\omega) \) represents the individual consumption of the (horizontally differentiated) product variety \( \omega \in \Omega_k \) in industry \( k \) that is produced and supplied to consumers, \( \Omega_k \) is the set of product varieties in industry \( k \) that are produced and \( \sigma > 1 \) describes the elasticity of substitution across product varieties within an industry. As it is assumed that the elasticity of substitution within firms is the same as the elasticity of substitution across firms, a within-firm (demand) cannibalization effect is ex-ante excluded\(^{28}\).

To solve the consumption problem, each individual maximizes the utility subject to the budget constraint \( \gamma_k y = \int_{\Omega_k} p_k(\omega) q_k(\omega) d\omega \), with \( \gamma_k \) as industry \( k \)’s budget share (\( \sum_k \gamma_k = 1 \)) and \( p_k(\omega) \) as the price of the product variety \( \omega \in \Omega_k \), and thereby derives the individual product demand, which can be aggregated over the \( L \) individuals to get the aggregate demand for the product variety \( \omega \in \Omega_k \):

\[
x_k(\omega) = A_k p_k(\omega)^{-\sigma}, \tag{4.3}
\]

with

\[
A_k \equiv L \gamma_k y P_k^{\sigma-1}.
\]

as the residual demand of industry \( k \), whereas \( P_k \equiv \left[ \int_{\Omega_k} p_k(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \) thereby represents the CES price index of industry \( k \).\(^{29}\)

### Production and Supply

The market structure of the economy is characterized by a monopolistic competition between firms with a mass of them in each industry, and a sufficiently large mass of atomless and ex ante identical firms always try to enter the economy and to start their business. Each active firm produces a continuum of product varieties with labor as the standard factor of production, for which the price (wage) equals \( w \). The production process thereby reveals a flexible manufacturing technology, which goes conceptually back to Prahalad and Hamel (1990) and finds its application in models of multi-product firms in international trade: Eckel and Neary (2010), Qiu and Zhou (2013), Mayer et al. (2014), Arkolakis et al. (2015), Eckel et al. (2015, 2016), Eckel and Irlacher (2017), Flach and Irlacher (2018) and Herzig (2019a) (Chapter 5).\(^{30}\) Due to this technology, the unit production costs \( h(\omega) \) of variety \( \omega \) exhibit the properties that \( \frac{\partial h(\omega)}{\partial \omega} > 0 \) and \( h(0) = 1 \) hold, i.e. each firm has a core competency variety \( \omega = 0 \) with minimal unit costs and the production of any other variety requires higher unit costs, whereas varieties can be ordered regarding their firm-specific efficiency in production and therefore varieties further away from the core competency variety can only be produced with increasingly higher unit costs.\(^{31}\)

Pivotal to the model, each multi-product firm is endowed with a fixed amount of a resource or capability \( O \), which reduces the firm’s costs in the production process and is called organizational capital. It can be split up as well as reallocated across the products that are produced by the firm. The more organizational capital a firm assigns to one of its products, the lower the production costs of this product.

By assumption, each firm is only active in one industry \( k \). Upon entry, a firm pays a fixed cost \( f_E \) in domestic labor units and discovers the productivity \( \frac{1}{z} \) of its core competency variety, i.e. its specific ability of making use of labor, where \( z \) is drawn from a distribution \( B(z) \), with a probability distribution function \( b(z) \) and support on \([0, \bar{z}]\). Only a mass \( N_k \) of firms pays the fixed cost of entry and free entry drives the expected profits down to zero. Therefore, the per capita income \( y \) equals the wage rate \( w \).

Firms face and solve a two-stage production problem: In the first stage, the firm decides of whether to pay a fixed industry penetration cost \( F_p^k \) in domestic labor units and, in case of doing so, whereas only a subset of the firms pays the fixed cost, the firm discov-

\(^{29}\)See the Appendix D.1 for the details of derivation.

\(^{30}\)Besides the international trade literature, the industrial organization literature also deals with flexible manufacturing in the production process, see e.g. Eaton and Schmitz (1994).

\(^{31}\)In a more general definition of flexible manufacturing, Mingrom and Roberts (1990) link the technology to the firm’s ability to quickly adjust to market conditions. In the literature, it is also related to the firm’s ability of introducing more varieties, of reducing delivery times (Tseng (2004)) and of changing production scale with only minor adjustment costs (Gal-Or (2002)). Empirical evidence in support of the concept of flexible manufacturing is provided by García-Marín and Voigtlander (2017).
4.3. MODEL

ers its endowment with the organizational capital $O$, which is drawn from a distribution $G_z(O)$, with a probability distribution function $g_z(O)$ and support on $[0, O_{\text{max}}, z]$. In the second stage, given $z$ and $O$, the firm decides about (i) the number of products that it produces, which is called the firm’s product scope $\delta_k(z, O)$, (ii) the allocation of its organizational capital $o_k(\omega)$ across the products that are produced by the firm and (iii) the price $p_k(\omega, z, O)$ that the firm charges for each produced product variety $\omega \in [0, \delta_k(z, O)]$. Firms in the production process can be uniquely identified by their (random drawn) type $(z, O)$, consisting of the productivity of their core competency variety and their organizational capital endowment.

In the production process, the marginal production costs for the product variety $\omega$ in industry $k$ are given by

$$c_k(\omega, z) = awzn_k(\omega) = awzh_k(\omega)o_k(\omega)^{-\theta}, \quad (4.4)$$

with $a > 0$ as the (constant) cost parameter and $n_k(\omega) \equiv h_k(\omega)o_k(\omega)^{-\theta}$ as the composite cost component of the product variety $\omega$ in industry $k$. The unit production costs of product variety $\omega$ in industry $k$, $h_k(\omega)$, thereby reflect the flexible manufacturing technology and $o_k(\omega)$ describes the allocation of the organizational capital to product variety $\omega$ in industry $k$, while $\theta > 0$ simply denotes some (fixed) parameter.

The multiplicative specification of the composite cost component $n_k(\omega)$ of variety $\omega$ in industry $k$ implies that any increase in the unit production costs due to the production of some variety with less competency increases the marginal costs:

$$\frac{\partial c_k(\omega, z)}{\partial h_k(\omega)} = awz \frac{\partial n_k(\omega)}{\partial h_k(\omega)} = awzo_k(\omega)^{-\theta} > 0, \quad (4.5)$$

whereas the cost increase appears to be smaller, the higher the organizational capital allocated to the corresponding product variety. In this way, organizational capital such as established business processes and knowledge works as some corporate resource which is able to mitigate the increase in production costs due to lacks of competency and production (labor) efficiency. With an increase in the allocated organizational capital, the firm can reduce the production costs of the corresponding product variety $\omega$:

$$\frac{\partial c_k(\omega, z)}{\partial o_k(\omega)} = awz \frac{\partial n_k(\omega)}{\partial o_k(\omega)} = -\theta awzh_k(\omega)o_k(\omega)^{-\theta-1} < 0. \quad (4.6)$$

The overall change in the marginal production costs due to the production of some variety

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32. In the international trade literature, models that incorporate more than one dimension of firm heterogeneity already exist: See Cherkashin et al. (2015) and Aw and Lee (2017).

33. In the basic model, costs are specified in a multiplicative way, but model extensions in the Appendix D.2 and the Appendix D.3 show alternative cost specifications and their implications.

34. In an extension, one can enhance the composite cost component by introducing a parameter for the firm’s ability of making use of the organizational capital. For the details and limitations of this approach, see the Appendix D.4.
with less competency is given by
\[
\frac{\partial c_k(\omega, z)}{\partial \omega} = awz \frac{\partial n_k(\omega)}{\partial \omega} = awz \left( \frac{\partial h_k(\omega)}{\partial \omega} o_k(\omega)^{-\theta} - \theta h_k(\omega) o_k(\omega)^{-\theta - 1} \frac{\partial o_k(\omega)}{\partial \omega} \right)
\] (4.7)

and thus depends on (the change in) the flexible manufacturing technology and the within-firm allocation of organizational capital across the product varieties. The marginal costs are increasing (decreasing) in the product varieties further away from the core competency if and only if the semi-elasticity of the unit production costs is larger (smaller) than the adjusted semi-elasticity of the organizational capital allocation:
\[
\frac{\partial c_k(\omega, z)}{\partial \omega} \gtrless 0 \iff \frac{\partial h_k(\omega)}{\partial \omega} h_k(\omega)^{-1} \gtrless \theta \frac{\partial o_k(\omega)}{\partial \omega} o_k(\omega)^{-1}.
\] (4.8)

For a firm endowed with some amount of organizational capital \( O \) and under the assumption that the quantity of organizational capital cannot be changed in the short to medium period, i.e. any investment in the firm’s organizational capital is excluded, the behavior of the firm is restricted by its resource constraint:
\[
\int_{\omega \in \{0, ..., \delta_k\}} o_k(\omega) d\omega \leq O.
\] (4.9)

The firm’s production problem can then be solved backwards: Starting with the second stage and the firm’s decision about the price \( p_k(\omega, z, O) \) that is charged for the product variety \( \omega \) in industry \( k \), the firm maximizes its overall profits
\[
\Pi_k(z, O) = \int_{0}^{\delta_k(z, O)} \left[ (p_k(\omega, z, O) - c_k(\omega, z, O)) x_k(\omega, z, O) - f_{k, \omega} \right] d\omega,
\] (4.10)

with \( \delta_k(z, O) \) as the number of product varieties that are produced by the firm with organizational capital \( O \) and a cost draw \( z \) in industry \( k \) (product scope), \( x_k(\omega, z, O) \) as the (received) demand for the product variety \( \omega \) in industry \( k \) and \( f_{k, \omega} \) as the fixed costs for the introduction of a new product variety (production and distribution adjustment costs).

As it is well established for the CES demand structure and a monopolistic competition, the profit-maximizing price emerges as some constant mark-up over the marginal production

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\[^{35}\text{In the economics and management literature, capabilities are described and characterized as an idiosyncratic, non-transferable and organizationally embedded input to the firm’s transformation process (Makadok (2001)) and in particular organizational capital as an input that cannot be bought “off the shelf” (Sutton (2012)): in this sense, capabilities are taken as intangible and intrinsic to the firm as well as a relatively stable outcome of a within-firm evolution process. Owing to their nature, changes in these capabilities by the firm’s people in authority are difficult, demanding and if at all possible only in the long term (Following a somewhat broader definition of capabilities, in which parts of those are embodied in the firm’s personnel, thereby becoming at least in parts fungible and tradeable for firms: “ [...] managers constitute one part of what are known as the capabilities of a firm. While managers can come and go, other aspects of firm-level capabilities are rooted in well-established processes, values, and culture that are slower to change.” (Teece (2016), p. 204).} \]
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costs (standard constant-markup pricing rule)\[36\]

\[ p_k(\omega, z, O) = \left( \frac{\sigma}{\sigma - 1} \right) c_k(\omega, z, O), \]  

(4.11)

whereas the markup solely depends on the within-industry elasticity of substitution across product varieties. The higher the elasticity of substitution across varieties, the smaller the markup:

\[ \frac{\partial (\frac{\sigma}{\sigma - 1})}{\partial \sigma} = \frac{\sigma - 1 - \sigma}{(\sigma - 1)^2} = -\frac{1}{(\sigma - 1)^2} < 0, \]  

(4.12)

i.e. in industries with product varieties that are more substitutable with each other (more homogeneous industries), the firms’ wedge between the marginal production costs and the charged prices is smaller and, for a given \( \omega, z \) and \( O \), prices are lower in more homogeneous industries. For the decision about the within-firm allocation of organizational capital \( o_k(\omega) \), the firm solves the following maximization problem:

\[
\begin{align*}
\max_{o_k(\omega)} \Pi_k(z, O) &= \int_0^{\delta_k(z,O)} \left( (p_k(\omega, z, O) - c_k(\omega, z, O)) x_k(\omega, z, O) - f_{k,\omega} \right) d\omega \\
\text{s.t.} \int_0^{\delta_k(z,O)} o_k(\omega)d\omega &\leq O,
\end{align*}
\]  

(4.13)

With the pricing rule and the product demand applied to the maximization problem in equation (4.13), it can be rewritten in the form

\[
\begin{align*}
\max_{o_k(\omega)} \Pi_k(z, O) &= \int_0^{\delta_k(z,O)} \left( \zeta_k(z) h_k(\omega)^{1-\sigma} o_k(\omega)^{\theta(\sigma - 1) - f_{k,\omega}} \right) d\omega \\
\text{s.t.} \int_0^{\delta_k(z,O)} o_k(\omega)d\omega &\leq O,
\end{align*}
\]  

with \( \zeta_k(z) \equiv \frac{1}{\sigma - 1} A_k \left( \frac{\sigma - 1}{\sigma} \right)^2 (awz)^{1-\sigma} = \frac{A_k}{\sigma} \left( \frac{1}{\sigma - 1} \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} > 0 \) as the variety-invariant constant in the profit function that is both industry- and firm-specific.

Due to the (strict) monotone increase of the profit function in the allocated organizational capital\[37\], the firm completely exploits its organizational capital and the resource constraint is therefore fulfilled with equality:

\[
\int_0^{\delta_k(z,O)} o_k(\omega)d\omega = O \quad \forall \delta_k(z, O). 
\]  

(4.14)

The profit maximization over \( o_k(\omega) \) then yields that

\[ o_k(\omega) = \left( \frac{1}{\theta(\sigma - 1)\zeta_k(z)} h_k(\omega)^{\sigma - 1}\lambda_k \right)^{\frac{1}{\sigma(\sigma - 1) - 1}} 
\]  

(4.15)

units of organizational capital are allocated to the product variety \( \omega \), with \( \lambda_k > 0 \) as the Lagrange multiplier of the maximization problem and hence representing the marginal

\[36\] See the Appendix D.5 for the details of derivation.

\[37\] See the Appendix D.6 for the proof.
profitability of the organizational capital. Given that a firm’s product scope is constant, the within-firm allocation of organizational capital across products depends quantitatively on firm characteristics ($z, O$), (the change in) the flexible manufacturing technology ($h(\omega)$, $\frac{\partial h(\omega)}{\partial \omega}$), where the latter is assumed to be always positive and therefore determines the production efficiency advantage of the firm’s core competency, industry characteristics ($\sigma, A_k, w$) and cost parameters ($a, \theta$): 

$$\frac{\partial o_k(\omega)}{\partial \omega} \bigg|_{\delta_k=\text{const.}} = \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \left( \frac{1}{\theta(\sigma - 1)\zeta_k(z)} - 1 \right) h_k(\omega)\lambda_k \frac{\sigma - 1}{\theta(\sigma - 1)^{\frac{1}{\sigma - 1}}} h_k(\omega) - 1 \frac{\partial h_k(\omega)}{\partial \omega},$$  

(4.16)

and qualitatively on the cost parameter ($\theta$) as well as the within-industry elasticity of substitution across product varieties ($\sigma$): In industries with a low elasticity of substitution across product varieties, i.e. $\sigma < 1 + \frac{1}{\theta}$ ((more) heterogeneous industries), multi-product firms allocate decreasingly less organizational capital to varieties further away from their core competency variety and thus concentrate in their organizational capital allocation on varieties close to their core competency variety:

$$\frac{\partial o_k(\omega)}{\partial \omega} \bigg|_{\delta_k=\text{const.}} < 0 \iff 1 < \sigma < 1 + \frac{1}{\theta},$$  

(4.17)

while in industries with more substitutable product varieties, i.e. $\sigma > 1 + \frac{1}{\theta}$ ((more) homogeneous industries), multi-product firms face a stronger competitive pressure from the demand, allocate increasingly more organizational capital to varieties further away from their core competency variety (fringe products) and therefore focus in their organizational capital allocation on varieties in which they have less competency or (labor) efficiency:

$$\frac{\partial o_k(\omega)}{\partial \omega} \bigg|_{\delta_k=\text{const.}} > 0 \iff \sigma > 1 + \frac{1}{\theta}.$$  

Therefore, organizational capital as a firm resource serves as some cost-reducing mean, whose deployment across the firm’s product mix depends on the industry-level demand conditions.

Analyzing the marginal production costs and taking into account the (endogenous) firm decision on organizational capital, the composite cost component $n_k(\omega)$ can be written in dependence on the unit production costs as:

$$n_k(\omega) = h_k(\omega)\omega_k(\omega)^{-\theta} = \psi_k(z)h_k(\omega)^{-\frac{1}{\sigma(\sigma - 1)^{\frac{1}{\sigma - 1}}}},$$  

(4.18)

with $\psi_k(z) \equiv \left( \frac{\theta(\sigma - 1)\zeta_k(z)}{\lambda_k} \right)^{\frac{\sigma}{\sigma - 1}} > 0$ as an industry-firm-specific constant of the composite cost component, thereby overall providing the firm’s (marginal) cost profile $c_k(\omega, z)$, which

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$^{38}$See the Appendix D.7 for the details of derivation.

$^{39}$See the Appendix D.7 for the details of derivation.

$^{40}$See the Appendix D.8 for the details of derivation.
4.3. MODEL

qualitatively equals the firm’s price profile $p_k(\omega, z, O)$, due to equation (4.11) and which is thus crucially characterized or determined by\footnote{See the Appendix D.8 for the details of derivation.}

$$\left. \frac{\partial n_k(\omega)}{\partial \omega} \right|_{\delta_k = \text{const.}} = -\frac{\psi_k(z)}{\theta(\sigma - 1) - 1} h_k(\omega)^{-\frac{\sigma(\sigma-1)}{\sigma}} \theta \left( \frac{\sigma - 1}{\sigma} - 1 \right) \frac{\partial h_k(\omega)}{\partial \omega}.$$ \hfill (4.19)

In industries with a low elasticity of substitution across product varieties (heterogeneous industries), multi-product firms concentrate on their competency and reveal an increasing cost profile across their product mix:

$$\left. \frac{\partial n_k(\omega)}{\partial \omega} \right|_{\delta_k = \text{const.}} > 0 \iff 1 < \sigma < 1 + \frac{1}{\theta}, \hfill (4.20)$$

while in industries with more substitutable product varieties (homogeneous industries), multi-product firms take their deficiency in production at the fringe of their product mix into account, allocate more organizational capital to those products and thereby over-compensate the increasing cost structure of the flexible manufacturing technology, finally resulting in a decreasing cost profile across their product mix:

$$\left. \frac{\partial n_k(\omega)}{\partial \omega} \right|_{\delta_k = \text{const.}} < 0 \iff \sigma > 1 + \frac{1}{\theta}. \hfill (4.21)$$

Multi-product firms with a higher productivity $\frac{1}{z}$ have lower marginal production costs $c_k(\omega, z)$ for some variety $\omega$, provided that the composite cost component $n_k(\omega)$ for the variety is kept constant: $\left. \frac{dc_k(\omega, z)}{dz} \right|_{n_k(\omega) = \text{const.}} = awn_k(\omega) > 0$. Taking into account that a change in the firm’s productivity also implies a change in the within-firm structure in terms of the organizational capital allocation, the change in the industry-firm-specific constant depends on the industry characteristic ($\sigma$), given that the firm’s product scope and the marginal profitability of organizational capital are kept constant:

$$\left. \frac{d\psi_k(z)}{dz} \right|_{\lambda_k = \text{const.}} = \frac{\theta}{\theta(\sigma - 1) - 1} \left( \frac{\theta(\sigma - 1)\zeta_k(z)}{\lambda_k} \right)^{\frac{\sigma(\sigma-1)}{\sigma} - 1} \theta(\sigma - 1) \frac{d\zeta_k(z)}{dz}$$

and

$$\frac{d\zeta_k(z)}{dz} = -A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (aw)^{1-\sigma} z^{-\sigma}.$$
for firms in industries with a low substitution elasticity: $\frac{dc_k(z,\omega)}{dz} = awn_k(\omega) + awz\frac{dn_k(\omega)}{dz} > 0$, that is, given the endowment with organizational capital and the product scope, more productive multi-product firms have lower marginal production costs. On the contrary, in industries with a high elasticity of substitution across product varieties (homogeneous industries), multi-product firms concentrate on their least efficient, i.e. fringe, products, thereby being able to increase the number of products produced (scope), which productive firms decrease the amount of organizational capital allocated to each product variety, thereby being able to increase the number of products produced (scope), which takes place at the discount of higher marginal production costs for the product varieties being already in the firm’s product portfolio.

Given the binding resource constraint of firms, $\int_0^{\delta_k(z,O)} o_k(\omega)d\omega = 0$, the marginal profitability of organizational capital takes the form:\footnote{See the Appendix D.9 for the details of derivation.}

$$\lambda_k(z, O, \delta_k(z, O)) = \theta(\sigma - 1)\zeta_k(z)H_k(\delta_k(z, O))^{1-\sigma}O^{\theta(\sigma-1)^{-1}}$$

(4.22)

with $H_k(\delta_k(z, O)) \equiv \left[\int_0^{\delta_k(z,O)} h_k(\omega)\frac{\sigma-1}{\theta(\sigma-1)^{-1}} d\omega\right]^{\frac{\theta(\sigma-1)^{-1}}{\sigma-1}}$ as an (aggregate) index of the flexible manufacturing technology. Thus, the firm’s marginal profitability of organizational capital depends on its productivity, its resource endowment as well as chosen scope, since the cost-reducing resource has to be split up and allocated across the products produced. Thereby, the variable that captures the additional profits of the last employed unit of organizational capital summarizes the within-firm processes and structures of the resource-endowed multi-product firms and thus its properties play a decisive role in their analysis.

Changes in the productivity, resource endowment and scope has the following implications for the marginal profitability of organizational capital:\footnote{See the Appendix D.9 for the details of derivation.}

$$\frac{\partial \lambda_k(z, O, \delta_k(z, O))}{\partial z} = \theta(\sigma - 1)H_k(\delta_k(z, O))^{1-\sigma}O^{\theta(\sigma-1)^{-1}}\frac{\partial \zeta_k(z)}{\partial z} < 0,$$

(4.23)

$$\frac{\partial \lambda_k(z, O, \delta_k(z, O))}{\partial O} = (\theta(\sigma - 1) - 1)\theta(\sigma - 1)\zeta_k(z)H_k(\delta_k(z, O))^{1-\sigma}O^{\theta(\sigma-1)^{-2}} \geq 0$$

(4.24)

and

$$\frac{\partial \lambda_k(z, O, \delta_k(z, O))}{\partial \delta_k(z, O)} = -\theta(\sigma - 1)^2\zeta_k(z)O^{\theta(\sigma-1)^{-1}}H_k(\delta_k(z, O))^{-\sigma}\frac{\partial H_k(\delta_k(z, O))}{\partial \delta_k(z, O)} \leq 0.$$

(4.25)
where \( \frac{\partial \lambda_k}{\partial k} \geq 0 \) \( \iff \) \( \sigma \geq 1 + \frac{1}{\theta} \) and \( \frac{\partial \lambda_k}{\partial \theta_k} \leq 0 \) \( \iff \) \( \sigma \geq 1 + \frac{1}{\theta} \) holds.

Taking the firm’s last decision in the second stage, i.e. the decision on the number of varieties produced (scope), the firm maximizes the profit function

\[
\Pi_k(z,O) = \int_0^{\delta_k(z,O)} \left[ \frac{1}{\zeta_k(z)} \frac{\theta(\sigma - 1)}{\theta(1) - 1} \lambda_k \frac{\theta(\sigma - 1)}{\theta(1) - 1} h_k(\omega) \frac{\sigma - 1}{\sigma - 1} h_k(\delta_k(z,O)) \right] d\omega,
\]

to derive its choice, which is implicitly given by the condition\(^{44}\)

\[
\text{cost of the marginal variety}
\]

\[
\text{revenue of the marginal variety}
\]

\[
\begin{array}{c}
\left( \frac{1}{\zeta_k(z)} \frac{\theta(\sigma - 1)}{\theta(1) - 1} \lambda_k \frac{\theta(\sigma - 1)}{\theta(1) - 1} h_k(\delta_k(z,O)) \frac{\sigma - 1}{\sigma - 1} \right) \\
+ \left( \frac{1}{\zeta_k(z)} \frac{\theta(\sigma - 1)}{\theta(1) - 1} \frac{\theta(\sigma - 1)}{\theta(1) - 1} \lambda_k \frac{\theta(\sigma - 1)}{\theta(1) - 1} H_k(\delta_k(z,O)) \frac{\sigma - 1}{\sigma - 1} \right) \frac{\partial \lambda_k}{\partial \delta_k(z,O)}
\end{array}
\]

Multi-product firms expand their scope up to the point where the (additional) revenue of the marginal variety equals the revenue effect of the scope adjustment on all the intra-marginal varieties (dis-economies of scope - IMRE: \( \sigma < 1 + \frac{1}{\theta} \); \( \frac{\partial \lambda_k}{\partial k} > 0 \) and IMRE\(<\) 0; \( \sigma > 1 + \frac{1}{\theta} \); \( \frac{\partial \lambda_k}{\partial \theta_k} < 0 \) and IMRE\(<\) 0) due to the necessary reallocation of organizational capital across product varieties and the (additional) fixed cost of the marginal variety. For the optimal scope, the internalization of the within-firm reallocation of resources drives a wedge between the revenue and the fixed cost of the marginal variety, whose equalization often states the optimal scope condition in multi-product firm models \textit{without} resources.\(^{45}\)

Due to the existence of the dis-economies of scope in form of a resource cannibalization effect and its associated performance implications\(^{46}\), the marginal variety of a resource-endowed multi-product firm is required to generate a higher revenue than the corresponding variety of firms for which the effect does not exist and, under the assumption that the revenue of the marginal variety decreases in scope, which is unambiguously the case for \( \sigma < 1 + \frac{1}{\theta} \), resource-endowed multi-product firms restrict their product scope and expand it less than

\(^{44}\)See the Appendix D.10 for the details of derivation.

\(^{45}\)See Qin and Zhou (2013) for a more abstract analysis of multi-product firms’ decisions, which encompasses some characteristic features of several models of multi-product firms in international trade.

\(^{46}\)Roberts and McEvily (2005) look at the diversification of firms and the implied leverage or cannibalization of resources due to a product-line expansion, depending on the properties of the resource usage of products.
firms without the resource cannibalization effect.\footnote{In contrast, the demand cannibalization effect that is analyzed in \cite{Eckel_and_Neary_2010} exerts a contracting implication on the product-intensive margin.}

The behavior of multi-product firms at the extensive margin is characterized by the equation on the marginal profitability of organizational capital \footnote{See the Appendix D.11 for the details of derivation.} and the equation on product scope \footnote{See the Appendix D.11 for the details of derivation.}. Their total differentiation provides insights in the firms’ adjustment of their product scope in response to changes in the two dimensions of firm heterogeneity, i.e. productivity and organizational capital.\footnote{See the Appendix D.11 for the details of derivation.}

The elasticity of the product scope with respect to organizational capital is therefore given by

\[
\frac{\delta_k d\ln \delta_k}{d\ln O} = -\frac{1}{\Delta} \theta(\sigma - 1) \rho_k(z), \tag{4.27}
\]

where \(\Delta \geq 0\) denotes the determinant of the coefficient matrix of the system of the total differentiations of the two fundamental equations that is written in matrix notation and \(\rho_k(z) \equiv \frac{1}{\sigma - 1} A_k^{-1} (\frac{\sigma - 1}{\sigma})^{-\sigma} (awz)^{\sigma - 1} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \geq 0\). For \(\sigma < 1 + \frac{1}{\theta}\), the determinant \(\Delta\) is unambiguously negative \((\Delta < 0)\), while it is positive or negative \((\Delta \gtrless 0)\) for \(\sigma > 1 + \frac{1}{\theta}\):

\[
\Delta \gtrless 0 \Leftrightarrow \frac{1}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\partial h_k(\delta_k)}{\partial \delta_k} \gtrless \theta \left( \frac{h_k(\delta_k)}{H_k(\delta_k)} \right)^{\frac{\sigma - 1}{\sigma - 1 - 1}} ,
\]

where \(\bar{\varepsilon}_{h_k(\delta_k)}\) denotes the semi-elasticity of the flexible manufacturing technology at the firm’s marginal variety. Accordingly, the determinant \(\Delta\) is negative \((\Delta < 0)\) for firms with a small scope, i.e. \(\bar{\varepsilon}_{h_k(\delta_k)} < \theta(\theta(\sigma - 1) - 1) \left( \frac{h_k(\delta_k)}{H_k(\delta_k)} \right)^{\frac{\sigma - 1}{\sigma - 1 - 1}}\), and positive \((\Delta > 0)\) for firms with a large scope, i.e. \(\bar{\varepsilon}_{h_k(\delta_k)} > \theta(\theta(\sigma - 1) - 1) \left( \frac{h_k(\delta_k)}{H_k(\delta_k)} \right)^{\frac{\sigma - 1}{\sigma - 1 - 1}}\). In heterogeneous industries, multi-product firms that are endowed with a cost-reducing resource operate with a larger product scope when they are endowed with a larger amount of organizational capital. Empirical support for the theoretical result of a positive relationship between the organizational capital (endowment) and the product scope of multi-product firms is derived in the Section 4.4.2. In homogeneous industries, multi-product firms that are endowed with a cost-reducing resource and that operate only with a small scope have a (relatively) larger product scope when they are endowed with a larger amount of organizational capital, while multi-product firms that are endowed with a cost-reducing resource and that operate already with a large scope have a (relatively) smaller product scope when they are endowed with a larger amount of organizational capital.
The elasticity of the product scope with respect to productivity is then given by

\[
\frac{\delta_k d \ln \delta_k}{d \ln z} = \frac{1}{\Delta} (\sigma - 1) \rho_k(z).
\] (4.28)

In heterogeneous industries, multi-product firms that are endowed with a cost-reducing resource have a larger product scope when they are more productive. Empirical support for the theoretical result of a positive relationship between the productivity and the product scope of multi-product firms is documented in the Section 4.4.2. In homogeneous industries, multi-product firms that are endowed with a cost-reducing resource and that operate only with a small scope have a (relatively) larger product scope when they are more productive, while multi-product firms that are endowed with a cost-reducing resource and that operate already with a large scope have a (relatively) smaller product scope when they are more productive.

As multi-product firms are especially characterized by the existence of multiple margins, in particular and already analyzed the product-extensive margin, i.e. the number of products produced \(\delta_k(z, O)\), but also the product-intensive margin, i.e. the output of each product \(x_k(\omega, z, O)\) as well as the firm-intensive margin, i.e. the overall output of the firm \(X_k(z, O)\), the relationships between these margins describe the unique constitution and structure of the multi-product firms in the bi-parametric nature of the model. The overall output of a firm with organizational capital \(O\) and productivity \(\frac{1}{z}\) being active in industry \(k\) (firm-intensive margin)

\[
X_k(z, O) = \int_0^{\delta_k(z, O)} x_k(\omega, z, O) d\omega
\]

can be written in the form

\[
X_k(z, O) = A_k^{-\frac{\theta + 1}{(\sigma - 1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{(1+\theta)}{\pi(\sigma-1)-1}} (awz)^{\frac{\sigma}{\pi(\sigma-1)-1}} \lambda_k^{\frac{\theta}{\pi(\sigma-1)-1}}
\]

\[
\times \theta^{-\frac{\theta}{\pi(\sigma-1)-1}} \int_0^{\delta_k(z, O)} h_k(\omega)^{\frac{\sigma}{\pi(\sigma-1)-1}} d\omega,
\] (4.29)

whereas

\[
x_k(\omega, z, O) = A_k^{-\frac{\theta + 1}{(\sigma - 1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{(1+\theta)}{\pi(\sigma-1)-1}} (awz)^{\frac{\sigma}{\pi(\sigma-1)-1}} \lambda_k^{\frac{\theta}{\pi(\sigma-1)-1}}
\]

\[
\times \theta^{-\frac{\theta}{\pi(\sigma-1)-1}} h_k(\omega)^{\frac{\sigma}{\pi(\sigma-1)-1}}
\] (4.30)

thereby describes the product-intensive margin.

The correlation between the product-extensive margin and the product-intensive margin

\[\text{See the Appendix D.11 for the details of derivation.}\]

\[\text{See the Appendix D.12 for the details of derivation.}\]
is given by\(^{52}\)

\[
\frac{\partial x_k(\omega, z, O)}{\partial \delta_k(z, O)} = \frac{\theta \sigma}{\theta (\sigma - 1) - 1} A_k^{\frac{\theta + 1}{\sigma (\sigma - 1) - 1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma (1 + \theta)}{\sigma (\sigma - 1) - 1}} (awz)^{\frac{\sigma}{\sigma (\sigma - 1) - 1} - 1} \theta^k_{\sigma (\sigma - 1) - 1} h_k(\omega)^{\frac{\sigma}{\sigma (\sigma - 1) - 1}} \times \lambda_k^{\frac{\sigma + 1}{\sigma (\sigma - 1) - 1}} \frac{\partial \lambda_k}{\partial \delta_k},
\]

and thus the two margins are negatively correlated for the multi-product firms, independent of the kind of industry in which these firms operate (\(\sigma < 1 + \frac{1}{\theta} \): \(\frac{\partial \lambda_k}{\partial \delta_k} > 0\) and \(\frac{\partial \lambda_k}{\partial \delta_k} < 0\); \(\sigma > 1 + \frac{1}{\theta} \): \(\frac{\partial \lambda_k}{\partial \delta_k} < 0\) and \(\frac{\partial \lambda_k}{\partial \delta_k} > 0\)). Any increase in the product scope requires a reallocation of organizational capital from the intra-marginal to the marginal varieties, which implies a reduction in the output of all the varieties. Empirical support for the theoretical result of a negative correlation is provided by [Elliott and Virakul (2010)] and [Navarro (2012)].

Any change in the product-extensive margin has both a direct and indirect effect on the firm-intensive margin:\(^{53}\)

\[
\frac{\partial X_k(z, O)}{\partial \delta_k(z, O)} = \begin{cases} 
A_k^{\frac{\theta + 1}{\sigma (\sigma - 1) - 1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma (1 + \theta)}{\sigma (\sigma - 1) - 1}} (awz)^{\frac{\sigma}{\sigma (\sigma - 1) - 1} - 1} \theta^k_{\sigma (\sigma - 1) - 1} h_k(\delta_k)^{\frac{\sigma}{\sigma (\sigma - 1) - 1}} \\
+ A_k^{\frac{\theta + 1}{\sigma (\sigma - 1) - 1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma (1 + \theta)}{\sigma (\sigma - 1) - 1}} \left( \frac{\sigma}{\sigma - 1} - 1 \right)^{\frac{\sigma}{\sigma (\sigma - 1) - 1}} (awz)^{\frac{\sigma}{\sigma (\sigma - 1) - 1} - 1} \lambda_k^{\frac{\sigma (\sigma - 1) - 1}{\sigma (\sigma - 1) - 1} - 1} \frac{\partial \lambda_k}{\partial \delta_k} \\
\times \theta^{-\frac{\theta}{\sigma (\sigma - 1) - 1}} \int_0^{\delta_k(z, O)} h_k(\omega)^{\frac{\sigma}{\sigma (\sigma - 1) - 1}} d\omega
\end{cases}
\]

The direct effect of an increase in the product scope is given by an increase in the overall output due to the (additional) output of the marginal variety, but the necessary reallocations of organizational capital from intra-marginal product varieties to the marginal variety lead to output reductions of the intra-marginal varieties, stating the indirect effect of the scope adjustment (dis-economies of scope - IMOE: \(\sigma < 1 + \frac{1}{\theta} \): \(\frac{\partial \lambda_k}{\partial \delta_k} > 0\) and IMOE < 0; \(\sigma > 1 + \frac{1}{\theta} \): \(\frac{\partial \lambda_k}{\partial \delta_k} < 0\) and IMOE < 0). Only the joint effect determines the overall adjustment of the firm-intensive margin due to changes of the product-extensive margin. The empirical evidence documents the (monotone) finding that the direct effect is larger than the indirect effect, as firm exports are positively correlated with the number of products exported by the firm [Iacovone and Javorcik (2008), Adalet (2009), Bernard et al. (2011), Berthou and Fontagné (2013) and Bernard et al. (2014)].

Multiplying each product’s output (product-intensive margin) by its price and summing up over all the products produced by a firm (product-extensive margin), the overall sales

\(^{52}\)See the Appendix D.12 for the details of derivation.
\(^{53}\)See the Appendix D.12 for the details of derivation.
(revenue) of a firm with productivity $\frac{1}{z}$ and organizational capital $O$ being active in industry $k$ can be written as:

$$R_k(z, O) = A_k \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (awz)^{1-\sigma} O^{\theta(\sigma-1)} H_k(\delta_k(z, O))^{-(\sigma-1)}$$

and the firm’s corresponding overall profits are given by:

$$\Pi_k(z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k (awz)^{1-\sigma} O^{\theta(\sigma-1)} H_k(\delta_k(z, O))^{-(\sigma-1)} - \delta_k(z, O)f_{k,\omega},$$

whereas each product’s profits take the form:

$$\pi_k(\omega, z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k (awz)^{1-\sigma} H_k(\delta_k(z, O))^{\theta(\sigma-1)} h_k(\omega) - f_{k,\omega}.$$

**Industry** Equilibrium

A resource-endowed multi-product firm is characterized by its productivity $\frac{1}{z}$, its endowment with organizational capital $O$ as well as its intensive margin (scale) $X_k(z, O)$ and extensive margin (scope) $\delta_k(z, O)$. Each entrant has to decide about production in industry $k$: Given $z$, an organizational capital threshold $O_k^*(z)$ exists at which the firm is indifferent to produce or not to produce its most favorable product variety. For the values of organizational capital above $O_k^*(z)$, it is profitable for the firm to produce at least one product variety and thus the firm is active in industry $k$. In contrast, firms with a productivity $\frac{1}{z}$ and a value of organizational capital below $O_k^*(z)$ are not able to profitably produce at least one product variety and thus these firms are not active in industry $k$. The most favorable product variety which will be produced first, and possibly solely, is always the firm’s core competency variety, given that the firm concentrates all its organizational capital on the production of a single product variety:

$$\pi_k(\omega, z, O)\big|_{o_k(\omega)=O} = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k (awz)^{1-\sigma} h_k(\omega)^{1-\sigma} O^{\theta(\sigma-1)} - f_{k,\omega}$$

and

$$\frac{\partial \pi_k(\omega, z, O)}{\partial \omega} \bigg|_{o_k(\omega)=O} = \frac{1 - \sigma}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k (awz)^{1-\sigma} O^{\theta(\sigma-1)} h_k(\omega)^{\theta(\sigma-1)} \frac{\partial h_k(\omega)}{\partial \omega} < 0.$$
and
\[
\pi_k(0, z, O)|_{o_k(0)=O} = \frac{1}{\sigma - 1} \left(\frac{\sigma - 1}{\sigma}\right)^\sigma A_k (awz)^{1-\sigma} O^{\theta(\sigma-1)} - f_{k,\omega},
\]
and the value of \(O\) for which the profits are equal to zero, i.e. \(\pi_k(0, z, O^\star_k(z))|_{o_k(0)=O} = 0\), given the productivity \(\frac{1}{z}\), represents the organizational capital threshold.\(^{57}\)

\[
O^\star_k(z) = \left(\frac{A_k}{(\sigma - 1)f_{k,\omega}}\right)^\frac{1}{\sigma(1-\sigma)} \left(\frac{\sigma - 1}{\sigma}\right)^\frac{\sigma}{1-\sigma} (awz)^{\frac{1}{\theta}},
\]
whereas it is decreasing in the productivity \(\frac{1}{z}\) and increasing in the elasticity of substitution across product varieties.\(^{58}\)

Given \(\sigma\), more productive firms need less organizational capital to be able to profitably produce and to be active in the industry (relationship of a substitutability of the two dimensions of firm heterogeneity), while firms in more homogeneous industries need more organizational capital to be able to profitably produce and to be active in these industries, given \(\frac{1}{z}\).

Besides the decision on production in industry \(k\) based on the organizational capital threshold \(O^\star_k(z)\), each firm has to decide of whether to enter the industry: A firm with cost draw \(z\) only enters the industry \(k\) as long as its expected profits over the possible realizations of the organizational capital (endowment) exceed the fixed industry penetration cost \(F^p_k\), whereas the firm’s expected profits are given by
\[
E[\Pi_k(z)] = \int_{O^\star_k(z)}^{O_{\max,z}} \Pi_k(z, O)g_z(O)dO.
\]
The marginal firm in industry \(k\) with cost draw \(z^\star_k\) is then the industry’s least productive firm which is indifferent to enter or not to enter the industry as its expected profits are exactly equal to the fixed industry penetration cost, i.e.
\[
E[\Pi_k(z^\star_k)] = F^p_k.
\]

Figure 4.1 illustrates the entry and production decisions of a set of firms with some values of productivity \(\frac{1}{z}\) and organizational capital \(O\) in industry \(k\).

For a firm with cost draw \(z > z^\star_k\), it is not profitable to enter the industry and thus the firm does not pay the fixed cost \(F^p_k\). In contrast, a firm with cost draw \(z < z^\star_k\) pays the fixed cost \(F^p_k\) and discovers its organizational capital draw \(O\). In case of a draw \(O < O^\star_k(z)\), the firm does not produce any variety. If its draw is above the threshold, i.e. \(O > O^\star_k(z)\), the firm produces and has a positive scope and scale.

A firm pays the fixed cost \(f_E\) if its expected profits over the possible realizations of the

\(^{57}\)See the Appendix D.13 for the details of derivation.

\(^{58}\)See the Appendix D.13 for the proof.

\(^{59}\)As it turns out, firms have both a positive scale and a positive scope only in heterogeneous industries. See the Appendix D.14 for the proof.
productivity $\frac{1}{z}$, 

$$\Pi_k = B(z^*_k) \int_0^{z^*_k} E[\Pi_k(z)] \mu(z) dz,$$

with $\mu(z)$ as the distribution of cost draws conditional on $z$ being below the threshold $z_k^*$

$$\mu(z) = \begin{cases} \frac{b(z)}{B(z^*_k)} & \text{if } z < z_k^* \\ 0 & \text{otherwise} \end{cases}$$

exceed the fixed cost of entry. Free entry implies that the firm’s expected profits are exactly equal to the fixed cost of entry:

$$\Pi_k = w f_E.$$ 

Figure 4.1: Entry and Production Decisions of Firms in Industry $k$ in the Closed Economy

According to the equilibrium concept that is presented in Cherkashin et al. (2015), the mass of firms $N_k$ that are active in industry $k$ is computed by integrating the probabilities over the gray area in Figure 4.1:

$$N_k = N \int_0^{z_k^*} \int_{O_k(z)}^{O_{\text{max}}(z)} dG_z(O) dB(z)$$

and the total revenues are given by

$$T_k = N \int_0^{z_k^*} \int_{O_k(z)}^{O_{\text{max}}(z)} R_k(z, O) g_z(O) b(z) dO dz.$$

In equilibrium, goods markets clear, i.e.

$$T_k = \phi_k w L,$$
with $\phi_k$ as the share of the budget that is spent on product varieties of industry $k$, and
the equilibrium allocation is characterized by the firms’ decisions on prices $p_k(\omega, z, O)$ and
scope $\delta_k(z, O)$, free entry as well as the goods markets clearing.

4.3.2 Open Economy

The following sub-section opens the economy up to international trade. For the implementation, it is assumed that the world consists of $I$ countries ($i, j \in \{1, \ldots, I\}$) between which multi-product firms trade goods. Firstly, the preferences and demand in (a representative) country $j$, secondly, the production and supply with which country $j$ is internationally served, and thirdly, the resulting equilibrium are considered. Fourthly, changes in the cost parameters of international trade, in particular the transportation costs and the variety introduction costs, and their implications on the firms’ extensive and intensive margins of trade are analyzed.

Preferences and Demand

Country $j$ consists of $L_j$ identical individuals (consumers, workers) with CES preferences
and a per capita income $y_j$. The (upper-tier) utility (function) is given by

$$U_j = \sum_{k=0}^{1} U_{k,j},$$

where $k \in \{0, \ldots, 1\}$ represents an industry and $U_{k,j}$ denotes the utility index of industry
$k$ in country $j$, which takes the form

$$U_{k,j} = \left[ \sum_{i=1}^{I} \int_{\Omega_{k,ij}} p_{k,ij}(\omega) q_{k,ij}(\omega) d\omega \right]^{\frac{1}{\sigma}},$$

where $q_{k,ij}(\omega)$ represents the individual consumption of the (horizontally differentiated) product variety $\omega \in \Omega_{k,ij}$ in industry $k$ that is produced in country $i$ and shipped (exported) to consumers in country $j$, $\Omega_{k,ij}$ is the set of product varieties in industry $k$ that are produced in country $i$ and supplied to country $j$ and $\sigma > 1$ describes the elasticity of substitution across product varieties within an industry. As it is done in the closed economy, the elasticity of substitution within firms is assumed to be the same as the elasticity of substitution across firms and therefore a within-firm (demand) cannibalization effect is ex-ante excluded.

To solve the consumption problem, each individual in country $j$ maximizes the utility subject to the budget constraint

$$\gamma_{k,j}y_j = \sum_{i=1}^{I} \int_{\Omega_{k,ij}} p_{k,ij}(\omega) q_{k,ij}(\omega) d\omega,$$
with $\gamma_{k,j}$ as industry $k$’s budget share in country $j$ ($\sum_k \gamma_{k,j} = 1$) and $p_{k,ij}(\omega)$ as the price of the product variety $\omega \in \Omega_{k,ij}$ in industry $k$ that is produced in country $i$ and supplied to country $j$, and thereby derives the individual product demand, which can be aggregated over the $L_j$ individuals to get the aggregate (country-wide) demand for the product variety $\omega \in \Omega_{k,ij}$ in industry $k$ that is produced in country $i$ and supplied to country $j$:

$$x_{k,ij}(\omega) = A_{k,j}p_{k,ij}(\omega)^{-\sigma}$$

(4.34)

with

$$A_{k,j} \equiv L_j\gamma_{k,j}y_jP_{k,j}^{\sigma - 1}$$

as the residual demand of industry $k$ in country $j$, whereas $P_{k,j} \equiv \left[\sum_{i=1}^{I_k} \int_{\Omega_{k,ij}} p_{k,ij}(\omega)^{1-\sigma} d\omega \right]^{1-\sigma}$ thereby represents the CES price index of industry $k$ in country $j$.

### Production and Supply

The production and supply of product varieties by multi-product firms is basically the same as it is in the closed economy, just replicated for each country out of the set of $I$ countries and allowing for the firms’ possibility of serving foreign countries: The market structure of each country is characterized by a monopolistic competition with a mass of firms being available in each industry. Within an industry, each firm produces a continuum of product varieties with labor as the standard factor of production and a flexible manufacturing technology. Upon entry, firms pay a fixed cost $f_E$ in domestic labor units and draw a cost parameter $z$ from a distribution $B_i(z)$, with a probability distribution function $b_i(z)$ and support on $[0, \bar{z}_i]$. In this way, $\frac{1}{z}$ represents the productivity of the firm’s core competency variety. Pivotal to the model, each firm is endowed with a fixed amount of a cost-reducing resource that is called organizational capital.

Taken together, the firm’s productivity $\frac{1}{z}$ and its (endowment with) destination-specific organizational capital $O$ constitute the two dimensions of firm heterogeneity in the model and, on their way to production and supply, firms face a two-stage (production) problem with decisions on the (destination-specific) price, the (destination-specific) organizational capital allocation and the (destination-specific) scope as well as the realization of the organizational capital (endowment) that is drawn from a distribution $G_z(O)$, with a probability distribution function $g_z(O)$ and support on $[0, O_{max,z}]$.

To export product varieties to a destination $j$, a firm in country $i$ has to pay iceberg transportation costs $\tau_{ij} > 1$, with $\tau_{ii} = 1$. The marginal production costs for a product variety $\omega$ in industry $k$ that is produced in country $i$ and supplied to country $j$ are then given by

$$c_{k,ij}(\omega, z) = \tau_{ij}aw_i z n_{k,ij}(\omega) = \tau_{ij}aw_i z h_k(\omega) o_{k,ij}(\omega)^{-\theta},$$

(4.35)

with $a$ as a (constant) cost parameter, $w_i$ as the wage in country $i$, and $n_{k,ij}(\omega) \equiv$ 

---

60See the Appendix D.15 for the details of derivation.
\( h_k(\omega) o_{k,ij}(\omega)^{-\theta} \) as the composite cost component. Thereby, \( h_k(\omega) \) denotes the unit production costs of the product variety \( \omega \) and represents the flexible manufacturing technology, with \( \frac{\partial h_k(\omega)}{\partial \omega} > 0 \) and \( h_k(0) = 1 \), and \( o_{k}(\omega) \) denotes the organizational capital allocated to the product variety \( \omega \). In addition to the variable costs, firms have to cover the variety introduction costs \( f_{ij}^{k,\omega} \) for each produced product variety, whereas these costs are source- and destination-specific, as they represent production and distribution adjustment costs which may vary across the countries due to the different needs to invest in the customization of the product varieties in terms of demand and dissemination.

As in the closed economy, the firm’s production problem can be solved backwards, starting with the second stage and subsequent going further to the first stage. Given the CES demand and the monopolistic competition as the market structure, each firm chooses by the profit maximization a price which is a constant markup over the marginal production costs (standard constant-markup pricing rule)\(^{61}\)

\[
p_{ij}(\omega, z, O) = \frac{\sigma}{\sigma-1} c_{ij}(\omega, z, O). \tag{4.36}
\]

With higher iceberg transportation costs \( \tau_{ij} \), a firm charges a higher price for the product variety that is shipped from country \( i \) to country \( j \). As in the closed economy, each firm completely exploits its organizational capital and, in order to maximize the firm profits, the organizational capital is allocated across the product varieties within the firm according to\(^{62}\)

\[
o_{k,ij}(\omega) = \frac{1}{\theta(\sigma-1)\zeta_{k,ij}(z)} h_k(\omega)^{\sigma-1} \lambda_{k,ij} \left( \frac{1}{\theta(\sigma-1)\zeta_{k,ij}(z)} h_k(\omega)^{\sigma-1} \lambda_{k,ij} \right)^{\sigma-1}, \tag{4.37}
\]

with \( \zeta_{k,ij}(z) \equiv \frac{1}{\sigma-1} A_{k,j} \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (aw_{i}z\tau_{ij})^{1-\sigma} = \frac{A_{k,j}}{\sigma} \left( \frac{1}{aw_{i}z\tau_{ij}} \frac{\sigma-1}{\sigma} \right)^{\sigma-1} > 0 \). Given that a firm’s product scope is constant, multi-product firms in heterogeneous industries allocate decreasingly less organizational capital to varieties further away from their core competency variety and thus concentrate in their organizational capital allocation on varieties close to their core competency variety. In contrast, multi-product firms in homogeneous industries face a stronger competitive pressure from the demand, allocate increasingly more organizational capital to varieties further away from their core competency variety (fringe products) and therefore focus in their organizational capital allocation on varieties in which they have less competency or (labor) efficiency.

Given the binding resource constraint for each firm, the marginal profitability of organizational capital is given by\(^{63}\)

\[
\lambda_{k,ij}(z, O, \delta_{k,ij}(z, O)) = \theta(\sigma-1)\zeta_{k,ij}(z)H_k(\delta_{k,ij}(z, O))^{1-\sigma} O^\theta(\sigma-1)-1. \tag{4.38}
\]

As a firm’s final decision in its two-stage production problem, the number of product

\[^{61}\text{See the Appendix D.16 for the details of derivation.}\]

\[^{62}\text{See the Appendix D.17 for the details of derivation.}\]

\[^{63}\text{See the Appendix D.18 for the details of derivation.}\]
varieties that the firm with productivity \( \frac{1}{z} \) and organizational capital \( O \) being active in industry \( k \) produces in country \( i \) and exports to country \( j \), i.e. the (export) product scope \( \delta_{k,ij}(z, O) \), arises as a result of the firm’s profit maximization and is implicitly expressed by the condition\(^{64} \)

\[
\eta_{k,ij}(z) \lambda_{k,ij}(z, O, \delta_{k,ij}(z, O)) \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} h_k(\delta_{k,ij}(z, O)) \frac{\sigma - 1}{\eta(\sigma - 1)} \\
+ \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \lambda_{k,ij}(z, O, \delta_{k,ij}(z, O)) H_k(\delta_{k,ij}(z, O)) \frac{\sigma - 1}{\eta(\sigma - 1)} \\
\times \frac{\partial \lambda_{k,ij}(z, O, \delta_{k,ij}(z, O))}{\partial \delta_{k,ij}(z, O)} = f_{k,ij}^{\omega},
\]

(4.39)

with

\[
\eta_{k,ij}(z) \equiv \left( \frac{1}{\zeta_{k,ij}(z) \frac{1}{\eta(\sigma - 1)} \theta(\sigma - 1)} \right)^{\frac{\theta(\sigma - 1)}{\eta(\sigma - 1) - 1}} > 0.
\]

(Industry) Equilibrium

The equilibrium concept is exactly the same as it is in the closed economy: An organizational capital threshold \( O^*_{k,ij}(z) \) exists for which firms are indifferent of whether to produce or not to produce their most favorable product variety for country \( j \). The threshold takes the form\(^{65} \)

\[
O^*_{k,ij}(z) = \left( \frac{A_{k,j}}{(\sigma - 1) f_{k,ij}^{\omega}} \right)^{\frac{1}{1(\sigma - 1)}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{\eta(\sigma - 1)}} (\tau_{ij} a w_i z)^{\frac{1}{2}}.
\]

(4.40)

In strict correspondence to the closed economy, the destination-specific sales (revenue) of a firm with productivity \( \frac{1}{z} \) and organizational capital \( O \) being active in industry \( k \) can be written as

\[
R_{k,ij}(z, O) = A_{k,j} \left( \frac{\sigma}{\sigma - 1} \right)^{1 - \sigma} (\tau_{ij} a w_i z)^{1 - \sigma} O^\theta(\sigma - 1) H_k(\delta_{k,ij}(z, O))^{-(\sigma - 1)}
\]

and, with the organizational capital threshold in equation (4.40), these sales (the revenue) can be rewritten as

\[
R_{k,ij}(z, O) = \left( \frac{\sigma}{\sigma - 1} \right)^{1 - \sigma} O^*_{k,ij}(z)^{\theta(\sigma - 1)} (\sigma - 1) f_{k,ij}^{\omega} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} O^\theta(\sigma - 1) H_k(\delta_{k,ij}(z, O))^{-(\sigma - 1)}
\]

and simplified as

\[
R_{k,ij}(z, O) = \sigma O^*_{k,ij}(z)^{\theta(\sigma - 1)} f_{k,ij}^{\omega} O^\theta(\sigma - 1) H_k(\delta_{k,ij}(z, O))^{-(\sigma - 1)}.
\]

(4.41)

\(^{64}\)See the Appendix D.19 for the details of derivation.

\(^{65}\)See the Appendix D.20 for the details of derivation.
Similarly, the condition on the destination-specific product scope of a firm with productivity \( \frac{1}{z} \) and organizational capital \( O \) being active in industry \( k \) in equation (4.39) can be rewritten by using the organizational capital threshold in equation (4.40) as:

\[
h_k(\delta_{k,ij})^{\frac{1-\sigma}{\sigma(\sigma-1)}} = (1 - \theta(\sigma - 1)) H_k(\delta_{k,ij})^{-\frac{\theta(\sigma-1)^2}{\sigma(\sigma-1)}} O_k^{*}(z)^{\theta(1-\sigma)} O^{\theta(\sigma-1)}. \tag{4.42}
\]

With equation (4.42) the destination-specific firm sales (revenue) in equation (4.41) take the form:

\[
R_k(\omega, z, O) = \sigma \int_{O_{\text{max}}(z)}^{O_{\text{max}}(z)} \Pi_k(\omega, z, O) g_z(O) dO.
\]

A firm pays the fixed cost \( f_E \) if its expected profits over the possible realizations of the productivity \( \frac{1}{z} \) across all destinations,

\[
\Pi_k = \sum_{j=1}^{l} B_i(z_{k,ij}) \int_{0}^{z_{k,ij}} E[\Pi_k(\omega, z, O)] \mu_i(z) dz,
\]

with \( \mu_i(z) \) as the distribution of cost draws conditional on \( z \) being below the threshold \( z_{k,ij}^{*} \):

\[
\mu_i(z) = \begin{cases} 
\frac{B_i(z)}{B_i(z_{k,ij})} & \text{if } z < z_{k,ij}^{*} \\
0 & \text{otherwise},
\end{cases}
\]

exceed the fixed cost of entry. Free entry implies that the firm’s expected profits are exactly equal to the fixed cost of entry:

\[
\Pi_k = w_i f_E.
\]

See the Appendix D.20 for the details of derivation.
According to Cherkashin et al. (2015), the mass of firms $N_{k,ij}$ that are active in industry $k$, located in country $i$ and serve country $j$ is computed by integrating the probabilities over the gray area in Figure 4.2:

$$N_{k,ij} = N_i \int_0^{z_{k,ij}^*} \int_{O_{k,ij}}^{O_{\text{max},z}} dG_z(O)dB_i(z)$$

and the total revenues are given by

$$T_{k,ij} = N_i \int_0^{z_{k,ij}^*} \int_{O_{k,ij}}^{O_{\text{max},z}} R_{k,ij}(z,O)g_z(O)b_i(z)dOdz.$$ (4.44)

In equilibrium, goods market clear, i.e.

$$\sum_{i=1}^{I} T_{k,ij} = \phi_{k,j}w_jL_j,$$

with $\phi_{k,j}$ as the share of the budget that is spent in country $j$ on product varieties of industry $k$ and the equilibrium allocation is characterized by the firms’ decisions on prices $p_{k,ij}(\omega,z,O)$ and scope $\delta_{k,ij}(z,O)$, free entry as well as the goods markets clearing.

![Figure 4.2: Entry and Production Decisions of Firms in Industry $k$ in the Open Economy](image)

**Comparative Statics**

Any change in the cost parameters of international trade, in particular the iceberg transportation costs $\tau_{ij}$ and the variety introduction costs $f_{k,ij}$, has some implications on the multi-product firm’s margins of trade: the intensive margin, i.e. (export) sales (revenue), as well as the extensive margin, i.e. (export) product scope. Moreover, these adjustments at the firm level are accompanied by reallocations of organizational capital across the product.
varieties within the firms.

The elasticity of the (export) product scope with respect to $\tau_{ij}$ is given by

$$
\frac{\delta_{k,ij} d \ln \delta_{k,ij}}{d \ln \tau_{ij}} = \frac{1}{\Delta} (\sigma - 1) \rho_{k,ij} (z),
$$

where $\Delta \geq 0$ denotes the determinant of the coefficient matrix of the system of the total differentiations of the two fundamental equations that is written in matrix notation and $\rho_{k,ij} (z) \equiv \frac{1}{\sigma - 1} A_{k,j} \left( \frac{\sigma - 1}{\sigma} \right) \tau_{ij}^{\sigma - 1} (\tau_{ij} a w_i z)^{\sigma - 1} - \theta H_k (\delta_{k,ij})^{1 - \sigma} \lambda_k^{1 - \sigma} \theta O^{\alpha (\sigma - 1) - 1} > 0$. Multi-product firms in heterogeneous industries increase their product scope in response to a trade liberalization, i.e. a reduction in the iceberg transportation costs $\tau_{ij}$, while they reduce their product scope in response to a trade de-liberalization, i.e. an increase in the iceberg transportation costs $\tau_{ij}$. Under the assumption that the iceberg transportation costs $\tau_{ij}$ are increasing in the (geographical) distance between the source and destination countries and thus differently interpreted, multi-product firms export more products to destinations that are closer to their location.

Alternatively and providing further insights into the adjustment of multi-product firms at their extensive margin in response to transportation costs changes, the elasticity of the (export) product scope with respect to $\tau_{ij}$ can be computed as

$$
\frac{\delta_{k,ij} d \ln \delta_{k,ij}}{d \ln \tau_{ij}} = - \frac{(\sigma - 1)(1 - \theta (\sigma - 1))^{1/\sigma - 1} \tau_{ij}^{-1} Oh_k (\delta_{k,ij})^{-1/\sigma} H_k (\delta_{k,ij})^{1/\sigma - 1} - \theta (\sigma - 1) H_k (\delta_{k,ij})^{1/\sigma - 1} + \theta \frac{\partial h_k (\omega)}{\partial \delta_{k,ij}}}{\theta (\sigma - 1)^{-1} h_k (\delta_{k,ij})^{1/\sigma - 1} - 1}.
$$

Once again, multi-product firms in heterogeneous industries increase (reduce) their product scope in response to a trade (de-)liberalization in terms of $\tau_{ij}$. However, given a (standard) functional form specification for the unit production costs, i.e. $h_k (\omega) = \exp (\mu_k \omega)$, with $\mu_k > 0$ as a (industry-specific) parameter, these multi-product firms do not uniformly adjust their product scope: At their extensive margin, firms with a larger scope respond less to trade shocks. As product varieties that are further away from the core competency variety are produced at a smaller scale and thus contribute less to the overall output and sales of a firm, multi-product firms with a large scope are relatively less sensitive to changes in trade costs with respect to the production decision of their product varieties at the margin and thus respond at their extensive margin less to trade shocks.

Along with the adjustment at the extensive margin, multi-product firms reallocate organizational capital across the product varieties in response to trade costs changes. Thereby,

67See the Appendix D.21.1 for the proof.
68Theoretically, multi-product firms with a small scope in homogeneous industries increase their product scope in response to a trade liberalization, while they reduce their product scope in response to a trade de-liberalization. However, multi-product firms with a large scope in homogeneous industries reduce their product scope in response to a trade liberalization, while they increase their product scope in response to a trade de-liberalization.
69See the Appendix D.21.1 for the details of derivation.
70See the Appendix D.21.1 for the proof. Theoretically, the result is ambiguous for multi-product firms in homogeneous industries.
the elasticity of the organizational capital allocation with respect to \( \tau_{ij} \) is given by\(^{72}\)

\[
\frac{d \ln o_{k,ij}(\omega)}{d \ln \tau_{ij}} = -\frac{1}{\Delta} (\sigma - 1) \rho_{k,ij}(z) \left( \frac{h_k(\delta_{k,ij})}{H_k(\delta_{k,ij})} \right)^{\frac{\sigma - 1}{\sigma - 1 - \theta}},
\]

where \( \Delta \geq 0 \) denotes the determinant of the coefficient matrix of the system of the total differentiations of the two fundamental equations that is written in matrix notation and \( \rho_{k,ij}(z) \equiv \frac{1}{\sigma - 1} A_{k,ij}^{-1} \left( \frac{\sigma - 1}{\sigma} \right) (\tau_{ij}a w_i z)^{\sigma - 1} - \theta H_k(\delta_{k,ij})^{1 - \sigma} \lambda_{k,ij}^{-1} \theta \theta^{(\sigma - 1) - 1} \). Multi-product firms in heterogeneous industries increase their product scope and thus allocate less organizational capital to each (intra-marginal) product variety in response to a trade liberalization, i.e. a reduction in the iceberg transportation costs, while they reduce their product scope and thus allocate more organizational capital to each (intra-marginal) product variety in response to a trade de-liberalization, i.e. an increase in the iceberg transportation costs.\(^{72}\)

As adjustments at the extensive margin and the within-firm allocation of organizational capital take place, changes in the cost parameters of international trade give also rise to adjustments at the intensive margin. The elasticity of the overall (export) sales (revenue) of a firm with respect to the variety introduction costs \( f_{k,ij}^{ij} \) is given by\(^{73}\)

\[
\frac{d \ln R_{k,ij}(z, O)}{d \ln f_{k,ij}^{ij}} = \begin{cases} \text{intensive margin} & \frac{(1 - \theta(\sigma - 1))\varepsilon H_k(\delta_{k,ij})}{\varepsilon h_k(\delta_{k,ij}) + \theta(\sigma - 1)\varepsilon H_k(\delta_{k,ij})}, \\ \text{extensive margin} & \end{cases}
\]

where \( \varepsilon H_k(\delta_{k,ij}) \equiv -\frac{\delta_{k,ij}}{H_k(\delta_{k,ij})} \frac{\partial H_k(\delta_{k,ij})}{\partial \delta_{k,ij}} > 0 \) denotes the absolute value of the elasticity of the flexible manufacturing technology index with respect to scope and \( \varepsilon h_k(\delta_{k,ij}) \equiv -\frac{\delta_{k,ij}}{h_k(\delta_{k,ij})} \frac{\partial h_k(\delta_{k,ij})}{\partial \delta_{k,ij}} > 0 \) describes the elasticity of the unit production costs with respect to scope. Any change in the (destination-specific) variety introduction costs affects the overall sales (revenue) of a firm only through the extensive margin of trade. As the variety introduction costs decrease, the overall sales (revenue) of multi-product firms in heterogeneous industries increase, while they decrease in response to an increase in the variety introduction costs.\(^{74}\)

Differently, the elasticity of the overall (export) sales (revenue) of a firms with respect

\(^{71}\)See the Appendix D.21.2 for the details of derivation.

\(^{72}\)Theoretically, multi-product firms with a small scope in homogeneous industries allocate less organizational capital to each (intra-marginal) product variety in response to a trade liberalization, while they allocate more organizational capital to each (intra-marginal) product variety in response to a trade de-liberalization. However, multi-product firms with a large scope in homogeneous industries allocate more organizational capital to each (intra-marginal) product variety in response to a trade liberalization, while they allocate less organizational capital to each (intra-marginal) product variety in response to a trade de-liberalization.

\(^{73}\)See the Appendix D.21.3 for the details of derivation.

\(^{74}\)Theoretically, as the variety introduction costs decrease, the overall sales (revenue) of multi-product firms in homogeneous industries decrease, while they increase in response to an increase in the variety introduction costs.
to the iceberg transportation costs \( \tau_{ij} \) is given by \(^{75}\)

\[
\frac{d \ln R_{k,ij}(z,O)}{d \ln \tau_{ij}} = \text{intensive margin } - (\sigma - 1) \frac{(1 - \theta(\sigma - 1)) \varepsilon_{Hk}(\delta_{k,ij})}{\varepsilon_{hk}(\delta_{k,ij}) + \theta(\sigma - 1) \varepsilon_{Hk}(\delta_{k,ij})},
\]


\[
\text{extensive margin }
\]

where \( \varepsilon_{Hk}(\delta_{k,ij}) \equiv -\frac{\delta_{k,ij}}{H_k(\delta_{k,ij})} \frac{\partial H_k(\delta_{k,ij})}{\partial \delta_{k,ij}} > 0 \) denotes the elasticity of the flexible manufacturing technology index with respect to scope and \( \varepsilon_{hk}(\delta_{k,ij}) \equiv \frac{\delta_{k,ij}}{h_k(\delta_{k,ij})} \frac{\partial h_k(\delta_{k,ij})}{\partial \delta_{k,ij}} > 0 \) describes the elasticity of the unit production costs with respect to scope. Any change in the (destination-specific) transportation costs affects the overall sales (revenue) of a firm through both the extensive and intensive margin, whereas the change through the intensive margin is independent of the firm’s product scope and thus uniform across firms. As the transportation costs decrease, the overall sales (revenue) of multi-product firms in heterogeneous industries increase, while they decrease in response to an increase in the transportation costs \(^{76}\).

Multi-product firms in heterogeneous industries increase (decrease) their overall (export) sales (revenue) in response to a trade (de-)liberalization both in terms of \( f_{ij}^{k,\omega} \) and \( \tau_{ij} \). However, given a (standard) functional form specification for the unit production costs, i.e. \( h_k(\omega) = \exp(\mu_k(\omega)) \), with \( \mu_k > 0 \) as a (industry-specific) parameter, these multi-product firms do not uniformly adjust their export sales (revenue): At their intensive margin, firms with a larger scope respond less to trade shocks \(^{77}\). As product varieties that are further away from the core competency variety are produced at a smaller scale and thus contribute less to the overall output and sales of a firm, multi-product firms with a large scope are relatively less sensitive to changes in trade costs with respect to the output decision of their product varieties at the margin and thus respond at their intensive margin less to trade shocks.

With changes in the cost parameters of international trade, in particular the iceberg transportation costs \( \tau_{ij} \) and the variety introduction costs \( f_{ij}^{k,\omega} \), adjustments do not only take place at the firm level, but also at the aggregate (industry) level. Taking the total derivative of equation (4.44) while keeping the mass of entrants \( N_i \) constant, provides a decomposition of the change in industry \( k \)’s total exports of goods from country \( i \) to country \( j \) into its margins \(^{75}\) the representation below is given for the case of a change in the iceberg transportation costs \( \tau_{ij} \), whereas such a representation is likewise possible for a change in the variety introduction costs \( f_{ij}^{k,\omega} \):

\[
\frac{dT_{k,ij}}{dz_{k,ij}} = \frac{\partial T_{k,ij}}{\partial z_{k,ij}} dz_{k,ij} \frac{dT_{k,ij}}{dO_{k,ij}^*(z)} dO_{k,ij}^*(z) + \frac{\partial T_{k,ij}}{\partial \tau_{ij}} d\tau_{ij}
\]

\(^{75}\)See the Appendix D.21.3 for the details of derivation.

\(^{76}\)Theoretically, as \( -1 < \frac{(1 - \theta(\sigma - 1)) \varepsilon_{Hk}(\delta_{k,ij})}{\varepsilon_{hk}(\delta_{k,ij}) + \theta(\sigma - 1) \varepsilon_{Hk}(\delta_{k,ij})} < 0 \) holds and the transportation costs decrease, the overall sales (revenue) of multi-product firms in homogeneous industries increases, while they decrease in response to an increase in the transportation costs.

\(^{77}\)See the Appendix D.21.3 for the proof. Theoretically, the result is ambiguous for multi-product firms in homogeneous industries.
4.3. MODEL

\[
\begin{align*}
&T_{k,ij} = N_i \left[ \int_{z^*_k,ij}^{\max,z} R_{k,ij}(z^*_k,ij,O)g_z(O) b_i(z^*_k,ij)dO \right] dz^*_k,ij \\
&- N_i \left[ \int_0^{z^*_k,ij} R_{k,ij}(z,O^*_k,ij(z))g_z(O^*_k,ij(z))b_i(z)dz \right] dO^*_k,ij(z) \\
&+ N_i \left[ \int_0^{z^*_k,ij} \int_{O^*_{k,ij}(z)}^{\max,z} \frac{\partial R_{k,ij}(z,O)}{\partial \tau_{ij}} g_z(O)b_i(z)dOdz \right] d\tau_{ij} \\
\end{align*}
\]

(4.45)

Figure 4.3: Adjustments in Industry \( k \) due to a Change in the Trade Cost Parameters

The first component of equation (4.45) describes the change in total exports due to the change in the cost cutoff \( z^*_k,ij \) and thus the set of entrants. Any increase in the cost cutoff (e.g. from \( z^*_k,ij \) to \( z^*_{k,ij}' \) in Figure 4.3) causes the total exports to increase by the amount of exports of firms at the cost cutoff (extensive margin). The second component of equation (4.45) represents the change in total exports due to the change in the organizational capital threshold \( O^*_k,ij \) and thus the set of active firms. Any decrease in the organizational capital cutoff (e.g. from \( O^*_k,ij \) to \( O^*_{k,ij}' \) in Figure 4.3) causes the total exports to increase by the amount of exports of firms at the organizational capital cutoff (extensive margin). Finally, the third component of equation (4.45) describes the change in total exports due to the change in the iceberg transportation costs (or the variety introduction costs) by itself. Any decrease in the iceberg transportation costs (or the variety introduction costs) causes the total exports to increase by the amount of export increases of the active multi-product firms in the heterogeneous industries (intensive margin), which can be further decomposed...
into the changes in the exports at the firms’ intensive margin and the exports at the firms’ extensive margin.

4.4 Empirical Support

The following section provides both (i) empirical support for the (general) relevance of organizational capital as an input in the production process of firms (indirect empirical support for the developed theory of multi-product firms in international trade) and (ii) empirical support for some of the basic predictions of the developed theory of multi-product firms (direct empirical support for the developed theory of multi-product firms in international trade). Any empirical exercise that attempts to verify the theory’s predictions on the internal structure of multi-product firms faces the difficulty of a shortage of the required firm-product-level data, which reveal the allocation of organizational capital across the products within firms. Therefore, the empirical verification of the theory is limited to the exercises that are undertaken in this section.

4.4.1 Organizational Capital

Given the concept of organizational capital and its treatment in the empirical literature over time, a central question remains finally unresolved: Does organizational capital matter for the performance of firms? The existing studies on that question suffer from serious shortcomings in their conceptional designs or data applications, which considerably restrict the validity of the findings that the studies generate as solutions to the question and that constitute the current state of research: While Lev and Radhakrishnan (2005) do not employ a capitalized measure for the organizational capital, as it becomes prevalent for the other important intangible asset of a firm, in particular research and development, and make use of an ordinary least squares (OLS) estimator which is imperfect in its performance of estimating production functions, De and Dutta (2007) apply merely some part of the capitalized income statement item that is generally used as a proxy for organizational capital and restrict their study to a single developing country. In the most recent attempt, Tronconi and Vittucci Marzetti (2011) also employ an OLS estimator which reveals an imperfect performance in the estimation of production functions.

The estimation of firm-level production functions by applying an OLS estimator to a panel of (continuing) firms in a regression can be associated with several difficulties, but the estimator’s imperfect performance is especially based on two biases. The selection bias, which appears in the case of no allowance for the entry and exit of firms (balanced...
and the endogeneity bias that occurs in the case of a correlation of the error term and the independent variables. The source of the endogeneity bias is quite often the simultaneity in the sense that the firm’s input choice depends on some determinants that are unobserved to the econometrician but observed by the firm, in particular the (expected) productivity of the firm, much more than it is driven by the omitted variables (model misspecification) or the measurement errors. Several methods to deal with the endogeneity bias exist: Parametric (fixed effects as well as instrumental variables and generalized method of moments (GMM)), non-parametric (index numbers and data envelopment analysis) and semi-parametric (algorithm developed by Olley and Pakes (1996), algorithm developed by Levinsohn and Petrin (2003)) estimators. The main idea of the semi-parametric estimators is that, under certain theoretical and statistical assumptions, one can “invert” the optimal input decisions of the firms to essentially enable the econometrician to “observe” the so far unobserved productivity shocks. To implement this idea, Olley and Pakes (1996) identify the conditions under which the firm-level investment, conditional on the capital stock, can be taken as a strictly increasing function of a scalar, firm-level, unobserved productivity shock, in this way getting able to develop an algorithm with which one can estimate firm-level production functions in an unbiased way by “controlling for” the unobserved productivity shocks, while conditioning on a non-parametric representation of the inverse function. Instead of inverting an investment demand function, Levinsohn and Petrin (2003) select an intermediate input demand function but otherwise follow the same steps of the proposed algorithm.

While De and Dutta (2007) use a system GMM estimator to avoid the endogeneity bias which may appear in the case of using an OLS estimator for the production function estimation, both Lev and Radhakrishnan (2005) and Tronconi and Vittucci Marzetti (2011) indeed employ an OLS estimator and, while being aware of the problem, their studies suffer from inadequately taking care of the imminence of the endogeneity bias of the estimator: Both studies make use of lagged independent variables in the OLS estimation to instrument for the independent and possibly endogenous variables, but recent studies by Reed (2015) and Bellemare et al. (2017) show that this strategy is not able to fulfil its purpose and to avoid the endogeneity bias.

In an empirical sidestep, this section removes the shortcomings of the previous studies and thus robustly tests and thereby verifies the relevance of the organizational capital as an input for the production process and output of firms by picking up on the most recent study conducted by Tronconi and Vittucci Marzetti (2011) and using COMPUSTAT - Capital IQ - Global and COMPUSTAT - Capital IQ - North-America as the databases, which provide unbalanced panel data on the firm level for the periods 2000-2007 as well as 2010-2017 and the regions Asia, Europe and North-America. Accordingly, this section works in total with six samples: Two for Asia with firms in 15 and 16 countries respectively
CHAPTER 4. MULTI-PRODUCT FIRMS AND ORGANIZATIONAL CAPITAL

(“Asian samples”: 2000-2007: A-S1, and 2010-2017: A-S2), two for Europe with firms in 32 countries\(^{83}\) (”European samples”: 2000-2007: E-S1, and 2010-2017: E-S2) and two for North-America with firms in two countries (Canada and the USA) (“North-American samples”: 2000-2007: NA-S1, and 2010-2017: NA-S2). This section takes into account (i) more recent time periods (2000-2007 and especially 2010-2017), (ii) more regions of firm (headquarters) location (Asia, Europe and North America), as well as applies (iii) more measures for the dependent variable (revenue and value added), and (iv) more recent robust estimation techniques (accounting for issues of endogeneity in the production function estimation) than the previous studies. Consequently, this section constitutes a considerable step on the way of validating the fundamental importance of organizational capital for the production process of firms.

Empirical Methodology

This section estimates firm-level production functions that include four inputs, in particular physical capital (K), labor (L), research and development (RD) and organizational capital (OC), and occur in two functional specifications: (i) Cobb-Douglas:

\[
q_{it} = \alpha_i + \beta_k k_{it} + \beta_l l_{it} + \beta_{rd} rd_{it} + \beta_{oc} oc_{it}
\]  

(4.46)

and (ii) translog, which relaxes the assumptions of constant output elasticities and constant unit elasticity of substitution for inputs and represent thereby a more flexible form:

\[
q_{it} = \alpha_i + \beta_k k_{it} + \beta_l l_{it} + \beta_{rd} rd_{it} + \beta_{oc} oc_{it} + \gamma_k k_{it}^2 + \gamma_l l_{it}^2 + \gamma_{rd} rd_{it}^2 + \gamma_{oc} oc_{it}^2 + \gamma_{kl} k_{it} l_{it} + \gamma_{kr} k_{it} rd_{it} + \gamma_{kr} k_{it} oc_{it} + \gamma_{lr} l_{it} rd_{it} + \gamma_{lr} l_{it} oc_{it} + \gamma_{rd} rd_{it} oc_{it},
\]  

(4.47)

where variables in lower case letters indicate logs, \(i\) represents some firm and \(t\) some time. The output variable \(q_{it}\) describes the revenue of firm \(i\) at time \(t\) and \(\alpha_i\) captures unobservable differences in production efficiency. In the COMPUSTAT - Capital IQ database, irrelevant whether the Global or North-America section is made use of, revenue is identified by yearly total revenue (\(REVT\)), the physical capital variable by yearly net total property, plant and equipment (\(PPENT\)) and the labour variable by yearly number of employees (\(EMP\)).

To generate stock variables for the other inputs, in particular research and development as well as organizational capital, in the analysis, flow variables that can be interpreted as investments into these intangible assets are capitalized by using the perpetual inventory method. For research and development, the database provides data on research and development expenses (\(XRD\)) and these are capitalized with the assumption of a depreciation rate of 20% and a growth rate of 8%, so that the result identifies the revenue and development variable. For organizational capital, following the main approach in the literature, while being aware of its shortcomings (Section 4.2.2), the organizational capital variable

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\(^{83}\)For the details on the countries that are covered by the samples: See the Appendix E.2.
is identified by the capitalized income statement item “selling, general and administrative (SGA) expenses” \((XSGA)\), whose data in the raw, i.e. not capitalized, form are supplied by the database, as a proxy for organizational capital. Once again, this item by itself includes several expenses that can generate or enhance organizational capital, e.g. employee training costs, brand enhancement activities, payments to systems and strategy consultants and IT outlays. The capitalization of the income statement item is conducted by using a capitalization rate of 20%, a depreciation rate of 10% and a growth rate of 6%.

The production functions are estimated by using an \textit{OLS estimator} and a \textit{semi-parametric estimation algorithm}, in particular the method by Olley and Pakes (1996). In the case of the latter approach, investments as the “control variable” are identified by yearly capital expenditures \((CAPX)\). With the OLS estimator, the production functions are estimated both in levels and first differences (FDs). In the case of the estimations in levels, sector and country-group dummies are applied in the regressions. The countries in each sample are thereby grouped according to their per capita income in a given year, which leads on to groups of homogeneous countries. The estimations are also conducted in first differences to eliminate any firm-specific unobserved heterogeneity, as far as it is correctly assumed that firm heterogeneity is constant over time.

All the estimations are separately undertaken for \textit{two sub-samples} of each sample: The sub-sample that only comprises firms that incur research and development (R&D) expenditures and thus have an R&D stock in a given year, in short referred to as R&D firms, and the sub-sample that only comprises firms that do not incur R&D expenditures and thus do not have an R&D stock in a given year, in short referred to as non-R&D firms. This dichotomy in each sample is mainly motivated by the empirically documented fact that R&D firms differ from non-R&D firms in their performance. Applied to this section, which deals with multi-product firms that undertake some R&D by assumption, as they innovate and introduce products to the market, R&D firms and the singularities in their performance are of special interest. For the R&D firms, the input elasticities are estimated both with the exclusion and the inclusion of organizational capital, thereby illustrating the miss-measurement of the elasticities if organizational capital is omitted from the production function.

The following analysis concentrates on the period 2010-2017. Firstly, the three samples, each for Europe, North-America and Asia, and the firms that are covered by them are described, secondly, the production functions are estimated, and thirdly, robustness checks are conducted. For the details on the samples and the estimation results for the previous period 2000-2007: See the \textit{Appendix E.5} and \textit{Appendix E.6}.

\textsuperscript{84}For the details on the sector classification: See the \textit{Appendix E.3}\textsuperscript{85}For the details on the construction of the country-group dummies: See the \textit{Appendix E.4}\textsuperscript{86}For an overview about the literature on the elasticity of R&D and the rate of return to R&D: See Hall et al. (2010). For an overview about as well as a meta-analysis of this literature: See Ugur et al. (2016). For an overview about the literature on the relationship between R&D (investment) and productivity: Hall (2011) and Mohnen and Hall (2013).
Descriptive Statistics

The analysis of the contribution of organizational capital to the performance of firms is based on three samples on the period 2010-2017, each for Europe, North-America and Asia: E-S2, NA-S2 and A-S2. All these samples cover medium- and large-sized firms, i.e. firms with at least 175 employees.

**European Sample on the Period 2010-2017 (E-S2)**

The sample of European firms on the period 2010-2017 consists of 1,963 firms in 2017 whose headquarters are located each in one of 32 countries in Europe and which operate each in one of 10 sectors.

![Figure 4.4: Distribution of Firms in the European Sample across the Countries (2017)](image)

![Figure 4.5: Distribution of Firms in the European Sample across the Sectors (2017)](image)
4.4. EMPIRICAL SUPPORT

While about half (2017: 50.7%) of the firms in the sample are located either in Great Britain, France or Germany, almost half (2017: 46.8%) of the firms operate in the manufacturing sector that thereby represents by far the largest sector in the sample, followed by the services sector in which 20.2% (2017) of the firms operate and the transportation sector in which 13.2% (2017) of the firms operate (Figure 4.4 and Figure 4.5).

As compared with E-S1, the share of firms that are active in the manufacturing sector decreases by 1.1%ps (2007: 47.9%), while the share of firms that are active in the services sector and the share of firms that are active in the transportation sector increases by 0.6%ps and 0.7%ps, respectively (2007: 19.6% and 12.5%, respectively). Over the 10-years period, the manufacturing sector contracts, while the services sector and the transportation sector expand (“structural change”).

Notes: Firms in the sample are grouped according to their number of employees, which yields employment categories (on the left: category “175-749”: firms with at least 175 but less than 750 employees, category “750-2,499”: firms with at least 750 but less than 2,500 employees, category “2,500-7,499”: firms with at least 2,500 but less than 7,500 employees, etc.). Higher-ranked employment categories include firms with a larger workforce.

Figure 4.6: Distribution of Non-R&D Firms and R&D Firms in the European Sample across the Employment Categories (2017)

The firms with less than 2,500 employees represent about half (2017: 52%) of the firms in the sample and the firms in the two highest-ranked employment categories with at least 15,000 employees account for 17% (2017) of the firms, both types together constituting about two-thirds of the sample population of firms, whereas R&D firms are incrementally more present among the firms in the higher-ranked employment categories: The proportion of R&D firms in the employment categories increases when moving along the categorization of the firm size from firms with a workforce at a small to medium scale to firms with a workforce at a medium to large scale, and the proportion increases once more when moving further to firms with a workforce at a very large scale (Figure 4.6).

Comparing E-S2 with E-S1, the firm size distribution at the end of each sample changes in the way that the lowest-ranked employment category and the two highest-ranked employ-
ment categories contract, while the other categories expand as their weights are measured in percentage terms of each sample’s population of firms and their changes are measured in terms of percentage points. Over the 10-years period, the total mass of firms in the sample shifts to some degree to the middle of the firm size distribution (“de-polarization”).

Figure 4.7: Distribution of R&D and Organizational Capital Stocks in the European Sample across the Sectors (Mean Values, 2017)

Figure 4.8: Distribution of R&D and Organizational Capital Stocks in the European Sample across the Sectors (Median Values, 2017)

For all the employment categories, the share of R&D firms in the employment categories increases, but the increase is the largest in the two lowest-ranked employment categories
with 13.8%ps for the lowest- and 16.9%ps for the second lowest-ranked category as well as in the highest-ranked employment category with 5.4%ps.

The firms in the consumer durables, health care and chemicals sectors are on average equipped with the largest R&D stocks across the sectors\(^{87}\), while the firms in the chemicals, consumer durables, telephone and television as well as utilities sectors have on average the largest organizational capital stocks (Figure 4.7).

Analyzing the input distributions within the sectors, the (strictly positive) differences between the mean and median values for the R&D and organizational capital stocks for all the sectors indicate that many firms with only small stocks and some firms with large stocks coexist in all the sectors, whereas the right-skewed distribution varies in its shape across the sectors (Figure 4.8).

**North-American Sample on the Period 2010-2017 (NA-S2)**

The sample of North-American firms on the period 2010-2017 consists of 2,737 firms in 2017 whose headquarters are located each either in the United States or Canada and which operate each in one of 10 sectors.

While almost nine out of ten firms in the sample (2017: 87.2%) are located in the United States, about one-third (2017: 37.1%) of the firms operate in the manufacturing sector that thereby represents by far the largest sector in the sample, followed by the services sector in which 18.6% (2017) of the firms operate and the finance sector in which 15.9% (2017) of the firms operate (Figure 4.9 and Figure 4.10).

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\(^{87}\)For the details on the alternative sector classification: See the Appendix E.3.
operates in the finance sector, and, as compared with NA-S1, the manufacturing sector contracts as the share of firms that are active in it decreases by 4.3%ps (2007: 41.4%) over the period of 10 years.

![Figure 4.10: Distribution of Firms in the North-American Sample across the Sectors (2017)](image)

![Figure 4.11: Distribution of Non-R&D Firms and R&D Firms in the North-American Sample across the Employment Categories (2017)](image)

Notes: Firms in the sample are grouped according to their number of employees, which yields employment categories (on the left: category “175-749”: firms with at least 175 but less than 750 employees, category “750-2,499”: firms with at least 750 but less than 2,500 employees, category “2,500-7,499”: firms with at least 2,500 but less than 7,500 employees, etc.). Higher-ranked employment categories include firms with a larger workforce.

The firms with less than 2,500 employees represent about half (2017: 51.4%) of the firms in the sample and the firms in the two highest-ranked employment categories with at least 15,000 employees account for 16.5% (2017) of the firms, both types together constituting
about two-thirds of the sample population of firms and being similar to the European sample, whereas R&D firms are more present among the firms with a workforce at the small to medium scale, especially at the medium scale, and the very large scale (Figure 4.11).

Figure 4.12: Distribution of R&D and Organizational Capital Stocks in the North-American Sample across the Sectors (Mean Values, 2017)

Figure 4.13: Distribution of R&D and Organizational Capital Stocks in the North-American Sample across the Sectors (Median Values, 2017)

As compared with the European sample, the proportion of R&D firms in the employment categories is smaller for all the employment categories.
Comparing NA-S2 with NA-S1, the firm size distribution at the end of each sample changes in the way that the lowest-ranked employment category contracts, while the other categories expand as their weights are measured in percentage terms of each sample’s population of firms and their changes are measured in terms of percentage points. Over the 10-years period, the total mass of firms in the sample shifts to some degree upwards along the categorization of the firm size.

For some of the employment categories, the share of R&D firms in the employment categories increases, while it decreases for others, but the decrease is the largest in the lowest-ranked employment category with 9.6%ps and the increase is the largest in the middle-ranked employment categories with 3.5%ps for the third lowest- and 6.2%ps for the third highest-ranked category.

The firms in the health care, consumer durables and business equipment sectors are on average equipped with the largest R&D stocks across the sectors, while the firms in the telephone and television, trade and services, chemicals, consumer non-durables and health care sectors have on average the largest organizational capital stocks (Figure 4.12).

Analyzing the input distributions within the sectors, the (strictly positive) differences between the mean and median values for the R&D and organizational capital stocks for almost all the sectors indicate that many firms with only small stocks and some firms with large stocks coexist in all the sectors, whereas the right-skewed distribution varies in its shape across the sectors (Figure 4.13).

**Asian Sample on the Period 2010-2017 (A-S2)**

The sample of Asian firms on the period 2010-2017 consists of 6,198 firms in 2017 whose headquarters are located each in one of 16 countries in Asia and which operate each in one of 10 sectors.

![Figure 4.14: Distribution of Firms in the Asian Sample across the Countries (2017)](image)
While about three-fourths (2017: 75.6%) of the firms in the sample are located either in Japan or China, almost two-thirds (2017: 60.8%) of the firms operate in the manufacturing sector that thereby represents by far the largest sector in the sample, followed by the services sector in which 12.9% (2017) of the firms operate and the transportation sector in which 7.5% (2017) of the firms operate (Figure 4.14 and Figure 4.15).

Figure 4.15: Distribution of Firms in the Asian Sample across the Sectors (2017)

As compared with the European and North-American sample, a larger share of firms
is active in the manufacturing sector, while a smaller share operates in the services sector, and, as compared with A-S1, the manufacturing sector expands as the share of firms that are active in it increases by 4.5%ps (2007: 56.3%) over the period of 10 years.

The firms with less than 2,500 employees represent almost two-thirds (2017: 63.7%) of the firms in the sample and the firms in the two highest-ranked employment categories with at least 15,000 employees account for 8% (2017) of the firms, both types together constituting almost three-fourths of the sample population of firms and being different to the European and North-American samples with a larger proportion of firms with a workforce at a small scale and a smaller proportion of firms with a workforce at a large scale, whereas R&D firms are less present among the firms with a workforce at the very small scale (Figure 4.16). As compared with the European and North-American samples, the share of R&D firms in the employment categories is the highest for all the employment categories among the three samples.

Comparing A-S2 with A-S1 and being similar to the North-American sample, the firm size distribution at the end of each sample changes in the way that the lowest-ranked employment category contrasts, while the other categories expand as their weights are measured in percentage terms of each sample’s population of firms and their changes are measured in terms of percentage points. Over the 10-years period, the total mass of firms in the sample shifts to some degree upwards along the categorization of the firm size.

![Figure 4.17: Distribution of R&D and Organizational Capital Stocks in the Asian Sample across the Sectors (Mean Values, 2017)](image)

For the two highest-ranked employment categories, the share of R&D firms in the employment categories decreases, while it increases for all the others.

The firms in the consumer durables, health care, business equipment as well as chemicals sectors are on average equipped with the largest R&D stocks across the sectors, while the
firms in the telephone and television, consumer durables as well as energy, oil, gas and coal sectors have on average the largest organizational capital stocks (Figure 4.17).

Analysing the input distributions within the sectors, the (strictly positive) differences between the mean and median values for the R&D and organizational capital stocks for almost all the sectors indicate that many firms with only small stocks and some firms with large stocks coexist in all the sectors, whereas the right-skewed distribution varies in its shape across the sectors (Figure 4.18).

The magnitude of organizational capital in 2017 as one of the inputs in the firms’ production process is large across all the samples, although substantial differences exist across the regions (Europe, North-America and Asia) as well as the firm types (non-R&D firms and R&D firms) (Table 4.1): In the European sample, the median value of organizational capital is about 79.44 million EUR for the non-R&D firms and about twice as large (160.77) for the R&D firms. In the latter case, it is about 4.5 times larger than R&D (34.97) and yet almost 1.5 times larger than physical capital (114.9).

The median value of organizational capital is even larger in the North-American sample, in fact substantially by about 104% for the non-R&D firms and 68% for the R&D firms: 161.85 million USD for the non-R&D firms and 269.45 million USD for the R&D firms. In the latter case, organizational capital is about twice as large as both R&D (135.31) and physical capital (136.9).

In the Asian sample, organizational capital is the smallest among the three samples, all the more in the case in which Hong Kong, Japan, Korea, Singapore and Taiwan (HJKST as the country group’s corresponding acronym) as the countries with the highest levels of the Human Development Index (HDI) are excluded from the sample and only the countries

![Figure 4.18: Distribution of R&D and Organizational Capital Stocks in the Asian Sample across the Sectors (Median Values, 2017)](image)
at a lower development status in life expectancy, education and per capita income remain: With HJKST, the median value of organizational capital is about 45.24 million USD for the non-R&D firms, which accounts for about 28% and 57% of its median value in the North-American and European sample, respectively, and yet it is about 1.5 times larger (67.29) for the R&D firms, which amounts to about 25% and 42% of its median value in the North-American and European sample, respectively.

By excluding HJKST, the value decreases to about 20.78 million USD for the non-R&D firms, which represents about 13% and 26% of its median value in the North-American and European sample, respectively, and about 49.22 million USD for the R&D firms, which accounts for about 18% and 31% of its median value in the North-American and European sample, respectively.

<table>
<thead>
<tr>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-S2(^a)</td>
<td>E-S2(^b)</td>
</tr>
<tr>
<td>w/o HJKST</td>
<td>w/ HJKST</td>
</tr>
<tr>
<td>K</td>
<td>95.16227</td>
</tr>
<tr>
<td>L(^c)</td>
<td>1.9965</td>
</tr>
<tr>
<td>RD</td>
<td>22.82059</td>
</tr>
<tr>
<td>OC</td>
<td>49.22255</td>
</tr>
</tbody>
</table>

Notes: \(^a\): sample measured in million USD, \(^b\): sample measured in million EUR, \(^c\): employment measured in thousands, and HJKST: acronym for Hong Kong, Japan, Korea, Singapore and Taiwan.

Table 4.1: Physical Capital, Labor, Research and Development as well as Organizational Capital in the Three Samples and the Two Sub-samples (R&D and Non-R&D Firms) of Each Sample and 2017 (Median Values)

<table>
<thead>
<tr>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-S2</td>
<td>E-S2</td>
</tr>
<tr>
<td>w/o HJKST</td>
<td>w/ HJKST</td>
</tr>
<tr>
<td>K</td>
<td>3.56</td>
</tr>
<tr>
<td>L</td>
<td>2.69</td>
</tr>
<tr>
<td>RD</td>
<td>10.74</td>
</tr>
<tr>
<td>OC</td>
<td>7.70</td>
</tr>
</tbody>
</table>

Notes: Measured in percentage and HJKST: acronym for Hong Kong, Japan, Korea, Singapore and Taiwan.

Table 4.2: Growth of Physical Capital, Labor, Research and Development as well as Organizational Capital between 2016 and 2017 in the Three Samples and the Two Sub-samples (R&D and Non-R&D Firms) of Each Sample (Median Values)
For the R&D firms, the difference between the firms in the Asian and North-American sample is especially large in their equipment with the intangible assets, R&D and organizational capital, compared to physical capital and labour. While physical capital and labour in their median values represent about 70% to 72% and 69% to 79% of their values in the North-American sample, respectively, R&D and organizational capital in their median values only represent about 16% to 17% and 18% to 25% of their values in the North-American sample, respectively.

Organizational capital is always smaller than physical capital in the case of the non-R&D firms, where its median value accounts for about 27% of the median value of physical capital in the Asian sample without HJKST, for about 61% in the Asian sample with HJKST and for about 84% in the European and 89% in the North-American sample, and smaller than physical capital in the case of the R&D firms in the Asian sample, whereas its median value amounts to a larger share (52% and 68%, respectively) of the median value of physical capital than for the non-R&D firms in the Asian sample.

Overall, R&D firms possess in the median values more organizational capital than non-R&D firms and North-American firms possess more organizational capital than European firms and those in turn more than Asian firms. Compared with the year 2007, the median value of organizational capital approximately doubles on average for the non-R&D firms across the samples, less for the Asian sample with HJKST (increase by about 63%), averagely for the North-American sample (increase by about 97%) and more for the European sample as well as the Asian sample without HJKST (each revealing an increase by about 123%). For the R&D firms, the median value of organizational capital increases on average by 69%, but a large variance exists across the samples: While the median value of organizational capital increases in the Asian sample without HJKST and the North-American sample by about 165% and 116%, respectively, it increases in the European sample by only about 6% and even decreases in the Asian sample with HJKST by about 11%.

The dynamics of the four production inputs in the period 2016-2017 shows that organizational capital has the highest median value of the growth rate among all the inputs for the European sample and almost all the inputs for the other samples, even so its level is already high (Table 4.2). Only R&D grows in the median value at a larger rate than the intangible asset.

Overall, organizational capital as an input in the production process seems to be both a large and very dynamic asset whose relevance in the firm needs to be verified. Compared with the period 2006-2007, the median value of the growth rate of organizational capital is smaller for almost all the samples and sub-samples, with the only exception for the R&D firms in the Asian sample with HJKST. The dynamics or accumulation of organizational capital decelerates in the final one-year period of the samples over the 10 years.

Given the descriptive statistics on E-S2, NA-S2 and A-S2, the following estimations are evidently based on comprehensive data material which allows to draw meaningful conclusions on the general relevance of organizational capital in the firms’ production process.
Empirical Results

The relevance of the organizational capital as an input for the production process and output of firms is tested and verified by estimating production functions both in the Cobb-Douglas and in the translog specification, equation (4.46) and equation (4.47), respectively, for the three samples E-S2, NA-S2 and A-S2.

For the European sample on the period 2010-2017, the estimated output elasticity of organizational capital is both substantial and significant in the case of the application of the OLS estimator (Table 4.3 for the Cobb-Douglas specification and Table 4.4 for the translog specification).

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(w/o OC)</td>
<td></td>
</tr>
<tr>
<td>Levels FDs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0.29***</td>
<td>0.19***</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>L</td>
<td>0.62***</td>
<td>0.47***</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>RD</td>
<td>0.07***</td>
<td>0.31***</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>OC</td>
<td>0.21***</td>
<td>0.25***</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Obs.</td>
<td>1,138</td>
<td>825</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9280</td>
<td>0.8196</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

Table 4.3: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the European Sample

The resulting point estimates in the range between 0.21 and 0.51 are not only significant in both specifications when estimating the production functions in levels for the year 2017, but remain significant for the estimations in first differences for the period 2016-2017, with a smaller significance in the latter case for R&D firms in the Cobb-Douglas specification. The estimated output elasticity of organizational capital is always larger in the translog specification than in the Cobb-Douglas specification, it is always larger than the output elasticity of R&D and the latter decreases in its value if organizational capital is additionally taken into account in the production function estimation.

In the case of employing the Olley and Pakes (1996) estimation procedure instead of the OLS estimator to deal with and solve for its associated endogeneity bias (Table 4.5), the estimated output elasticity of organizational capital is still significant and at least as large as it is in the case of the OLS estimator, with resulting point estimates in the range between 0.23 and 0.65, which provides an unambiguous and robust indication of the existence of an
additional relevant input besides physical capital, labour and R&D.

Table 4.4: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the European Sample

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th></th>
<th>Non-R&amp;D firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Translog$^a$ (w/o OC)</td>
<td>Translog$^a$</td>
<td>Translog$^a$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.28***</td>
<td>0.07**</td>
<td>0.24***</td>
<td>0.08**</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
<td>0.65***</td>
<td>0.39***</td>
<td>0.49***</td>
<td>0.31***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>RD</td>
<td>0.07***</td>
<td>0.14***</td>
<td>0.05***</td>
<td>0.07***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>OC</td>
<td></td>
<td>0.24***</td>
<td>0.51***</td>
<td>0.41***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.02)</td>
<td>(0.07)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Obs.</td>
<td>1,138</td>
<td>1,112</td>
<td>1,138</td>
<td>1,111</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9340</td>
<td>0.3567</td>
<td>0.9454</td>
<td>0.4229</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and $^a$: evaluation at the sample medians.

Table 4.5: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the European Sample on the Period 2010-2017

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th></th>
<th>Non-R&amp;D firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog$^a$</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.07</td>
<td>0.21***</td>
<td>0.30***</td>
<td>-0.13**</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.02)</td>
<td>(0.08)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>L</td>
<td>0.64***</td>
<td>0.50***</td>
<td>0.76***</td>
<td>0.95***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>RD</td>
<td>0.05***</td>
<td>0.02***</td>
<td>0.17***</td>
<td>0.54***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.04)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>OC</td>
<td>0.23***</td>
<td></td>
<td>0.65***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td></td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>7,654</td>
<td>7,654</td>
<td>7,654</td>
<td>7,654</td>
</tr>
<tr>
<td>Groups</td>
<td>1,138</td>
<td>1,138</td>
<td>1,138</td>
<td>1,138</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and $^a$: evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.5: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the European Sample on the Period 2010-2017

Compared with the output elasticity estimations for E-S1, organizational capital gains in importance over the 10-years period as its estimated output elasticity increases in its value with only a few exceptions in the variety of specifications, in particular in the case of the OLS estimator for non-R&D firms in the Cobb-Douglas as well as the translog

For the North-American sample on the period 2010-2017, the estimated output elasticity of organizational capital is both substantial and significant in the case of the application of the OLS estimator (Table 4.6 for the Cobb-Douglas specification and Table 4.7 for the translog specification).

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas (w/o OC)</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.32***</td>
<td>0.16***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>L</td>
<td>0.56***</td>
<td>0.58***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>RD</td>
<td>0.13***</td>
<td>0.15**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>OC</td>
<td>0.41***</td>
<td>0.75***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Obs.</td>
<td>1,111</td>
<td>1,107</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9218</td>
<td>0.4332</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

Table 4.6: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the North-American Sample

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Translog&quot; (w/o OC)</td>
<td>Translog&quot;</td>
</tr>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.30***</td>
<td>0.14***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>L</td>
<td>0.58***</td>
<td>0.56***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>RD</td>
<td>0.15***</td>
<td>0.30***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>OC</td>
<td>0.43***</td>
<td>0.82***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Obs.</td>
<td>1,111</td>
<td>1,107</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9272</td>
<td>0.4512</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and " evaluation at the sample medians.

Table 4.7: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the North-American Sample
The resulting point estimates in the range between 0.41 and 1.01 are not only significant in both specifications when estimating the production functions in levels for the year 2017, but remain significant for the estimations in first differences for the period 2016-2017.

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Translog(^a)</th>
<th>Non-R&amp;D firms</th>
<th>Translog(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog(^a)</td>
<td>Cobb-Douglas</td>
<td>Translog(^a)</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.16***</td>
<td>0.23***</td>
<td>0.25***</td>
<td>0.22***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.07)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>L</td>
<td>0.54***</td>
<td>0.38***</td>
<td>0.57***</td>
<td>0.46***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>RD</td>
<td>0.09***</td>
<td>-0.05***</td>
<td>0.40***</td>
<td>0.05***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.14)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>OC</td>
<td>0.40***</td>
<td>0.53***</td>
<td>0.46***</td>
<td>0.51***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Obs.</td>
<td>7,786</td>
<td>7,786</td>
<td>7,786</td>
<td>11,782</td>
</tr>
<tr>
<td>Groups</td>
<td>1,111</td>
<td>1,111</td>
<td>1,111</td>
<td>1,682</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and \(^a\): evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.8: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the North-American Sample on the Period 2010-2017

The estimated output elasticity of organizational capital is for almost all cases larger in the translog specification than in the Cobb-Douglas specification, with the only exception for the non-R&D firms in levels estimation, it is always larger than the output elasticity of R&D and the latter decreases in its value, becomes negative and insignificant if organizational capital is additionally taken into account in the production function estimation.

In the case of employing the Olley and Pakes (1996) estimation procedure instead of the OLS estimator to deal with and solve for its associated endogeneity bias (Table 4.8), the estimated output elasticity of organizational capital is still significant and substantial, as the resulting point estimates in the range between 0.40 and 0.53 are larger than the corresponding output elasticity of any other input.

Compared with the output elasticity estimations for NA-S1, organizational capital gains in importance for some parts over the 10-years period as its estimated output elasticity increases in its value for some sub-samples and specifications, in particular in the case of the OLS estimator for non-R&D firms in the Cobb-Douglas specification and for all firms in the translog specification as well as in the case of the Olley and Pakes (1996) estimation procedure for R&D firms in the translog specification. Compared with the output elasticity estimations for E-S2, organizational capital has a greater impact on production outcome as its estimated output elasticity is for almost all cases larger in the North-American sample, with the only exceptions in the case of the Olley and Pakes (1996) estimation procedure for R&D as well as non-R&D firms in the translog specification.
For the Asian sample on the period 2010-2017, the estimated output elasticity of organizational capital is both substantial and significant in the case of the application of the OLS estimator (Table 4.9 for the Cobb-Douglas specification and Table 4.10 for the translog specification).

### Table 4.9: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the Asian Sample

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th></th>
<th>Non-R&amp;D firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas (w/o OC)</td>
<td>Cobb-Douglas</td>
<td></td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.30***</td>
<td>0.11***</td>
<td>0.21***</td>
<td>0.06**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
<td>0.56***</td>
<td>0.49***</td>
<td>0.33***</td>
<td>0.36***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.07)</td>
<td>(0.02)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>RD</td>
<td>0.11***</td>
<td>0.15***</td>
<td>0.02***</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>OC</td>
<td></td>
<td></td>
<td>0.47***</td>
<td>1.32***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.02)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Obs.</td>
<td>4,210</td>
<td>4,148</td>
<td>4,210</td>
<td>4,148</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.8143</td>
<td>0.2580</td>
<td>0.8578</td>
<td>0.3868</td>
</tr>
</tbody>
</table>

**Notes:** Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

Table 4.10: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the Asian Sample

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th></th>
<th>Non-R&amp;D firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Translog$^a$ (w/o OC)</td>
<td>Translog$^a$</td>
<td></td>
<td>Translog$^a$</td>
</tr>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.31***</td>
<td>0.14***</td>
<td>0.24***</td>
<td>0.07**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
<td>0.53***</td>
<td>0.50***</td>
<td>0.33***</td>
<td>0.35***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.07)</td>
<td>(0.02)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>RD</td>
<td>0.14***</td>
<td>0.33***</td>
<td>0.04***</td>
<td>0.07**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>OC</td>
<td></td>
<td></td>
<td>0.43***</td>
<td>1.26***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.02)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Obs.</td>
<td>4,210</td>
<td>4,148</td>
<td>4,210</td>
<td>4,148</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.8249</td>
<td>0.2867</td>
<td>0.8642</td>
<td>0.4188</td>
</tr>
</tbody>
</table>

**Notes:** Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and $^a$: evaluation at the sample medians.

The resulting point estimates in the range between 0.43 and 1.32 are not only significant...
in both specifications when estimating the production functions in levels for the year 2017, but remain significant for the estimations in first differences for the period 2016-2017.

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th></th>
<th>Non-R&amp;D firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog\textsuperscript{a}</td>
<td>Cobb-Douglas</td>
<td>Translog\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.10\textsuperscript{**}</td>
<td>0.13\textsuperscript{***}</td>
<td>0.35\textsuperscript{***}</td>
<td>0.41\textsuperscript{***}</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>L</td>
<td>0.41\textsuperscript{***}</td>
<td>0.24\textsuperscript{***}</td>
<td>0.37\textsuperscript{***}</td>
<td>-0.13\textsuperscript{**}</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>0.08\textsuperscript{***}</td>
<td>-0.02\textsuperscript{***}</td>
<td>0.07\textsuperscript{***}</td>
<td>-0.17\textsuperscript{***}</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.004)</td>
<td>(0.01)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>OC</td>
<td>0.54\textsuperscript{***}</td>
<td>0.10\textsuperscript{*}</td>
<td>0.10\textsuperscript{*}</td>
<td>0.54\textsuperscript{***}</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.06)</td>
<td>(0.01)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Obs.</td>
<td>21,923</td>
<td>21,923</td>
<td>21,923</td>
<td>21,923</td>
</tr>
<tr>
<td>Groups</td>
<td>4,210</td>
<td>4,210</td>
<td>4,210</td>
<td>4,210</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: \textsuperscript{*} 10\%, \textsuperscript{**} 5\%, and \textsuperscript{***} 1\%, and \textsuperscript{a}: evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.11: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the Asian Sample on the Period 2010-2017

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th></th>
<th>Non-R&amp;D firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog\textsuperscript{a}</td>
<td>Cobb-Douglas</td>
<td>Translog\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.14\textsuperscript{***}</td>
<td>0.21\textsuperscript{***}</td>
<td>0.33\textsuperscript{***}</td>
<td>0.30\textsuperscript{***}</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.00004)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>L</td>
<td>0.32\textsuperscript{***}</td>
<td>0.20\textsuperscript{***}</td>
<td>0.20\textsuperscript{***}</td>
<td>0.21\textsuperscript{***}</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.00002)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>0.13\textsuperscript{***}</td>
<td>0.01\textsuperscript{***}</td>
<td>0.06\textsuperscript{***}</td>
<td>0.03\textsuperscript{***}</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.004)</td>
<td>(0.00002)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>OC</td>
<td>0.47\textsuperscript{***}</td>
<td>0.46\textsuperscript{***}</td>
<td>0.46\textsuperscript{***}</td>
<td>0.46\textsuperscript{***}</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.01)</td>
<td>(0.004)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Obs.</td>
<td>16,247</td>
<td>16,247</td>
<td>16,247</td>
<td>16,247</td>
</tr>
<tr>
<td>Groups</td>
<td>3,128</td>
<td>3,128</td>
<td>3,128</td>
<td>3,128</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: \textsuperscript{*} 10\%, \textsuperscript{**} 5\%, and \textsuperscript{***} 1\%, and \textsuperscript{a}: evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.12: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the Asian Sample on the Period 2010-2017 (Only Manufacturing Firms)

The estimated output elasticity of organizational capital is for almost all cases larger in the Cobb-Douglas specification than in the translog specification, with the only exception for the non-R&D firms in levels estimation, it is always larger than the output elasticity of
CHAPTER 4. MULTI-PRODUCT FIRMS AND ORGANIZATIONAL CAPITAL

R&D and the latter decreases in its value, in part becomes negative and less significant or insignificant if organizational capital is additionally taken into account in the production function estimation.

In the case of employing the Olley and Pakes (1996) estimation procedure instead of the OLS estimator to deal with and solve for its associated endogeneity bias (Table 4.11), the estimated output elasticity of organizational capital is for almost all cases still significant and substantial, as the resulting point estimates in the range between 0.36 and 0.54 are larger than the corresponding output elasticity of any other input, with only a single exception: The output elasticity is smaller and less significant for R&D firms in the translog specification.

By restricting the sample to firms that operate in the manufacturing sector, which is motivated by the fact that this chapter has these firms in mind when conducting the analysis, and replicating the estimations, both a substantial value and high significance of the output elasticity of organizational capital are restored (Table 4.12).

<table>
<thead>
<tr>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cobb-Douglas (w/o OC)</strong></td>
<td><strong>Cobb-Douglas</strong></td>
</tr>
<tr>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.29***</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
<td>0.58***</td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>0.10***</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>OC</td>
<td>0.46***</td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.10)</td>
</tr>
</tbody>
</table>

Obs. 2,176 2,133 2,176 2,133 895 895
R-sq. 0.7567 0.2666 0.8000 0.4125 0.7812 0.2673

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

Table 4.13: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the Asian Sample without Hong Kong, Japan, Korea, Singapore and Taiwan

 Compared with the output elasticity estimations for A-S1, organizational capital gains in importance for some parts over the 10-years period as its estimated output elasticity increases in its value for some sub-samples and specifications, in particular in the case of the OLS estimator for R&D as well as non-R&D firms in the Cobb-Douglas specification and first-differences estimation and for R&D firms in the translog specification and first-differences estimation as well as non-R&D firms in the translog specification and levels estimation. Compared with the output elasticity estimations for E-S2, organization capital has a greater impact on production outcome as its estimated output elasticity is for almost all cases larger in the Asian sample, with the only exceptions in the case of the Olley

The countries in Asia that are covered by the Asian sample as they represent the locations of the headquarters of the firms in the sample reveal a heterogeneity in their economic as well as social development status, as distinguished from the countries in the European and North-American samples, and so belong to two different groups of a classification that is based on the Human Development Index (HDI) which represents a combined measure of the life expectancy, education and per capita income level in a country: A group of countries that feature low (0.555-0.699) or medium (0.700-0.799) levels of the HDI and a group of countries that feature high (0.800-1.000) levels of the HDI. To be able to disentangle whether the observed differences in the estimated output elasticity of organizational capital across the European, North-American and Asian samples go back to differences in the geographical location of the firm or the development status of the countries or country groups in which the firm is located, the estimations are also conducted on the Asian sample without the countries in the high-level group, namely Hong Kong, Japan, Korea, Singapore and Taiwan (HJKST as the country group’s acronym) (Table 4.13 for the Cobb-Douglas specification, Table 4.14 for the translog specification and Table 4.15 for the method by Olley and Pakes (1996)).

<table>
<thead>
<tr>
<th>R&amp;D firms</th>
<th>Translog(^a) (w/o OC)</th>
<th>Translog(^a)</th>
<th>Non-R&amp;D firms</th>
<th>Translog(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.27***</td>
<td>0.11***</td>
<td>0.23***</td>
<td>0.06*</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>L</td>
<td>0.54***</td>
<td>0.45***</td>
<td>0.34***</td>
<td>0.33***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.06)</td>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>0.17***</td>
<td>0.47***</td>
<td>0.08***</td>
<td>0.10**</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>OC</td>
<td>0.39***</td>
<td>1.32***</td>
<td>0.55***</td>
<td>0.99***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.09)</td>
<td>(0.04)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Obs.</td>
<td>2,176</td>
<td>2,133</td>
<td>2,176</td>
<td>2,133</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.7719</td>
<td>0.3121</td>
<td>0.8080</td>
<td>0.4442</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and \(^a\): evaluation at the sample medians.

Table 4.14: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the Asian Sample without Hong Kong, Japan, Korea, Singapore and Taiwan

For the Asian sample with HJKST and without HJKST, the estimated output elasticity of organizational capital is quite similar in its value and identical in its significance in the case of the OLS estimation in both the Cobb-Douglas and translog specification,

88 For the details on the countries’ HDI levels: See the Appendix E.1 and Appendix E.2
with deviations of less than or about 15% in the restricted sample, while the estimated coefficients on the other inputs change in part by a larger magnitude, especially those on R&D, which suggests that the geographical location of the firm and regional idiosyncrasies play a larger role for the impact of organizational capital on the firm outcome than the development status of the location of the firm. In the case of the estimation procedure by Olley and Pakes (1996), the estimated output elasticity is quite similar for the two samples in the Cobb-Douglas specification, but a sizeable upward deviation in its value and a higher significance for R&D firms exist in the restricted sample for the estimation in the translog specification, which suggests that the development status of the location of the firm has some influence on the impact of organizational capital. Alternatively, as the estimates are closer to the estimates for the Asian sample with HJKST and only manufacturing firms (Table 4.12), with deviations of less than 10%, the restriction of the sample to countries with a low or medium level of the HDI may also be correlated with a change in the sectoral composition of the sample, which suggests that the sectoral membership of the firm has some influence on the impact of organizational capital.

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog(^a)</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.08***</td>
<td>0.23***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>L</td>
<td>0.53***</td>
<td>0.26***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>0.08***</td>
<td>0.02**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>OC</td>
<td>0.48***</td>
<td>0.42***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.08)</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Obs.</th>
<th>Groups</th>
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</thead>
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</tr>
<tr>
<td>Non-R&amp;D firms</td>
<td>7,460</td>
<td>2,176</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: * 10%, ** 5%, and *** 1%, and \(^a\): evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.15: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the Asian Sample without Hong Kong, Japan, Korea, Singapore and Taiwan on the Period 2010-2017

Summarizing, even so the empirical validation approach reveals some variability in the relevance of organizational capital over time and regions, it turns out that organizational capital appears to be a relevant input in the firms’ production process with a positive and significant output elasticity.

Given the estimation results on E-S2, NA-S2 and A-S2, the integration of organizational capital into the theory of multi-product firms in international trade is both justified and required to get a realistic representation of the internal structure of multi-product firms.
4.4. EMPIRICAL SUPPORT

Robustness Checks

The estimations in this section serve by themselves as robustness checks for the outcomes of the previous studies, but they can in turn be backed up by a series of further robustness checks, whereas each suspends a single condition of the setting in which the estimations are conducted as well as their results are derived and is thereby able to clarify their dependency on the setting’s condition. In particular, the robustness checks consider the question of whether and to which degree the findings still hold if (i) the parameters with which the capitalization of the income statement item is conducted change (computation of organizational capital), (ii) the sample is restricted to a specific sector (sample), (iii) the output variable is captured by a different term (output variable) and (iv) the estimation method that is able to account for the endogeneity bias changes (estimation method).

<table>
<thead>
<tr>
<th>R&amp;D firms</th>
<th>Cobb-Douglas (w/o OC)</th>
<th>Cobb-Douglas</th>
<th>Non-R&amp;D firms</th>
<th>Cobb-Douglas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.29***</td>
<td>0.06*</td>
<td>0.25***</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>L</td>
<td>0.62***</td>
<td>0.43***</td>
<td>0.47***</td>
<td>0.40***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.07)</td>
<td>(0.03)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>RD</td>
<td>0.07***</td>
<td>0.11***</td>
<td>0.05***</td>
<td>0.09***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>OC</td>
<td>0.22***</td>
<td>0.19***</td>
<td>0.31***</td>
<td>0.24***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.08)</td>
<td>(0.03)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Obs.</td>
<td>1,138</td>
<td>1,112</td>
<td>1,138</td>
<td>1,111</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9280</td>
<td>0.3442</td>
<td>0.9377</td>
<td>0.3798</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

Table 4.16: Robustness: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the European Sample (Capitalization Rate of 10% and Depreciation Rate of 20% for the Organizational Capital Computation)

All the robustness checks that are depicted in this section are conducted for the European sample on the period 2010-2017. The results of the robustness checks for the other samples are not shown in this section, but are available upon request. They allow a similar conclusion.

First, the capitalization of the SGA expenses (XSQA) as the income statement item, which yields the proxy for the organizational capital, is conducted with a capitalization rate of 10%, instead of 20% as in the baseline specification, and a depreciation rate of 20%, instead of 10% as in the baseline specification, and the estimations are subsequently undertaken for the modified organizational capital computation (Table 4.16) for the Cobb-Douglas specification, Table 4.17 for the translog specification and Table 4.18 for the method by
CHAPTER 4. MULTI-PRODUCT FIRMS AND ORGANIZATIONAL CAPITAL


<table>
<thead>
<tr>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translog(^a) (w/o OC)</td>
<td>Translog(^a)</td>
</tr>
<tr>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.28***</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
<td>0.65***</td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>RD</td>
<td>0.07***</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>OC</td>
<td>0.26***</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>Obs.</td>
<td>1,138</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9340</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and \(^a\): evaluation at the sample medians.

Table 4.17: Robustness: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the European Sample (Capitalization Rate of 10% and Depreciation Rate of 20% for the Organizational Capital Computation)

<table>
<thead>
<tr>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobb-Douglas</td>
<td>Translog(^a)</td>
</tr>
<tr>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.07</td>
</tr>
<tr>
<td>(0.07)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>L</td>
<td>0.64***</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>RD</td>
<td>0.05***</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>OC</td>
<td>0.24***</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Obs.</td>
<td>7,654</td>
</tr>
<tr>
<td>Groups</td>
<td>1,138</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and \(^a\): evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.18: Robustness: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the European Sample on the Period 2010-2017 (Capitalization Rate of 10% and Depreciation Rate of 20% for the Organizational Capital Computation)

Even so the organizational capital decreases both in its absolute quantity and its quantity relative to the other inputs (decrease in its median value for the non-R&D firms by
about 69% and for the R&D firms by about 70% its output elasticity is still significant, with an even higher significance in the case of the OLS estimator for the R&D firms in the Cobb-Douglas specification and first-differences estimation, and the value of the elasticity is quite similar to the one in the baseline specification, with either no changes or up- as well as downward deviations between about 2% and about 12%.

Overall, the relevance of organizational capital is statistically verified independent of the choice of the capitalization parameters.

Second, the sample on which the production functions are estimated is restricted to the manufacturing sector, as it is motivated by the fact that this chapter has the firms in this sector in mind when conducting the analysis, and the estimations are subsequently undertaken for the modified sample (Table 4.19 for the Cobb-Douglas specification, Table 4.20 for the translog specification and Table 4.21 for the method by Olley and Pakes (1996)).

<table>
<thead>
<tr>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobb-Douglas (w/o OC)</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.33***</td>
</tr>
<tr>
<td>(0.02)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>L</td>
<td>0.60***</td>
</tr>
<tr>
<td>(0.03)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>RD</td>
<td>0.07***</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>OC</td>
<td>0.60***</td>
</tr>
<tr>
<td>(0.07)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>Obs.</td>
<td>725</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9419</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

Table 4.19: Robustness: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the European Sample (Only Manufacturing Firms)

For manufacturing firms only, the output elasticity of organizational capital is still significant, with a smaller significance in the case of the OLS estimator for the R&D firms in the Cobb-Douglas specification and first-differences estimation as well as in the case of the Olley and Pakes (1996) estimation procedure for the non-R&D firms in the translog specification, and the value of the elasticity is for most of the cases at least as large as it is in the baseline specification, with the only exception in the case of the Olley and Pakes (1996) estimation procedure for the R&D firms in the translog specification, where it has

89 Under the modified capitalization parameters, the median value of organizational capital for the non-R&D firms amounts to 24.64 million EUR, while for the R&D firms, it amounts to 49.03 million EUR (and cf. Table 4.1).
a lower level with a deviation of about 55%, and otherwise either no changes or upward deviations between about 17% and about 113%.

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Translog(a) (w/o OC)</td>
<td>Translog(a)</td>
</tr>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.31***</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>L</td>
<td>0.62***</td>
<td>0.36***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>RD</td>
<td>0.07***</td>
<td>0.18***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>OC</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>725</td>
<td>715</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9479</td>
<td>0.3780</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and \(a\): evaluation at the sample medians.

Table 4.20: Robustness: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the European Sample (Only Manufacturing Firms)

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog(a)</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.03</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>L</td>
<td>0.57***</td>
<td>0.39***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>RD</td>
<td>0.06***</td>
<td>0.03***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>OC</td>
<td>0.27***</td>
<td>0.29***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Obs.</td>
<td>5,052</td>
<td>5,052</td>
</tr>
<tr>
<td>Groups</td>
<td>725</td>
<td>725</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and \(a\): evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.21: Robustness: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the European Sample on the Period 2010-2017 (Only Manufacturing Firms)

Overall, differences in the value of the estimated output elasticity of organizational capital exist across the sectors, but the relevance of organizational capital is statistically verified independent of the sample choice.
### 4.4. EMPIRICAL SUPPORT

#### Table 4.22: Robustness: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the European Sample (Value Added as the Output Variable)

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms (w/o OC)</th>
<th></th>
<th></th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Levels FDs</td>
<td>Levels FDs</td>
<td>Levels FDs</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0.27*** 0.02</td>
<td>0.24*** 0.02</td>
<td>0.21*** 0.16*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.02) (0.07)</td>
<td>(0.02) (0.07)</td>
<td>(0.02) (0.09)</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>0.58*** 0.51***</td>
<td>0.47*** 0.48***</td>
<td>0.54*** 0.37***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.03) (0.08)</td>
<td>(0.04) (0.09)</td>
<td>(0.03) (0.07)</td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td>0.11*** 0.04</td>
<td>0.09*** 0.02</td>
<td>0.19*** 0.32*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01) (0.06)</td>
<td>(0.01) (0.06)</td>
<td>(0.02) (0.11)</td>
<td></td>
</tr>
<tr>
<td>OC</td>
<td>0.17*** 0.21**</td>
<td>0.19*** 0.32*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.03) (0.11)</td>
<td>(0.03) (0.08)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>1,009 988</td>
<td>1,009 987</td>
<td>718 714</td>
<td></td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.8946 0.1057</td>
<td>0.9003 0.1170</td>
<td>0.8264 0.1521</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

#### Table 4.23: Robustness: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2017) and First Differences (FDs, Years 2016-2017) for the European Sample (Value Added as the Output Variable)

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms (w/o OC)</th>
<th>Translog(^a) (w/o OC)</th>
<th>Translog(^a)</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Levels FDs</td>
<td>Levels FDs</td>
<td>Levels FDs</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>0.26*** 0.03</td>
<td>0.23*** 0.03</td>
<td>0.21*** 0.20**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.02) (0.06)</td>
<td>(0.02) (0.06)</td>
<td>(0.02) (0.11)</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>0.58*** 0.42***</td>
<td>0.44*** 0.35***</td>
<td>0.49*** 0.43***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04) (0.06)</td>
<td>(0.04) (0.06)</td>
<td>(0.03) (0.11)</td>
<td></td>
</tr>
<tr>
<td>RD</td>
<td>0.14*** 0.17***</td>
<td>0.12*** 0.11*</td>
<td>0.27*** 0.58***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.01) (0.06)</td>
<td>(0.01) (0.06)</td>
<td>(0.03) (0.10)</td>
<td></td>
</tr>
<tr>
<td>OC</td>
<td>0.20*** 0.46***</td>
<td></td>
<td>0.27*** 0.58***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.03) (0.10)</td>
<td></td>
<td>(0.03) (0.15)</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>1,009 988</td>
<td>1,009 987</td>
<td>718 714</td>
<td></td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9012 0.1239</td>
<td>0.9082 0.1472</td>
<td>0.8430 0.1984</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and \(^a\): evaluation at the sample medians.

Third, for the output variable, revenue is substituted by value added. Following thereby Imrohoroğlu and Tüzel (2014), value added is computed as the sales minus the materials. Sales is identified by yearly net sales or turnover (\textit{SALE}). Materials in turn is computed as the total expenses minus the labour expenses, which are identified by staff expense - wages and salaries (\textit{XSTFWS}). Total expenses in turn are computed as the sales minus the income, which is identified by operating income before depreciation and amortization.
Overall, value added is therefore given by the income plus the labour expenses and identified by operating income before depreciation and amortization plus staff expense - wages and salaries and the estimations are subsequently undertaken for the modified output variable (Table 4.22 for the Cobb-Douglas specification, Table 4.23 for the translog specification and Table 4.24 for the method by Olley and Pakes (1996)).

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.09</td>
<td>0.19***</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
<td>0.59***</td>
<td>0.51***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>RD</td>
<td>0.08***</td>
<td>0.06***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>OC</td>
<td>0.14***</td>
<td>0.25***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Obs.</td>
<td>6,716</td>
<td>6,716</td>
</tr>
<tr>
<td>Groups</td>
<td>1,009</td>
<td>1,009</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and <sup>a</sup>: evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.24: Robustness: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the European Sample on the Period 2010-2017 (Value Added as the Output Variable)

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.18***</td>
<td>0.21***</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>L</td>
<td>0.39***</td>
<td>0.30***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>RD</td>
<td>0.05***</td>
<td>0.03***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>OC</td>
<td>0.18***</td>
<td>0.48***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Obs.</td>
<td>5,844</td>
<td>5,844</td>
</tr>
<tr>
<td>Groups</td>
<td>875</td>
<td>875</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and <sup>a</sup>: evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table 4.25: Robustness: Output Elasticity Estimations with the Method by Levinsohn and Petrin (2003) for the European Sample on the Period 2010-2017
For value added as the output variable, the output elasticity of organizational capital is still significant, with a smaller significance in the case of the OLS estimator for the non-R&D firms in the Cobb-Douglas specification and first-differences estimation, and the value of the elasticity is for most of the cases smaller than it is in the baseline specification, with the only exception in the case of the OLS estimator for the non-R&D firms in both the Cobb-Douglas specification and translog specification and first-differences estimation, where it has a higher level with a deviation of 28% and 45%, respectively, and otherwise either no changes or downward deviations between about 4% and about 62%. However, the output elasticity is still quite substantial as it almost never has the lowest level among the inputs.

Overall, even so differences in the value of the estimated output elasticity of organizational capital exist across the output variables, the relevance of organizational capital is statistically verified independent of the choice of the output variable.

Forth, the estimations are conducted with the method by Levinsohn and Petrin (2003), instead of the method by Olley and Pakes (1996). Being structurally identical, the methods only differ in the choice of the “control variable”, based on Levinsohn and Petrin (2003)’s calculus that their variable is more likely to be strictly monotonic in the unobserved firm productivity, as it is required by its theoretical foundation: While Olley and Pakes (1996) use investments, Levinsohn and Petrin (2003) employ intermediate inputs or materials. These are computed as the production expenses, which are identified by cost of goods sold (COGS), minus the labour expenses, which are identified by staff expense - wages and salaries (XSTFWS). The estimations are subsequently undertaken with the modified estimation method (Table 4.25).

For the Levinsohn and Petrin (2003) procedure as the estimation method, the output elasticity of organizational capital is still significant and the value of the elasticity is for most of the cases smaller than it is in the baseline specification, with the only exception for the non-R&D firms in the translog specification, where it has a higher level with a deviation of about 22%, and otherwise downward deviations between about 22% and about 45%. However, the output elasticity is still quite substantial as it never has the lowest level among the inputs.

Overall, even so differences in the value of the estimated output elasticity of organizational capital exist across the estimation procedures, the relevance of organizational capital is statistically verified independent of the choice of the estimation method.

Summarizing, the main finding of this chapter’s empirical task passes through the robustness checks and thus the empirical sidestep provides strong evidence for the statement of the literature of organizational capital to which this section contributes with the analysis: Organizational capital is a relevant and thus non-negligible input in the production process of firms.

Given the empirically significant and robust finding, organizational capital ought to be included into the theory of multi-product firms in international trade.
4.4.2 Organizational Capital, Productivity and Product Scope

In an empirical exercise, this section tests and thereby verifies some of the basic predictions of the developed theory on multi-product firms that are characterized by two dimensions of firm heterogeneity, i.e. productivity and organizational capital: In heterogeneous industries, multi-product firms with a higher productivity and multi-product firms with more organizational capital have a larger product scope. Using Orbis-Amadeus as the databases, which provide unbalanced panel data on the firm level for the period 2007-2017, simple regressions on the firm characteristics are undertaken for a single point in time (2017):

\[ \ln (ps_f) = a_0 + a_1 \ln (fc_f) + \varepsilon_f , \]

where \( ps_f \) denotes the product scope of firm \( f \), \( fc_f \) represents the respective characteristic of firm \( f \), in particular productivity and organizational capital, and \( \varepsilon_f \) denotes the error term. In the Orbis-Amadeus database, product scope is identified by the number of products that firm \( f \) produces, whereas products are defined as 5-digit Standard Industrial Classification (SIC) categories. To obtain both the productivity variable and the organizational capital variable, data in the Orbis-Amadeus database are matched with data in the COMPUSTAT - Capital IQ - Global database according to the firms’ identification codes. Productivity is then computed with the Olley and Pakes (1996) procedure as the estimation method\(^90\) and organizational capital is identified by the capitalized income statement item “selling, general and administrative (SGA) expenses” \((XSGA)\) as a proxy for organizational capital. The capitalization of the income statement item is conducted by using the perpetual inventory method with a capitalization rate of 20%, a depreciation rate of 10% and a growth rate of 6%\(^91\). The sample is restricted to the manufacturing sector in France, Germany and the United Kingdom and covers in total 847 large- and medium-sized firms.

<table>
<thead>
<tr>
<th></th>
<th>Machinery</th>
<th>Textiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>( TFP_f )</td>
<td>0.036***</td>
<td>0.021*</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>( OC_f )</td>
<td>0.043***</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>Model Fit ((R^2))</td>
<td>0.89</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.79</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%.

Table 4.26: Regressions - Correlation between the firm product scope and total factor productivity \((TFP_f)\) or organizational capital \((OC_f)\)

The regression is separately undertaken for the industries which are defined as 2-digit SIC categories and its results for the machinery industry (SIC code: 35) as well as textiles

\(^90\)See Section 4.4.1 for the details of the method.

\(^91\)See Section 4.4.1 for the details of the method.
industry (SIC code: 23) are presented by Table 4.26. Verifying the model’s basic predictions, firms in a heterogeneous industry, i.e. machinery, that have a higher productivity or more organizational capital typically have a larger product scope, with the respective elasticities varying between 3.6% and 4.3%. In contrast, for a homogeneous industry, i.e. textiles, the positive and statistically (high) significant relationships do not hold anymore, as predicted by the model.

As a robustness check, the regression is one by one conducted with (i) productivity estimations based on the Levinsohn and Petrin (2003) procedure and (ii) organizational capital computations based on the capitalization of the SGA expenses (XSGA) with varying capitalization and depreciation rates (10% instead of 20% as well as 20% instead of 10%). In addition, the regression is undertaken for (iii) different periods (2000-2010 and 2005-2015) and points in time (2010 and 2015). All the results of the alternative specifications do not vary systematically from the presented baseline specification and are thus not further documented here.

Summarizing, the empirical exercise on manufacturing firms in three European countries verifies the model’s basic predictions that firms in heterogeneous industries that have a higher productivity or more organizational capital have a larger product scope, in contrast to firms in homogeneous industries for which the relationship does not hold.

4.5 Conclusion

Based on the traditional resource-based theory of firms, this chapter develops a theory of multi-product firms with both dis-economies of scope and asymmetric products in international trade. Firms with a flexible manufacturing technology in production are endowed with some production cost-reducing resource, called organizational capital, which is (in the short run) for each firm limited in volume and for which firms have to decide over its allocation across the product mix. In (sufficiently) heterogeneous industries, multi-product firms concentrate in their organizational capital allocation on varieties close to their core competency variety and operate with a larger product scope when they are endowed with a larger amount of organizational capital. Furthermore, more productive multi-product firms have a larger product scope. Opening up to international trade, firms adjust their allocation of organizational capital, their intensive and extensive margin in response to changes in the cost parameters of trade. A trade liberalization induces the resource-endowed multi-product firms to increase their product scope, to allocate less organizational capital to each intra-marginal product and to increase their overall sales (revenue). Under some (standard) functional specification for the production costs, firms with a larger scope respond less to trade shocks, both at the extensive and intensive margin.

For firm-level data on production factors and performance, some of the basic predictions of the theory are consistent with the provided empirical evidence and organizational capital in general is shown to be a relevant factor in the production process of firms, which provides
(i) direct and (ii) indirect empirical support for the developed theory of multi-product firms in international trade.
D.1 Product Variety Demand

Each individual maximizes the utility subject to the budget constraint and thereby derives the individual demand for the product variety $\omega \in \Omega_k$. To implement this technically, the Lagrange function $L$ is established and its derivative with respect to $q_k(\omega)$ is set equal to zero, as it is required by the Lagrange theorem to compute the optimum of the objective function:

$$L = \left[ \int_{\Omega_k} q_k(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} + \lambda_k \left[ \gamma_k y - \int_{\Omega_k} p_k(\omega)q_k(\omega)d\omega \right],$$

with $\lambda_k > 0$ as the (industry-specific) Lagrange multiplier, and

$$\frac{\partial L}{\partial q_k(\omega)} = \frac{\sigma}{\sigma-1} \left[ \int_{\Omega_k} q_k(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma-1}{\sigma}} - \frac{1}{\sigma} q_k(\omega)^{\frac{\sigma-1}{\sigma}-1} - \lambda_k p_k(\omega) \overset{!}{=} 0,$$

which yields

$$q_k(\omega)^{-\frac{1}{\sigma}} \left[ \int_{\Omega_k} q_k(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{1}{\sigma-1}} = \lambda_k p_k(\omega).$$

By making use of the utility index $U_k$, the first-order condition can be rewritten as

$$q_k(\omega)^{-\frac{1}{\sigma}} U_k^{\frac{1}{\sigma}} = \lambda_k p_k(\omega) \iff q_k(\omega) = U_k^{\frac{1}{\sigma}} \lambda_k^{-\sigma} p_k(\omega)^{-\sigma}. \quad (D.1)$$

To determine a term for $U_k \lambda_k^{-\sigma}$, $q_k(\omega)$ by itself is plugged into the budget constraint

$$\gamma_k y = \int_{\Omega_k} p_k(\omega)q_k(\omega)d\omega = U_k \lambda_k^{-\sigma} \int_{\Omega_k} p_k(\omega)^{1-\sigma}d\omega,$$

and the latter rearranged to get

$$\gamma_k y \left[ \int_{\Omega_k} p_k(\omega)^{1-\sigma}d\omega \right]^{-1} = U_k \lambda_k^{-\sigma},$$
while its substitution back into the right equation of (D.1) yields
\[ q_k(\omega) = \gamma_k y \left[ \int_{\Omega_k} p_k(\omega)^{1-\sigma} d\omega \right]^{-1} p_k(\omega)^{-\sigma}. \]

Aggregating the individual demand over the \( L \) individuals provides the aggregate demand for the product variety \( \omega \in \Omega_k \), \( x_k(\omega) = L q_k(\omega) \),
\[ x_k(\omega) = L \gamma_k y \left[ \int_{\Omega_k} p_k(\omega)^{1-\sigma} d\omega \right]^{-1} p_k(\omega)^{-\sigma}, \]

which can be written as
\[ x_k(\omega) = L \gamma_k y P_k^{\sigma-1} p_k(\omega)^{-\sigma}, \]
with \( P_k \equiv \left[ \int_{\Omega_k} p_k(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \) as the CES price index of industry \( k \), and, by summarizing the variety-invariant parameters and aggregates, written as
\[ x_k(\omega) = A_k p_k(\omega)^{-\sigma}, \] (D.2)

with \( A_k \equiv L \gamma_k y P_k^{\sigma-1} \) as the residual demand of industry \( k \).

## D.2 Additive Cost Specification

For an additive specification, the marginal production costs of variety \( \omega \) take the form
\[ c(\omega, z) = awz n(\omega) \quad \text{with} \quad n(\omega) \equiv h(\omega) + o(\omega)^{-\theta} \] (D.3)
and have the properties that they are increasing in the unit production costs
\[ \frac{\partial c(\omega, z)}{\partial h(\omega)} = awz \frac{\partial n(\omega)}{\partial h(\omega)} = awz > 0, \] (D.4)
decreasing in the allocated organizational capital
\[ \frac{\partial c(\omega, z)}{\partial o(\omega)} = awz \frac{\partial n(\omega)}{\partial o(\omega)} = -\theta awzo(\omega)^{-\theta-1} < 0 \] (D.5)
and their overall change due to the production of some variety with less competency
\[ \frac{\partial c(\omega, z)}{\partial \omega} = awz \frac{\partial n(\omega)}{\partial \omega} = awz \left( \frac{\partial h(\omega)}{\partial \omega} - \theta o(\omega)^{-\theta-1} \frac{\partial o(\omega)}{\partial \omega} \right) \] (D.6)
D.3. MULTIPLICATIVE COST SPECIFICATION WITH SPILLOVERS

depends on the change in the flexible manufacturing technology and the adjusted semi-elasticity of the organizational capital allocation:

\[ \frac{\partial c(\omega, z)}{\partial \omega} \gtrless 0 \iff \frac{\partial h(\omega)}{\partial \omega} \gtrless 0 \theta o(\omega)^{-1} \frac{\partial o(\omega)}{\partial \omega} o(\omega)^{-\theta}. \]  (D.7)

D.3 Multiplicative Cost Specification with Spillovers

In a study on US manufacturing firms over the period 1982-2011, Chen and Inklaar (2016) do not find any productivity spillovers of organizational capital between firms, but the existence of spillovers between products within a firm is not examined by the authors and thus can not be rejected by their work. For a multiplicative specification with product-level spillovers of organizational capital, the marginal production costs of variety \( \omega \) can take the form

\[ c(\omega, z) = awzn(\omega) + \tau O_{-\omega} \text{ with } n(\omega) = h(\omega) o(\omega)^{-\theta} \]  (D.8)

and with \( O_{-\omega} \equiv \int_{i \in \{0, \ldots, \delta\} \setminus \{\omega, \ldots, \delta\}} o(i)^{-\theta} di \) as the aggregate organizational capital allocated to varieties closer to the core competency than variety \( \omega \) by itself as well as \( \tau \in (0, 1) \) as the parameter of the strength of the organizational capital spillovers. Equation (D.8) can be rewritten in the way that

\[ c(\omega, z) = (awzh(\omega) - \tau) o(\omega)^{-\theta} + \tau \int_{0}^{\omega} o(i)^{-\theta} di \]  (D.9)

and the marginal production costs therefore have the properties that they are increasing in the unit production costs

\[ \frac{\partial c(\omega, z)}{\partial h(\omega)} = awzo(\omega)^{-\theta} > 0, \]  (D.10)

decreasing in the allocated organizational capital

\[ \frac{\partial c(\omega, z)}{\partial o(\omega)} = -\theta (awzh(\omega) - \tau) o(\omega)^{-\theta-1} - \tau o(\omega)^{-\theta-1} = -\theta awzh(\omega) o(\omega)^{-\theta-1} < 0 \]  (D.11)

and their overall change due to the production of some variety with less competency is given by

\[ \frac{\partial c(\omega, z)}{\partial \omega} = -\theta (awzh(\omega) - \tau) o(\omega)^{-\theta-1} \frac{\partial o(\omega)}{\partial \omega} + o(\omega)^{-\theta} awz \frac{\partial h(\omega)}{\partial \omega} + \tau o(\omega)^{-\theta}, \]  (D.12)

which can be simplified to

\[ \frac{\partial c(\omega, z)}{\partial \omega} = \left( -\theta (awzh(\omega) - \tau) o(\omega)^{-1} \frac{\partial o(\omega)}{\partial \omega} + awz \frac{\partial h(\omega)}{\partial \omega} + \tau \right) o(\omega)^{-\theta}. \]
Therefore, the overall change in the marginal production costs depends on the adjusted change in the flexible manufacturing technology, the adjusted semi-elasticity of the organizational capital allocation and the spillovers parameter:

\[
\frac{\partial c(\omega, z)}{\partial \omega} \overset{\text{LHS}}{=} \frac{\partial h(\omega)}{\partial \omega} \overset{\text{RHS}}{=} \theta a w z h(\omega) \frac{\partial o(\omega)}{\partial \omega} o(\omega)^{-1} - \tau \left( \theta \frac{\partial o(\omega)}{\partial \omega} o(\omega)^{-1} + 1 \right) . \tag{D.13}
\]

With \( \tau = 0 \), the cost specification reduces to the simple multiplicative cost specification, but it differs in the case that \( \tau \in (0, 1) \) holds, i.e. the existence of spillovers of organizational capital:

1. In case of an increasing allocation of organizational capital, \( \frac{\partial o(\omega)}{\partial \omega} > 0 \), term (1) and term (2) are positive and the spillovers by themselves make it more likely that LHS > RHS holds and that an increasing cost profile is given. Therefore, the spillovers structurally counteract the (decreasing) cost profile that is implied by the organizational capital allocation.

2. In case of a decreasing allocation of organizational capital, \( \frac{\partial o(\omega)}{\partial \omega} < 0 \), term (1) is always negative and the sign of term (2) depends on the magnitude of the decrease in the allocation of organizational capital:

For the case that (i) the decrease is only limited, i.e. \( \theta \frac{\partial o(\omega)}{\partial \omega} o(\omega)^{-1} \in (-1, 0) \), term (2) is positive and the spillovers just reinforce the increasing cost profile. However, for the case that (ii) the decrease is not limited, i.e. \( \theta \frac{\partial o(\omega)}{\partial \omega} o(\omega)^{-1} \in (-\infty, -1) \), term (2) is negative and the spillovers by themselves make it more likely that LHS < RHS holds and that a decreasing cost profile is given, which is even more likely in case of stronger spillovers, i.e. the larger the spillovers parameter \( \tau \). Therefore, the spillovers structurally counteract the (increasing) cost profile that is implied by the organizational capital allocation.

In sum, spillovers of organizational capital have counteracting effects on the cost profile that is implied by the organizational capital allocation, partly provided that the allocation of organizational capital is sufficiently strong in its change across the product varieties.

### D.4 Organizational Efficiency

By incorporating a firm’s ability of making use of the organizational capital, which is called organizational efficiency \( \tilde{\theta} \) and which can be related to the production efficiency \( \frac{1}{z} \) for making use of labor, the composite cost component can be written in the form

\[
n_k(\omega, \tilde{\theta}) = h_k(\omega) o_k(\omega)^{-\tilde{\theta}} \tag{D.14}
\]
with $\tilde{\theta} \in [0,1]$. Since a higher organizational efficiency $\tilde{\theta}$ should be associated with lower production costs for some allocated amount of organizational capital, it is required that

$$\frac{\partial c_k(\omega, z, \tilde{\theta})}{\partial \tilde{\theta}} = \frac{\partial c_k(\omega, z, \tilde{\theta})}{\partial \ln c_k(\omega, z, \tilde{\theta})} = -c_k(\omega, z, \tilde{\theta}) \ln o_k(\omega) \leq 0,$$

which is satisfied for the case that $o_k(\omega) \geq 1$. Therefore, the marginal production costs in case of an organizational efficiency parameter $\tilde{\theta}$ are written as

$$c_k(\omega, z, \tilde{\theta}) = \begin{cases} \text{awzn}_k(\omega, \tilde{\theta}) = \text{awzh}_k(\omega) o_k(\omega) - \tilde{\theta} & \text{for } o_k(\omega) \geq 1 \\ +\infty & \text{otherwise} \end{cases} \quad (D.15)$$

### D.5 Price Setting

To set the optimal price, the firm solves the following profit maximization problem:

$$\max_{p_k(\omega, z, O)} \Pi_k(z, O) = \int_0^{\delta_k(z, O)} [(p_k(\omega, z, O) - c_k(\omega, z, O)) x_k(\omega, z, O) - f_{k,\omega}] d\omega.$$

With $x_k(\omega, z, O) = A_k p_k(\omega, z, O)^{-\sigma}$ as the (received) product demand, the profit function can be written as

$$\Pi_k(z, O) = \int_0^{\delta_k(z, O)} [(p_k(\omega, z, O) - c_k(\omega, z, O)) A_k p_k(\omega, z, O)^{-\sigma} - f_{k,\omega}] d\omega$$

and its derivative with respect to $p_k(\omega, z, O)$ is set equal to zero:

$$\frac{\partial \Pi_k(z, O)}{\partial p_k(\omega, z, O)} = A_k p_k(\omega, z, O)^{-\sigma} + (p_k(\omega, z, O) - c_k(\omega, z, O)) (-\sigma) A_k p_k(\omega, z, O)^{-\sigma-1} \overset{=} 0.$$

Rearranging the first-order condition to isolate the price yields

$$p_k(\omega, z, O) = \frac{\sigma}{\sigma-1} c_k(\omega, z, O). \quad (D.16)$$

### D.6 Profit Function

The profit function of a firm with $z$ and $O$, $\Pi_k(z, O)$, is (strictly) monotonically increasing in the allocated organizational capital $o_k(\omega)$. It is given by

$$\Pi_k(z, O) = \int_0^{\delta_k(z, O)} [\zeta_k(z) h_k(\omega)^{1-\sigma} o_k(\omega)^{\theta(\sigma-1)} - f_{k,\omega}] d\omega \quad (D.17)$$
with \( \zeta_k(z) \equiv \frac{A_k}{\sigma} \left( \frac{1}{\sigma} \right)^{\sigma-1} > 0 \). The first and second derivatives with respect to \( o_k(\omega) \) are then be written as

\[
\frac{\partial \Pi_k(z, O)}{\partial o_k(\omega)} = \zeta_k(z) h_k(\omega)^{1-\sigma} \theta(\sigma - 1) o_k(\omega)^{\theta(\sigma-1)-1} > 0 \quad \text{and} \quad (D.18)
\]

\[
\frac{\partial^2 \Pi_k(z, O)}{\partial o_k(\omega)^2} = \zeta_k(z) h_k(\omega)^{1-\sigma} \theta(\sigma - 1) (\theta(\sigma - 1) - 1) o_k(\omega)^{\theta(\sigma-1)-2} \geq 0, \quad (D.19)
\]

respectively. Depending on the elasticity of substitution across product varieties within the industries, the second derivative is positive or negative and the profit function therefore convex or concave in the organizational capital allocation. For a sufficiently high elasticity of substitution (i.e. \( \sigma > 1 + \frac{1}{\theta} \)), the profit function is convex in \( o_k(\omega) \), while for industries with less substitutable product varieties (i.e. \( \sigma < 1 + \frac{1}{\theta} \)), the second derivative becomes negative and the profit function concave in the organizational capital allocation. The shape of the profit function and therefore industry properties have important implications for the chosen structure of the organizational capital allocation by multi-product firms.

**D.7 Organizational Capital Allocation**

To choose the optimal organizational capital allocation, the firm solves the following profit maximization problem:

\[
\max_{o_k(\omega)} \Pi_k(z, O) = \int_{0}^{\delta_k(z, O)} \left[ \zeta_k(z) h_k(\omega)^{1-\sigma} o_k(\omega)^{\theta(\sigma-1) - f_k(\omega)} \right] d\omega \quad \text{s.t.} \quad \int_{0}^{\delta_k(z, O)} o_k(\omega) d\omega = O.
\]

To implement this technically, the Lagrange function \( L \) is established and its derivative with respect to \( o_k(\omega) \) is set equal to zero, as it is required by the Lagrange theorem to compute the optimum of the objective function:

\[
L = \int_{0}^{\delta_k(z, O)} \left[ \zeta_k(z) h_k(\omega)^{1-\sigma} o_k(\omega)^{\theta(\sigma-1) - f_k(\omega)} \right] d\omega + \lambda_k \left[ O - \int_{0}^{\delta_k(z, O)} o_k(\omega) d\omega \right]
\]

with \( \lambda_k > 0 \) as the (industry-specific) Lagrange multiplier and

\[
\frac{\partial L}{\partial o_k(\omega)} = \zeta_k(z) h_k(\omega)^{1-\sigma} \theta(\sigma - 1) o_k(\omega)^{\theta(\sigma-1)-1} - \lambda_k \frac{1}{\theta(\sigma - 1) \zeta_k(z) h_k(\omega)^{\sigma-1}} = 0.
\]

Rearranging the first-order condition to isolate the organizational capital allocation yields

\[
o_k(\omega) = \left( \frac{1}{\theta(\sigma - 1) \zeta_k(z) h_k(\omega)^{\sigma-1} \lambda_k} \right)^{\frac{1}{\theta(\sigma-1)-1}}. \quad (D.20)
\]
Under the assumption that the product scope $\delta_k$ is kept constant, the derivative of equation (D.20) with respect to $\omega$ is given by

$$\frac{\partial o_k(\omega)}{\partial \omega} \bigg|_{\delta_k=\text{const.}} = \frac{1}{\theta(\sigma - 1) - 1} \left( \frac{1}{\theta(\sigma - 1)\zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right) \left( \frac{1}{\theta(\sigma - 1)\zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right)^{\frac{1}{\theta(\sigma - 1) - 1}} h_k(\omega)^{-\frac{1}{\theta(\sigma - 1) - 1}} \frac{\partial h_k(\omega)}{\partial \omega}.$$

Simplifying the derivative then yields

$$\frac{\partial o_k(\omega)}{\partial \omega} \bigg|_{\delta_k=\text{const.}} = \sigma - 1 \frac{1}{\theta(\sigma - 1) - 1} \left( \frac{1}{\theta(\sigma - 1)\zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right) \left( \frac{1}{\theta(\sigma - 1)\zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right)^{\frac{1}{\theta(\sigma - 1) - 1}} h_k(\omega)^{-\frac{1}{\theta(\sigma - 1) - 1}} \frac{\partial h_k(\omega)}{\partial \omega}.$$

### D.8 (Marginal) Cost Profile

With the organizational capital allocation $o_k(\omega)$ in equation (4.15), the composite cost component can be written as

$$n_k(\omega) = h_k(\omega) o_k(\omega)^{-\theta} = h_k(\omega) \left( \frac{1}{\theta(\sigma - 1)\zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right)^{-\frac{1}{\theta(\sigma - 1) - 1}}.$$

Simplifying this equation then yields

$$n_k(\omega) = \left( \frac{\theta(\sigma - 1)\zeta_k(z)}{\lambda_k} \right)^{\frac{\theta}{\theta(\sigma - 1) - 1}} h_k(\omega)^{-\frac{1}{\theta(\sigma - 1) - 1}} = \psi_k(z) h_k(\omega)^{-\frac{1}{\theta(\sigma - 1) - 1}}, \quad (D.21)$$

with $\psi_k(z) \equiv \left( \frac{\theta(\sigma - 1)\zeta_k(z)}{\lambda_k} \right)^{\frac{\theta}{\theta(\sigma - 1) - 1}} > 0$ as the variety-invariant constant of the composite cost component.

Under the assumption that the product scope $\delta_k$ is kept constant, the derivative of equation (D.21) with respect to $\omega$ is given by

$$\frac{\partial n_k(\omega)}{\partial \omega} \bigg|_{\delta_k=\text{const.}} = \psi_k(z) \left( -\frac{1}{\theta(\sigma - 1) - 1} \right) h_k(\omega)^{-\frac{1}{\theta(\sigma - 1) - 1} - 1} \frac{\partial h_k(\omega)}{\partial \omega},$$

which simplifies to

$$\frac{\partial n_k(\omega)}{\partial \omega} \bigg|_{\delta_k=\text{const.}} = -\psi_k(z) \theta(\sigma - 1) - 1 \frac{\partial h_k(\omega)}{\partial \omega}.$$
D.9 Marginal Profitability of Organizational Capital

With the organizational capital allocation \( o_k(\omega) \) in equation (4.15), the resource constraint that is binding can be written as

\[
\int_0^{\delta_k(z,O)} \left( \frac{1}{\theta(\sigma - 1) \zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right)^{\frac{1}{(\sigma - 1)-1}} d\omega = O
\]

and rearranging yields

\[
\lambda_k^{\frac{1}{(\sigma - 1)-1}} \left( \frac{1}{\theta(\sigma - 1) \zeta_k(z)} \right)^{\frac{1}{(\sigma - 1)-1}} \int_0^{\delta_k(z,O)} h_k(\omega)^{\sigma - 1} d\omega = O.
\]

To present the marginal profitability of organizational capital in a more compact way, an (aggregate) index of the flexible manufacturing technology is defined:

\[
H_k(\delta_k(z,O)) \equiv \left[ \int_0^{\delta_k(z,O)} h_k(\omega)^{\sigma - 1} \right]^{\frac{1}{\theta(\sigma - 1)-1}}.
\]

Its derivative with respect to \( \delta_k(z,O) \) is then given by

\[
\frac{\partial H_k(\delta_k(z,O))}{\partial \delta_k(z,O)} = \frac{\theta(\sigma - 1) - 1}{\sigma - 1} \int_0^{\delta_k(z,O)} h_k(\omega)^{\sigma - 1} d\omega
\]

and can be simplified to

\[
\frac{\partial H_k(\delta_k(z,O))}{\partial \delta_k(z,O)} = \frac{\theta(\sigma - 1) - 1}{\sigma - 1} H_k(\delta_k(z,O))^{\frac{\theta(\sigma - 1) - \sigma}{\sigma - 1 - 1}} h_k(\delta_k(z,O))^{\frac{\sigma - 1}{\sigma - 1 - 1}},
\]

with the result that the relation

\[
\frac{\partial H_k(\delta_k(z,O))}{\partial \delta_k(z,O)} \geq 0 \iff \sigma \geq 1 + \frac{1}{\theta}
\]

holds.

Deploying the flexible manufacturing technology index in equation (D.22) provides

\[
\lambda_k^{\frac{1}{(\sigma - 1)-1}} \left( \frac{1}{\theta(\sigma - 1) \zeta_k(z)} \right)^{\frac{1}{(\sigma - 1)-1}} H_k(\delta_k(z,O))^{\frac{\sigma - 1}{\sigma - 1 - 1}} = O
\]

and finally by rearranging

\[
\lambda_k(z,O, \delta_k(z,O)) = \theta(\sigma - 1) \zeta_k(z) H_k(\delta_k(z,O))^{1 - \sigma} O^{\theta(\sigma - 1)-1}.
\]

The derivatives of the marginal profitability of organizational capital with respect to its
arguments are given by
\[
\frac{\partial \lambda_k(z, O, \delta_k(z, O))}{\partial z} = \theta(\sigma - 1) H_k(\delta_k(z, O))^{1-\sigma} O^{\theta(\sigma - 1) - 1} \frac{\partial \zeta_k(z)}{\partial z},
\]
\[
\frac{\partial \lambda_k(z, O, \delta_k(z, O))}{\partial O} = (\theta(\sigma - 1) - 1) \theta(\sigma - 1) \zeta_k(z) H_k(\delta_k(z, O))^{1-\sigma} O^{\theta(\sigma - 1) - 2}
\]
and
\[
\frac{\partial \lambda_k(z, O, \delta_k(z, O))}{\partial \delta_k(z, O)} = -\theta(\sigma - 1)^2 \zeta_k(z) O^{\theta(\sigma - 1) - 1} H_k(\delta_k(z, O))^{-\sigma} \frac{\partial H_k(\delta_k(z, O))}{\partial \delta_k(z, O)},
\]
with
\[
\frac{\partial \lambda_k}{\partial z} < 0 \quad \text{and} \quad \frac{\partial \lambda_k}{\partial O} \gtrless 0 \iff \sigma \gtrless 1 + \frac{1}{\theta}
\]
as well as
\[
\frac{\partial \lambda_k}{\partial \delta_k} \gtrless 0 \iff \sigma \lesssim 1 + \frac{1}{\theta}.
\]

D.10 Product Scope Choice

To choose the optimal product scope, the firm solves the following profit maximization problem:
\[
\max_{\delta_k(z, O)} \Pi_k(z, O) = \int_0^{\delta_k(z, O)} \left[ \zeta_k(z) h_k(\omega)^{1-\sigma} o_k(\omega)^\theta(\sigma - 1) - f_{k, \omega} \right] d\omega.
\]

With the organizational capital allocation \( o_k(\omega) \) in equation (4.15), the profit function can be written as
\[
\Pi_k(z, O) = \int_0^{\delta_k(z, O)} \left[ \zeta_k(z) h_k(\omega)^{1-\sigma} \left( \frac{1}{\theta(\sigma - 1) \zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right)^{\theta(\sigma - 1) - 1} - f_{k, \omega} \right] d\omega
\]
and simplifying yields
\[
\Pi_k(z, O) = \int_0^{\delta_k(z, O)} \left[ \left( \frac{1}{\zeta_k(z) \theta(\sigma - 1)} \right)^{\theta(\sigma - 1) - 1} \lambda_k^{\theta(\sigma - 1) - 1} h_k(\omega)^{\sigma - 1} - f_{k, \omega} \right] d\omega.
\]
The derivative of the profit function with respect to \( \delta_k(z, O) \) is set equal to zero:

\[
\frac{\partial \Pi_k(z, O)}{\partial \delta_k(z, O)} = \left( \frac{1}{\zeta_k(z)^{\frac{1}{\sigma-1}} \theta(\sigma - 1)} \right)^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} \lambda_k^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} h_k(\delta_k(z, O))^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} f_{k, \omega} + \int_0^{\delta_k(z, O)} \left[ \left( \frac{1}{\zeta_k(z)^{\frac{1}{\sigma-1}} \theta(\sigma - 1)} \right)^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} h_k(\omega)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \lambda_k^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} \frac{\partial \lambda_k}{\partial \delta_k(z, O)} \right] d\omega \equiv 0
\]

and, by rearranging, simplifying as well as deploying the flexible manufacturing technology index, the first-order condition takes the form

\[
\left( \frac{1}{\zeta_k(z)^{\frac{1}{\sigma-1}} \theta(\sigma - 1)} \right)^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} \lambda_k^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} h_k(\delta_k(z, O))^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} \theta(\sigma - 1) = \frac{\partial \lambda_k}{\partial \delta_k(z, O)}
\]

\[
= f_{k, \omega}.
\]

**D.11 Firms’ Behavior at the Extensive Margin**

The marginal profitability of organizational capital is given by

\[
\lambda_k = \theta(\sigma - 1) \zeta_k(z) H_k(\delta_k)^{1-\sigma} O^{\theta(\sigma - 1) - 1}
\]

(D.24)

and the product scope is implicitly determined by

\[
\eta_k(z) \lambda_k^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} \left( h_k(\delta_k)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} + \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} H_k(\delta_k)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} \lambda_k^{-1} \frac{\partial \lambda_k}{\partial \delta_k} \right) = f_{k, \omega},
\]

(D.25)

with

\[
\eta_k(z) \equiv \left( \frac{1}{\zeta_k(z)^{\frac{1}{\sigma-1}} \theta(\sigma - 1)} \right)^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} > 0.
\]

With \( \zeta_k(z) = \frac{1}{\sigma - 1} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \), equation (D.24) can be rewritten as

\[
\lambda_k = \theta(\sigma - 1) \frac{1}{\sigma - 1} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} H_k(\delta_k)^{1-\sigma} O^{\theta(\sigma - 1) - 1}
\]

(D.26)

and equation (D.25) can be rewritten as

\[
\left( \frac{1}{\sigma - 1} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \right)^{-\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} \theta^{-\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} (\sigma - 1) \lambda_k^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}}
\]
\[ f_k, \omega = \left( h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} + \frac{\partial_1(\sigma-1)}{\theta(\sigma-1)-1} H_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \lambda_k^{-1} \frac{\partial h_k}{\partial_0} \right) = f_k, \omega. \] (D.27)

Plugging \( \frac{\partial_0 h_k(\delta_k)}{\partial_0} \) and \( \frac{\partial H_k(\delta_k)}{\partial_0} \) into equation (D.27) and simplifying it yields

\[ \left( A_k \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \right)^{-\frac{1}{\sigma-1}} \theta^{-\frac{\theta(\sigma-1)}{\sigma-1}} \lambda_k^{-1} \frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1} \left( h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \right) + \theta h_k(\delta_k)^{\frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1}} \lambda_k^{-1} \theta(\sigma-1) \zeta(z) O^{\theta(\sigma-1)-1} (1-\sigma) \]

\[ \times \theta(\sigma-1)-1 \frac{1}{\sigma-1} h_k(\delta_k)^{\frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1}} h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \right) = f_k, \omega, \]

which can be further rewritten by using \( \zeta(z) = \frac{1}{\sigma-1} A_k \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \):

\[ \left( A_k \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \right)^{-\frac{1}{\sigma-1}} \theta^{-\frac{\theta(\sigma-1)}{\sigma-1}} \lambda_k^{-1} \frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1} \left( h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \right) + \theta h_k(\delta_k)^{\frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1}} \lambda_k^{-1} \theta(\sigma-1) \frac{1}{\sigma-1} A_k \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \]

\[ \times O^{\theta(\sigma-1)-1} (1-\sigma) h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \right) = f_k, \omega. \] (D.28)

Rearranging and simplifying equation (D.28) further yields

\[ \left( A_k \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \right)^{-\frac{1}{\sigma-1}} \theta^{-\frac{\theta(\sigma-1)}{\sigma-1}} \lambda_k^{-1} \frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1} h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \]

\[ \times \left( 1 + \theta h_k(\delta_k)^{\frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1}} \lambda_k^{-1} \theta A_k \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} O^{\theta(\sigma-1)-1} (1-\sigma) \right) = f_k, \omega, \]

\[ \left( A_k \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \right)^{-\frac{1}{\sigma-1}} \theta^{-\frac{\theta(\sigma-1)}{\sigma-1}} A_k \left( \frac{\sigma-1}{\sigma} \right)^{\sigma} (awz)^{1-\sigma} \lambda_k^{-1} \frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1} h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \]

\[ \times \left( \frac{1}{\sigma-1} A_k^{-1} \left( \frac{\sigma-1}{\sigma} \right)^{-\sigma} (awz)^{1-\sigma} - \theta h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \lambda_k^{-1} \theta O^{\theta(\sigma-1)-1} \right) = f_k, \omega, \]

and finally

\[ \frac{\theta(\sigma-1)-2}{\theta(\sigma-1)-1} \sum_{d=0}^{\theta(\sigma-1)-2} \theta^{\frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1}} \lambda_k^{-1} \frac{\theta(\sigma-1)-1}{\theta(\sigma-1)-1} h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}}. \]
The total differentiation of equation (D.30) takes the general form

\[ \frac{\partial}{\partial \ln \lambda_k} \left( \ln \lambda_k \right) = \frac{\partial \ln \lambda_k}{\partial \ln z} \frac{\partial \ln z}{\partial \ln O} + \frac{\partial \ln \lambda_k}{\partial \ln O} \frac{\partial \ln O}{\partial \ln \lambda_k} \]

with

\[ \frac{\partial \ln \lambda_k}{\partial \ln z} = \frac{\partial \ln \lambda_k}{\partial \lambda_k} \frac{\partial \lambda_k}{\partial z} \frac{\partial z}{\partial \ln z} = \frac{z \partial \ln \lambda_k}{\lambda_k \partial z}, \]

\[ \frac{\partial \ln \lambda_k}{\partial \ln O} = \frac{O}{\lambda_k} \left( \theta A_k \left( \frac{\sigma - 1}{\sigma} \right) \right) \]

and

\[ \frac{\partial \ln \lambda_k}{\partial \ln \delta_k} = \frac{\partial \ln \lambda_k}{\partial \lambda_k} \frac{\partial \lambda_k}{\partial \delta_k} \frac{\partial \delta_k}{\partial \ln \delta_k} = \frac{\delta_k \partial \ln \lambda_k}{\lambda_k \partial \delta_k}. \]

The total differentiation of equation (D.30) takes the general form

\[ \frac{1}{\sigma - 1} A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} \theta H_k(\delta_k)^{1 - \sigma} \theta O^{\sigma(\sigma - 1) - 1} = f_{k,\omega}. \]
With
\[ H_k(\delta_k) = \left[ \int_0^{\delta_k} h_k(\omega)^{\frac{\sigma-1}{\sigma-1}} d\omega \right]^{\frac{\theta(\sigma-1)-1}{\sigma-1}} \]
and
\[ \frac{\partial H_k(\delta_k)}{\partial \delta_k} = \frac{\theta(\sigma-1)-1}{\sigma-1} H_k(\delta_k)^{\frac{\theta(\sigma-1)-1}{\sigma-1}} h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \]
equation (D.35) simplifies to
\[ \frac{\partial \ln \lambda_k}{\partial \ln \delta_k} = (1 - \sigma) \frac{\delta_k}{H_k(\delta_k)} \frac{\theta(\sigma-1)-1}{\sigma-1} H_k(\delta_k)^{\frac{1}{\sigma-1}} h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \]
and
\[ \frac{\partial \ln \lambda_k}{\partial \ln \delta_k} = -(\theta(\sigma-1) - 1)\delta_k H_k(\delta_k)^{\frac{1}{\sigma-1}} h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} \]
\[ \text{(D.36)} \]
Plugging equation (D.33), equation (D.34) and equation (D.36) into equation (D.32) provides the total differentiation of the equation that describes the marginal profitability of organizational capital:
\[ d \ln \lambda_k = -(\sigma - 1) d \ln z + (\theta(\sigma - 1) - 1) d \ln O \]
\[ - (\theta(\sigma - 1) - 1)\delta_k H_k(\delta_k)^{\frac{1}{\sigma-1}} h_k(\delta_k)^{\frac{\sigma-1}{\sigma-1}} d \ln \delta_k. \]
\[ \text{(D.37)} \]
The total differentiation of equation (D.31) takes the general form
\[ d \ln \delta_k = \frac{\partial \ln \delta_k}{\partial \ln z} d \ln z + \frac{\partial \ln \delta_k}{\partial \ln O} d \ln O + \frac{\partial \ln \delta_k}{\partial \ln \lambda_k} d \ln \lambda_k, \]
\[ \text{(D.38)} \]
with
\[ \frac{\partial \ln \delta_k}{\partial \ln z} = \frac{\partial \ln \delta_k}{\partial \delta_k} \frac{\partial \delta_k}{\partial z} \frac{z}{\delta_k} \frac{\partial z}{\partial \ln z} = \frac{z}{\delta_k} \frac{\partial \delta_k}{\partial z} = \frac{z}{\delta_k} \left( -\frac{\partial F/\partial z}{\partial F/\partial \delta_k} \right), \]
\[ \frac{\partial \ln \delta_k}{\partial \ln z} = \frac{\partial}{\partial \ln z} \left( z \left( \frac{\delta_k}{\delta_k} \right)^{\frac{1}{\sigma}} \frac{\theta(\sigma-1)}{\theta(\sigma-1)} \frac{(azw)^{\sigma-1} h_k(\delta_k)^{\sigma-1} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{\theta(\sigma-1-1)} + A_k^{\frac{\theta(\sigma-1-2)}{\theta(\sigma-1-1)}} (azw)^{\sigma-1} \lambda_k^{\theta(\sigma-1-1)}}{\frac{\delta_k}{\delta_k} \frac{\theta(\sigma-1-2)}{\theta(\sigma-1-1)} h_k(\delta_k)^{\sigma-1} \lambda_k^{\theta(\sigma-1-1)} - \frac{\delta_k}{\delta_k} \frac{\theta(\sigma-1-2)}{\theta(\sigma-1-1)} h_k(\delta_k)^{\sigma-1} \lambda_k^{\theta(\sigma-1-1)}} \right) \]
\[ \frac{\partial \ln \delta_k}{\partial \ln z} = -1 - \sigma(z) \frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = 0, \]

with \( p_k(z) \equiv \frac{1}{\sigma(z)} A_k (\sigma(z)^{-1} - \frac{\partial \ln \delta_k}{\partial \ln \lambda_k} - \frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = 0). \]

\[ \frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = 0 \]

\[ \frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = 0 \]

\[ \frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = 0 \]
\[
\frac{\partial \ln \delta_k}{\partial \ln O} = \frac{O}{\delta_k} \left( \frac{\theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta(\theta(\sigma - 1) - 1) \sigma^{(\sigma - 1) - 2}}{\sigma - 1 - \theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\partial h_k(\delta_k)}{\partial \delta_k} \left( \frac{1}{\sigma - 1} A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right) + \theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{-1} \frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\sigma - 1}{\lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1}} \right) \right),
\]

and

\[
\frac{\delta_k \partial \ln \delta_k}{\partial \ln O} = \frac{\theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1}}{\sigma - 1 - \theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\partial h_k(\delta_k)}{\partial \delta_k} \rho_k(z) + \theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{-1} \frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\sigma - 1}{\lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1}} \right),
\]

\[
\frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = \frac{\partial \ln \delta_k}{\partial \delta_k} \frac{\partial \lambda_k}{\partial \ln \lambda_k} = \frac{\lambda_k}{\delta_k} \frac{\partial \lambda_k}{\partial \ln \lambda_k} = \frac{\lambda_k}{\delta_k} \left( \frac{\partial F/\partial \lambda_k}{\delta F/\delta \delta_k} \right),
\]

\[
\frac{\partial \ln \lambda_k}{\partial \ln \lambda_k} = \frac{\lambda_k}{\delta_k} \left( - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right) + A_k^{\theta(\sigma - 1) - 2} \left( \frac{\sigma - 1}{\sigma} \right) \left( \frac{\partial h_k(\delta_k)}{\partial \delta_k} \right) \left( \frac{1}{\sigma - 1} A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} \right) \right)
\]

\[
\frac{\partial \ln \lambda_k}{\partial \ln \lambda_k} = \frac{\lambda_k}{\delta_k} \left( - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right) + A_k^{\theta(\sigma - 1) - 2} \left( \frac{\sigma - 1}{\sigma} \right) \left( \frac{\partial h_k(\delta_k)}{\partial \delta_k} \right) \left( \frac{1}{\sigma - 1} A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} \right) \right)
\]

\[
\frac{\partial \delta_k}{\partial \ln \lambda_k} = \frac{\lambda_k}{\delta_k} \left( - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right) + A_k^{\theta(\sigma - 1) - 2} \left( \frac{\sigma - 1}{\sigma} \right) \left( \frac{\partial h_k(\delta_k)}{\partial \delta_k} \right) \left( \frac{1}{\sigma - 1} A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} \right) \right)
\]
\[
\frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = \frac{\lambda_k}{\delta_k} \left( - \frac{\theta(\sigma-1)}{\theta(\sigma-1)-1} \lambda_k^{-1} \left( \frac{1}{\sigma-1} A_k^{-1} \left( \frac{\sigma-1}{\sigma} \right)^{-\sigma} (awz)^{\sigma-1} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^\theta(\sigma-1)-1 \right) + \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-2} \theta O^\theta(\sigma-1)-1 \right) \right)
\]

\[
\frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = \frac{\lambda_k}{\delta_k} \left( - \frac{\theta(\sigma-1)}{\theta(\sigma-1)-1} \lambda_k^{-1} \left( \frac{1}{\sigma-1} A_k^{-1} \left( \frac{\sigma-1}{\sigma} \right)^{-\sigma} (awz)^{\sigma-1} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^\theta(\sigma-1)-1 \right) + \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-2} \theta O^\theta(\sigma-1)-1 \right) \right)
\]

\[
\frac{\delta_k \partial \ln \delta_k}{\partial \ln \lambda_k} = - \frac{\theta(\sigma-1)}{\theta(\sigma-1)-1} \left( \frac{1}{\sigma-1} A_k^{-1} \left( \frac{\sigma-1}{\sigma} \right)^{-\sigma} (awz)^{\sigma-1} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^\theta(\sigma-1)-1 \right) + \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-2} \theta O^\theta(\sigma-1)-1
\]

and

\[
\frac{\partial \ln \delta_k}{\partial \ln \lambda_k} = - \frac{\theta(\sigma-1)}{\theta(\sigma-1)-1} \rho_k(z) + \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^\theta(\sigma-1)-1
\]

Plugging equation (D.39), equation (D.40) and equation (D.41) into equation (D.38) provides the total differentiation of the equation that...
describes the product scope of multi-product firms:

\[
\left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\partial h_k(\delta_k)}{\partial \delta_k} \rho_k(z) + \theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{-\frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1}} h_k(\delta_k)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} \lambda_k^{-1} 1^{\theta(\sigma - 1) - 1} \right) \delta_k d \ln \delta_k
\]

\[= - \left( \frac{-(\sigma - 1)(\theta(\sigma - 1) - 2)}{\theta(\sigma - 1) - 1} \rho_k(z) + A_k^{-1} \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \delta_k \right)^{-\sigma} \right) d \ln z + \theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} d \ln O
\]

\[\cdot \left( \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) + \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right) d \ln \lambda_k. \quad (D.42)
\]

Rearranging equation (D.37) and equation (D.42) and writing the resulting equation system in matrix notation yields

\[
\begin{bmatrix}
1 \\
\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) + \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1}
\end{bmatrix}
\begin{bmatrix}
(\theta(\sigma - 1) - 1) H_k(\delta_k)^{\frac{1-\sigma}{\theta(\sigma - 1) - 1}} h_k(\delta_k)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} \\
\frac{\sigma - 1}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\partial h_k(\delta_k)}{\partial \delta_k} \rho_k(z) + \theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{-\frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1}} h_k(\delta_k)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} \lambda_k^{-1} 1^{\theta(\sigma - 1) - 1}
\end{bmatrix}
\begin{bmatrix}
\delta_k d \ln \delta_k
\end{bmatrix}
\times
\begin{bmatrix}
\frac{d \ln \lambda_k}{\delta_k d \ln \delta_k}
\end{bmatrix}
= - \left( \frac{-(\sigma - 1)(\theta(\sigma - 1) - 2)}{\theta(\sigma - 1) - 1} \rho_k(z) + A_k^{-1} \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \delta_k \right)^{-\sigma} \right) d \ln z + \left[ \theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right] d \ln O.
\]

The determinant of the coefficient matrix \( \Delta \) is computed as

\[
\Delta = \frac{\sigma - 1}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\partial h_k(\delta_k)}{\partial \delta_k} \rho_k(z) + \theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{-\frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1}} h_k(\delta_k)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} \lambda_k^{-1} 1^{\theta(\sigma - 1) - 1} - (\theta(\sigma - 1) - 1) H_k(\delta_k)^{\frac{1-\sigma}{\theta(\sigma - 1) - 1}} \\
\times h_k(\delta_k)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}} \left( \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) + \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right)
\]

and has the property that

\[
\Delta \geq 0
\]
\[
\begin{align*}
\Leftrightarrow & \quad \frac{\sigma - 1}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\partial h_k(\delta_k)}{\partial \delta_k} \rho_k(z) + \theta(\theta(\sigma - 1) - 1) H_k(\delta_k) - \frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} - (\theta(\sigma - 1) - 1) H_k(\delta_k) \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \\
& \quad \times h_k(\delta_k)^{-1} \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) - (\theta(\sigma - 1) - 1) H_k(\delta_k) \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \theta H_k(\delta_k)^{1 - \sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \geq 0,
\end{align*}
\]

which can be simplified to

\[
\Leftrightarrow \quad \frac{1}{\theta(\sigma - 1) - 1} h_k(\delta_k)^{-1} \frac{\partial h_k(\delta_k)}{\partial \delta_k} \geq \theta \left( \frac{h_k(\delta_k)}{H_k(\delta_k)} \right)^{\frac{\sigma - 1}{\theta(\sigma - 1) - 1}}.
\]

The elasticity of the product scope with respect to organizational capital can then be computed as

\[
\frac{\delta_k d\ln \delta_k}{d\ln O} = \frac{1}{\Delta} \left[ \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) + \theta H_k(\delta_k)^{1 - \sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \theta(\sigma - 1) - 1 \right] \left( \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) + \theta H_k(\delta_k)^{1 - \sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right),
\]

which simplifies to

\[
\frac{\delta_k d\ln \delta_k}{d\ln O} = \frac{1}{\Delta} \left( \theta(\theta(\sigma - 1) - 1) H_k(\delta_k)^{1 - \sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} - (\theta(\sigma - 1) - 1) \left( \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) + \theta H_k(\delta_k)^{1 - \sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right) \right)
\]

and

\[
\frac{\delta_k d\ln \delta_k}{d\ln O} = \frac{1}{\Delta} \theta(\sigma - 1) \rho_k(z).
\]

The elasticity of the product scope with respect to productivity can then be computed as

\[
\frac{\delta_k d\ln \delta_k}{d\ln z} = \frac{1}{\Delta} \left[ \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) + \theta H_k(\delta_k)^{1 - \sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \frac{\theta(\sigma - 1) - 2}{\theta(\sigma - 1) - 1} \rho_k(z) - \lambda_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} \left( \alpha w z \right)^{\sigma - 1} \right].
\]
which simplifies to

\[
\frac{\delta_k d \ln \delta_k}{d \ln z} = \frac{1}{\triangle} \left( (\sigma - 1)(\sigma - 1) - 2 \right) \rho_k(z) - A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} + (\sigma - 1) \left( \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \rho_k(z) + \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right)
\]

and

\[
\frac{\delta_k d \ln \delta_k}{d \ln z} = \frac{1}{\triangle} \left( 2(\sigma - 1) \rho_k(z) - A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} + (\sigma - 1) \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right).
\]

With \( \rho_k(z) \equiv \frac{1}{\sigma - 1} A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \), equation (D.44) can be written as

\[
\frac{\delta_k d \ln \delta_k}{d \ln z} = \frac{1}{\triangle} \left( 2(\sigma - 1) \rho_k(z) - A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} + A_k^{-1} \left( \frac{\sigma - 1}{\sigma} \right)^{-\sigma} (awz)^{\sigma - 1} - (\sigma - 1) \rho_k(z) \right)
\]

and simplified to

\[
\frac{\delta_k d \ln \delta_k}{d \ln z} = \frac{1}{\triangle} (\sigma - 1) \rho_k(z).
\]

(D.45)
D.12 Margins of Multi-product Firms

The overall output of a firm with productivity $\frac{1}{z}$ and organizational capital $O$ being active in industry $k$ (firm-intensive margin) is given by

$$X_k(z, O) = \int_0^{\delta_k(z,O)} x_k(\omega, z, O)d\omega. \quad (D.46)$$

With the (aggregate) demand $x_k(\omega) = A_k p_k(\omega)^{-\sigma}$, equation (D.46) can be written as

$$X_k(z, O) = \int_0^{\delta_k(z,O)} A_k p_k(\omega)^{-\sigma} d\omega$$

and it can be further rewritten by using firstly the price choice of the firm,

$$X_k(z, O) = \int_0^{\delta_k(z,O)} A_k \left( \frac{\sigma}{\sigma - 1} c_k(\omega, z, O) \right)^{-\sigma} d\omega,$$

secondly the marginal production costs of the firm,

$$X_k(z, O) = \int_0^{\delta_k(z,O)} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (aw z n_k(\omega))^{-\sigma} d\omega,$$

and as well as thirdly the organizational capital allocation of the firm,

$$X_k(z, O) = \int_0^{\delta_k(z,O)} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (aw z)^{-\sigma} (h_k(\omega) a_k(\omega)^{-\theta})^{-\sigma} d\omega,$$

whose simplified equivalent can be further rewritten by using $\zeta_k(z) \equiv \frac{A_k}{\sigma} \left( \frac{1}{aw z} \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1}$ as

$$X_k(z, O) = \int_0^{\delta_k(z,O)} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (aw z)^{-\sigma} \left( \frac{1}{\theta(\sigma - 1) \zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right)^{\frac{\sigma}{\sigma - 1}} d\omega,$$

simplifying equation (D.47) yields

$$X_k(z, O) = \int_0^{\delta_k(z,O)} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} \left( \frac{1}{aw z} \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \left( \frac{\theta(\sigma - 1)}{\lambda_k} \right)^{-\frac{\sigma}{\sigma - 1}} h_k(\omega)^{\frac{\sigma}{\sigma - 1}} d\omega. \quad (D.47)$$

Simplifying equation (D.47) yields

$$X_k(z, O) = \int_0^{\delta_k(z,O)} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \left( \frac{\theta(\sigma - 1)}{\lambda_k} \right)^{-\frac{\sigma}{\sigma - 1}} h_k(\omega)^{\frac{\sigma}{\sigma - 1}} d\omega,$$

$$X_k(z, O) = \int_0^{\delta_k(z,O)} A_k \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \left( \frac{\theta(\sigma - 1)}{\lambda_k} \right)^{-\frac{\sigma}{\sigma - 1}} h_k(\omega)^{\frac{\sigma}{\sigma - 1}} d\omega.$$
\[ X_k(z, O) = \int_0^{\delta_k(z, O)} A_k^{-\frac{\theta+1}{\theta(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\theta(\sigma-1)-1}} (awz)^{\frac{\sigma}{\theta(\sigma-1)-1}} \lambda_k^{\frac{\theta \sigma}{\theta(\sigma-1)-1}} \times \theta^{-\frac{\theta \sigma}{\theta(\sigma-1)-1}} h_k(\omega)^{\frac{\sigma}{\theta(\sigma-1)-1}} d\omega \]

and
\[ X_k(z, O) = A_k^{-\frac{\theta+1}{\theta(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\theta(\sigma-1)-1}} (awz)^{\frac{\sigma}{\theta(\sigma-1)-1}} \lambda_k^{\frac{\theta \sigma}{\theta(\sigma-1)-1}} \times \theta^{-\frac{\theta \sigma}{\theta(\sigma-1)-1}} \int_0^{\delta_k(z, O)} h_k(\omega)^{\frac{\sigma}{\theta(\sigma-1)-1}} d\omega. \]

The output of each product (product-intensive margin) is thus given by
\[ x_k(\omega, z, O) = A_k^{-\frac{\theta+1}{\theta(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\theta(\sigma-1)-1}} (awz)^{\frac{\sigma}{\theta(\sigma-1)-1}} \lambda_k^{\frac{\theta \sigma}{\theta(\sigma-1)-1}} \times \theta^{-\frac{\theta \sigma}{\theta(\sigma-1)-1}} h_k(\omega)^{\frac{\sigma}{\theta(\sigma-1)-1}} \]

and its derivate with respect to the product-extensive margin is computed as
\[ \frac{\partial x_k(\omega, z, O)}{\partial \delta_k(z, O)} = \frac{\theta \sigma}{\theta(\sigma-1)-1} A_k^{-\frac{\theta+1}{\theta(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\theta(\sigma-1)-1}} (awz)^{\frac{\sigma}{\theta(\sigma-1)-1}} \theta^{-\frac{\theta \sigma}{\theta(\sigma-1)-1}} h_k(\omega)^{\frac{\sigma}{\theta(\sigma-1)-1}} \times \lambda_k^{\frac{\theta \sigma}{\theta(\sigma-1)-1}} - \frac{1}{\theta(\sigma-1)-1} \frac{\partial \lambda_k}{\partial \delta_k} \]

and
\[ \frac{\partial x_k(\omega, z, O)}{\partial \delta_k(z, O)} = \frac{\theta \sigma}{\theta(\sigma-1)-1} A_k^{-\frac{\theta+1}{\theta(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\theta(\sigma-1)-1}} (awz)^{\frac{\sigma}{\theta(\sigma-1)-1}} \theta^{-\frac{\theta \sigma}{\theta(\sigma-1)-1}} h_k(\omega)^{\frac{\sigma}{\theta(\sigma-1)-1}} \times \lambda_k^{\frac{\theta \sigma}{\theta(\sigma-1)-1}} \frac{\partial \lambda_k}{\partial \delta_k}. \] (D.48)

The derivative of the firm-intensive margin with respect to the product-extensive margin is computed as
\[ \frac{\partial X_k(z, O)}{\partial \delta_k(z, O)} = A_k^{-\frac{\theta+1}{\theta(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\theta(\sigma-1)-1}} (awz)^{\frac{\sigma}{\theta(\sigma-1)-1}} \lambda_k^{\frac{\theta \sigma}{\theta(\sigma-1)-1}} \theta^{-\frac{\theta \sigma}{\theta(\sigma-1)-1}} h_k(\delta_k)^{\frac{\sigma}{\theta(\sigma-1)-1}} \]
\[ + A_k^{-\frac{\theta+1}{\theta(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\theta(\sigma-1)-1}} (awz)^{\frac{\sigma}{\theta(\sigma-1)-1}} \left( \frac{\theta \sigma}{\theta(\sigma-1)-1} \right) \lambda_k^{\frac{\theta \sigma}{\theta(\sigma-1)-1}} - \frac{1}{\theta(\sigma-1)-1} \frac{\partial \lambda_k}{\partial \delta_k} \]
\[ \times \theta^{-\frac{\theta \sigma}{\theta(\sigma-1)-1}} \int_0^{\delta_k(z, O)} h_k(\omega)^{\frac{\sigma}{\theta(\sigma-1)-1}} d\omega. \]
and

\[
\frac{\partial X_k(z, O)}{\partial \delta_k(z, O)} = A_k^{-\frac{\sigma + 1}{\sigma(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\sigma(\sigma-1)-1}} (awz)^{\frac{\theta}{\sigma(\sigma-1)-1}} \lambda_k^{\frac{\theta}{\sigma(\sigma-1)-1}} h_k(\delta_k)^{\frac{\theta}{\sigma(\sigma-1)-1}}
\]

\[
+ A_k^{-\frac{\sigma + 1}{\sigma(\sigma-1)-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma(1+\theta)}{\sigma(\sigma-1)-1}} \left( \frac{\theta}{\theta(\sigma - 1) - 1} \right) (awz)^{\frac{\theta}{\sigma(\sigma-1)-1}} \lambda_k^{\frac{\theta}{\sigma(\sigma-1)-1}} \frac{\partial \lambda_k}{\partial \delta_k} \frac{\theta}{\sigma(\sigma-1)-1} \int_0^{\delta_k(z, O)} h_k(\omega)^{\frac{\theta}{\sigma(\sigma-1)-1}} d\omega.
\]

The overall sales (revenue) of a firm with productivity \( \frac{1}{2} z \) and organizational capital \( O \) being active in industry \( k \) are given by

\[
R_k(z, O) = \int_0^{\delta_k(z, O)} r_k(\omega, z, O) d\omega,
\]

with \( r_k(\omega, z, O) = p_k(\omega, z, O) x_k(\omega, z, O) \) as the sales (revenue) of product variety \( \omega \), and therefore by

\[
R_k(z, O) = \int_0^{\delta_k(z, O)} p_k(\omega, z, O) x_k(\omega, z, O) d\omega. \tag{D.49}
\]

With the (aggregate) demand \( x_k(\omega) = A_k p_k(\omega)^{-\sigma} \), equation (D.49) can be written as

\[
R_k(z, O) = \int_0^{\delta_k(z, O)} p_k(\omega, z, O) A_k p_k(\omega, z, O)^{-\sigma} d\omega
\]

and its simplified equivalent can be further rewritten by using firstly the price choice of the firm,

\[
R_k(z, O) = \int_0^{\delta_k(z, O)} A_k \left( \frac{\sigma}{\sigma - 1} c_k(\omega, z, O) \right)^{1-\sigma} d\omega,
\]

secondly the marginal production costs of the firm,

\[
R_k(z, O) = \int_0^{\delta_k(z, O)} A_k \left( \frac{\sigma}{\sigma - 1} (awzn_k(\omega))^{1-\sigma} \right) d\omega
\]

and

\[
R_k(z, O) = \int_0^{\delta_k(z, O)} A_k \left( \frac{\sigma}{\sigma - 1} (awz)^{1-\sigma} (h_k(\omega) o_k(\omega)^{-\theta})^{1-\sigma} \right) d\omega,
\]

as well as thirdly the organizational capital allocation of the firm,

\[
R_k(z, O) = \int_0^{\delta_k(z, O)} A_k \left( \frac{\sigma}{\sigma - 1} (awz)^{1-\sigma} h_k(\omega)^{1-\sigma} \left( \frac{1}{\theta(\sigma - 1) \zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right) \right)^{\frac{\theta}{\sigma(\sigma-1)-1}} d\omega,
\]
and secondly the marginal production costs of the firm, whose simplified equivalent can be further rewritten by using \( \lambda_k = \theta(\sigma-1)\zeta_k(z)H_k(\delta_k)^{1-\sigma}O^\theta(\sigma-1)^{-1} \) as
\[
R_k(z, O) = \int_0^{\delta_k(z,O)} A_k \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (awz)^{1-\sigma} h_k(\omega) \frac{\sigma}{\pi(\sigma-1)^{-1}} \left( H_k(\delta_k)^{1-\sigma}O^\theta(\sigma-1)^{-1} \right)^{\frac{\theta(\sigma-1)}{\pi(\sigma-1)^{-1}}} d\omega
\]
and
\[
R_k(z, O) = A_k \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (awz)^{1-\sigma} H_k(\delta_k)^{-\frac{\theta(\sigma-1)^2}{\pi(\sigma-1)^{-1}}} O^\theta(\sigma-1)
\times \int_0^{\delta_k(z,O)} h_k(\omega) \frac{\sigma}{\pi(\sigma-1)^{-1}} d\omega.
\]
Simplifying equation (D.50) yields
\[
R_k(z, O) = A_k \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (awz)^{1-\sigma} H_k(\delta_k)^{-\frac{\theta(\sigma-1)^2}{\pi(\sigma-1)^{-1}}} O^\theta(\sigma-1)H_k(\delta_k)^{\frac{\sigma}{\pi(\sigma-1)^{-1}}}
\]
and
\[
R_k(z, O) = A_k \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} (awz)^{1-\sigma} O^{\theta(\sigma-1)}H_k(\delta_k(z, O))^{-(\sigma-1)}.
\]
The profits of product variety \( \omega \) are given by
\[
\pi_k(\omega, z, O) = (p_k(\omega, z, O) - c_k(\omega, z, O)) x_k(\omega, z, O) - f_{k,\omega}.
\]
With the (aggregate) demand \( x_k(\omega) = A_kp_k(\omega)^{-\sigma} \), equation (D.51) can be written as
\[
\pi_k(\omega, z, O) = (p_k(\omega, z, O) - c_k(\omega, z, O)) A_k p_k(\omega, z, O)^{-\sigma} - f_{k,\omega}
\]
and it can be further rewritten by using firstly the price choice of the firm,
\[
\pi_k(\omega, z, O) = \left( \frac{\sigma}{\sigma - 1} c_k(\omega, z, O) - c_k(\omega, z, O) \right) A_k \left( \frac{\sigma}{\sigma - 1} c_k(\omega, z, O) \right)^{-\sigma} - f_{k,\omega}
\]
and
\[
\pi_k(\omega, z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^\sigma A_k c_k(\omega, z, O)^{1-\sigma} - f_{k,\omega},
\]
secondly the marginal production costs of the firm,
\[
\pi_k(\omega, z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^\sigma A_k (awz)^{1-\sigma} n_k(\omega)^{1-\sigma} - f_{k,\omega}
\]
and
\[
\pi_k(\omega, z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^\sigma A_k (awz)^{1-\sigma} (h_k(\omega)\phi_k(\omega)^{-\theta})^{1-\sigma} - f_{k,\omega},
\]
as well as thirdly the organizational capital allocation of the firm,

\[
\pi_k(\omega, z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k(awz)^{1-\sigma} h_k(\omega)^{1-\sigma} \left( \frac{1}{\theta(\sigma - 1)\zeta_k(z)} h_k(\omega)^{\sigma - 1} \lambda_k \right)^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} - f_{k,\omega},
\]

whose simplified equivalent can be further rewritten by using \( \lambda_k = \theta(\sigma - 1)\zeta_k(z)H_k(\delta_k)^{1-\sigma}O^{\theta(\sigma - 1) - 1} \) as

\[
\pi_k(\omega, z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k(awz)^{1-\sigma} h_k(\omega)^{\sigma - 1} \left( H_k(\delta_k)^{1-\sigma}O^{\theta(\sigma - 1) - 1} \right)^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} - f_{k,\omega}.
\]

Rearranging equation (D.52) yields

\[
\pi_k(\omega, z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k(awz)^{1-\sigma} H_k(\delta_k(z, O))^{-\frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1}} O^{\theta(\sigma - 1) - 1} h_k(\omega)^{\sigma - 1} - f_{k,\omega}.
\]

The overall profits of a firm with productivity \( \frac{1}{2} \) and organizational capital \( O \) being active in industry \( k \) are given by

\[
\Pi_k(z, O) = \int_0^{\delta_k(z,O)} \pi_k(\omega, z, O) d\omega,
\]

\[
\Pi_k(z, O) = \int_0^{\delta_k(z,O)} \left( \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k(awz)^{1-\sigma} H_k(\delta_k)^{-\frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1}} \right) O^{\theta(\sigma - 1) - 1} h_k(\omega)^{\sigma - 1} - f_{k,\omega} d\omega,
\]

and

\[
\Pi_k(z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k(awz)^{1-\sigma} H_k(\delta_k)^{-\frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1}} O^{\theta(\sigma - 1)} \int_0^{\delta_k(z,O)} h_k(\omega)^{\sigma - 1} d\omega - \delta_k f_{k,\omega}.
\]

Simplifying equation (D.53) yields

\[
\Pi_k(z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k(awz)^{1-\sigma} H_k(\delta_k)^{-\frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1}} O^{\theta(\sigma - 1)} H_k(\delta_k)^{\frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1}} - \delta_k f_{k,\omega}
\]

and

\[
\Pi_k(z, O) = \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} A_k(awz)^{1-\sigma} O^{\theta(\sigma - 1)} H_k(\delta_k(z, O))^{-(\sigma - 1)} - \delta_k(z, O) f_{k,\omega}.
\]
D.13 Organizational Capital Threshold

The organizational capital threshold $O^*_k(z)$ is characterized by the condition

$$\pi_k(0, z, O^*_k(z))|_{\alpha(0) = 0} = 0,$$

which takes the form

$$\frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right) A_k (awz)^{1-\sigma} (O^*_k(z))^\theta(\sigma-1) - f_{k,\omega} = 0$$

and can be rearranged to isolate $O^*_k(z)$ as

$$O^*_k(z) = \left( \frac{A_k}{(\sigma - 1)f_{k,\omega}} \right)^{\frac{1}{\theta(1-\sigma)}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{\theta(1-\sigma)}} (awz)^{\frac{1}{\theta}}.$$

The organizational capital threshold $O^*_k(z)$ is decreasing in the firm productivity $\frac{1}{z}$,

$$\frac{\partial O^*_k(z)}{\partial z} = \frac{1}{\theta^2} \left( \frac{A_k}{(\sigma - 1)f_{k,\omega}} \right)^{\frac{1}{\theta(1-\sigma)}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{\theta(1-\sigma)}} (aw)^{\frac{1}{\theta}} z^{\frac{1}{\theta} - 1} > 0,$$

and increasing in the elasticity of substitution across product varieties, whereas the logarithm of the threshold can be written as

$$\ln O^*_k(z) = \frac{1}{\theta(1-\sigma)} (\ln A_k - \ln ((\sigma - 1)f_{k,\omega})) + \frac{\sigma}{\theta(1-\sigma)} (\ln(\sigma - 1) - \ln \sigma) + \frac{1}{\theta} \ln(awz)$$

and its derivative with respect to $\ln \sigma$ is given by

$$\frac{\partial \ln O^*_k(z)}{\partial \ln \sigma} = \frac{\partial \ln O^*_k(z)}{\partial \sigma} \frac{\partial \sigma}{\partial \ln \sigma}$$

and

$$\frac{\partial \ln O^*_k(z)}{\partial \ln \sigma} = \sigma \left( \frac{\theta}{(\theta(1-\sigma))^2} (\ln A_k - \ln ((\sigma - 1)f_{k,\omega})) - \frac{1}{\theta(1-\sigma)} (\frac{f_{k,\omega}}{(\sigma - 1)f_{k,\omega}}) \right) + \frac{\theta(1-\sigma) + \theta \sigma}{(\theta(1-\sigma))^2} (\ln(\sigma - 1) - \ln \sigma) + \frac{\sigma}{\theta(1-\sigma)} \left( \frac{1}{\sigma - 1} - \frac{1}{\sigma} \right).$$

(D.54)

Simplifying equation (D.54) yields

$$\frac{\partial \ln O^*_k(z)}{\partial \ln \sigma} = \sigma \left( \frac{1}{\theta(1-\sigma)^2} \ln \left( \frac{A_k}{(\sigma - 1)f_{k,\omega}} \right) + \frac{1}{\theta(1-\sigma)^2} + \frac{1}{\theta(1-\sigma)^2} \ln \left( \frac{\sigma - 1}{\sigma} \right) \right)$$

$$+ \frac{\sigma}{\theta(1-\sigma)} \frac{\sigma - (\sigma - 1)}{\sigma(\sigma - 1)}.$$
\[
\frac{\partial \ln O^*(z)}{\partial \ln \sigma} = \frac{\sigma}{\theta (1 - \sigma)^2} \left( \ln \left( \frac{A_k}{(\sigma - 1)f_{k,\omega}} \right) + \ln \left( \frac{\sigma - 1}{\sigma} \right) \right) > 0.
\]

### D.14 Multi-product Firms in Heterogeneous Industries

With the organizational capital threshold

\[
O_k^*(z) = \left( \frac{A_k}{(\sigma - 1)f_{k,\omega}} \right)^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{1}{\theta(\sigma - 1) - 1}} (awz)^{\frac{\sigma - 1}{\sigma}}
\]

the equation that characterizes the product scope (equation (D.31)),

\[
A_k^{\frac{\theta(\sigma - 1) - 2}{\theta(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma(\sigma - 1) - 2\sigma}{\theta(\sigma - 1) - 1}} (awz)^{\frac{(1-\sigma)(\theta(\sigma - 1) - 2)}{\theta(\sigma - 1) - 1}} \theta^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} h_k(\delta_k) \frac{\sigma - 1}{\theta(\sigma - 1) - 1}
\]

\[
\times \left( \frac{1}{\sigma - 1} A_k^{-\frac{1}{\sigma}} (awz)^{-\frac{\sigma - 1}{\sigma}} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right) = f_{k,\omega},
\]

can be written as

\[
A_k^{\frac{\theta(\sigma - 1) - 2}{\theta(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma(\sigma - 1) - 2\sigma}{\theta(\sigma - 1) - 1}} (awz)^{\frac{(1-\sigma)(\theta(\sigma - 1) - 2)}{\theta(\sigma - 1) - 1}} \theta^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} h_k(\delta_k) \frac{\sigma - 1}{\theta(\sigma - 1) - 1}
\]

\[
\times \left( \frac{1}{\sigma - 1} \frac{1}{(\sigma - 1)f_{k,\omega}} O_k^*(z)^{\theta(\sigma - 1) - 1} - \theta H_k(\delta_k)^{1-\sigma} \lambda_k^{-1} \theta O^{\theta(\sigma - 1) - 1} \right) = f_{k,\omega}.
\]

With the marginal profitability of organizational capital \( \lambda_k(z, O, \delta_k(z, O)) \) in equation (4.22) equation (D.55) can be further rewritten as

\[
A_k^{\frac{\theta(\sigma - 1) - 2}{\theta(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma(\sigma - 1) - 2\sigma}{\theta(\sigma - 1) - 1}} (awz)^{\frac{(1-\sigma)(\theta(\sigma - 1) - 2)}{\theta(\sigma - 1) - 1}} \theta^{\frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1}} \left( \theta(\sigma - 1) \zeta_k(z) H_k(\delta_k)^{1-\sigma} \right.
\]

\[
\times O^{\theta(\sigma - 1) - 1} h_k(\delta_k) \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \left( \frac{1}{\sigma - 1} \frac{1}{(\sigma - 1)f_{k,\omega}} O_k^*(z)^{\theta(\sigma - 1) - 1} - \theta H_k(\delta_k)^{1-\sigma} \theta O^{\theta(\sigma - 1) - 1} \right) = f_{k,\omega},
\]

which can be simplified to get

\[
A_k^{\frac{\theta(\sigma - 1) - 2}{\theta(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma(\sigma - 1) - 2\sigma}{\theta(\sigma - 1) - 1}} (awz)^{\frac{(1-\sigma)(\theta(\sigma - 1) - 2)}{\theta(\sigma - 1) - 1}} \left( \theta(\sigma - 1) \zeta_k(z) H_k(\delta_k)^{1-\sigma} \right.
\]

\[
\times O^{\theta(\sigma - 1) - 1} h_k(\delta_k) \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \left( \frac{1}{\sigma - 1} \frac{1}{(\sigma - 1)f_{k,\omega}} O_k^*(z)^{\theta(\sigma - 1) - 1} - \theta H_k(\delta_k)^{1-\sigma} \theta O^{\theta(\sigma - 1) - 1} \right) = f_{k,\omega}.
\]
\[-\theta H_k(\delta_k)^{1-\sigma} \theta^{-1}(\sigma - 1)^{-1} \zeta_k(z)^{-1} H_k(\delta_k)^{\sigma-1} \theta) = f_{k,\omega}\]

and rewritten by using \( \zeta_k(z) = \frac{1}{\sigma - 1} A_k \left( \frac{\sigma - 1}{\sigma} \right)^\sigma (awz)^{1-\sigma} \) as

\[
A_k^{\theta(\sigma - 1) - 2} \left( \frac{\sigma - 1}{\sigma} \right)^{\theta(\sigma - 1) - 2\sigma} (awz)^{(1-\sigma)(\theta(\sigma - 1) - 2)} (\sigma - 1)^{\theta(\sigma - 1) - 1} \left( \frac{1}{\sigma - 1} A_k \left( \frac{\sigma - 1}{\sigma} \right)^\sigma \right) \times (awz)^{1-\sigma} H_k(\delta_k)^{-\theta(\sigma - 1)} \theta H_k(\delta_k)^{\sigma-1} \theta) = f_{k,\omega}.
\]

Simplifying equation (D.56) yields

\[
A_k^2 \left( \frac{\sigma - 1}{\sigma} \right)^{2\sigma} (awz)^{-\theta(\sigma - 1) + 2} \theta H_k(\delta_k)^{-\theta(\sigma - 1)} \theta H_k(\delta_k)^{\sigma-1} \theta) \times \left( \frac{1}{(\sigma - 1)^2 f_{k,\omega}} O_k^*(z)^{\theta(\sigma - 1)} - \frac{1}{(\sigma - 1) f_{k,\omega}} A_k \left( \frac{\sigma - 1}{\sigma} \right)^\sigma (awz)^{1-\sigma} \right) = f_{k,\omega},
\]

which can be rewritten by using the organizational capital threshold,

\[
O_k^*(z) = \left( \frac{A_k}{(\sigma - 1) f_{k,\omega}} \right)^{\frac{1}{\sigma(\sigma - 1)}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{\sigma(\sigma - 1)}} (awz)^{\frac{1}{\sigma}},
\]

as

\[
A_k^2 \left( \frac{\sigma - 1}{\sigma} \right)^{2\sigma} (awz)^{-2(\sigma - 1) \theta(\sigma - 1)} H_k(\delta_k)^{-\theta(\sigma - 1)} \theta H_k(\delta_k)^{\sigma-1} \theta) \times \left( \frac{1}{(\sigma - 1)^2 f_{k,\omega}} O_k^*(z)^{\theta(\sigma - 1)} - \frac{\theta}{(\sigma - 1) f_{k,\omega}} O_k^*(z)^{\theta(\sigma - 1)} \right) = f_{k,\omega},
\]

and finally simplifying equation (D.57) yields

\[
\left( A_k \left( \frac{\sigma - 1}{\sigma} \right)^\sigma (awz)^{1-\sigma} \right)^2 H_k(\delta_k)^{-\theta(\sigma - 1)} \theta H_k(\delta_k)^{\sigma-1} \theta) \times \frac{1 - \theta(\sigma - 1)}{\sigma - 1} \frac{1}{(\sigma - 1)^2 f_{k,\omega}} O_k^*(z)^{\theta(\sigma - 1)} = f_{k,\omega}.
\]

The overall sales (revenue) of a firm with productivity \( \frac{1}{z} \) and organizational capital \( O \) being active in industry \( k \) are given by

\[
R_k(z, O) = A_k \left( \frac{\sigma - 1}{\sigma} \right)^{1-\sigma} (awz)^{1-\sigma} \theta H_k(\delta_k)^{-(\sigma - 1)}. \]
With the organizational capital threshold

\[ O^*_k(z) = \left( \frac{A_k}{(\sigma - 1)f_{k,\omega}} \right)^{\frac{1}{\sigma - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma - 1}{\sigma - 1}} (awz)^\frac{1}{\sigma}, \]

equation (D.59) can be rewritten as

\[ R_k(z, O) = \frac{\sigma}{\sigma - 1} O^*_k(z)^{\theta(1-\sigma)}(\sigma - 1)f_{k,\omega}O^{\theta(\sigma - 1)}H_k(\delta_k)^{-(\sigma - 1)}, \]

and

\[ R_k(z, O) = \sigma O^*_k(z)^{\theta(1-\sigma)}f_{k,\omega}O^{\theta(\sigma - 1)}H_k(\delta_k)^{-(\sigma - 1)}, \]

while equation (D.58) can be rewritten as

\[ (\sigma - 1)f_{k,\omega}O^*_k(z)^{\theta(1-\sigma)}H_k(\delta_k)^{\frac{\theta(\sigma - 1)^2}{\sigma(\sigma - 1)^{-1}}} \]

\[ \times \frac{1-\theta(\sigma - 1)}{\sigma - 1}(\sigma - 1)f_{k,\omega}O^*_k(z)^{\theta(\sigma - 1)} = f_{k,\omega}, \]

which can be simplified and rearranged as

\[ O^*_k(z)^{\theta(1-\sigma)}f_{k,\omega}O^{\theta(\sigma - 1)} = \frac{f_{k,\omega}}{1-\theta(\sigma - 1)}H_k(\delta_k)^{\frac{\theta(\sigma - 1)^2}{\sigma(\sigma - 1)^{-1}}} H_k(\delta_k)^{\frac{\theta(\sigma - 1)^2}{\sigma(\sigma - 1)^{-1}}}. \]

Plugging equation (D.61) into equation (D.60) yields

\[ R_k(z, O) = \sigma \frac{f_{k,\omega}}{1-\theta(\sigma - 1)}H_k(\delta_k)^{\frac{1-\sigma}{\sigma(\sigma - 1)^{-1}}} H_k(\delta_k)^{\frac{\theta(\sigma - 1)^2}{\sigma(\sigma - 1)^{-1}}} H_k(\delta_k)^{-(\sigma - 1)}, \]

which can be simplified as

\[ R_k(z, O) = \frac{\sigma}{1-\theta(\sigma - 1)}f_{k,\omega}H_k(\delta_k)^{\frac{1-\sigma}{\sigma(\sigma - 1)^{-1}}} H_k(\delta_k)^{\frac{\theta(\sigma - 1)^2}{\sigma(\sigma - 1)^{-1}}} \]

and

\[ R_k(z, O) = \frac{\sigma}{1-\theta(\sigma - 1)}f_{k,\omega} \left( \frac{h_k(\delta_k(z, O))}{H_k(\delta_k(z, O))} \right)^{\frac{1-\sigma}{\sigma(\sigma - 1)^{-1}}}. \]

\section*{D.15 Product Variety Demand in the Open Economy}

Each individual in country \( j \) maximizes the utility subject to the budget constraint and thereby derives the individual demand for the product variety \( \omega \in \Omega_{k,ij} \) that is produced in country \( i \) and supplied to country \( j \). To implement this technically, the Lagrange function \( \mathcal{L} \) is established and its derivative with respect to \( q_{k,ij}(\omega) \) is set equal to zero, as it is
required by the Lagrange theorem to compute the optimum of the objective function:

\[
L = \left[ \sum_{i=1}^{I} \int_{\Omega_{k,ij}} q_{k,ij}(\omega)^{\frac{1}{\sigma}-1} d\omega \right]^{\frac{\sigma}{\sigma-1}} + \lambda_{k,j} \left[ \gamma_{k,j} y_j - \sum_{i=1}^{I} \int_{\Omega_{k,ij}} p_{k,ij}(\omega)q_{k,ij}(\omega)d\omega \right],
\]

with \( \lambda_{k,j} > 0 \) as the (industry- and country-specific) Lagrange multiplier, and

\[
\frac{\partial L}{\partial q_{k,ij}(\omega)} = \frac{\sigma}{\sigma - 1} \left[ \sum_{i=1}^{I} \int_{\Omega_{k,ij}} q_{k,ij}(\omega)^{\frac{1}{\sigma}-1} d\omega \right]^{\frac{\sigma - 1}{\sigma}} q_{k,ij}(\omega)^{\frac{1}{\sigma}-1} - \lambda_{k,j} p_{k,ij}(\omega) = 0,
\]

which yields

\[
q_{k,ij}(\omega)^{\frac{1}{\sigma}} \left[ \sum_{i=1}^{I} \int_{\Omega_{k,ij}} q_{k,ij}(\omega)^{\frac{1}{\sigma}} d\omega \right]^{\frac{1}{\sigma-1}} = \lambda_{k,j} p_{k,ij}(\omega).
\]

By making use of the utility index \( U_{k,j} \), the first-order condition can be rewritten as

\[
q_{k,ij}(\omega)^{-\frac{1}{\sigma}} U_{k,j}^{\frac{1}{\sigma}} = \lambda_{k,j} p_{k,ij}(\omega) \Leftrightarrow q_{k,ij}(\omega) = U_{k,j}^{\frac{1}{\sigma}} \lambda_{k,j}^{-\alpha} p_{k,ij}(\omega)^{-\alpha}.
\]  \hspace{1cm} (D.62)

To determine a term for \( U_{k,j}^{\frac{1}{\sigma}} \lambda_{k,j}^{-\alpha} q_{k,ij}(\omega) \) by itself is plugged into the budget constraint

\[
\gamma_{k,j} y_j = \sum_{i=1}^{I} \int_{\Omega_{k,ij}} p_{k,ij}(\omega)q_{k,ij}(\omega)d\omega = U_{k,j}^{\frac{1}{\sigma}} \lambda_{k,j}^{-\alpha} \sum_{i=1}^{I} \int_{\Omega_{k,ij}} p_{k,ij}(\omega)^{1-\sigma} d\omega
\]

and the latter rearranged to get

\[
\gamma_{k,j} y_j \left[ \sum_{i=1}^{I} \int_{\Omega_{k,ij}} p_{k,ij}(\omega)^{1-\sigma} d\omega \right]^{-1} = U_{k,j}^{\frac{1}{\sigma}} \lambda_{k,j}^{-\alpha},
\]

while its substitution back into the right equation of (D.62) yields

\[
q_{k,ij}(\omega) = \gamma_{k,j} y_j \left[ \sum_{i=1}^{I} \int_{\Omega_{k,ij}} p_{k,ij}(\omega)^{1-\sigma} d\omega \right]^{-1} p_{k,ij}(\omega)^{-\sigma}.
\]

Aggregating the individual demand over the \( L_j \) individuals provides the aggregate (country-wide) demand for the product variety \( \omega \in \Omega_{k,ij} \) that is produced in country \( i \) and supplied to country \( j \), \( x_{k,ij}(\omega) = L_j q_{k,ij}(\omega) \),

\[
x_{k,ij}(\omega) = L_j \gamma_{k,j} y_j \left[ \sum_{i=1}^{I} \int_{\Omega_{k,ij}} p_{k,ij}(\omega)^{1-\sigma} d\omega \right]^{-1} p_{k,ij}(\omega)^{-\sigma},
\]

which can be written as

\[
x_{k,ij}(\omega) = L_j \gamma_{k,j} y_j p_{k,ij}^{\alpha-1} p_{k,ij}(\omega)^{-\sigma},
\]
with \( P_{k,j} \equiv \left[ \sum_{i=1}^{I} \int_{0}^{\Omega} p_{k,ij}(\omega)^{1-\sigma} d\omega \right] ^{\frac{1}{1-\sigma}} \) as the CES price index of industry \( k \) in country \( j \), and, by summarizing the variety-invariant parameters and aggregates, written as

\[
x_{k,ij}(\omega) = A_{k,j} p_{k,ij}(\omega)^{-\sigma},
\]

with \( A_{k,j} \equiv L_j \gamma_{k,j} y_j P_{k,j}^{\sigma-1} \) as the residual demand of industry \( k \) in country \( j \).

### D.16 Price Setting in the Open Economy

To set the optimal price, the firm solves the following profit maximization problem:

\[
\max_{p_{k,ij}(\omega,z,O)} \Pi_{k,ij}(z,O) = \int_{0}^{\delta_{k,ij}(z,O)} \left[ (p_{k,ij}(\omega,z,O) - c_{k,ij}(\omega,z,O)) x_{k,ij}(\omega,z,O) - f_{ij} \right] d\omega.
\]

With \( x_{k,ij}(\omega,z,O) = A_{k,j} p_{k,ij}(\omega,z,O)^{-\sigma} \) as the (received) product demand, the profit function can be written as

\[
\Pi_{k,ij}(z,O) = \int_{0}^{\delta_{k,ij}(z,O)} \left[ (p_{k,ij}(\omega,z,O) - c_{k,ij}(\omega,z,O)) A_{k,j} p_{k,ij}(\omega,z,O)^{-\sigma} - f_{ij} \right] d\omega
\]

and its derivative with respect to \( p_{k,ij}(\omega,z,O) \) is set equal to zero:

\[
\frac{\partial \Pi_{k,ij}(z,O)}{\partial p_{k,ij}(\omega,z,O)} = A_{k,j} p_{k,ij}(\omega,z,O)^{-\sigma} + (p_{k,ij}(\omega,z,O) - c_{k,ij}(\omega,z,O)) \times (-\sigma) A_{k,j} p_{k,ij}(\omega,z,O)^{-\sigma-1} = 0.
\]

Rearranging the first-order condition to isolate the price yields

\[
p_{k,ij}(\omega,z,O) = \frac{\sigma}{\sigma - 1} c_{k,ij}(\omega,z,O).
\]

### D.17 Organizational Capital Allocation in the Open Economy

To choose the optimal organizational capital allocation, the firm solves the following profit maximization problem:

\[
\max_{o_{k,ij}(\omega)} \Pi_{k,ij}(z,O) = \int_{0}^{\delta_{k,ij}(z,O)} \left[ \xi_{k,ij}(z) h_k(\omega)^{1-\sigma} a_{k,ij}(\omega)^{\theta(\sigma - 1)} - f_{ij} \right] d\omega
\]

s.t. \( \int_{0}^{\delta_{k,ij}(z,O)} o_{k,ij}(\omega) d\omega = O. \)
To implement this technically, the Lagrange function $L$ is established and its derivative with respect to $o_{k,ij}(\omega)$ is set equal to zero, as it is required by the Lagrange theorem to compute the optimum of the objective function:

$$L = \int_0^{\delta_{k,ij}(z,O)} \left[ \zeta_{k,ij}(z) h_k(\omega)^{1-\sigma} o_{k,ij}(\omega)^{\theta(\sigma-1)} - f_k^{ij} \right] d\omega + \lambda_{k,ij} \left[ O - \int_0^{\delta_{k,ij}(z,O)} o_{k,ij}(\omega)d\omega \right]$$

with $\lambda_{k,ij} > 0$ as the (industry- and country-specific) Lagrange multiplier and

$$\frac{\partial L}{\partial o_{k,ij}(\omega)} = \zeta_{k,ij}(z) h_k(\omega)^{1-\sigma} \theta(\sigma - 1) o_{k,ij}(\omega)^{\theta(\sigma-1)-1} - \lambda_{k,ij} = 0.$$

Rearranging the first-order condition to isolate the organizational capital allocation yields

$$o_{k,ij}(\omega) = \left( \frac{1}{\theta(\sigma - 1)\zeta_{k,ij}(z)} h_k(\omega)^{\sigma-1} \lambda_{k,ij} \right)^{1/(\sigma-1)}. \quad (D.65)$$

Under the assumption that the product scope $\delta_{k,ij}(z, O)$ is kept constant, the derivative of equation (D.65) with respect to $\omega$ is given by

$$\frac{\partial o_{k,ij}(\omega)}{\partial \omega} \bigg|_{\delta_{k,ij}=\text{const.}} = \frac{1}{\theta(\sigma - 1) - 1} \left( \frac{1}{\theta(\sigma - 1)\zeta_{k,ij}(z)} h_k(\omega)^{\sigma-1} \lambda_{k,ij} \right)^{1/(\sigma-1)-1} \times \frac{1}{\theta(\sigma - 1)\zeta_{k,ij}(z)} \lambda_{k,ij}(\sigma - 1) h_k(\omega)^{\sigma-2} \frac{\partial h_k(\omega)}{\partial \omega}.$$

Simplifying the derivative then yields

$$\frac{\partial o_{k,ij}(\omega)}{\partial \omega} \bigg|_{\delta_{k,ij}=\text{const.}} = \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \left( \frac{1}{\theta(\sigma - 1)\zeta_{k,ij}(z)} h_k(\omega)^{\sigma-1} \lambda_{k,ij} \right)^{1/(\sigma-1)-1} h_k(\omega)^{-1} \frac{\partial h_k(\omega)}{\partial \omega}.$$

### D.18 Marginal Profitability of Organizational Capital in the Open Economy

With the organizational capital allocation $o_{k,ij}(\omega)$ in equation (4.37), the resource constraint that is binding can be written as

$$\int_0^{\delta_{k,ij}(z,O)} h_k(\omega)^{1-\sigma} \lambda_{k,ij} \left( \frac{1}{\theta(\sigma - 1)\zeta_{k,ij}(z)} \right)^{1/(\sigma-1)-1} d\omega = O$$

and rearranging yields

$$\lambda_{k,ij} \left( \frac{1}{\theta(\sigma - 1)\zeta_{k,ij}(z)} \right)^{1/(\sigma-1)-1} \int_0^{\delta_{k,ij}(z,O)} h_k(\omega)^{\sigma-1} d\omega = O. \quad (D.66)$$
To present the marginal profitability of organizational capital in a more compact way, an (aggregate) index of the flexible manufacturing technology is defined:

\[
H_k(\delta_{k,ij}(z,O)) \equiv \int_0^{\delta_{k,ij}(z,O)} h_k(\omega)^{\frac{\sigma-1}{\sigma-1}} d\omega.
\]

Its derivative with respect to \(\delta_{k,ij}(z,O)\) is then given by

\[
\frac{\partial H_k(\delta_{k,ij}(z,O))}{\partial \delta_{k,ij}(z,O)} = \frac{\theta(\sigma - 1)}{\sigma - 1} \left[ \int_0^{\delta_{k,ij}(z,O)} h_k(\omega)^{\frac{\sigma-1}{\sigma-1}} d\omega \right] h_k(\delta_{k,ij}(z,O))^{\frac{\sigma-1}{\sigma-1}},
\]

and can be simplified to

\[
\frac{\partial H_k(\delta_{k,ij}(z,O))}{\partial \delta_{k,ij}(z,O)} = \frac{\theta(\sigma - 1)}{\sigma - 1} H_k(\delta_{k,ij}(z,O))^{\frac{\sigma-1}{\sigma-1}} h_k(\delta_{k,ij}(z,O))^{\frac{\sigma-1}{\sigma-1}},
\]

with the result that the relation

\[
\frac{\partial H_k(\delta_{k,ij}(z,O))}{\partial \delta_{k,ij}(z,O)} \geq 0 \iff \sigma \geq 1 + \frac{1}{\theta}
\]

holds.

Deploying the flexible manufacturing technology index in equation (D.66) provides

\[
\lambda_{k,ij}(z,O,\delta_{k,ij}(z,O)) = \theta(\sigma - 1) H_k(\delta_{k,ij}(z,O))^{\frac{\sigma-1}{\sigma-1}} = O
\]

and finally by rearranging

\[
\lambda_{k,ij}(z,O,\delta_{k,ij}(z,O)) = \theta(\sigma - 1) H_k(\delta_{k,ij}(z,O))^{\frac{\sigma-1}{\sigma-1}} = O
\]

(D.67)

**D.19 Product Scope Choice in the Open Economy**

To choose the optimal product scope, the firm solves the following profit maximization problem:

\[
\max_{\delta_{k,ij}(z,O)} \Pi_{k,ij}(z,O) = \int_0^{\delta_{k,ij}(z,O)} \left[ \zeta_{k,ij}(z) h_k(\omega)^{1-\sigma} o_{k,ij}(\omega)^{\theta(\sigma-1)} - f_{k,ij}^{ij} \right] d\omega.
\]

With the organizational capital allocation \(o_{k,ij}(\omega)\) in equation (4.37), the profit function can be written as

\[
\Pi_{k,ij}(z,O) = \int_0^{\delta_{k,ij}(z,O)} \left[ \zeta_{k,ij}(z) h_k(\omega)^{1-\sigma} \left( \frac{1}{\theta(\sigma - 1) \zeta_{k,ij}(z)} h_k(\omega)^{\sigma-1} \lambda_{k,ij} \right)^{\frac{\theta(\sigma-1)}{\sigma-1}} - f_{k,ij}^{ij} \right] d\omega
\]
and simplifying yields

\[ \Pi_{k,ij}(z, O) = \int_0^{\delta_{k,ij}(z, O)} \left[ \left( \frac{1}{\zeta_{k,ij}(z)^{\theta(\sigma - 1)}} \right)^{\frac{\theta(\sigma - 1)}{\pi(\sigma - 1)}} \lambda_{k,ij}^{\frac{\theta(\sigma - 1)}{\pi(\sigma - 1)}} h_k(\omega)^{\frac{\pi(\sigma - 1)}{\pi(\sigma - 1)}} - f_{k,\omega}^{ij} \right] d\omega. \]

The derivative of the profit function with respect to \( \delta_{k,ij}(z, O) \) is set equal to zero:

\[ \frac{\partial \Pi_{k,ij}(z, O)}{\partial \delta_{k,ij}(z, O)} = \left( \frac{1}{\zeta_{k,ij}(z)^{\pi(\sigma - 1)}} \right)^{\frac{\theta(\sigma - 1)}{\pi(\sigma - 1)}} \lambda_{k,ij}^{\frac{\theta(\sigma - 1)}{\pi(\sigma - 1)}} h_k(\delta_{k,ij}(z, O))^{rac{\pi(\sigma - 1)}{\pi(\sigma - 1)}} - f_{k,\omega}^{ij} + \int_0^{\delta_{k,ij}(z, O)} \left[ \left( \frac{1}{\zeta_{k,ij}(z)^{\pi(\sigma - 1)}} \right)^{\frac{\theta(\sigma - 1)}{\pi(\sigma - 1)}} h_k(\omega)^{\frac{\pi(\sigma - 1)}{\pi(\sigma - 1)}} \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \right] \frac{\partial \lambda_{k,ij}}{\partial \delta_{k,ij}(z, O)} d\omega = 0 \]

and, by rearranging, simplifying as well as deploying the flexible manufacturing technology index, the first-order condition takes the form

\[ \frac{1}{\zeta_{k,ij}(z)^{\pi(\sigma - 1)}} \lambda_{k,ij}^{\frac{\theta(\sigma - 1)}{\pi(\sigma - 1)}} h_k(\delta_{k,ij}(z, O))^{rac{\pi(\sigma - 1)}{\pi(\sigma - 1)}} + \left( \frac{1}{\zeta_{k,ij}(z)^{\pi(\sigma - 1)}} \right)^{\frac{\theta(\sigma - 1)}{\pi(\sigma - 1)}} h_k(\delta_{k,ij}(z, O))^{rac{\pi(\sigma - 1)}{\pi(\sigma - 1)}} \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \frac{\partial \lambda_{k,ij}}{\partial \delta_{k,ij}(z, O)} = f_{k,\omega}^{ij}. \]

**APPENDIX D.**

**D.20 Organizational Capital Threshold in the Open Economy**

The organizational capital threshold \( O_{k,ij}^*(z) \) is characterized by the condition

\[ \pi_{k,ij}(0, z, O_{k,ij}^*(z))|_{O_{k,ij}(0)=O} = 0, \]

which takes the form

\[ \frac{1}{\sigma - 1} \left( \frac{\sigma - 1}{\sigma} \right)^\sigma A_{k,j} (\tau_{ij} a \omega_i z)^{1-\sigma} (O_{k,ij}^*(z))^{\theta(\sigma - 1)} - f_{k,\omega}^{ij} = 0 \]

and can be rearranged to isolate \( O_{k,ij}^*(z) \) as

\[ O_{k,ij}^*(z) = \left( \frac{A_{k,j}}{(\sigma - 1) f_{k,\omega}^{ij}} \right)^{\frac{1}{\pi(1-\sigma)}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\pi(1-\sigma)}{\pi(1-\sigma)}} (\tau_{ij} a \omega_i z)^{\frac{1}{\sigma}}. \]
With the organizational capital threshold, 

\[ O_{k,ij}^*(z) = \left( \frac{A_{k,j}}{(\sigma - 1) f_{k,ij}^{ij}} \right)^{\frac{1}{\sigma - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{(\sigma - 1)}} (\tau_{ij} a w_i z)^{\frac{1}{\sigma}}, \]

the equation that characterizes the product scope (equation (4.39)),

\[ A_{k,j}^{\frac{\sigma}{(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{(\sigma - 1) - 1}} (\tau_{ij} a w_i z)^{\frac{1}{\sigma} - 1} \theta H_k(\delta_{k,ij})^{1 - \sigma} \lambda_{k,ij}^{\sigma - 1} \theta O^{\sigma - 1} = f_{k,ij}^{ij}, \]

can be written as

\[ A_{k,j}^{\frac{\sigma}{(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{(\sigma - 1) - 1}} (\tau_{ij} a w_i z)^{\frac{1}{\sigma} - 1} \theta H_k(\delta_{k,ij})^{1 - \sigma} \lambda_{k,ij}^{\sigma - 1} \theta O^{\sigma - 1} = f_{k,ij}^{ij}. \]

With the marginal profitability of organizational capital \( \lambda_{k,ij}(z, O, \delta_{k,ij}(z, O)) \) in equation (4.38) equation (D.68) can be further rewritten as

\[ A_{k,j}^{\frac{\sigma}{(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{(\sigma - 1) - 1}} (\tau_{ij} a w_i z)^{\frac{1}{\sigma} - 1} \theta H_k(\delta_{k,ij})^{1 - \sigma} \lambda_{k,ij}^{\sigma - 1} \theta O^{\sigma - 1} = f_{k,ij}^{ij}, \]

which can be simplified to get

\[ A_{k,j}^{\frac{\sigma}{(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{(\sigma - 1) - 1}} (\tau_{ij} a w_i z)^{\frac{1}{\sigma} - 1} \theta H_k(\delta_{k,ij})^{1 - \sigma} \lambda_{k,ij}^{\sigma - 1} \theta O^{\sigma - 1} = f_{k,ij}^{ij}, \]

and rewritten by using \( \lambda_{k,ij}(z) = \frac{1}{\sigma - 1} \lambda_{k,ij}^{\sigma - 1} \) as

\[ A_{k,j}^{\frac{\sigma}{(\sigma - 1) - 1}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{(\sigma - 1) - 1}} (\tau_{ij} a w_i z)^{\frac{1}{\sigma} - 1} \theta H_k(\delta_{k,ij})^{1 - \sigma} \lambda_{k,ij}^{\sigma - 1} \theta O^{\sigma - 1} = f_{k,ij}^{ij}. \]
By finally using the organizational capital threshold, equation (D.71) can be rewritten as
\[ O^*_{k,ij}(z) = \left( \frac{A_{k,j}}{(\sigma - 1) f^{ij}_{k,\omega}} \right)^{\frac{1}{\alpha(\sigma - 1)}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{\alpha(1 - \sigma)}} (\tau_{ij} aw_i z)^{\frac{1}{2}}. \]

Simplifying equation (D.69) yields
\[ A^2_{k,j} \left( \frac{\sigma - 1}{\sigma} \right)^{2\sigma} (\tau_{ij} aw_i z) - \theta (\sigma - 1)^{-1} \left( \frac{1}{\sigma - 1} A_{k,j} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (\tau_{ij} aw_i z)^{1 - \sigma} \right) = f^{ij}_{k,\omega}, \]
which can be rewritten by using the organizational capital threshold,
\[ O^*_{k,ij}(z) = \left( \frac{A_{k,j}}{(\sigma - 1) f^{ij}_{k,\omega}} \right)^{\frac{1}{\alpha(\sigma - 1)}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{\alpha(1 - \sigma)}} (\tau_{ij} aw_i z)^{\frac{1}{2}}. \]

and simplifying equation (D.70) yields
\[ \left( A_{k,j} \left( \frac{\sigma - 1}{\sigma} \right)^{\sigma} (\tau_{ij} aw_i z)^{1 - \sigma} \right)^2 H_k(\delta_{k,ij})^{\frac{\theta(\sigma - 1)^2}{\alpha(\sigma - 1)}} O^*_{k,ij}(z)^{\theta(\sigma - 1)} \]
\[ \times \frac{1 - \theta (\sigma - 1)}{\sigma - 1} \frac{1}{(\sigma - 1) f^{ij}_{k,\omega}} O^*_{k,ij}(z)^{\theta(\sigma - 1)} = f^{ij}_{k,\omega}. \]

By finally using the organizational capital threshold,
\[ O^*_{k,ij}(z) = \left( \frac{A_{k,j}}{(\sigma - 1) f^{ij}_{k,\omega}} \right)^{\frac{1}{\alpha(\sigma - 1)}} \left( \frac{\sigma - 1}{\sigma} \right)^{\frac{\sigma}{\alpha(1 - \sigma)}} (\tau_{ij} aw_i z)^{\frac{1}{2}}. \]

equation (D.71) can be rewritten as
\[ \left( (\sigma - 1) f^{ij}_{k,\omega} O^*_{k,ij}(z)^{\theta(1 - \sigma)} \right)^2 H_k(\delta_{k,ij})^{\frac{\theta(\sigma - 1)^2}{\alpha(\sigma - 1)}} O^*_{k,ij}(z)^{\theta(\sigma - 1)} \]
\[ \times \frac{1 - \theta (\sigma - 1)}{\sigma - 1} \frac{1}{(\sigma - 1) f^{ij}_{k,\omega}} O^*_{k,ij}(z)^{\theta(\sigma - 1)} = f^{ij}_{k,\omega}. \]
and simplifying equation (D.72) yields

$$h_k(\delta_{k,ij})^{1-\sigma} = (1 - \theta(\sigma - 1))H_k(\delta_{k,ij})^{-\frac{\theta(\sigma-1)^2}{\theta(\sigma-1)-1}}O_{k,ij}^* (z)^{\theta(1-\sigma)}O^{\theta(\sigma-1)}.$$

### D.21 Comparative Statics

#### D.21.1 Product Scope

The change in the product scope in response to any change in the iceberg transportation costs $\tau_{ij}$ is qualitatively as well as quantitatively exactly equal to the change in the product scope in response to a change in the firm’s cost draw $z$ of the same size as the change in the iceberg transportation costs $\tau_{ij}$. Therefore, in strict correspondence to the computations in the closed economy, the elasticity of the product scope in response to a change in the iceberg transportation costs is given by

$$\frac{\delta_{k,ij} d\ln \delta_{k,ij}}{d\ln \tau_{ij}} = \frac{1}{\Delta}(\sigma - 1)\rho_{k,ij}(z),$$

with $\Delta \geq 0$ as the determinant of the coefficient matrix of the system of the total differentiations of the two fundamental equations (equation (4.38) and equation (4.39)) that is written in matrix notation and $\rho_{k,ij}(z) \equiv \frac{1}{\sigma-1} A_{k,ij}^{-1} \left(\frac{\sigma-1}{\sigma}\right) \sigma^{-1}(\tau_{ij}a \nu z)^{\sigma-1} - \theta h_k(\delta_{k,ij})^{1-\sigma} \lambda_{k,ij}^{-1} \theta O^{\theta(\sigma-1)-1}$.

The total differentiation of equation (D.42) takes the general form

$$\frac{d\ln \delta_{k,ij}}{d\ln \tau_{ij}} = \frac{\partial \ln \delta_{k,ij}}{\partial \ln \tau_{ij}} + \frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^*} \frac{d\ln O_{k,ij}^*(z)}{d\ln \tau_{ij}},$$

with

$$\frac{\partial \ln \delta_{k,ij}}{\partial \ln \tau_{ij}} = \frac{\partial \ln \delta_{k,ij}}{\partial \ln \tau_{ij}} \frac{\partial \delta_{k,ij}}{\partial \tau_{ij}} = \frac{\tau_{ij}}{\delta_{k,ij}} \frac{\partial \delta_{k,ij}}{\partial \tau_{ij}} = \frac{\tau_{ij}}{\delta_{k,ij}} \left(\frac{\partial O_{k,ij}^*}{\partial \delta_{k,ij}}\right) = 0,$$

$$\frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^*(z)} = \frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^*} \frac{\partial O_{k,ij}^*(z)}{\partial \ln O_{k,ij}^*} = \frac{O_{k,ij}^*(z)}{\delta_{k,ij}} \frac{\partial O_{k,ij}^*(z)}{\partial \delta_{k,ij}} = \frac{O_{k,ij}^*(z)}{\delta_{k,ij}} \left(\frac{\partial O_{k,ij}^*}{\partial \delta_{k,ij}}\right).$$

Thus,

$$\frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^*(z)} = \frac{O_{k,ij}^*(z)}{\delta_{k,ij}} \left(-\frac{1}{\sigma-1} h_k(\delta_{k,ij})^{-\frac{\theta(\sigma-1)^2}{\theta(\sigma-1)-1}} H_k(\delta_{k,ij})^{-\frac{\theta(\sigma-1)^2}{\theta(\sigma-1)-1}} - \frac{1}{\sigma-1} h_k(\delta_{k,ij})^{-\frac{\theta(\sigma-1)^2}{\theta(\sigma-1)-1}} - (1 - \theta(\sigma - 1)) O_{k,ij}^*(z)^{\theta(1-\sigma)}\right).$$

Finally,

$$\frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^*(z)} = \frac{1}{\delta_{k,ij}} \left(-\frac{1}{\sigma-1} h_k(\delta_{k,ij})^{-\frac{\theta(\sigma-1)^2}{\theta(\sigma-1)-1}} H_k(\delta_{k,ij})^{-\frac{\theta(\sigma-1)^2}{\theta(\sigma-1)-1}} - \frac{1}{\sigma-1} h_k(\delta_{k,ij})^{-\frac{\theta(\sigma-1)^2}{\theta(\sigma-1)-1}} - (1 - \theta(\sigma - 1)) O_{k,ij}^*(z)^{\theta(1-\sigma)}\right).$$
\[ O^{\theta(\sigma-1)} \left( -\frac{\theta(\sigma-1)^2}{\theta(\sigma-1)-1} \right) H_k(\delta_{k,ij}) \theta^{(\sigma-1)^2} (1 - \frac{\theta(\sigma-1)-1}{\sigma-1} H_k(\delta_{k,ij}) \theta^{(\sigma-1)-1} h_k(\delta_{k,ij}) \theta^{(\sigma-1)-1}) \right)^{\sigma-1}.

\[
\frac{\partial \ln \delta_{k,ij}}{\partial \ln O^*_{k,ij}(z)} = \frac{1}{\delta_{k,ij}} \left( 1 - \frac{\theta(\sigma-1)}{\sigma-1} I_k(\delta_{k,ij}) \theta^{(\sigma-1)^2} (1 - \frac{\theta(\sigma-1)-1}{\sigma-1} H_k(\delta_{k,ij}) \theta^{(\sigma-1)-1} h_k(\delta_{k,ij}) \theta^{(\sigma-1)-1}) \right)^{\sigma-1}.
\]

\[
\frac{\partial \ln \delta_{k,ij}}{\partial \ln O^*_{k,ij}(z)} = \frac{1}{\delta_{k,ij}} \left( 1 - \frac{\theta(\sigma-1)}{\sigma-1} I_k(\delta_{k,ij}) \theta^{(\sigma-1)^2} (1 - \frac{\theta(\sigma-1)-1}{\sigma-1} H_k(\delta_{k,ij}) \theta^{(\sigma-1)-1} h_k(\delta_{k,ij}) \theta^{(\sigma-1)-1}) \right)^{\sigma-1}.
\]

Plugging equation (D.74), equation (D.75) and equation (D.76) into equation (D.73) provides the total differentiation of the equation that describes the product scope of multi-product firms:

\[
\frac{d \ln \delta_{k,ij}}{d \ln \tau_{ij}} = \frac{1}{\theta_{ij} - \frac{\sigma-1}{\theta(\sigma-1)-1} I_k(\delta_{k,ij}) \theta^{(\sigma-1)^2} (1 - \frac{\theta(\sigma-1)-1}{\sigma-1} H_k(\delta_{k,ij}) \theta^{(\sigma-1)-1} h_k(\delta_{k,ij}) \theta^{(\sigma-1)-1}) \right)^{\sigma-1}}.
\]

With the (standard) functional specification \( h_k(\omega) = \exp(\mu_k \omega) \), with \( \mu_k > 0 \) as a (industry-
specific) parameter, equation (D.77) takes the form
\[
-\frac{\delta_{k,ij}d\ln \delta_{k,ij}}{d\ln \tau_{ij}} = \frac{(\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right)}{-\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \exp \left( \frac{\mu_k \delta_{k,ij}}{\theta(\sigma - 1) - 1} \right) \mu_k \exp (\mu_k \delta_{k,ij})} \\
+ \theta(\sigma - 1) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}.
\]

(D.78)

The derivative of equation (D.78) with respect to \( \delta_{k,ij} \) has the property that
\[
\frac{\partial}{\partial \delta_{k,ij}} \left( -\frac{\delta_{k,ij}d\ln \delta_{k,ij}}{d\ln \tau_{ij}} \right) \geq 0 \iff \\
\left( (\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \\
\times \left( (\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \geq 0,
\]

\[
\left( (\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \\
\times \left( (\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \geq 0,
\]

\[
\left( (\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \\
\times \left( (\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \geq 0,
\]

\[
\left( (\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \\
\times \left( (\sigma - 1)(1 - \theta(\sigma - 1))^{\frac{1}{\theta(\sigma - 1) - 1}} \tau_{ij}^{-1} O \exp \left( -\frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \geq 0.
\]
\[
\times \left[ \int_0^\delta \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) - \left( (\sigma - 1)(1 - \theta(\sigma - 1)) \right)^{\frac{1}{\theta(\sigma - 1)} - 1} \tau_{ij}^{-1} O
\]
\[
\times \exp \left( \frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^\delta \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \left( - \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \right)
\]
\[
\times \exp \left( \left( \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \right) \mu_k \delta_{k,ij} \right) \mu_k \exp \left( \mu_k \delta_{k,ij} \right) \left[ \frac{1 - \sigma}{\theta(\sigma - 1) - 1} - 1 \right] + \theta(\sigma - 1) \left[ \int_0^\delta \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \geq 0,
\]
\[
\left( (\sigma - 1)(1 - \theta(\sigma - 1)) \right)^{\frac{1}{\theta(\sigma - 1)} - 1} \tau_{ij}^{-1} O \exp \left( \frac{1}{\theta} \mu_k \delta_{k,ij} \right) \left[ \int_0^\delta \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right)
\]
\[
\times \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \right) \exp \left( \left( \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \right) \mu_k \delta_{k,ij} \right) \mu_k \exp \left( \mu_k \delta_{k,ij} \right) - (\sigma - 1) \mu_k
\]
\[
\times \left[ \int_0^\delta \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} + \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \exp \left( \left( \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \right) \mu_k \delta_{k,ij} \right)
\]
\[
\times \mu_k \exp \left( \mu_k \delta_{k,ij} \right) \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^\delta \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
- \theta(\sigma - 1) \left[ \int_0^\delta \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-2} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right)
\]
\[
- \left( \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \right)^2 \exp \left( \left( \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \right) \mu_k \delta_{k,ij} \right) \mu_k \exp \left( \mu_k \delta_{k,ij} \right)
\]
\[
+ \theta(\sigma - 1) \left[ \int_0^\delta \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-2} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \right) \geq 0,
\]
Taking the logarithm of the organizational capital allocation in equation (4.37),

\[
\left(\sigma - 1\right)(1 - \theta(\sigma - 1))^{\frac{1}{\sigma - 1}} \tau_{ij}^{-1} O \exp \left(-\frac{1}{\theta} \mu_k \delta_{k,ij}\right) \left[\int_0^{\delta_{k,ij}} \exp \left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1}
\]

\[
\times \left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} - \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \right)^2 \exp \left(\frac{1 - \sigma}{\theta(\sigma - 1) - 1} - 1\right) \mu_k \delta_{k,ij}
\]

\[
\times \mu_k \mu_k \exp \left(\mu_k \delta_{k,ij}\right) - (\sigma - 1) \mu_k \left[\int_0^{\delta_{k,ij}} \exp \left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1} + \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k
\]

\[
\times \left[\int_0^{\delta_{k,ij}} \exp \left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1} \gtrless 0,
\]

and

\[
\left(\sigma - 1\right)(1 - \theta(\sigma - 1))^{\frac{1}{\sigma - 1}} \tau_{ij}^{-1} O \exp \left(-\frac{1}{\theta} \mu_k \delta_{k,ij}\right) \left[\int_0^{\delta_{k,ij}} \exp \left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1}
\]

\[
\times \frac{\sigma - 1}{\theta(\sigma - 1) - 1}^2 \exp \left(\frac{1 - \sigma}{\theta(\sigma - 1) - 1} - 1\right) \mu_k \delta_{k,ij} \mu_k \mu_k \exp \left(\mu_k \delta_{k,ij}\right)
\]

\[
+ (\sigma - 1) \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \left[\int_0^{\delta_{k,ij}} \exp \left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1} \gtrless 0,
\]

D.21.2 Organizational Capital Allocation

Taking the logarithm of the organizational capital allocation in equation (4.37)

\[
o_{k,ij}(\omega) = \left(\frac{1}{\theta(\sigma - 1) \zeta_{k,ij}(\omega)} h_k(\omega)\right)^{\frac{1}{\sigma - 1}} \lambda_{k,ij},
\]

(D.79)

yields

\[
\ln o_{k,ij}(\omega) = \frac{1}{\theta(\sigma - 1) - 1} ((\sigma - 1) \ln (h_k(\omega)) + \ln (\lambda_{k,ij}) - \ln (\theta(\sigma - 1))
\]

\[
- \left(\ln \left(\frac{1}{\sigma - 1}\right) + \ln (A_{k,j}) + \sigma \ln \left(\frac{\sigma - 1}{\sigma}\right) + (1 - \sigma) (\ln (aw_{ij}) + \ln (\tau_{ij}))\right).
\]
The total differentiation of equation (D.79) takes the general form

\[
\frac{\partial \ln o_{k,ij}(\omega)}{\partial \ln \tau_{ij}} = \frac{\partial \ln o_{k,ij}(\omega)}{\partial \ln \tau_{ij}} + \frac{\partial \ln o_{k,ij}(\omega)}{\partial \ln \lambda_{k,ij}} \frac{d \ln \lambda_{k,ij}}{d \ln \tau_{ij}},
\]

with

\[
\frac{\partial \ln o_{k,ij}(\omega)}{\partial \ln \tau_{ij}} = \frac{\sigma - 1}{\theta(\sigma - 1) + 1 - \frac{1}{\theta(\sigma - 1)}}
\]

and

\[
\frac{\partial \ln o_{k,ij}(\omega)}{\partial \ln \lambda_{k,ij}} = \frac{1}{\theta(\sigma - 1) + 1 - \frac{1}{\theta(\sigma - 1)}}.
\]

The total differentiation of equation (4.38) takes the general form

\[
\frac{d \ln \lambda_{k,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln \lambda_{k,ij}}{\partial \ln \lambda_{k,ij}} + \frac{\partial \ln \lambda_{k,ij}}{\partial \ln \delta_{k,ij}} \frac{d \ln \delta_{k,ij}}{d \ln \tau_{ij}},
\]

with

\[
\frac{\partial \ln \lambda_{k,ij}}{\partial \ln \tau_{ij}} = 1 - \sigma,
\]

\[
\frac{\partial \ln \lambda_{k,ij}}{\partial \ln \delta_{k,ij}} = \frac{\delta_{k,ij}}{\lambda_{k,ij}} \left( \theta(\sigma - 1)\zeta_{k,ij}(z)(1 - \sigma)H_k(\delta_{k,ij}) - \frac{1}{\lambda_{k,ij}} \frac{\partial H_k(\delta_{k,ij})}{\partial \delta_{k,ij}} \right),
\]

\[
\frac{d \ln \lambda_{k,ij}}{d \ln \tau_{ij}} = 1 - \sigma + (1 - \sigma) \frac{1}{\Delta \delta_{k,ij}} \left( \rho_{k,ij}(z) \right).
\]

Plugging equation (D.84), equation (D.85) and equation (D.86) into equation (D.83) provides the total differentiation of the equation that describes the marginal profitability of organizational capital:

\[
\frac{d \ln \lambda_{k,ij}}{d \ln \tau_{ij}} = 1 - \sigma + (1 - \sigma) \frac{1}{\Delta \delta_{k,ij}} \left( \rho_{k,ij}(z) \right).
\]

and

\[
\frac{d \ln \lambda_{k,ij}}{d \ln \tau_{ij}} = 1 - \sigma - (\sigma - 1)^2 \frac{1}{\Delta \delta_{k,ij}} \left( \rho_{k,ij}(z) \right).
\]

Plugging equation (D.81), equation (D.82) and equation (D.87) into equation (D.80) provides the total differentiation of the equation that describes the organizational capital allocation of multi-product firms:

\[
\frac{d \ln \lambda_{k,ij}}{d \ln \tau_{ij}} = \frac{\sigma - 1}{\theta(\sigma - 1) + 1 - \frac{1}{\theta(\sigma - 1)}} + \frac{1}{\theta(\sigma - 1) + 1 - \frac{1}{\theta(\sigma - 1)}} \frac{d \ln \lambda_{k,ij}}{d \ln \tau_{ij}}.
\]
and

$$ \frac{d \ln o_{k,ij}(\omega)}{d \ln \tau_{ij}} = - \frac{(\sigma - 1)^2}{\theta(\sigma - 1) - 1} \frac{1}{H_k(\delta_{k,ij})} \frac{\partial H_k(\delta_{k,ij})}{\partial \delta_{k,ij}} \frac{1}{\Delta} \rho_{k,ij}(z), $$

which can be simplified to get

$$ \frac{d \ln o_{k,ij}(\omega)}{d \ln \tau_{ij}} = - \frac{1}{\Delta} (\sigma - 1) \rho_{k,ij}(z) \left( \frac{h_k(\delta_{k,ij})}{H_k(\delta_{k,ij})} \right)^{1/(\sigma - 1)}. $$

### D.21.3 Firm Sales (Revenue)

Taking the logarithm of the overall sales (revenue) of a firm in equation (4.43)

$$ R_{k,ij}(z, O) = \frac{\sigma}{1 - \theta(\sigma - 1)} f_{k,\omega}^{ij} \left( \frac{h_k(\delta_{k,ij})}{H_k(\delta_{k,ij})} \right)^{1/(\sigma - 1)}, $$

yields

$$ \ln R_{k,ij}(z, O) = \ln (\sigma) - \ln (1 - \theta(\sigma - 1)) + \ln \left( f_{k,\omega}^{ij} \right) + \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \left( \ln (h_k(\delta_{k,ij})) - \ln (H_k(\delta_{k,ij})) \right). $$

The total differentiation of equation (D.88) takes the general form

$$ \frac{d \ln R_{k,ij}(z, O)}{d \ln f_{k,\omega}^{ij}} = \frac{\partial \ln R_{k,ij}(z, O)}{\partial \ln f_{k,\omega}^{ij}} + \frac{\partial \ln R_{k,ij}(z, O)}{\partial \ln \delta_{k,ij}} \frac{d \ln \delta_{k,ij}}{d \ln f_{k,\omega}^{ij}}, $$

with

$$ \frac{\partial \ln R_{k,ij}(z, O)}{\partial \ln f_{k,\omega}^{ij}} = 1, $$

$$ \frac{\partial \ln R_{k,ij}(z, O)}{\partial \ln \delta_{k,ij}} = \frac{\partial \ln R_{k,ij}(z, O)}{\partial \delta_{k,ij}} \frac{\partial \delta_{k,ij}}{\partial \ln \delta_{k,ij}} = \frac{\partial \ln R_{k,ij}(z, O)}{\partial \delta_{k,ij}} $$

and

$$ \frac{\partial \ln R_{k,ij}(z, O)}{\partial \ln \delta_{k,ij}} = \delta_{k,ij} \frac{1 - \sigma}{\theta(\sigma - 1) - 1} \left( \frac{1}{h_k(\delta_{k,ij})} \frac{\partial h_k(\delta_{k,ij})}{\partial \delta_{k,ij}} - \frac{1}{H_k(\delta_{k,ij})} \frac{\partial H_k(\delta_{k,ij})}{\partial \delta_{k,ij}} \right). $$

The total differentiation of equation (4.42) takes the general form

$$ \frac{d \ln \delta_{k,ij}}{d \ln f_{k,\omega}^{ij}} = \frac{\partial \ln \delta_{k,ij}}{\partial \ln f_{k,\omega}^{ij}} + \frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^{*}(z)} \frac{d \ln O_{k,ij}^{*}(z)}{d \ln f_{k,\omega}^{ij}}, $$

with

$$ \frac{\partial \ln \delta_{k,ij}}{\partial \ln f_{k,\omega}^{ij}} = \frac{\partial \ln \delta_{k,ij}}{\partial \delta_{k,ij}} \frac{\partial \delta_{k,ij}}{\partial f_{k,\omega}^{ij}} \frac{f_{k,\omega}^{ij}}{\delta_{k,ij}} \frac{\partial \delta_{k,ij}}{\partial f_{k,\omega}^{ij}} = \delta_{k,ij} \frac{f_{k,\omega}^{ij}}{\delta_{k,ij}} \frac{f_{k,\omega}^{ij}}{\delta_{k,ij}} \frac{- \partial F/\partial f_{k,\omega}^{ij}}{- \partial F/\partial \delta_{k,ij}} = 0, $$

$$ \frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^{*}(z)} = \frac{\partial \ln \delta_{k,ij}}{\partial \delta_{k,ij}} \frac{\partial \delta_{k,ij}}{\partial O_{k,ij}^{*}(z)} \frac{\partial O_{k,ij}^{*}(z)}{\partial \ln O_{k,ij}(z)} \frac{\partial \ln O_{k,ij}(z)}{\partial \delta_{k,ij}} = \frac{O_{k,ij}^{*}(z)}{\delta_{k,ij}} \frac{\partial \ln O_{k,ij}(z)}{\partial \delta_{k,ij}} = \frac{O_{k,ij}^{*}(z)}{\delta_{k,ij}} \frac{- \partial F/\partial O_{k,ij}^{*}(z)}{- \partial F/\partial \delta_{k,ij}}. $$
\[
\frac{\partial \ln \delta_{k,ij}}{\partial \ln \mathcal{O}^*_{k,ij}(z)} = \frac{O^*_{k,ij}(z)}{\delta_{k,ij}} \left( \frac{-(1 - \theta(\sigma - 1))\mathcal{H}_k(\delta_{k,ij})}{\theta(\sigma - 1)} - \frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1} \frac{\partial \delta_{k,ij}}{\partial \mathcal{O}^*_{k,ij}} \right) \theta(1 - \sigma),
\]

\[
\frac{\partial \ln \delta_{k,ij}}{\partial \ln \mathcal{O}^*_{k,ij}(z)} = \frac{O^*_{k,ij}(z)}{\delta_{k,ij}} \left( \frac{-(1 - \theta(\sigma - 1))\mathcal{H}_k(\delta_{k,ij})}{\theta(\sigma - 1) - 1} - \frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1} \frac{\partial \delta_{k,ij}}{\partial \mathcal{O}^*_{k,ij}} \right) \theta(1 - \sigma) \theta(\sigma - 1) - 1
\]

\[
\frac{\partial \ln \delta_{k,ij}}{\partial \ln \mathcal{O}^*_{k,ij}(z)} = \frac{O^*_{k,ij}(z)}{\delta_{k,ij}} \left( \frac{-(1 - \theta(\sigma - 1))\mathcal{H}_k(\delta_{k,ij})}{\theta(\sigma - 1) - 1} - \frac{\theta(\sigma - 1)^2}{\theta(\sigma - 1) - 1} \frac{\partial \delta_{k,ij}}{\partial \mathcal{O}^*_{k,ij}} \right) \theta(1 - \sigma) \theta(\sigma - 1)
\]
\[ \frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^*(z)} = \frac{1}{\delta_{k,ij}} \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \]  
\[ \frac{1}{\partial \ln O_{k,ij}^*(z)} = \frac{1}{\partial \ln O_{k,ij}^*(z)} \]  
\[ \frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^*(z)} = \frac{1}{\delta_{k,ij}} \frac{\theta(\sigma - 1)}{\theta(\sigma - 1) - 1} \]  
\[ \frac{\partial \ln O_{k,ij}^*(z)}{\partial \ln f_{k,\omega}^{ij}} = \frac{\partial \ln O_{k,ij}^*(z)}{\partial \ln f_{k,\omega}^{ij}} = \frac{1}{\theta(\sigma - 1)}. \]  

Plugging equation (D.93), equation (D.94) and equation (D.95) into equation (D.89) provides the total differentiation of the equation that describes the product scope of multi-product firms:

\[ \frac{d \ln \delta_{k,ij}}{d \ln f_{k,\omega}^{ij}} = \frac{1}{\theta(\sigma - 1)} \frac{\theta(\sigma - 1) - 1}{\epsilon h_k(\delta_{k,ij}) + \theta(\sigma - 1)\epsilon H_k(\delta_{k,ij})}, \]  
with \( \epsilon h_k(\delta_{k,ij}) = \frac{\delta_{k,ij}}{h_k(\delta_{k,ij}) - \delta_{k,ij}} > 0 \) as the elasticity of the unit production costs with respect to scope and \( \epsilon H_k(\delta_{k,ij}) = -\frac{\delta_{k,ij}}{H_k(\delta_{k,ij})} > 0 \) as the elasticity of the flexible manufacturing technology index with respect to scope, and

\[ \frac{d \ln \delta_{k,ij}}{d \ln f_{k,\omega}^{ij}} = \frac{1}{(\sigma - 1) \epsilon h_k(\delta_{k,ij}) + \theta(\sigma - 1)\epsilon H_k(\delta_{k,ij})}. \]  

Plugging equation (D.90), equation (D.91) and equation (D.96) into equation (D.89) provides the total differentiation of the equation that describes the overall sales (revenue) of multi-product firms:

\[ \frac{d \ln R_{k,ij}(z, O)}{d \ln f_{k,\omega}^{ij}} = 1 + \frac{1 - \sigma}{\theta(\sigma - 1) - 1} (\epsilon h_k(\delta_{k,ij}) + \epsilon H_k(\delta_{k,ij})) \frac{1}{\theta(\sigma - 1) - 1}, \]  
\[ \frac{d \ln R_{k,ij}(z, O)}{d \ln f_{k,\omega}^{ij}} = 1 - \frac{\epsilon h_k(\delta_{k,ij}) + \epsilon H_k(\delta_{k,ij})}{\epsilon h_k(\delta_{k,ij}) + \theta(\sigma - 1)\epsilon H_k(\delta_{k,ij})}, \]  
and

\[ \frac{d \ln R_{k,ij}(z, O)}{d \ln f_{k,\omega}^{ij}} = 0 + \frac{(1 - \theta(\sigma - 1))\epsilon H_k(\delta_{k,ij})}{\epsilon h_k(\delta_{k,ij}) + \theta(\sigma - 1)\epsilon H_k(\delta_{k,ij})}. \]  
The total differentiation of equation (D.88) takes the general form

\[ \frac{d \ln R_{k,ij}(z, O)}{d \ln \tau_{ij}} = \frac{\partial \ln R_{k,ij}(z, O)}{\partial \ln \tau_{ij}} + \frac{\partial \ln R_{k,ij}(z, O)}{\partial \ln \tau_{ij}} \frac{d \ln \delta_{k,ij}}{d \ln \tau_{ij}}, \]  

with

\[ \frac{\partial \ln R_{k,ij}(z, O)}{\partial \ln \tau_{ij}} = 0. \]  

(D.98)
The total differentiation of equation (D.42) takes the general form
\[
\frac{d \ln \delta_{k,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln \delta_{k,ij}}{\partial \ln \tau_{ij}} + \frac{\partial \ln \delta_{k,ij}}{\partial \ln O_{k,ij}^*} \frac{d \ln O_{k,ij}^*(z)}{d \ln \tau_{ij}}, \tag{D.99}
\]
with
\[
\frac{\partial \ln \delta_{k,ij}}{\partial \ln \tau_{ij}} = \frac{\partial \ln \delta_{k,ij}}{\partial \tau_{ij}} \frac{\partial \tau_{ij}}{\partial \ln \tau_{ij}} = \frac{\tau_{ij}}{\delta_{k,ij}} \frac{\delta_{k,ij}}{\tau_{ij}} \left( - \frac{\partial F/\partial \tau_{ij}}{\partial \delta_{k,ij}} \right) = 0 \tag{D.100}
\]
and
\[
\frac{d \ln O_{k,ij}^*(z)}{d \ln \tau_{ij}} = \frac{\partial \ln O_{k,ij}^*(z)}{\partial \ln \tau_{ij}} = \frac{1}{\theta}. \tag{D.101}
\]
Plugging equation (D.100), equation (D.94) and equation (D.101) into equation (D.99) provides the total differentiation of the equation that describes the product scope of multi-product firms:
\[
\frac{d \ln \delta_{k,ij}}{d \ln \tau_{ij}} = \frac{1}{\theta} \frac{1}{\delta_{k,ij}} h_k(\delta_{k,ij})^{\frac{1}{\theta}} \left( \theta(\sigma - 1) - 1 \right) + \frac{\partial h_k(\delta_{k,ij})}{\partial \delta_{k,ij}} - \theta(\sigma - 1) h_k(\delta_{k,ij})^{\frac{1}{\theta}} \frac{\partial h_k(\delta_{k,ij})}{\partial \delta_{k,ij}} \tag{D.102}
\]
Plugging equation (D.98), equation (D.91) and equation (D.102) into equation (D.97) provides the total differentiation of the equation that describes the overall sales (revenue) of multi-product firms:
\[
\frac{d \ln R_{k,ij}(z, O)}{d \ln \tau_{ij}} = \frac{1}{\theta(\sigma - 1) - 1} \left( \varepsilon_h(\delta_{k,ij}) + \varepsilon_H(\delta_{k,ij}) \right) \frac{1}{\theta} \frac{1}{\delta_{k,ij}} \left( \theta(\sigma - 1) - 1 \right) + \frac{\partial h_k(\delta_{k,ij})}{\partial \delta_{k,ij}} - \theta(\sigma - 1) h_k(\delta_{k,ij})^{\frac{1}{\theta}} \frac{\partial h_k(\delta_{k,ij})}{\partial \delta_{k,ij}} \tag{D.102}
\]

\[
\frac{d \ln R_{k,ij}(z, O)}{d \ln \tau_{ij}} = \frac{\sigma \varepsilon_h(\delta_{k,ij}) - \varepsilon_h(\delta_{k,ij}) + \sigma^2 \theta \varepsilon_H(\delta_{k,ij}) - \sigma^2 \theta \varepsilon_H(\delta_{k,ij}) - 2 \sigma \theta \varepsilon_H(\delta_{k,ij})}{\varepsilon_h(\delta_{k,ij}) + \theta(\sigma - 1) \varepsilon_H(\delta_{k,ij})} \tag{D.102}
\]

\[
\frac{d \ln R_{k,ij}(z, O)}{d \ln \tau_{ij}} = \frac{\sigma - 1 \varepsilon_h(\delta_{k,ij}) + \theta(\sigma - 1)^2 \varepsilon_H(\delta_{k,ij}) - (\sigma^2 \theta - 2 \sigma \theta + \theta) \varepsilon_H(\delta_{k,ij})}{\varepsilon_h(\delta_{k,ij}) + \theta(\sigma - 1) \varepsilon_H(\delta_{k,ij})} \tag{D.102}
\]
The fundamental expression in the overall (export) sales (revenue) elasticities then takes the form:

\[ -\frac{d\ln R_{k,ij}(z, O)}{d\ln \tau_{ij}} = (\sigma - 1) + (\sigma - 1) \frac{(1 - \theta(\sigma - 1))\varepsilon_{h_k} + \theta(\sigma - 1)\varepsilon_{H_k}}{\varepsilon_{h_k} + \varepsilon_{H_k}}. \]

With the (standard) functional specification \( h_k(\omega) = \exp(\mu_k \omega) \), with \( \mu_k > 0 \) as a (industry-specific) parameter, the elasticity of the unit production costs with respect to scope takes the form:

\[ \varepsilon_{h_k} = \frac{\delta_{k,ij}}{h_k(\delta_{k,ij})} \frac{\partial h_k(\delta_{k,ij})}{\partial \delta_{k,ij}} = \frac{\delta_{k,ij}}{\exp(\mu_k \delta_{k,ij})} \mu_k \exp(\mu_k \delta_{k,ij}) = \mu_k \delta_{k,ij} \]

and the elasticity of the flexible manufacturing technology index with respect to scope is then given by

\[ \varepsilon_{H_k} = -\frac{\delta_{k,ij}}{H_k(\delta_{k,ij})} \frac{\partial H_k(\delta_{k,ij})}{\partial \delta_{k,ij}} = -\frac{\delta_{k,ij}}{\exp(\mu_k \omega) \frac{1}{\theta(\sigma - 1) - 1} \int_0^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) d\omega} \frac{\theta(\sigma - 1) - 1}{\sigma - 1} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \]

and

\[ \varepsilon_{H_k} = \frac{\theta(\sigma - 1) - 1}{\sigma - 1} \frac{\delta_{k,ij}}{\int_0^{\delta_{k,ij}} \exp(\mu_k \omega) \frac{1}{\theta(\sigma - 1) - 1} d\omega} \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \frac{\theta(\sigma - 1) - 1}{\sigma - 1} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \]

The fundamental expression in the overall (export) sales (revenue) elasticities then takes the form:

\[ \frac{(1 - \theta(\sigma - 1))\varepsilon_{h_k}}{\varepsilon_{h_k} + \theta(\sigma - 1)\varepsilon_{H_k}} = \frac{(\theta(\sigma - 1) - 1)^2 \delta_{k,ij}}{\sigma - 1} \frac{\exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right)}{\int_0^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) d\omega}, \]

and

\[ \frac{(1 - \theta(\sigma - 1))\varepsilon_{H_k}}{\varepsilon_{h_k} + \theta(\sigma - 1)\varepsilon_{H_k}} = \frac{(\theta(\sigma - 1) - 1)^2}{\sigma - 1} \frac{\exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right)}{\int_0^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) d\omega} \]

whereas the derivative of equation (D.103) with respect to \( \delta_{k,ij} \) has the property that

\[ \partial \frac{(1 - \theta(\sigma - 1))\varepsilon_{H_k}}{\varepsilon_{h_k} + \theta(\sigma - 1)\varepsilon_{H_k}} \partial \delta_{k,ij} \geq 0 \quad \Leftrightarrow \quad \left(\frac{(\theta(\sigma - 1) - 1)^2}{\sigma - 1} \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \int_0^{\delta_k} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \right)^{-1} \]
\[\frac{\left(\theta(\sigma - 1) - 1\right)^2}{\sigma - 1} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \quad (-1) \quad \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-2}\]

\[\times \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \left(\mu_k - \theta(\sigma - 1)\right) \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right)\]

\[\times \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1} - \left(\frac{\theta(\sigma - 1) - 1}{\sigma - 1}\right)^2 \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right)\]

\[\times \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1} \left(-\theta(\sigma - 1) - 1\right) \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k\]

\[\times \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \quad (-1) \quad \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-2}\]

\[\times \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \geq 0,\]

\[\left(\frac{\theta(\sigma - 1) - 1}{\sigma - 1}\right)^2 \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1}\]

\[\times \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \left(\mu_k - \theta(\sigma - 1)\right) \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right)\]

\[\times \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1}\]

\[-\left(\frac{\theta(\sigma - 1) - 1}{\sigma - 1}\right)^2 \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1}\]

\[\times \left(-\theta(\sigma - 1)\mu_k \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1}\right)\]

\[+ \theta(\sigma - 1) - 1 \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-2}\]

\[\times \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \geq 0,\]

\[-\left(-\sigma - 1\right) \mu_k \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-1}\]

\[+ \left(\theta(\sigma - 1) - 1\right) \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij}\right) \left[\int_{0}^{\delta_{k,ij}} \exp\left(\frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega\right) d\omega\right]^{-2}\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left( -\frac{\theta(\sigma - 1) - 1}{\sigma - 1} \right) \left( \mu_k - \theta(\sigma - 1) - 1 \right) \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
- \left( \frac{\theta(\sigma - 1) - 1}{\sigma - 1} \right)^2 \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\times \left( -\theta(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right)
\]
\[
+ \theta(\sigma - 1) - 1 \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-2}
\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\left( -(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \right)
\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\left( -(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \right)
\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\left( -(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \right)
\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\left( -(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \right)
\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\left( -(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \right)
\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\left( -(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \right)
\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\left( -(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \right)
\]
\[
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1}
\]
\[
\left( -(\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \right) \right)
\]
\[
\left( - (\sigma - 1) \mu_k \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \right] \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-1} \\
+ (\theta(\sigma - 1) - 1) \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left[ \int_0^{\delta_{k,ij}} \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \omega \right) d\omega \right]^{-2} \\
\times \exp \left( \frac{\sigma - 1}{\theta(\sigma - 1) - 1} \mu_k \delta_{k,ij} \right) \left( - \frac{\theta(\sigma - 1) - 1}{\sigma - 1} \mu_k \right) \geq 0.
\]
Appendix E

E.1 COMPUSTAT - Capital IQ - Global: Countries in Asia

The countries in Asia which are ultimately covered by the samples whose firm-level data are compiled by COMPUSTAT - Capital IQ - Global are the following, whereas each is listed with its ISO 3166-1 ALPHA-3 country code and its level of the Human Development Index (HDI) that is published in 2018 (unless otherwise indicated) in parentheses: Bangladesh (BGD, 0.608), Cambodia (KHM, 0.582), China (CHN, 0.752), Hong Kong (HKG, 0.933), India (IND, 0.640), Indonesia (IDN, 0.694), Japan (JPN, 0.909), Korea (KOR, 0.903), Malaysia (MYS, 0.798), Pakistan (PAK, 0.562), Philippines (PHL, 0.699), Singapore (SGP, 0.932), Sri Lanka (LKA, 0.770), Taiwan (TWN, 0.885 [2017]), Thailand (THA, 0.755) and Vietnam (VNM, 0.694).

E.2 COMPUSTAT - Capital IQ - Global: Countries in Europe

The countries in Europe which are ultimately covered by the samples whose firm-level data are compiled by COMPUSTAT - Capital IQ - Global are the following, whereas each is listed with its ISO 3166-1 ALPHA-3 country code and its level of the Human Development Index (HDI) that is published in 2018 (unless otherwise indicated) in parentheses: Austria (AUT, 0.908), Belgium (BEL, 0.916), Bulgaria (BGR, 0.813), Croatia (HRV, 0.831), Cyprus (CYP, 0.869), Czech Republic (CZE, 0.888), Denmark (DNK, 0.929), Estonia (EST, 0.871), Finland (FIN, 0.920), France (FRA, 0.901), Germany (DEU, 0.936), Great Britain (GBR, 0.922), Greece (GRC, 0.870), Hungary (HUN, 0.838), Iceland (ISL, 0.935), Ireland (IRL, 0.938), Israel (ISR, 0.903), Italy (ITA, 0.880), Latvia (LVA, 0.847), Lithuania (LTU, 0.858), Luxembourg (LUX, 0.904), Malta (MLT, 0.878), the Netherlands (NLD, 0.931), Norway (NOR, 0.953), Poland (POL, 0.865), Portugal (PRT, 0.847), Romania (ROU, 0.811), Slovakia (SVK, 0.855), Slovenia (SVN, 0.896), Spain (ESP, 0.891), Sweden (SWE, 0.933) and Switzerland (CHE, 0.944).
E.3 Empirical Methodology - Sector Classification

The sectors are constructed as groups of 2-digit SIC codes and are so the following: Agriculture (SIC codes 01-09), mining (SIC codes 10-14), construction (SIC codes 15-19), manufacturing (SIC codes 20-39), transportation (SIC codes 40-49), wholesale trade (SIC codes 50-51), retail trade (SIC codes 52-59), finance (SIC codes 60-69), services (SIC codes 70-90), public administration (SIC codes 91-99).

Following Fama and French (1997), an alternative sector classification is applied: The sectors are constructed as groups of 4-digit SIC codes and are so the following: Consumer non-durables: food, tobacco, textiles, apparel, leather, and toys (SIC codes 0100-0999, 2000-2399, 2700-2749, 2770-2799, 3100-3199, 3940-3989), consumer durables: cars, TVs, furniture, and household appliances (SIC codes 2500-2519, 2590-2599, 3630-3659, 3710-3711, 3714, 3716, 3750-3751, 3792, 3900-3939, 3990-3999), manufacturing: machinery, trucks, planes, office furniture, paper, and commercial printing (SIC codes 2520-2589, 2600-2699, 2750-2769, 3000-3099, 3200-3569, 3580-3629, 3700-3709, 3712-3713, 3715, 3717-3749, 3752-3791, 3793-3799, 3830-3839, 3860-3899), energy, oil, gas, and coal extraction and products (SIC codes 1200-1399, 2900-2999), chemicals and allied products (SIC codes 2800-2829, 2840-2899), business equipment: computers, software, and electronic equipment (SIC codes 3570-3579, 3660-3692, 3694-3699, 3810-3829, 7370-7379), telecom, telephone, and television transmission (SIC codes 4800-4899), utilities (SIC codes 4900-4949, wholesale, retail, and some services (laundries, repair shops) (SIC codes 5000-5999, 7200-7299, 7600-7699), health care, medical equipment, and drugs (SIC codes 2830-2839, 3693, 3840-3859, 8000-8099), money, and finance (SIC codes 6000-6999), others: mines, construction, building materials, transportation, hotels, business services, and entertainment (all other SIC codes).

E.4 Empirical Methodology - Country-group Dummies

For the European samples:
The groups on which the country-group dummies for 2007 are constructed are the following:

- Group 1: Macedonia, Serbia, Ukraine, Bulgaria, Romania, Poland, Latvia, Lithuania and Hungary
- Group 2: Croatia, Slovakia, Estonia, Malta, Portugal, Czech Republic, Slovenia, Greece and Spain
- Group 3: Cyprus, Italy, France, Great Britain, Israel, Belgium, Germany, Finland, Denmark and Austria
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- Group 4: Sweden, Iceland, the Netherlands, Ireland, Switzerland, Norway, Liechtenstein, Monaco and Luxembourg

The groups on which the country-group dummies for 2017 are constructed are the following:

- Group 1: Ukraine, Serbia, Macedonia, Bulgaria, Croatia, Romania, Latvia, Greece and Hungary
- Group 2: Poland, Estonia, Portugal, Slovakia, Lithuania, Cyprus, Slovenia, Czech Republic and Spain
- Group 3: Italy, Malta, France, Great Britain, Finland, Belgium, Israel, Sweden, Germany and Denmark
- Group 4: the Netherlands, Austria, Iceland, Norway, Switzerland, Ireland, Liechtenstein, Luxembourg and Monaco

For the Asian samples:
The groups on which the country-group dummies for 2007 are constructed are the following:

- Group 1: Bangladesh, Cambodia and Vietnam
- Group 2: Pakistan, India and Sri Lanka
- Group 3: Philippines, Indonesia and China
- Group 4: Thailand, Taiwan and Malaysia
- Group 5: Korea, Hong Kong, Japan and Singapore

The groups on which the country-group dummies for 2017 are constructed are the following:

- Group 1: Cambodia, Bangladesh and Pakistan
- Group 2: India, Vietnam and Philippines
- Group 3: Indonesia, Sri Lanka and Thailand
- Group 4: Taiwan, China and Malaysia
- Group 5: Korea, Japan, Hong Kong and Singapore

E.5 Descriptive Statistics

All the samples cover medium- and large-sized firms, i.e. firms with at least 175 employees.
E.5.1 European Sample on the Period 2000-2007 (E-S1)

The sample of European firms on the period 2000-2007 consists of 1,225 firms in 2007 whose headquarters are located each in one of 32 countries in Europe and which operate each in one of 10 sectors.

While almost half (2007: 47.2%) of the firms in the sample are located either in Great Britain, Germany or France, almost half (2007: 47.9%) of the firms operate in the manufacturing sector that thereby represents by far the largest sector in the sample, followed by the services sector in which 19.6% (2007) of the firms operate and the transportation sector in which 12.5% (2007) of the firms operate (Figure E.1 and Figure E.2).
The firms with less than 2,500 employees represent about half (2007: 51.1%) of the firms in the sample and the firms in the two highest-ranked employment categories with at least 15,000 employees account for 21.8% (2007) of the firms, whereas R&D firms are incrementally more present among the firms in the higher-ranked employment categories:

![Figure E.3: Distribution of Non-R&D Firms and R&D Firms in the European Sample across the Employment Categories (2007)](image)

**Notes:** Firms in the sample are grouped according to their number of employees, which yields employment categories (on the left: category “175-749”: firms with at least 175 but less than 750 employees, category “750-2,499”: firms with at least 750 but less than 2,500 employees, category “2,500-7,499”: firms with at least 2,500 but less than 7,500 employees, etc.). Higher-ranked employment categories include firms with a larger workforce.

![Figure E.4: Distribution of R&D and Organizational Capital Stocks in the European Sample across the Sectors (Mean Values, 2007)](image)

**Figure E.4:** Distribution of R&D and Organizational Capital Stocks in the European Sample across the Sectors (Mean Values, 2007)

The proportion of R&D firms in the employment categories increases when moving along the categorization of the firm size from firms with a workforce at a small to medium
scale to firms with a workforce at a medium to large scale, and the proportion increases once more when moving further to firms with a workforce at a very large scale (Figure E.3).

The firms in the consumer durables, health care and chemicals sectors are on average equipped with the largest R&D stocks across the sectors, while the firms in the chemicals, consumer durables, health care as well as telephone and television sectors have on average the largest organizational capital stocks (Figure E.4).

Figure E.5: Distribution of R&D and Organizational Capital Stocks in the European Sample across the Sectors (Median Values, 2007)

Analyzing the input distributions within the sectors, the (strictly positive) differences between the mean and median values for the R&D and organizational capital stocks for all the sectors indicate that many firms with only small stocks and some firms with large stocks coexist in all the sectors, whereas the right-skewed distribution varies in its shape across the sectors (Figure E.5).

E.5.2 North-American Sample on the Period 2000-2007 (NA-S1)

The sample of North-American firms on the period 2000-2007 consists of 3,281 firms in 2007 whose headquarters are located each either in the United States or Canada and which operate each in one of 10 sectors.

While about nine out of ten firms in the sample (2007: 92.4%) are located in the United States, a proportion between one-third and half (2007: 41.4%) of the firms operate in the manufacturing sector that thereby represents by far the largest sector in the sample, followed by the services sector in which 18.1% (2007) of the firms operate and the finance sector in which 15.1% (2007) of the firms operate (Figure E.6 and Figure E.7).
The firms with less than 2,500 employees represent a proportion between half and two-thirds (2007: 58.2%) of the firms in the sample and the firms in the two highest-ranked employment categories with at least 15,000 employees account for 13.5% (2007) of the firms, whereas R&D firms are more present among the firms with a workforce at the medium scale [Figure E.8].

The firms in the consumer durables, health care and business equipment sectors are on average equipped with the largest R&D stocks across the sectors, while the firms in the telephone and television, consumer durables as well as trade and services sectors have on
average the largest organizational capital stocks (Figure E.9).

Analyzing the input distributions within the sectors, the (strictly positive) differences between the mean and median values for the R&D and organizational capital stocks for all the sectors indicate that many firms with only small stocks and some firms with large stocks coexist in all the sectors, whereas the right-skewed distribution varies in its shape across the sectors (Figure E.10).

Notes: Firms in the sample are grouped according to their number of employees, which yields employment categories (on the left: category “175-749”: firms with at least 175 but less than 750 employees, category “750-2,499”: firms with at least 750 but less than 2,500 employees, category “2,500-7,499”: firms with at least 2,500 but less than 7,500 employees, etc.). Higher-ranked employment categories include firms with a larger workforce.

Figure E.8: Distribution of Non-R&D Firms and R&D Firms in the North-American Sample across the Employment Categories (2007)

Figure E.9: Distribution of R&D and Organizational Capital Stocks in the North-American Sample across the Sectors (Mean Values, 2007)
E.5. DESCRIPTIVE STATISTICS

Figure E.10: Distribution of R&D and Organizational Capital Stocks in the North-American Sample across the Sectors (Median Values, 2007)

E.5.3 Asian Sample on the Period 2000-2007 (A-S1)

The sample of Asian firms on the period 2000-2007 consists of 3,937 firms in 2007 whose headquarters are located each in one of 15 countries\(^1\) in Asia and which operate each in one of 10 sectors.

Figure E.11: Distribution of Firms in the Asian Sample across the Countries (2007)

While almost two-thirds (2007: 65.4\%) of the firms in the sample are located in Japan, about half (2007: 56.3\%) of the firms operate in the manufacturing sector that thereby

\(^1\)Compared to A-S2, firms in Korea do not appear in A-S1.
represents by far the largest sector in the sample, followed by the services sector in which 13.1% (2007) of the firms operate and the retail trade sector in which 8% (2007) of the firms operate [Figure E.11 and Figure E.12].

![Figure E.12: Distribution of Firms in the Asian Sample across the Sectors (2007)](image1)

The firms with less than 2,500 employees represent about two-thirds (2007: 69.4%) of the firms in the sample and the firms in the two highest-ranked employment categories with at least 15,000 employees account for 6.2% (2007) of the firms, whereas R&D firms
are more present among the firms with a workforce at the medium scale than at the small scale and more present among the firms with a workforce at the large scale than at the medium scale (Figure E.13).

Figure E.14: Distribution of R&D and Organizational Capital Stocks in the Asian Sample across the Sectors (Mean Values, 2007)

Figure E.15: Distribution of R&D and Organizational Capital Stocks in the Asian Sample across the Sectors (Median Values, 2007)

The firms in the consumer durables, telephone and television as well as health care sectors are on average equipped with the largest R&D stocks across the sectors, while the firms in the telephone and television, utilities and consumer durables sectors have on average the largest organizational capital stocks (Figure E.14).
Analyzing the input distributions within the sectors, the (strictly positive) differences between the mean and median values for the R&D and organizational capital stocks for all the sectors indicate that many firms with only small stocks and some firms with large stocks coexist in all the sectors, whereas the right-skewed distribution varies in its shape across the sectors. [Figure E.15].

### E.5.4 Input Levels and Growth Rates

|                | R&D firms | | | Non-R&D firms | | | |
|----------------|-----------|---|---|----------------|---|---|
|                | A-S1<sup>a</sup> | E-S1<sup>b</sup> | NA-S1<sup>a</sup> | A-S1<sup>a</sup> | E-S1<sup>b</sup> | NA-S1<sup>a</sup> |
|                | w/o HJKST | w/ HJKST | w/o HJKST | w/ HJKST | w/o HJKST | w/ HJKST |
| K              | 53.60111  | 92.52966 | 235.85 | 53.3855 | 26.46432 | 42.68395 | 68.17936 | 91.611 |
| L<sup>c</sup>  | 1.6635  | 1.3655  | 4.75 | 1.374 | 0.943 | 0.875 | 1.4815 | 1.88 |
| RD             | 1.018829 | 12.52332 | 49.48603 | 67.66069 | - | - | - | - |
| OC             | 18.57816 | 75.55068 | 151.8951 | 125.0278 | 9.310842 | 27.80197 | 35.64378 | 82.19669 |

**Notes:**<sup>a</sup>: sample measured in million USD, <sup>b</sup>: sample measured in million EUR, <sup>c</sup>: employment measured in thousands, and HJKST: acronym for Hong Kong, Japan, Korea, Singapore and Taiwan.

Table E.1: Physical Capital, Labor, Research and Development as well as Organizational Capital in the Three Samples and the Two Sub-samples (R&D and Non-R&D Firms) of Each Sample and 2007 (Median Values)

|                | R&D firms | | | Non-R&D firms | | | |
|----------------|-----------|---|---|----------------|---|---|
|                | A-S1<sup>a</sup> | E-S1 | NA-S1 | A-S1<sup>a</sup> | E-S1 | NA-S1 |
|                | w/o HJKST | w/ HJKST | w/o HJKST | w/ HJKST | w/o HJKST | w/ HJKST |
| K              | 21.03 | 0.62 | 3.75 | 6.83 | 13.19 | 4.90 | 5.79 | 7.87 |
| L              | 4.20 | 2.94 | 6.17 | 4.26 | 3.66 | 3.09 | 7.21 | 3.77 |
| RD             | 14.59 | 3.30 | 9.62 | 8.42 | - | - | - | - |
| OC             | 13.18 | 4.31 | 8.41 | 8.06 | 10.92 | 7.73 | 8.77 | 8.39 |

**Notes:** Measured in percentage and HJKST: acronym for Hong Kong, Japan, Korea, Singapore and Taiwan.

Table E.2: Growth of Physical Capital, Labor, Research and Development as well as Organizational Capital between 2006 and 2007 in the Three Samples and the Two Sub-samples (R&D and Non-R&D Firms) of Each Sample (Median Values)
### E.6 Empirical Results

#### E.6.1 E-S1: European Sample (2000-2007)

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>Levels FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.34***</td>
<td>0.09*</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>L</td>
<td>0.63***</td>
<td>0.68***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>RD</td>
<td>0.05***</td>
<td>0.08***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>OC</td>
<td>0.11***</td>
<td>0.11***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Obs.</td>
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<td>558</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9449</td>
<td>0.5009</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

Table E.3: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2007) and First Differences (FDs, Years 2006-2007) for the European Sample

<table>
<thead>
<tr>
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<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Translog(^a)</td>
<td>Translog(^a)</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>Levels FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.33***</td>
<td>0.13***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>L</td>
<td>0.66***</td>
<td>0.53***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>RD</td>
<td>0.04***</td>
<td>0.13***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>OC</td>
<td>0.17***</td>
<td>0.20***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Obs.</td>
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<td>558</td>
</tr>
<tr>
<td>R-sq.</td>
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<td>0.5266</td>
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</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and \(^a\): evaluation at the sample medians.

Table E.4: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2007) and First Differences (FDs, Years 2006-2007) for the European Sample
### Table E.5: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the European Sample on the Period 2000-2007

**R&D firms**

<table>
<thead>
<tr>
<th></th>
<th>Cobb-Douglas</th>
<th>Translog&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.26***</td>
<td>0.30***</td>
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<tr>
<td></td>
<td>(0.08)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
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<td>0.53***</td>
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<tr>
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<td>(0.02)</td>
</tr>
<tr>
<td>RD</td>
<td>0.05***</td>
<td>0.02***</td>
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<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
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<tr>
<td>OC</td>
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<td>Groups</td>
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</table>

**Non-R&D firms**

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<thead>
<tr>
<th></th>
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<th>Translog&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
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<td>w/ OC</td>
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<td>0.20**</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>L</td>
<td>0.42***</td>
<td>0.38***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>RD</td>
<td>0.35***</td>
<td>0.37***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>OC</td>
<td>0.35***</td>
<td>0.37***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.14)</td>
</tr>
</tbody>
</table>

**Notes:** Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and ^a: evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

### E.6.2 NA-S1: North-American Sample (2000-2007)

**Table E.6: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2007) and First Differences (FDs, Years 2006-2007) for the North-American Sample**

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas (w/o OC)</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.32***</td>
<td>0.17***</td>
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<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
<td>0.58***</td>
<td>0.44***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.05)</td>
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<tr>
<td>RD</td>
<td>0.10***</td>
<td>0.23***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.04)</td>
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<tr>
<td>OC</td>
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<td>Obs.</td>
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<td>1,391</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9172</td>
<td>0.4690</td>
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</table>

**Notes:** Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and estimations in levels with sector and country-group dummies.

Table E.6: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2007) and First Differences (FDs, Years 2006-2007) for the North-American Sample
### E.6. EMPIRICAL RESULTS

#### Table E.7: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2007) and First Differences (FDs, Years 2006-2007) for the North-American Sample

<table>
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<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Translog(a) (w/o OC)</td>
<td>Translog(a)</td>
</tr>
<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.28***</td>
<td>0.14***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>L</td>
<td>0.60***</td>
<td>0.51***</td>
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<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>RD</td>
<td>0.12***</td>
<td>0.29***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>OC</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
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<td>1,391</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.9220</td>
<td>0.4867</td>
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*Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and \(a\): evaluation at the sample medians.*

#### Table E.8: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the North-American Sample on the Period 2000-2007

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
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<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog(a)</td>
</tr>
<tr>
<td></td>
<td>w/ OC w/ OC</td>
<td>w/ OC w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.27***</td>
<td>0.31***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>L</td>
<td>0.63***</td>
<td>0.45***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>RD</td>
<td>0.08***</td>
<td>-0.07***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.01)</td>
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<tr>
<td>OC</td>
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<td>(0.02)</td>
<td>(0.10)</td>
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<td>Obs.</td>
<td>9,586</td>
<td>9,586</td>
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<td>Groups</td>
<td>1,402</td>
<td>1,402</td>
</tr>
</tbody>
</table>

*Notes: Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and \(a\): evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.*

Table E.8: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the North-American Sample on the Period 2000-2007
### E.6.3 A-S1: Asian Sample (2000-2007)

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas (w/o OC)</td>
<td>Cobb-Douglas</td>
</tr>
<tr>
<td><strong>K</strong></td>
<td>0.45***</td>
<td>0.29***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>0.42***</td>
<td>0.50***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
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<tr>
<td><strong>RD</strong></td>
<td>0.10***</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
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<tr>
<td><strong>OC</strong></td>
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<td>(0.02)</td>
<td>(0.07)</td>
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<td><strong>Obs.</strong></td>
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<td>2,312</td>
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<tr>
<td><strong>R-sq.</strong></td>
<td>0.8497</td>
<td>0.8944</td>
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</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and<sup>a</sup>: evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.

Table E.10: Output Elasticity Estimations with the OLS Estimator and the Cobb-Douglas Specification in Levels (Year 2007) and First Differences (FDs, Years 2006-2007) for the Asian Sample
### Table E.11: Output Elasticity Estimations with the OLS Estimator and the Translog Specification in Levels (Year 2007) and First Differences (FDs, Years 2006-2007) for the Asian Sample

<table>
<thead>
<tr>
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<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Translog(^a) (w/o OC)</td>
<td>Translog(^a)</td>
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<tr>
<td></td>
<td>Levels</td>
<td>FDs</td>
</tr>
<tr>
<td>K</td>
<td>0.45***</td>
<td>0.21***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>L</td>
<td>0.44***</td>
<td>0.43***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>RD</td>
<td>0.09***</td>
<td>0.18***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>OC</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>2,312</td>
<td>2,240</td>
</tr>
<tr>
<td>R-sq.</td>
<td>0.8621</td>
<td>0.3903</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, estimations in levels with sector and country-group dummies, and \(^a\): evaluation at the sample medians.

### Table E.12: Output Elasticity Estimations with the Method by Olley and Pakes (1996) for the Asian Sample on the Period 2000-2007

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D firms</th>
<th>Non-R&amp;D firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cobb-Douglas</td>
<td>Translog(^a)</td>
</tr>
<tr>
<td></td>
<td>w/o OC</td>
<td>w/ OC</td>
</tr>
<tr>
<td>K</td>
<td>0.03</td>
<td>0.22***</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>L</td>
<td>0.42***</td>
<td>0.24***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>RD</td>
<td>0.08***</td>
<td>-0.05***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>OC</td>
<td>0.57***</td>
<td>0.65***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Obs.</td>
<td>14,641</td>
<td>14,641</td>
</tr>
<tr>
<td>Groups</td>
<td>2,312</td>
<td>2,312</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses, significance levels: * 10%, ** 5% and *** 1%, and \(^a\): evaluation at the sample medians and application of the Ackerberg et al. (2015) correction.
Chapter 5

Multi-product Exporters in a World with Heterogeneous Countries and Non-homothetic Preferences

5.1 Introduction

With the increasing availability of disaggregate data, three facts about both countries and firms robustly appear in the empirical literature of international trade: First, countries with a higher per capita income import more product varieties from other countries. [Hummels and Klenow (2002)] analyze the trade flows between 110 exporting and 59 importing countries by decomposing each country’s trade into an extensive margin (number of product varieties and number of markets) and an intensive margin (value of each product) as well as the quality of exports (using available quantity data). In addition to the result that the extensive margin accounts for a substantial share of the greater exports of larger economies, they find that countries with a higher per capita income not only import greater values, but even a larger number of product varieties. The extensive margin thereby accounts for a notable, compared to exports although smaller share of the greater imports. Applying data on bilateral trade flows in the analysis, parts of the empirical literature deal with the frequently observable absence of potential trade relations, which are called “zeros” due to their corresponding entries in tables of trade. [Baldwin and Harrigan (2011)] identify the

1In their baseline study, [Hummels and Klenow (2002)] identify products in the applied U.N. data by 6-digit Harmonized System (HS) classification categories and are therefore able to deal with 5,017 products. For a robustness check of their results, they also examine US import data from 119 countries in 13,386 10-digit HS classification categories as products.

2In contrast to the standard characterization of the extensive and intensive margins as absolute measures of the decomposed trade flows, [Hummels and Klenow (2002)] apply relative measures for those margins to take the respective country’s trade proportions and those of its traded products relative to the overall world into account. By doing this, the analysis puts weights on the absolute measures according to the country’s products’ quantitative importance in world trade. However, the relevant qualitative evidence for the margins is unchanged (shown by robustness checks), and therefore representable in the way above.

3See [Hummels and Klenow (2005)] for (export) data at the product level, [Bernard et al. (2007)] for trade data at the firm level and [Helpman et al. (2008)] for trade data at the country level.
real GDP per worker in the destination country as a significant determinant of zeros in U.S. export data, thereby leading up to the inference that the destination’s income level may play a significant role for both extensive margins of trade, i.e. whether a country is served with product varieties at all and whether the country imports a particular number of product varieties.

Second, imports and exports are concentrated among a relatively small number of firms, so that firms in form of “export superstars” exist and represent the “granular” components of the set of internationally active players. [Bernard et al. (2007) and Bernard et al. (2009) report a highly skewed distribution of trade among firms in the United States: In 2000, the top 1% of trading firms by value are responsible for over 80% of the value of total trade, and the top 10% of trading firms account for over 95% of the value of total trade. Importers reveal a similar, even though slightly smaller degree of concentration compared to exporters. For several European countries, Mayer and Ottaviano (2008) document comparable patterns: The top 1% of exporters in Germany account in 2003 for 59% of the total manufacturing exports, while the top 10% of exporters are responsible for 90% of exports. In France, the distribution of exports across firms is even more skewed: More than two-thirds (68%) of the total manufacturing exports go back to the top 1% of exporters and the top 10% of exporters carry responsibility for 94% of exports. Extending the analysis to a set of mostly developing countries, Freund and Pierola (2015) confirm the finding of a high concentration of exports among only a few firms, which they call “export superstars”: Across 32 countries and mostly for the period 2004-2008, the top firm on average accounts for 14% of a country’s total exports (17% of a country’s manufacturing exports), and the top five firms add up to 30%.

And third, multi-product firms are prevalent and dominant in international trade. With firm-product-level data for the United States, Bernard et al. (2007) show that in 2000 about 58% of the exporting firms export more than one product (multi-product exporters) and are responsible for more than 99% of the export value. Amador and Opromolla (2013) describe comparable figures for Portugal for the period 1996-2005: About 61% of the exporting firms are multi-product exporters and they account for about 92% of the export value. In a study on exporters in Belgium in 2005, Bernard et al. (2014) document that about 66% of the exporting firms export at least two products and make up the overwhelming share of the export value (about 98%).

The appearance of the last two facts in the empirical literature of international trade draws the attention to the relatively small set of pivotal firms in the international economy that produce and export a range of products, i.e. multi-product exporters, and are thereby theoretically show how heterogeneous and granular firms contribute to the existence of the extreme skewness in exporter size and of the zeros in international trade; see the Section 5.2.2. In their analysis, Bernard et al. (2007) identify products as 10-digit HS classification categories. Amador and Opromolla (2013) employ a 4-digit HS classification category as the definition reference for a product. Based on their export data, Bernard et al. (2014) define products as 8-digit Combined Nomenclature (CN) classification categories.
responsible for substantial shares of country-level exports, i.e. export superstars. Together with the first fact, the question arises as to which role these multi-product exporters may play in the variation of the trade flows’ extensive margin across countries. The present chapter therefore studies the export behavior of multi-product firms and analyses in particular in a non-standard but well-founded model the number of exported products (export scope) of multi-product firms across countries that differ in terms of their per capita income. With non-homotheticity in the preferences of consumers, the model thereby allows for income effects in demand, which represent a potential determinant of the firms’ decision about their export scope to significantly heterogeneous countries but are completely neglected by the standard models of multi-product firms in the theoretical literature of international trade. Further reflecting the real world, multi-product firms are modeled as “granular” components of the international economy, whose product portfolio and size are not of measure zero, leading to an oligopoly in the competition between firms and to a cannibalization effect within firms, likewise internalized by them while taking the decision about scope, and multi-product firms are as well modeled to be asymmetric or heterogeneous within themselves, in the sense of differentials across products within firms in several dimensions. Following the standard approach in the literature, called flexible manufacturing, each firm possess a core competency in the production of a single product variety and other varieties can only be produced by the firm at higher marginal costs, whereas those varieties are ordered regarding their firm-specific efficiency in production, with the core competency variety revealing minimal marginal costs in production and increasingly higher costs for varieties further away from the firm’s most efficient production option.

As the main insight to the raised question, the present chapter derives that multi-

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8Bernard et al. (forthcoming) provide empirical evidence that large shares of the multi-product firms in the manufacturing sector export products that they do not produce but source and call it “carry-along trade”. Recent work that deals with firms of a non-zero measure that potentially produce or export multiple products is given by Hottman et al. (2016) and Bernard et al. (2018). In the latter case, firms that participate in the international economy along multiple margins and account for large shares of aggregate trade are referred to as “global firms”. Parenti (2018) studies a market in which large and oligopolistic firms coexist with a monopolistically competitive fringe of small firms.

9In the real world, the heterogeneity of per capita income across countries is in fact significant in form of the existence of both rich and poor countries (Figure 5.1). For the global (interpersonal) distribution of income and its polarization: See e.g. Roope et al. (2018).


11For an overview about the application of an oligopoly as the market structure in the theory of international trade and its rise and fall in the literature over time, see Head and Spencer (2017).

12Empirical evidence for a skewed sales or export distribution across products within firms is given by a large set of studies on multi-product firms: See e.g. Goldberg et al. (2010b) for India, Bernard et al. (2010, 2011) and Liu (2010) for the US, Söderbom and Weng (2012) for China, Amador and Oprimolla (2013) for Portugal and Lopresti (2016) for the US.

13For the conceptual background of the flexible manufacturing, see Prahalad and Hamel (1990), and for its applications in the literature of multi-product firms in international trade, see Eckel and Neary (2010), Qiu and Zhou (2013), Mayer et al. (2014), Eckel et al. (2015) and Eckel et al. (2016). In a more general definition of flexible manufacturing, Milgrom and Roberts (1990) link the technology to the firm’s ability to quickly adjust to market conditions. In the broader literature, it is also related to the firm’s ability of introducing more varieties, of reducing delivery times (Tseng (2004)) and of changing production scale with only minor adjustment costs (Gal-Or (2002)).
product firms export more product varieties to countries with a higher per capita income and thus adjust their export scope but also their prices and markups in response to the income but also trade cost changes across the heterogeneous destinations, while the adjustments vary across the firms and their product varieties.

The remainder of the present chapter is organized as follows: The next section describes the literature in international trade and beyond which is related to the chapter’s issue and on which the chapter builds when trying to generate an answer to raised question. In the subsequent section, a non-standard model of multi-product firms in a world with heterogeneous countries and non-homothetic preferences is set up and analyzed both in general and with a cost specification, for which a comparative statics analysis in international trade is undertaken. The chapter then ends with a summarizing section on the main results of the consideration.

5.2 Related Literature

Three general strands of the literature are related to the present chapter: The first one deals with non-homothetic consumer preferences as well as income effects and their implications for the outcomes of the theoretical trade models in which they are applied. The second one looks at the granularity of economies and its contribution to the micro-macro nexus. And finally, the third strand of the literature analyses multi-product firms in international trade and makes some statements about their behavior.
5.2.1 Non-homotheticity and Income Effects - Demand-driven Trade

Traditional (“(neo)classical”) theories in international trade base their explanations of the exchange of goods between countries on (i) whose differences in production technologies or labor productivities and resource endowments: Ricardo (1817) and Heckscher and Ohlin (1919 and 1924)\(^{14}\) (inter-industry trade between countries, “old” trade theory) and (ii) the economies of scale in the production technologies of the firms and their imperfect competition: Krugman (1979, 1980)\(^{15}\) (intra-industry trade between countries, “new” trade theory). \(^{15}\) According to these theories, countries trade due to the associated gains that can be realized through the countries’ concentration in production on the narrower set of goods (“specialization”) in whose manufacturing (i) they are advantaged by supply or (ii) they exploit the scale economies and their export of these goods to other countries as well as their import of the goods produced by those other countries, overall resulting in an expansion of the countries’ consumption possibilities. Besides these supply-driven approaches, some demand-driven approaches exist that allow income to be a factor in the consumption allocation decision of individuals, thereby attribute income a crucial role in their theoretical structures of parts of the international economy and in particular identify the basis for trade in the similarities in income across the countries.

A first approach is the Prebisch-Singer (1962 and 1950) hypothesis\(^{16}\), which states that the price of primary commodities declines relative to the price of manufactured goods over the long term, implying a terms-of-trade deterioration of primary-commodity-based economies, which are mostly given by countries in the developing South of the world. Classical theorists by contrast would establish the opposite statement, i.e. an improvement of the terms of trade of primary commodities in time, since land and natural resource are only given in a limited and exhaustive supply, leading to an upward trend in their price. Considering not the side of the supply but demand, Singer and Prebisch argue that the income elasticity of demand varies across goods categories and in practice that the demand for manufactured goods is more elastic with respect to income than that for primary goods, especially food. As income rises over time, the demand for manufactured goods therefore increases more rapidly than the demand for primary goods, which leads to a shift of the relative demand away from the developing South, typically specialized in the cultivation, exploitation or production of goods with low income elasticities, and towards the industrialized North, typically specialized in the production of goods with high income elasticities. Due to the change in their demand for primary commodities and manufactured

\(^{14}\)The numbers in the parentheses represent the dates of publication of the most influential contributions for the development of the respective theory. For the references (reprints and translations): See Sraffa [ed., 1951] [On the Principles of Political Economy and Taxation (1817)] and Flam and Flanders [eds., 1991] [The Effect of Foreign Trade on the Distribution of Income (1919, pp. 43-69) and The Theory of Trade (1924, pp. 75-214)].

\(^{15}\)For the development of incorporating heterogeneous firms: See Melitz (2003) (“new new” trade theory).

\(^{16}\)Going back to the work by Singer (1950) and Prebisch (1962).
goods, prices adjust in the way stated by the hypothesis and the South’s economies’ terms of trade deteriorates with the decline in the relative price of their export goods. The South thereby benefits just a little from productivity improvements in its export sectors, i.e. in the sectors of primary commodities, because the increased purchasing power is spent mostly on the Northern goods.

The empirical evidence on the Prebisch-Singer hypothesis is only mixed. Based on both United Nations and World Bank data series (using the Sauerbeck price index for 37 primary commodities), Spraos (1980) analyses the long-run dynamics of the net barter terms of trade between primary commodities and manufactured goods and finds a deteriorating trend for the early period between 1876 and 1938. In the extended period from 1900 up to 1970, statistically significant evidence in favor of the Prebisch-Singer hypothesis can not be found by the study. By taking into account a structural instability in the parameters of the growth path for the identical data base, which is updated to the early 1980s, Sapsford (1985) however gets strong support for a deteriorating trend of the net barter terms of trade over the entire period of time (both, Spraos’ original and the extended one) and thereby the Prebisch-Singer hypothesis.

Grilli and Yang (1988) construct a novel price index of 24 primary commodities for the period 1900-1986 and detect that the relative prices of primary commodities fall on trend, inferring a deterioration in the terms of trade of commodity-exporting developing countries and therefore strongly supporting the Prebisch-Singer hypothesis.

Subsequent studies on the evidence of the demand-driven approach consider less the issue of improvements in the quality of the data sets on which they are based, but account more for econometric issues in their analysis, which follows the purpose of verifying a trend in the time series of relative prices. While the early studies mentioned above assume an underlying trend-stationary process and deduce misleading results in the case of a unit root, the body of succeeding studies described in the following allows for a difference-stationary process, in whose case of misspecification inefficiencies appear. A considerably more recent study conducted by Harvey et al. (2010) employs a new data set for primary commodity prices, initially applies trend-significance testing procedures that are robust to the existence of a unit root and thereby draws a mixed picture: By analysing 25 primary commodity price data series, partly going back as far as 1650, and addressing issues of the

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17 For an overview about the studies examining the Prebisch-Singer hypothesis: See Baffes and Etienne (2016). About half of the post-1980 studies supports the hypothesis, while the other half rejects it.
18 Prebisch (1962) by himself provides evidence in support of the hypothesis jointly worked out with Singer (1950) by employing data for the United Kingdom. Spraos (1980) summarizes the critics on and shortcomings of Prebisch (1962’s data choice and tries to develop a remedying approach on a more meaningful data base. His study confirms a deteriorating trend in the terms of trade detectable in the data, but only with a less distinct magnitude compared to Prebisch (1962).
19 Replicating Spraos (1980), the sub-periods before and after World War II reveal significant downward trends in the net barter terms of trade.
20 See e.g. Cuddington and Urzúa (1989), Bleaney and Greenaway (1993) and Kim et al. (2003) (all studies using some (reduced or extended) version of the Grilli and Yang data set). Those studies have found evidence against the Prebisch-Singer hypothesis.
21 Twelve series begin in the seventeenth century, three series begin in the eighteenth century, eight series
trend function (order of integration) as well as the existence of structural breaks, whose misspecification leads to further errors of inference, they detect no uniform long-term development, but eleven series of primary commodity relative prices reveal a significant downward trend over all or some fraction of the sample period. Ghoshray (2011) by contrast conducts a study that allows for up to two structural breaks in the unit root tests for 24 primary commodity prices (using an extended version of the Grilli and Yang data set for the period 1900-2003) and finds that eleven commodity prices are difference stationary, while the remaining thirteen prices are trend stationary with either one or two structural breaks and sixteen relative price series exhibit a negative trend over at least one period segment. In some further study provided by Arezki et al. (2014), the data set constructed by Harvey et al. (2010) is employed in time series stationary tests that allow for endogenous multiple structural breaks. And once again, the commodity price series thereby reveal no uniform long-term development, even so the majority of them shows a negative trend as hypothesized by Prebisch and Singer. Using a bivariate approach and bootstrap panel co-integration tests without structural breaks on a subsample of the extended Grilli-Yang data set for three commodity price indices, Di Iorio and Fachin (2018) find that the hypothesis is never confirmed for the period 1950-1980, but derive support for the entire period 1950-2011 for agricultural products, even so not for metal goods.

A second approach is given by the Linder (1961) hypothesis, which states that countries with similar levels of per capita income consume similar bundles of goods and trade more intensively with one another; in this way, stressing similarities in demand as trade determinants instead of differences in supply, as done by the classical theorists. They would establish the opposite statement: A larger difference in the capital-labor ratios of two countries and hence in the per capita incomes result in stronger specialization and more trade. Linder by contrast argues that a robust local demand for a good provides an incentive for the firms to invest in their productive capacity for it, and these undertaken investments lead in the longer term to a surplus of its supply over its demand and hence to exports to countries with some demand for the good that is not met by their own domestic production. Consequently, countries that share similar consumption patterns will trade intensively with each other. And finally, to the extent that the preferences are non-homothetic, an intensive trade between countries that have similar demand structures implies an intensive trade between countries that have similar levels of per capita income.

The empirical evidence on the Linder hypothesis is mixed, but in substantial parts supports it. Based on a total trade data set for 32 exporting countries and the years 1955/58, Linder by himself tests his hypothesis by initially compiling a matrix, in which both each of its rows and each of its columns represents one of these countries with arranging them according to the per capita income, in such a way that the countries with more similar per

\begin{footnotesize}
\begin{itemize}
\item[22]\footnotesize{Going back to the work by Linder (1961).}
\item[23]\footnotesize{For reviews of the early parts of the evidence: See Deardorff (1984) and Leamer and Levinsohn (1995).}
\end{itemize}
\end{footnotesize}
capita incomes are positioned closer together in the matrix, and that then contains the bi-
lateral average propensities to import, giving rise to the so-called import propensity matrix.
By approaching the northwest-southeast diagonal of this matrix from above or from below,
one ought to observe an increase in the bilateral propensities to find the Linder hypothesis
to be empirically validated. Illustrating the result for merely 14 of these countries, Linder
graphs for a given exporter and his trading partners the bilateral average propensities to
import in opposite to the per capita incomes of the importing countries. The resulting
graphs’ general tendency to peak near the per capita income of the exporter provides some
support for the Linder hypothesis.

Subsequent studies on the empirical validation of the hypothesized country-level posi-
tive relation between the bilateral trade intensity and the per capita income similarity in
principle follow Linder’s approach but employ more systematic econometric techniques.\textsuperscript{24}
To pursue that strategy, they largely adopt the average propensity to import as the mea-
sure for the bilateral trade intensity, rather than the actual volume of trade as a possible
alternative with the aim to eliminate size effects, and introduce the absolute difference in
the per capita income as the measure for the per capita income similarity.\textsuperscript{25} Utilizing Lin-
der’s total trade data set for 31 exporting countries, Sailors \textit{et al.} (1973) rank for a given
exporter import propensities and per capita income differences with his trading partners
and examine those measures’ relation by means of a Spearman rank correlation analysis.
They obtain a finding in favor of the Linder hypothesis for 16 countries, with a prepon-
derance of favorable results among those countries that are organized in the European
Economic Community. By eliminating moreover those countries that specialize in manu-
factures exports that require non-transportable natural resources or whose manufactures
exports represent only a small part of their total exports, the study is left with 25 countries
and in this sample support for the Linder hypothesis is provided for 14 countries. In a
comment to this study, Hoftyzer (1975) criticizes the non-consideration of the geographical
distance between the countries in the intended analysis of the determinants of bilateral
trade patterns, as it is possibly correlated with the per capita income difference as well
as a proxy for both transportation costs and trade horizons and thus may represent the
true explanatory variable for the bilateral trade intensities, but is so far hidden by the per
capita income difference as the single factor under consideration. He further demonstrates
the objection’s relevance through a regression analysis. In addition to this, the employ-
ment of Linder’s total trade data set gets criticized, as the Linder hypothesis is explicitly
formulated for trade of manufactured goods.

Responding to the above critique, Greytak and McHugh (1977) replicate the Spearman

\textsuperscript{24}In an alternative validation exercise that has not been established in the literature, Hufbauer (1970)
examines a commodity composition corollary of the Linder hypothesis by using a trade similarity index in
a regression analysis.

\textsuperscript{25}Besides the absolute difference in the per capita income, Kohlhagen (1977) applies the percentage
income overlap constructed by using country-level income distribution data as a measure for the per
capita income similarity in a regression analysis.
rank correlation analysis for a different trade data set, which includes 7 regions within the United States for the year 1963 and is restricted to manufactures, and moreover consider the geographical distance between the regions as another factor for the bilateral trade intensities. In this way, they derive some, although less significant, support for the Linder hypothesis for 3 regions, while the distance turns out to have in total more explanatory power than the per capita income difference in the determination of trade patterns of manufactured goods among these regions, as it is relevant for 6 regions. Qureshi et al. (1980) subsequently extend the last study’s treatment to trade between 10 regions in the United States and receive in a Kendall rank correlation analysis a finding along the same lines but even more pronounced, with virtually no support for the Linder hypothesis, in particular a weak significant supportive result for solely 1 region, and the distance as an important determinant of regional trade in manufactures and having in total more explanatory power than the per capita income difference in the determination of trade patterns, as it is relevant for 8 regions.

Subsequent studies on the empirical validation of the Linder hypothesis take the development so far accomplished by the literature, i.e. the adjustment of the data base as well as the consideration of the geographical distance, up and partly allow for more variation in the data base, but basically alter the econometric technique. While the early studies mentioned above examine the relation by way of a rank correlation analysis, the body of succeeding studies described in the following applies a regression analysis that involves geographical distance as a factor. An exceptionally early contribution by Fortune (1971) thereby investigates a data set of trade in finished manufactures for 23 exporting countries and the years 1966/67 and find support for the Linder hypothesis for 7 countries, whereas the per capita income similarity appears to be a better explanation for trade intensity between some of the wealthier countries. As it is also found in the studies with a rank correlation analysis above, distance plays a relevant role as it has some explanatory power in the case of 17 countries. Hirsch and Lev (1973) however alter the measures for trade intensity and per capita income similarity by employing industry-level exports and restricted differentials in the per capita income in form of an income ratio for 4 exporting countries, 5 industries and the year 1966. They detect some, although less significant, support for the Linder hypothesis in the case of 3 countries.

In an alternative approach to account for geographical distance, Kennedy and McHugh (1983) do not regress the values of the average propensities to import on the values of the absolute per capita income differences, but their respective changes with the aim to eliminate time-invariant factors, in particular distance, and thereby find for a data set on total trade and trade in manufactures for 1 exporting country, in particular the United States, and the non-consecutive years 1963, 1970 and 1976 no support at all for the Linder hypothesis. Based on a panel data set on total trade for a cross-section of 17 exporting countries and a time series consisting of the years 1974-1982, Thursby and Thursby (1987) conduct a regression analysis, which is grounded on a gravity equation and employs ab-
solute differences in the per capita income and exports as the respective measures for the components of the hypothesis. By obtaining a finding in favor of the Linder hypothesis for 14 countries, they provide strong support for the hypothesis that is under examination.

Greytak and Tuchinda (1990) generalize the analyses by allowing for linear and non-linear regression specifications as well as alternatives for the components’ measures. Besides the absolute differences in the per capita income as usual, a measure for the consumption similarity in form of Spearman rank correlations of the consumption vectors, and besides the average import propensities as usual, average export and total trade propensities are employed in the study. Given a data set on total trade for the federal states in the United States and the year 1963, they find in a linear (non-linear) specification with absolute differences in the per capita income and import propensities support for the Linder hypothesis for 18 (15) out of 35 (33) states, being a bit lower for the other specifications with the absolute differences in the per capita income, with the exception that the support substantially increases in the linear specification by applying total trade propensities, i.e. 22 out of 30 states. Using the rank correlations, similar results are derived, e.g. support for 15 (14) out of 31 (30) states for the linear (non-linear) specification with import propensities, while the support substantially increases in the non-linear specification with total trade (export) propensities, i.e. 37 (19) out of 41 (28) states. Once again, distance turns out as being a strong explanatory variable and, as a first appearance, being better captured by the non-linear specifications.

Based on a total trade data set for 24 exporting countries and the years 1983/84 as well as a gravity equation, Hanink (1990) does not regress import propensities but simply imports on the absolute differences in the per capita income and find some, although less significant, support for the Linder hypothesis for 8 countries, especially among the wealthier ones, but distance turns out to be an important factor of explaining trade for 22 countries. With the same measures, but a panel data set on total trade for 6 developing countries in Africa that export and the years 1984-1992, McPherson et al. (2001) detect by means of a fixed-effects Tobit estimation procedure that accounts for zeros in the bilateral trade flows a finding in favor of the Linder hypothesis in the case of 5 countries. In the most recent contribution, Choi (2002) conducts a further fixed-effects regression analysis, which is based on a gravity equation, with a panel data set for 63 countries and the non-consecutive years 1970, 1980, 1990 and 1992. But as the measures, the absolute differences in the per capita income over the sum of the per capita incomes and, besides the average import propensities, the exports over the sum of the incomes are employed in the study. For almost all periods, the pooled data and specifications, the Linder hypothesis is supported and distance appears once more as a significant factor for explaining trade.

In complementary strands of the literature, relations close to the one that is hypothesized by Linder are established: First, countries’ income levels do not only affect the quantity or volume of trade between them, as stated by the Linder hypothesis, but also the quality of the goods produced, consumed and exchanged, largely provided that variations
in unit values (prices) reflect differences in quality. While Schott (2004) and Hummels and Klenow (2005) discover that export prices within narrowly defined product categories are strongly positively correlated with the countries’ income per capita and Hallak and Schott (2011) find that richer countries specialize in quality production, Bils and Klenow (2001) and Hallak (2006) figure out that the demand for quality is strongly positively correlated with the household income, i.e. the richer households demand goods of higher quality. In their data on trade, Choi et al. (2009) detect an analogy to the Linder hypothesis that is based on product quality, whereas the distributions of income instead of its levels matter for it: Country pairs with more similar income distributions import baskets of goods of a more similar quality, as they have more similar import price distributions. Supported by the evidence on product quality and income levels, Hallak (2010) picks up on Linder’s original emphasis on product quality as the driving force of his hypothesis and thus formulates a “quality view” of the Linder hypothesis at the sectoral level of trade: Countries with more similar levels of per capita income trade more with each other as such country pairs demand and produce goods of similar quality. While the Linder hypothesis does not find empirical support at the aggregate level of trade on a sample of 64 countries in 1995, its quality version at the sectoral level of trade does hold while controlling for its inter-sectoral determinants. This discrepancy in the empirical support can thereby be rationalized by a cross-sector aggregation bias.

Second, a determinant of trade is not only given by the level of per capita income, but also by the within-country distribution of per capita income. Firstly, the income distribution of the importing country plays some role for the kind of its imported goods: Considering the trade patterns between developed and developing countries, Francois and Kaplan (1996) find that an increase in the per capita income and inequality in the developing countries lead them to shift their imports to differentiated, manufactured goods from developed countries. Dalgin et al. (2008) show that the import of luxuries (necessities) is positively (negatively) related to the importing country’s inequality. Secondly, as in the vein of the Linder hypothesis, the similarity between the trading partners, in this instance in terms of their income distributions, comes into play for the volume of trade between them: While Martínez-Zarzoso and Vollmer (2016) observe an analogy to the Linder hypothesis with the income distribution similarity as the determinant of bilateral trade volumes, i.e. the countries with more similar income distributions trade more with each other, Eppinger and Felbermayr (2015) find, in contrast to the Linder hypothesis, that differences in per capita incomes between two countries increase the bilateral trade, while differences in the income dispersions reduce it.

Going beyond the trade of goods between countries, Fajgelbaum et al. (2015) derive a Linder hypothesis for the horizontal foreign direct investment (FDI) in a model with non-homothetic preferences for quality and monopolistic competition. Taking into account both a purely demand-driven specialization and a proximity-concentration trade-off for the decision about how to serve foreign countries, either via exports or FDIs, they find that
FDI is more likely to occur between countries with similar per capita income levels and empirical evidence in its support is provided.

Theoretically, income effects as they are claimed and observed above are generated by non-homothetic consumer preferences. Preferences are called to be homothetic if they can be represented by an utility function that is homogeneous of degree one, i.e. with the property that increasing the quantities of consumption by some factor raises the level of utility out of them by the same factor. Each indifference curve is simply a magnified or reduced version of every other indifference curve. The feature of homotheticity consequently has the implication that the slope of the indifference curves is constant along the rays beginning at the origin. As a result, each income expansion curve with constant prices is given by a straight line and the proportional composition of the consumption basket is independent of the per capita income, i.e. constant expenditure shares across per capita income. Finally, all expenditure (income) elasticities of demand are equal to one.

A notable body of the empirical literature rejects the homotheticity of preferences, going far back up to Engel (1857). In a study on Belgian workers, he observes that the proportion of income spent on food falls as income rises, which becomes known as “Engel’s Law”. In this way, richer workers spend a smaller fraction of their income on their essentials. Houthakker (1957) checks for the robustness or universality of the Engel’s law across different sectors (food, housing, clothing and all other items) and countries and he observes by employing a double-logarithmic functional form that the total expenditure elasticities for food are all less than one, thereby confirming Engel’s law in the case of the observed countries. Furthermore, the elasticities for clothing are all larger than one and in most cases less than 1.5, while the elasticities for housing (miscellaneous expenditures) are mostly (all) below (above) one. Subsequent empirical studies confirm that the total expenditure (income) elasticities are not equal to one and vary across sectors and countries: Using an extended linear expenditure system, Lluch et al. (1977) discover that the elasticities for food are all less than one, while those for clothing and housing fall about evenly on either side of one. Theil and Clements (1987) however report income elasticities for food and clothing that are all less than one and elasticities for housing in most cases between one and 1.5.

Hunter and Markusen (1988) once again find for a linear expenditure system that the income elasticities vary between 0.45 and 1.91 across the categories of consumption goods, illustrating deviations from homotheticity that are statistically significant and in addition to it economically relevant for trade: They show that per capita income can operate as a basis for trade, which is totally neglected in the traditional theories in international trade that apply identical and homothetic preferences, and that its consideration in theory implies some significant changes in terms of the predicted volume of trade. Quantita-

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26See e.g. Deaton and Muellbauer (1980).
27In their exercise on trade indices, Ballance et al. (1985) also reject the hypothesis of homothetic preferences.
tively and on the basis of a linear expenditure system, Hunter (1991) detects that the non-homotheticity accounts for about one quarter of the inter-industry trade flows. The imposition of homothetic preferences instead reinforces trade directions and so increases net trade flows, providing the insight that the effect of non-homotheticity is a reduction of the volume of inter-industry trade, i.e. trade among countries with different per capita income levels.

Non-homotheticity and so income effects may thus provide a theoretical approach to close the existing gap of consistency between the traditional theories in international trade and the empirical facts on trade volumes and the factor content of trade. In particular, the empirical literature robustly documents that (i) the trade between countries in the North (i.e. North-North (N-N) trade) is large relative to the trade between countries in the North and the South (i.e. North-South (N-S) trade) (a “trade volume” variation, e.g. Deardorff (1984)) and (ii) the aggregate trade-to-GDP ratios are higher in countries with a higher per capita income (a “trade openness” variation, see the Figure 5.2), which would not be captured by the supply-driven trade theories. And finally, (iii) a “missing trade” puzzle (Trefler (1995)) exists: The factor content of trade is smaller than in the predictions made by the Heckscher-Ohlin-Vanek (HOV) model, i.e. the HOV model reveals some tendency to over-predict world trade.

Theoretically addressing the non-homotheticity’s potential to reconcile the traditional trade theories and the empirical evidence, Markusen (1986) constructs a model that incorporates on the one hand the central elements of the traditional theories in international trade, i.e. differences in the relative factor endowments and scale economies in form of monopolistic competition with product differentiation, with both being able to replicate the direction of trade observed in the empirical evidence, i.e. inter- and intra-industry trade, but additionally includes on the other hand a third element, i.e. non-homothetic preferences, with all three elements together being able to replicate both the direction and volume of trade. Other possible explanation approaches (trade frictions, i.e. higher North-South trade barriers, and a low total gross national product in the South) however seem to be partial but not complete solutions to the differences in the volumes between

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28In general, countries in the North are assumed to be already developed and characterized by a high per capita income (“rich”), while countries in the South are assumed to be still developing and characterized by a low per capita income (“poor”).

29In addition to the non-homothetic preferences that are omitted in the traditional trade theories, several other explanation approaches for the “missing trade” puzzle in the standard HOV models exist, in the way that each alleges one of the following other missing features whose consideration possibly brings theory and empirical evidence into accord: (Armington) Home biases in consumption, based on a limited substitutability between home and foreign goods (goods are differentiated due to their origins, economic or political reasons) and cross-country technology differences (Trefler (1995)), trade frictions (Davis and Weinstein (2001), Waugh (2010) finds systematically asymmetric trade frictions between rich and poor countries in the sense that poor countries face higher costs to export) and systematic differences in the price and the quality across countries (Schott (2004)). Evaluating the relative importance of those approaches in the puzzle’s explanation, Cassing and Nishioka (2015) find that preference biases between rich and poor countries explain a larger proportion of the missing factor trade than technology differences, whereas they do not distinguish preference differences from the non-homotheticity of preferences.
CHAPTER 5. MULTI-PRODUCT EXPORTERS AND NON-HOMOTHETICITY

Figure 5.2: Trade-to-GDP Ratios and Per Capita Income (GDP Per Capita (in Current US$) and Trade (% of GDP) (Both 2016), Excluding Hong Kong, Luxembourg and Singapore, Source: World Bank Database)

N-N and N-S trade. In the model’s framework, he finds that increases in the degree of non-homotheticity lead to reductions in the N-S inter-industry trade, but to increases in the N-N intra-industry trade, based on the assumption that a good’s capital (skilled-labor) intensity in production and its income elasticity of demand in consumption are positively correlated. Countries with a high per capita income, located in the North, are typically relatively capital (skilled-labor) abundant and both produce and export the capital-(skilled-labor-)intensive goods, but, more importantly, also consume these goods. The countries are thus relatively “specialized” in the production and consumption of the same goods. For these goods’ transactions across countries as a consequence, a disproportionate trade between and among the countries with a similar (high) per capita income in the North, i.e. a “Linder-type” trade, relative to the trade with the (poor) countries in the South, i.e. a “Heckscher-Ohlin-Vanek-type” trade, takes place, being consistent with the empirical evidence.

Fundamental to the model and its result is the directed product-level connection between the production and demand side mentioned above that is simply assumed in the model: The positive correlation between a good’s capital (skilled-labor) intensity in the production and its income elasticity in the consumption. While both Reimer and Hertel (2010) and Caron et al. (2014) do not find any significant correlation between a good’s capital intensity and its income elasticity, the latter document a positive correlation of more than 45% between a good’s skilled-labor intensity and its income elasticity, even when accounting for trade costs and a number of other factors, in particular cross-country differences in prices. Non-homotheticity, whose reasonableness as an assumption in the theory
is further emphasized by the documentation of large deviations of the income elasticity estimates from the unitary values as implied by homothetic preferences and a considerable variation of the income elasticity of demand across goods from different industries, however does not only help to give an explanation for the trade volume variation, but also for the trade openness variation and helps to solve in part the missing trade puzzle: The strong correlation between the factor content of production and consumption across countries can be explained in about half by trade costs, but non-homotheticity turns out to be as important quantitatively. So, non-homothetic preferences and the correlation respectively can reduce the HOV’s over-prediction of the variance of the net factor content of trade relative to that in the data by about 60%. As the countries tend to specialize in the consumption of the same goods that they are specialized in producing, they will source a larger share of their consumption from themselves and lower levels of trade-to-GDP ratios occur. For the trade openness variation though, its explanatory power is smaller than that of another channel: A positive sector-level correlation between the income elasticity and a sector’s tradability exists. The income-elastic goods that are produced by the high-income countries are systematically more tradable and thus a positive relationship between the per capita income and trade openness results.

Beyond both trade volumes and the factor content of trade, non-homotheticity has the potential to reconcile the traditional trade theories and the empirical evidence in terms of the positive correlation between an importer’s per capita income and the extensive margin of bilateral trade (positive importer income elasticity of the extensive margin of bilateral trade, see Section 1), as it is shown by Hepenstrick and Tarasov (2015) based on a Ricardian model with a continuum of goods and countries.

Besides the classical contributions by Markusen (1986) and Bergstrand (1990) that stick to CES preferences for differentiated products but introduce non-homotheticity through a homogeneous product with a minimum consumption requirement, non-homothetic preferences are applied in a range of models of international trade, coming up especially in the second decade of the 21st century and meanwhile setting up an alternative to or an enhancement of the traditional trade theories: Following the extensions of the Ricardian model, first Matsuyama (2000) incorporates non-homothetic preferences in form of consumption indivisibilities, representing an orthogonal version to the CES preferences, in a Ricardian model with a continuum of goods à la Dornbusch et al. (1977), while subsequently Fieler (2011) integrates constant relative income elasticity preferences, representing some generalized version of the CES preferences, and Hepenstrick and Tarasov (2015) incorporate preferences that allow for binding non-negativity constraints in a Ricardian model with a continuum of goods and countries à la Eaton and Kortum (2002). In a Heckscher-Ohlin as well as an imperfect competition model, Markusen (2013) applies a variation of Stone-Geary preferences and thereby generates some “generic” model that is able to replicate several empirical facts. Etro (2017) employs non-homothetic preferences in a Heckscher-Ohlin model with monopolistic competition. Moreover, Foellmi et al. (2018) incorporate non-
homothetic preferences in form of consumption indivisibilities in a model of monopolistic competition. In the tradition of the monopolistic competition models with heterogeneous firms à la Melitz (2003), Behrens et al. (2014) apply CARA preferences and Simonovska (2015) employs Stone-Geary preferences as alternative types of non-homothetic preferences in a heterogeneous firm model of trade. Fajgelbaum et al. (2011) and Fajgelbaum et al. (2015) incorporate vertical product differentiation in trade models with non-homothetic preferences. Applying indirectly additive preferences, Bertoletti et al. (2017) consider a monopolistic competition model with trade between two countries and homogeneous firms, while Bertoletti et al. (2018) open the model up to general trade with heterogeneous firms. In a normative analysis with an almost-ideal demand system (Deaton and Muellbauer (1980)), Fajgelbaum and Khandelwal (2016) consider the unequal gains from trade across heterogeneous consumers within countries.

5.2.2 Granularity

A recently established strand of literature takes the empirically documented distribution of firm sizes into account and focuses on the role of single firms for aggregate patterns and outcomes in macroeconomics and international trade, thereby putting forth the term “granularity” for the existence of a few large influential firms, the “granular” components (grains) of the economy.

In a conceptual contribution to the theoretical literature, Neary (2010) demonstrates how the feature of endogenous entry and exit can be integrated into a model of oligopoly, thereby retaining the existence of a small number of dominating firms in equilibrium (granularity), the strategic interaction between them and thus being distinguishable from models of perfect and monopolistic competition. Three mechanisms make this possible: Heterogeneous industries with a variation in fixed costs, a natural oligopoly as the case where the equilibrium number of firms does not increase as the market size rises and superstar firms as the outcome of a firm choice to become large which is associated with additional fixed costs.

Eaton et al. (2013) illustrate how to adjust a standard trade model with heterogeneous firms, which are conventionally treated as points on a continuum, to overcome its shortcomings in empirical consistency: With only an integer number of firms and therefore some degree of granularity in the economy, the model can account for both the extreme skewness in exporter size and zeros in bilateral trade.

30 A small number of larger firms coexists alongside a large number of smaller firms: Axtell (2001) documents for the United States that the distribution of firm sizes is described by a power law with an exponent close to one (Zipf’s law). Such a distribution can theoretically be generated by a model of proportional random growth, satisfying Gibrat’s law, which by itself leads to a log-normal distribution, with a small friction and some adding-up constraint for the total size of the system. For an introduction to the power laws in economics: See Gabaix (2016).

31 For the role of granularity in the comparative advantage: See Gaubert and Itskhoki (2016).

32 For the welfare consequences of deviations from a Pareto distribution in the firm productivity distribution in models of heterogeneous firms in international trade: See Bee and Schiavo (2018).
By providing a microfoundation for aggregate shocks, Gabaix (2011) shows in his seminal study that idiosyncratic shocks to the incompressible grains of economic activity, i.e. large firms, are able to explain a large part of the aggregate, i.a. business-cycle, fluctuations and do not average out in the aggregate as suggested by the central limit theorem, since this breaks down given the extreme fat-tailed distribution of firm sizes (granularity). And in fact, about one-third of the variations in output growth is found to be attributable to the idiosyncratic shocks to the largest 100 firms in the United States.

Di Giovanni and Levchenko (2012) apply this insight to patterns of international trade, by studying the role of large firms in explaining cross-country differences in aggregate volatility with respect to the country’s size and trade openness: Accounting for the empirical facts that smaller as well as more open countries reveal a higher volatility, idiosyncratic shocks to large firms account substantially for the aggregate volatility due to the very fat-tailed distribution of firm sizes and so the fact that the typical economy is dominated by a few very large firms. In smaller countries with fewer firms, shocks to the largest firms matter more, while trade openness increases the granularity of the economy as only the largest and most productive firms are selected and thus large firms get more important, thereby increasing aggregate volatility. For a sample of the 50 largest economies in the world, it turns out that a typical country that accounts for 0.5% of world GDP has an aggregate volatility that is two times higher than that of the largest world economy, the United States, and trade can increase the aggregate volatility by 15-20% in some small open economies.

Providing empirical evidence for the relevance of shocks to single firms for aggregate fluctuations, Di Giovanni et al. (2014) decompose the annual sales growth rate of an individual firm to a single sales market into three components, i.e. a macroeconomic shock, a sectoral shock and a firm-specific shock, and find for a data set capturing the universe of French firms for 1990-2007 that the firm-specific component contributes substantially and more to aggregate fluctuations than the other two components. Splitting firm sales up into domestic and export sales, the firm-specific component makes a larger contribution to the volatility of exports than that of overall sales, nevertheless substantially accounting for the volatility of aggregate domestic sales as well. A further decomposition of the firm-specific component into a direct effect, i.e. due to the fat-tailed distribution of firm sizes (granularity), idiosyncratic shocks to large firms directly contribute to aggregate fluctuations, and a linkages effect, i.e. due to input-output linkages across the economy, aggregate fluctuations are attributable to idiosyncratic shocks, provides a more comprehensive understanding of the at work. While empirical evidence for both effects can be provided, in particular firm-
specific shocks in more concentrated industries contribute more to aggregate volatility and sectors with stronger input-output linkages tend to exhibit a significantly greater correlation of firm-specific shocks, and the direct effect of firm shocks on aggregate fluctuations is quantitatively relevant, the linkages effect is approximately three times as important at the direct effect in driving aggregate volatility.

Friberg and Sanctuary (2016) replicate the previous study for the universe of Swedish firms for 1997-2008. Even so Sweden shows up as a more granular economy with a higher overall sales volatility and the firm-specific component once again accounts substantially for aggregate fluctuations, its contribution is lower in Sweden compared to France, in the way that both the firm-specific and other components contribute roughly equally to the aggregate volatility. As in the case for France, firm-specific shocks in Sweden are more important for the volatility of export sales than that of total sales.

5.2.3 Multi-product Firms

For the analysis of the behaviour of multi-product firms in international trade, the theoretical contributions in the literature typically take the case of homogeneous countries in the world. A heterogeneity in terms of per capita income, which is obviously observed in reality and captured in the present chapter, never plays a role, as standard models of multi-product firms in the literature of international trade ignore the income effects on the firms’ scope by employing either quasi-linear preferences (Feenstra and Ma (2008), Dhin-gra (2013), Mayer et al. (2014) and Cheng and Tabuchi (2018)), homothetic preferences (Bernard et al. (2011), Nocke and Yeaple (2014)) or by construction of the model (Eckel and Neary (2010)). However, Mayer et al. (2014) and Cheng and Tabuchi (2018) allow for a country heterogeneity in terms of its market size (and location), which is obviously the case in the real world, and thereby form an exception in the international trade literature. Mayer et al. (2014) illustrate how the market size and geography of trading partners affect both a firm’s exported product range and its exported product mix across market destinations. In a model of monopolistic competition and quadratic preferences, a tougher competition in a larger or more centrally-located export market induces a firm to cut its worst performing products from its export scope and to skew its export sales towards its better performing products. Both reallocations within firms lead to an increase in firm productivity. Considering a model of oligopolistic multi-product firms conducting trade between countries of different sizes, Cheng and Tabuchi (2018) show that more firms but fewer exporters exist in a larger country, and firms produce a wider range of products but

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36 Eckel and Neary (2010) assume a configuration of firms described as “large in the small, and small in the large”, i.e. a firm is large in the industry in which it is active, but small relative to the overall economy consisting of infinitely many industries.
With only a relatively small number of firms dominating international trade, as it can be observed in the real world, each of those firms has some market power and thus its decisions have some impact on the market aggregates, and likewise typically also on the firm’s own variables. In this way, it is imaginable and reasonable to think that any (additional) sales of a (new) product crowd out sales of the other existing products by the same firm, i.e. some negative within-firm demand spillover, called cannibalization (effect), exists. A within-firm cannibalization by itself is not a novel feature to the models of multi-product firms in international trade. However, it is generated in the present chapter by some novel mechanism: A first group of models (Baldwin and Gu, Eckel and Neary (2010), Eckel et al. (2015, 2016), Eckel and Irlacher (2017) and Flach and Irlacher (2018)) apply an oligopolistic or monopolistic market structure and some preferences that yield a utility function which all varieties enter symmetrically, independent of whether or not they are produced by the same firm. Cannibalization is thereby ultimately linked to the feature of large firms. In a second group of models (Dhingra (2013) and Hottman et al. (2016)) by contrast, a nested structure for the preferences is chosen which provides the case that the elasticity of substitution across varieties within firms and the elasticity of substitution across firms fall apart and differ as a matter of fact in the way that the first one is larger than the second one, following the idea that the within-firm cannibalization is linked to the higher substitutability of the varieties of the same firm compared to those of the competitors. The present chapter instead generates a within-firm cannibalization through the so far neglected income effect and in particular the adjustment of the marginal utility of income due to the additional supply of a large firm’s products. Its mechanism thus follows the first group.

Empirical contributions in the literature of international trade however generate insights into the export behavior of multi-product firms in the real world with heterogeneous countries and thereby point to asymmetries across countries, both the domestic market and the destination markets abroad. Iacovone and Javorcik (2010) detect that firms export only a fraction of their domestic scope and Mayer et al. (2014) are able to confirm their theoretical predictions by establishing the empirical result that firms skew their export sales towards their core products in destinations with a stronger competition. Furthermore, within-firm cannibalization does not occur: See Bernard et al. (2011), Qiu and Zhou (2013), Mayer et al. (2014), Nocke and Yeaple (2014) as well as Arkolakis et al. (2015). A single exception is given by Frenstra and Ma (2008).
asymmetries exist in the sense that the core varieties are exported across more destinations than non-core products, derived in a study by Arkolakis et al. (2015).

5.3 Model

The world consists of $I$ countries and each country $i \in \{1, \ldots, I\}$ is inhabited by $L_i$ identical individuals, who are endowed with quadratic preferences of consumption and a per capita income $E_i$. Thereby, the model assumes as intended a within-country homogeneity and an across-country heterogeneity by letting $E_i$ varying across the countries. In each country $i$, each firm $f$ produces a continuum of varieties and sells them to country $j \in \{1, \ldots, I\}$, with $\omega \in [0, \ldots, \delta_{f,ij}]$ as a product variety out of firm $f$’s set of produced varieties $\delta_{f,ij}$ (scope). Firms of non-zero measure compete in an oligopoly on the basis of the amount of outputs that they generate (Cournot competition), which results in a discrete number of homogeneous firms $m_i$ being active in country $i$. Serving a market in a country $j$ other than the one in which the firm is located (i.e. $j \neq i$) requires the coverage of some iceberg trade costs $\tau_{ij} > 1$, which reduce to one ($\tau_{ii} = 1$) in case of serving the domestic market.

5.3.1 Preferences and Demand

Each individual has quadratic preferences over the possibilities of consumption, which are expressed by the following utility (function) of a representative agent in country $j$:

$$U_j = \sum_{i=1}^{I} \sum_{f=1}^{m_i} \int_{0}^{\delta_{f,ij}} \left[ a q_{f,ij}(\omega) - \frac{1}{2} b q_{f,ij}^{2}(\omega) \right] d\omega,$$

with $a$ and $b$ as (constant) parameters and $q_{f,ij}(\omega)$ as the individual consumption of product variety $\omega$ produced by firm $f$ in country $i$ and shipped to country $j$. The utility maximization subject to each consumer’s budget constraint $\sum_{i=1}^{I} \sum_{f=1}^{m_i} \int_{0}^{\delta_{f,ij}} p_{f,ij}(\omega) q_{f,ij}(\omega) d\omega \leq E_j$ with $p_{f,ij}(\omega)$ as the price of variety $\omega$ produced by firm $f$ in country $i$ and shipped to country $j$ yields the following individual inverse demand:

$$p_{f,ij}(\omega) = \frac{1}{\lambda_j} (a - bq_{f,ij}(\omega))$$

with $\lambda_j$ as the Lagrange multiplier associated with the budget constraint, representing the marginal utility of income of consumers in country $j$. With plugging the individual inverse demand (equation 5.2) into the budget constraint of each individual, which holds for the consumption allocation with equality, an expression for the marginal utility of income can

39Models of oligopolistic firms tend to assume their homogeneity in terms of the cost structure due to issues of tractability. See e.g. Eckel and Neary (2010) and Ottaviano and Thisse (2011).

40As applied by Neary (2016) in a general equilibrium model of oligopoly.
be derived:

\[
\sum_{i=1}^{I} \sum_{f=1}^{m_i} \int_0^{\delta_{f,i,j}} \frac{(a - bq_{f,i,j}(\omega))}{\lambda_j} q_{f,i,j}(\omega) d\omega = E_j
\]

and

\[
\lambda_j = \frac{1}{E_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \int_0^{\delta_{f,i,j}} (a - bq_{f,i,j}(\omega)) q_{f,i,j}(\omega) d\omega.
\] (5.3)

\(\lambda_j\) is obviously decreasing in per capita income \(E_j\), i.e. \(\frac{\partial \lambda_j}{\partial E_j} < 0\): The richer consumers in country \(j\) are, the less utility they receive from the last unit of income. Furthermore, the marginal utility of income is increasing in the scope of each firm \(\delta_{f,i,j}\), i.e. \(\frac{\partial \lambda_j}{\partial \delta_{f,i,j}} > 0\), because the last unit of income is associated with a higher utility for a larger consumption choice. For a sufficiently small (large) consumption quantity \(q_{f,i,j}(\omega)\), the marginal utility of income is increasing (decreasing) in the quantity:

\[
\frac{\partial \lambda_j}{\partial q_{f,i,j}(\omega)} = \frac{1}{E_j} \frac{\partial}{\partial q_{f,i,j}(\omega)} [(a - bq_{f,i,j}(\omega)) q_{f,i,j}(\omega)] = \frac{1}{E_j} ((a - bq_{f,i,j}(\omega)) - bq_{f,i,j}(\omega)),
\]

\[
\frac{\partial \lambda_j}{\partial q_{f,i,j}(\omega)} \geq 0 \iff a - bq_{f,i,j}(\omega) \geq bq_{f,i,j}(\omega) \iff a \geq 2bq_{f,i,j}(\omega)
\]

and

\[
\frac{\partial \lambda_j}{\partial q_{f,i,j}(\omega)} \geq 0 \iff \frac{a}{2b} \geq q_{f,i,j}(\omega).
\]

Aggregating the individual demand over the \(L_j\) individuals gives the aggregate demand in country \(j\): \(x_{f,i,j}(\omega) = L_j q_{f,i,j}(\omega)\). To get a plausible model setting, the following assumption is made throughout the section:

\[
\frac{x_{f,i,j}(\omega)}{L_j} < \frac{a}{2b} \iff \frac{2}{a} x_{f,i,j}(\omega) < L_j \quad \forall \omega, \quad \text{(model assumption)}
\]

i.e. the destination country \(j\) is sufficiently large in terms of its population. The aggregate inverse demand takes the form

\[
p_{f,i,j}(\omega) = \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,i,j}(\omega) \right)
\] (5.4)

and the marginal utility of income is given by

\[
\lambda_j = \frac{1}{L_j E_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \int_0^{\delta_{f,i,j}} \left( a - \frac{b}{L_j} x_{f,i,j}(\omega) \right) x_{f,i,j}(\omega) d\omega.
\] (5.5)

Finally, the aggregate demand can be written as

\[
x_{f,i,j}(\omega) = \frac{L_j}{b} \left( a - \lambda_j p_{f,i,j}(\omega) \right).
\] (5.6)
The demand’s elasticity with respect to the price derives in its absolute value as

$$
\varepsilon_{f,ij}(\omega) = -\frac{\partial x_{f,ij}(\omega)}{\partial p_{f,ij}(\omega)} \frac{p_{f,ij}(\omega)}{x_{f,ij}(\omega)} = \frac{\lambda_j p_{f,ij}(\omega)}{a - \lambda_j p_{f,ij}(\omega)}
$$

and is increasing in the marginal utility of income,

$$
\frac{\partial \varepsilon_{f,ij}(\omega)}{\partial \lambda_j} = \frac{ap_{f,ij}(\omega)}{(a - \lambda_j p_{f,ij}(\omega))^2} > 0,
$$

i.e. the more the individuals in country $j$ value the last unit of income, the more they are concerned about its optimal usage in consumption and thus the more elastic is their demand with respect to changes in the price, while under the model assumption the price elasticity of demand decreases with the per capita income.

$$
\frac{d \varepsilon_{f,ij}(\omega)}{dE_j} = \frac{\partial \varepsilon_{f,ij}(\omega)}{\partial \lambda_j} \frac{d\lambda_j}{dE_j} < 0,
$$

i.e. the richer the individuals in country $j$, the less they value the last unit of income and thus the less elastic is their demand to changes in the price and those consumers therefore have a larger, or if at all, positive demand for varieties with a higher price.

### 5.3.2 Production and Supply


Due to this technology, the marginal production costs are increasing over the product varieties, while being minimal for some variety $\omega = 0$, called the firm’s core competency: $\frac{dc_{f,ij}(\omega)}{d\omega} > 0$ and $c_{f,ij}(0) = 1$, i.e. producing product varieties that are successively further away from the firm’s core competency variety and thus expanding the firm’s product portfolio is associated with increasingly higher marginal costs of production, which establishes for the firm a ladder of products depending on their decreasing efficiency in production.

Ultimately, this asymmetry in the firm’s production efficiency across its product varieties leads to a within-firm heterogeneity in the price and quantity dimensions.

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41 See the Appendix F.1 for the proof.

42 Note that the choke price of richer consumers in country $j$ is larger than that of poorer consumers in country $k$, i.e. $E_j > E_k$: $p_{f,ij}^{\text{max}} = \frac{a}{\lambda_j} > p_{f,ik}^{\text{max}} = \frac{a}{\lambda_k}$, given that $\frac{d\lambda_j}{dE_j} < 0$.

43 Besides the international trade literature, the industrial organization literature also deals with flexible manufacturing in the production process, see e.g. Eaton and Schmitt (1994).

44 Empirical evidence in support of the concept of flexible manufacturing is provided by Garcia-Marin and Voigtländer (2017).
which can be indeed observed in reality\textsuperscript{45}.

In its production, each firm has to take two decisions: Taking other firms’ choices as given, it simultaneously decides about the quantity produced $x_{f,ij}(\omega)$ for variety $\omega \in [0, \ldots, \delta_{f,ij}]$ and about the mass of varieties produced $\delta_{f,ij}$ for $j = \{1, \ldots, I\}$. These decisions are made based on the maximization of the firm’s profit function:

$$\Pi_{f,i} = \sum_{j=1}^{I} \int_{0}^{\delta_{f,ij}} \left( \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) - c_{f,ij}(\omega) \right) x_{f,ij}(\omega) \, d\omega - w_i F$$

with $\lambda_j$ given by equation (5.5), $w_i$ as the wage rate in country \textsuperscript{46} and $F$ as some fixed operation cost (measured in domestic labor units). The first-order condition with respect to the quantity $x_{f,ij}(\omega)$ is then given by\textsuperscript{47}

$$\frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) - \frac{1}{\lambda_j^2} \int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) \, d\omega \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} = c_{f,ij}(\omega), \quad (5.9)$$

where the first term on the left-hand side of the equation represents the marginal revenue and the term on the equation’s right-hand side the marginal cost. In the standard models of monopolistic competition with differentiated product varieties and increasing returns to scale\textsuperscript{48}, firms act as monopolists and set those two terms, marginal revenue and marginal cost, equal to each other to determine their profit-maximizing output quantity. As the entry to the economy is assumed to be free, which leads to zero long-run profits, and a sufficiently large number of firms exist in the economy, precluding strategic interaction between the economic agents, the effects of firms’ actions on aggregates are assumed away and thus firms do not take them into account while making their choices, especially not the effect of their quantity choice on the marginal utility of income ($\frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} = 0$) in the respective case of its relevance\textsuperscript{49}.

With only a few firms acting in the oligopolistic economy, each firm represents a substantial part of the economy, thus influences the economy’s aggregates, especially the marginal utility of income ($\frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} \neq 0$)\textsuperscript{50} and takes this into account while making its choices, leading to the first-order condition in equation (5.9) with the additional second term on its left-hand side. Provided that the destination country $j$ is sufficiently large in terms of its population (model assumption, i.e. $L_j > \frac{2b}{a} x_{f,ij}(\omega)$), an increase in the supply of variety $\omega$ raises the marginal utility of income: $\frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} > 0$, i.e. a consumer in country

\textsuperscript{45}See for the quantity (sales/exports) dimension e.g. Goldberg et al. (2010b), Bernard et al. (2010), Amador and Oproomolla (2013) and Lopresti (2016).

\textsuperscript{46}As labor is inelastically supplied by assumption, each firm takes the wage rate as exogenously given.

\textsuperscript{47}See the Appendix F.2 for the details of derivation.

\textsuperscript{48}Going back to the seminal contributions by Lancaster (1975, 1979), Spence (1976) and Dixit and Stiglitz (1977) as well as the model’s application to international trade by Krugman (1979).

\textsuperscript{49}Once again, in the present model, the marginal utility of income is in general relevant and thus the income effect is provided by the non-homotheticity of the preferences and the absence of a numéraire variety.

\textsuperscript{50}See the Appendix F.3 for the details of the response of the marginal utility with respect to the quantity.
$j$ receives more utility from the last unit of income obtained when facing a larger supply of a variety $\omega$. A higher marginal utility of income due to an increase in the supply of a variety $\omega$ reduces the demand for all the other product varieties (equation (5.6)) and the second term on the left-hand side of equation (5.9) which turns out to be negative, therefore represents a cannibalization effect within the multi-product firm: Through the income effect, any expansion of the supply of a product variety crowds out the demand of other varieties and can thus only be taken out at their expense. The firm internalizes this effect of cannibalization in its choice and adjusts its output of variety $\omega$, compared to the case of an absence of a cannibalization effect, by reducing it, thereby attenuating the cannibalization between product varieties and optimizing profits.

Given the market dominance of a firm $f$ located in country $i$ in country $j$ by $g_{f,ij}$, which is defined as a measure of the firm’s sales share in the country’s market and represents an index for its concentration or granularity, the first-order condition (equation (5.9)) can be rewritten:

$$\frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\delta_{f,ij}) \right) x_{f,ij}(\delta_{f,ij}) \left[ 1 - g_{f,ij} \right] = c_{f,ij}(\omega). \quad (5.10)$$

As it gets obvious, the standard first-order condition of the equalization of marginal revenue and marginal cost is adjusted by the firm’s market dominance: The larger and more dominant a firm is in terms of its sales share in a country’s market, the stronger is the cannibalization effect and the lower is the marginal revenue for some variety $\omega$. By internalizing this effect, the firm ends up with a smaller output amount chosen for the variety.

Regarding the firm’s second decision, the first-order condition with respect to the mass of varieties (scope) $\delta_{f,ij}$ is given by:

$$\left( \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\delta_{f,ij}) \right) - c_{f,ij}(\delta_{f,ij}) \right) x_{f,ij}(\delta_{f,ij})$$

$$= \frac{1}{\lambda_j} \int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \frac{\partial \lambda_j}{\partial \delta_{f,ij}}, \quad (5.11)$$

such that the firm chooses its scope in the way that the profit from the marginal variety (term on the left-hand side) is just equal to the variety’s sales-depressing effect due to the cannibalization on the firm’s intra-marginal varieties (term on the right-hand side). With the market dominance of the firm $f$ located in country $i$ in country $j$, given by $g_{f,ij}$, in hand, the first-order condition (equation 5.11) can be rewritten:

$$\frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\delta_{f,ij}) \right) x_{f,ij}(\delta_{f,ij}) \left[ 1 - g_{f,ij} \right] = c_{f,ij}(\delta_{f,ij}) x_{f,ij}(\delta_{f,ij}). \quad (5.12)$$

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51 See the Appendix F.4 for the details of derivation.
52 See the Appendix F.5 for the details of derivation.
53 See the Appendix F.3 for the details of the response of the marginal utility with respect to the scope.
54 Given the above assumption, i.e. $L_j > 2\frac{b}{a} x_{f,ij}(\omega)$, the marginal utility of income necessarily increases with an increase in the firm’s scope, i.e. $\frac{\partial \lambda_j}{\partial \delta_{f,ij}} > 0$.
55 See the Appendix F.6 for the details of derivation.
As it turns out, the firm expands its scope until the demand for the marginal variety is driven down to zero: 
\[ x_{f,ij}(\delta_{f,ij}) = 0, \] representing the solution to equation (5.12)\textsuperscript{55}

From the modified first-order condition for the marginal variety (equation (5.10)), one gets therefore an implicit solution for the firm’s decision about its scope:

\[ \frac{a}{\lambda_j} [1 - g_{f,ij}] = c_{f,ij}(\delta_{f,ij}). \] (5.13)

Due to the flexible manufacturing technology, the right-hand side is increasing in the mass of varieties \( \delta_{f,ij} \). With a constant market dominance of the firm \( g_{f,ij} \), the left-hand side is increasing in the per capita income of the destination market \( j \), \( E_j \), because the marginal utility of income \( \lambda_j \) decreases with \( E_j \). Thus, keeping the firm’s market dominance constant, the firm exports more varieties to the richer countries. This fact of a variation in the export scope across countries that are heterogeneous in terms of their per capita income is so far absent in and therefore novel with respect to the standard models of multi-product firms in international trade that ignore the effect of the export destination’s income on the firm’s scope.

In addition, for a given destination market, a firm with a larger market dominance faces a stronger cannibalization effect and therefore exports less varieties to the country, i.e. the firm exports a smaller scope. In multi-product firm models of monopolistic competition with \( g_{f,ij} = 0 \) by construction, this fact can not be reproduced.

The optimal output, price and markup profiles of the firm are then given by\textsuperscript{56}

\[ x_{f,ij}(\omega) = \frac{a L_j}{2b} \left( 1 - \frac{c_{f,ij}(\omega)}{c_{f,ij}(\delta_{f,ij})} \right), \] (5.14)

\[ p_{f,ij}(\omega) = \frac{1}{2[1 - g_{f,ij}]} \left( c_{f,ij}(\omega) + c_{f,ij}(\delta_{f,ij}) \right) \]

and

\[ p_{f,ij}(\omega) = \frac{1}{2[1 - g_{f,ij}]} \left( 1 + \frac{c_{f,ij}(\delta_{f,ij})}{c_{f,ij}(\omega}) \right) c_{f,ij}(\omega). \] (5.15)

The firm with a scope \( \delta_{f,ij} \) exhibits a decreasing output and an increasing price profile over the firm’s products ladder, such that it charges the lowest price for the variety in which it is most efficient in production (core variety) and sells it with the highest amount. Firms with a larger scope produce and sell more of each of their products, which indicates a positive correlation between the product-extensive and product-intensive margin at the firm level. Empirical evidence in line with this finding is provided by Elliott and Virakul (2010) for firm exports in Thailand, Goldberg et al. (2010b) for firm output in India, Liu (2010) for firm sales in the U.S., Bernard et al. (2011) for firm exports in the U.S. and Berthou and Fontagné (2013) for firm exports in France. In addition, a typical scale effect for the output profile exists: Firms export more of each of their products to markets with

\textsuperscript{55}See the Appendix F.7 for the proof.

\textsuperscript{56}See the Appendix F.8 for the details of derivation.
a larger size.

As it is shown in equation (5.15), the price for a variety \( \omega \) can be represented by the product of its marginal production cost and its markup. Markups thereby vary both across firms and across varieties within firms. Multi-product firms with a larger scope and a higher market dominance in the export destination are able to charge higher markups for their products: The cannibalization effect being relevant for those firms leads to some restriction of the firms’ scope and quantities, making possible the implementation of higher markups. Even so a firm charges the lowest price for its core variety among its products, the core variety’s markup is the highest and markups fall with the distance to the core variety (down to \( \frac{1}{1-g_{f,ij}} \) for the marginal variety, which is singularly determined by the market dominance of the firm). This finding of the model is consistent with the empirical evidence provided by De Loecker et al. (2016).

Given these insights, the export behavior of a multi-product firm towards countries that are heterogeneous in terms of their per capita income obviously follows a clear pattern which gets determined by the within-firm, i.e. product, as well as the country heterogeneity: While both consumers with a low and consumers with a high per capita income demand varieties with a low price, being the firm’s core products, only consumers with a higher per capita income have a larger, or if at all, positive demand for varieties with a higher price\(^{57}\), which are in turn those varieties in whose production the firm is less efficient in, i.e. its non-core or marginal varieties. Thus, taken this together, each firm exports its core and relatively cheap varieties to possibly all destinations, but its marginal varieties with the higher prices are only exported to the richer countries. Consequently, firms export their core products across more destinations than the non-core products; a finding of the model that is consistent with the empirical evidence provided by Arkolakis et al. (2015). Overall, this leads to export scopes which are shortened or extended by the firm along its products ladder depending on the country’s per capita income.

5.3.3 Multiplicative Cost Specification

A functional form specification for the marginal cost of production and delivery \( c_{f,ij}(\omega) \) provides deeper insights to the model:

\[
c_{f,ij}(\omega) = \tau_{ij} w_i z_{f,i} \omega^\eta,
\]

where \( \tau_{ij} \) represents the iceberg cost of transportation from country \( i \) to country \( j \) (with the standard assumption that \( \tau_{ii} = 1 \)), \( w_i \) the wage rate in the country of production \( i \), \( z_{f,i} \) the production efficiency of firm \( f \) in country \( i \) and \( \eta > 0 \) the elasticity of the marginal cost with respect to the distance to the core variety, i.e. \( \frac{\partial c_{f,ij}(\omega)}{\partial \omega} c_{f,ij}(\omega) = \eta \). As assumed by the flexible manufacturing technology, each firm has some lowest marginal cost of production

\(^{57}\)See the Section 5.3.1.
and delivery for its core variety ($\omega = 0$) and the marginal cost for other varieties increases with their distance to this variety. The rate of the increase is thereby captured by $\eta$.

Given the cost specification, the revenue of the firm from exports to country $j$ can be explicitly expressed in terms of its scope:

$$R_{f,ij} = \frac{a^2 \eta L_j \delta_{f,ij}}{2b \lambda_j (2\eta + 1)} = \frac{a}{2b (2\eta + 1)} \frac{\eta}{[1 - g_{f,ij}]} \delta_{f,ij}^\eta,$$

where the revenue depends overall on the demand parameters ($a$ and $b$), the supply parameter ($\eta$ as the marginal cost elasticity and representing the flexible manufacturing technology), both countries’ characteristics ($w_i$ as the wage rate in country $i$ and $L_j$ as the market size of the destination country $j$), the geographical distance between both countries ($\tau_{ij}$ as the iceberg transport cost) and the firm characteristics ($z_{f,i}$ as the firm’s efficiency in production, $g_{f,ij}$ as the market dominance of the firm in country $j$ and $\delta_{f,ij}$ as the firm’s export scope).

The firm’s export scope as the crucial variable of its choice portfolio is in general determined by the two countervailing effects: the income effect which provides incentives for the firm to expand its scope in case of a higher per capita income and the cannibalization effect which provides incentives for the firm to restrict or contract its scope due to the within-firm cannibalization. With the above cost specification, one gets an explicit expression for the firm’s export scope, disclosing those effects:

$$\delta_{f,ij} = \left[ \frac{4b}{a} \frac{1}{\tau_{ij} w_i z_{f,i}} \left[ 1 + \frac{1}{2\eta} \right] [1 - g_{f,ij}] E_j \right]^{\frac{1}{\eta}},$$

where $\delta_j = \sum_{i=1}^I \sum_{f=1}^{m_i} \delta_{f,ij}$ denotes the number of varieties available in country $j$. As it was already derived in case of the implicit solution for the export scope, but now being explicit, all else constant, firms export more varieties to richer countries, i.e. countries with a higher per capita income $E_j$. Consumers in those countries reveal a lower price elasticity of demand and are willing to purchase the more expensive goods. Multi-product firms thus export their core products to all the destinations, but their non-core products are only exported to the richer countries. In addition, comparing both the domestic and export scope and due to the transportation cost with $\tau_{ii} = 1$ and $\tau_{ij} > 1$, firms export only a fraction of their domestic scope to the destination abroad, given that all else is equal for both countries. Empirical evidence for this finding of the model is provided by Iacovone and Javorcik (2010).

For some constant number of varieties available in country $j$, any increase in the firm’s dominance in the market is associated with a reduction in its scope due to the cannibalization effect. However, it is possible to go one step further to get an even deeper insight:

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58 See the Appendix F.9 for the details of derivation.
59 See the Appendix F.10 for the details of derivation.
60 See the Section 5.3.1.
The market dominance of the firm \( f \) located in country \( i \) in country \( j \) can be expressed as the ratio of the firm’s export scope to the destination and the overall number of varieties available in country \( j \), i.e. \( g_{f,ij} = \frac{\delta_{f,ij}}{\delta_j} \). With this finding, the firm’s export scope (equation (5.18)) gets:

\[
\delta_{f,ij} = \left[ \frac{4b}{a} \frac{1}{\tau_{ij}w_i z_{f,i}} \left( 1 + 1 \right) \frac{1 - g_{f,ij}g_{f,ij}E_j}{\eta} \right]^{\frac{1}{\eta + 1}},
\]

(5.19)

which yields a non-linear relationship between the firm’s export scope and its market dominance: Small firms, i.e. firms that have a market dominance of less than 0.5, with a relatively weak within-firm cannibalization effect expand their scope with an increase in their market dominance. In contrast, firms with an already large market dominance, i.e. larger than 0.5, experience a strong cannibalization effect and thus a decrease in their export scope whenever their market dominance rises further.

To complement the analysis under the given cost specification, explicit solutions for the firm’s export costs and its operating export profits from the operation to country \( j \) are derived:

\[
C_{f,ij} = \frac{L_j}{2b} \frac{a\tau_{ij} w_i z_{f,i}}{\eta} \frac{\delta_{f,ij}^{\eta + 1}}{(\eta + 1)(2\eta + 1)}
\]

(5.20)

and

\[
\Pi_{f,ij} = \frac{L_j}{2b} \frac{a\tau_{ij} w_i z_{f,i}}{\eta} \frac{\delta_{f,ij}^{\eta + 1}}{(\eta + 1)(2\eta + 1)(1 - g_{f,ij})}\delta_{f,ij}^{\eta + 1}.
\]

(5.21)

Overall, the firm realizes profits out of its activities among the \( I \) countries:

\[
\Pi_{f,i} = \sum_{j=1}^{I} \Pi_{f,ij} - w_i F,
\]

(5.22)

whereas it has to pay some fixed operation cost \( F \) for the establishment of its production facilities in country \( i \), which is measured in domestic labour units and therefore weighted by the country’s wage rate \( w_i \).

With the above cost specification, the model additionally provides the following finding: First, the sales ratio of two varieties \( \omega \) and \( \omega' \) of firm \( f \) in country \( j \) can be expressed in terms of the firm’s export scope:

\[
\frac{r_{f,ij}(\omega)}{r_{f,ij}(\omega')} = \frac{1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta}}{1 - \left( \frac{\omega'}{\delta_{f,ij}} \right)^{2\eta}}.
\]

(5.23)

Easily verifiable, the sales ratio of two varieties \( \omega \) and \( \omega' \) with \( \omega < \omega' \) decreases with

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61 See the Appendix F.11 for the details of derivation.
62 See the Appendix F.12 for the proof.
63 See the Appendix F.13 for the details of derivation.
64 See the Appendix F.14 for the details of derivation.
65 See the Appendix F.14 for the proof.
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the firm’s export scope. As the competition increases and the firm’s market dominance decreases, the firm with a market dominance below 0.5 reduces its export scope and its sales ratio of varieties closer to the core competency to varieties being further away increases. Therefore, in case of an increased competition, multi-product firms alter their product mix by skewing their export sales towards their core products.

For its part, the model replicates with this finding a robust empirical fact, or regularity, of multi-product firms in international trade, which gets established in the literature.\footnote{For additional regularities of multi-product firms in international trade: See the Chapter 2.}

Being confronted with an increased competition, multi-product firms adjust their product mix at the extensive and intensive margin by dropping peripheral products and undertaking a within-firm reallocation, i.e. shifting their sales away from their peripheral products and concentrating more on their core products, leading to an increased skewness in the firms’ sales distribution across their products. First, Liu (2010) reports for the United States that a rising import competition induces multi-product firms to divest their peripheral products, whereas those products with weaker linkages to the core are more likely to be dropped, and to reallocate their product composition towards their core production, i.e. increase the sales share for the core products and decrease the sales share for the peripheral products. In a related study of the effects of an increased Chinese competition for Mexico, Iacovone et al. (2013) find an asymmetry at the extensive and intensive margins across the products of multi-product plants: Marginal products are more likely to be dropped and their sales compressed, while core products are relatively impervious to the competition surge, implying a within-firm reallocation in terms of market shares. Mayer et al. (2014) document for French exporters that those firms skew their export sales towards their better performing products in export markets with a tougher competition. In a follow-up study, Mayer et al. (2016) do not consider the variation in the firms’ export skewness across destinations, but in a given destination across time and find that positive demand shocks in export markets lead French multi-product firms to skew their export sales in those destinations towards their better performing products, in this way reacting to an increased competition.

5.3.4 **Equilibrium**

The equilibrium of the model is given by an allocation of the number of firms in each country $j \in \{1, \ldots, I\}$, $m_j$, the scope of each firm $f \in \{1, \ldots, m_i\}$ in country $i \in \{1, \ldots, I\}$, $\delta_{f,ij}$, and the per capita income of each country $j \in \{1, \ldots, I\}$, $E_j$, such that

(i) firms choose the mass of varieties they sell domestically and export,

(ii) free entry drives profits to zero, which implies $E_j = w_j$, and

(iii) labor and goods markets clear and trade is balanced.
5.3.5 Comparative Statics

Further following the chapter’s purpose of examining the behaviour of multi-product firms across countries that differ in terms of their per capita income, its model’s comparative statics provide additional insights. For the analysis of the changes in the endogenous variables, i.e. the price respectively the markup and the scope, in response to changes in the exogenous variables, i.e. the cost of transportation and the per capita income, the following system of model equations is used, whereas, once again, the cost specification in the Section 5.3.3 is applied:

\[ p_{f,ij}(\omega) = \frac{\tau_{ij}w_i z_{f,i}}{2 (1 - g_{f,ij})} \left( \omega^n + \delta_{f,ij}^n \right), \]  
(5.24)

\[ \mu_{f,ij}(\omega) = \frac{1}{2 (1 - g_{f,ij})} \left( 1 + \frac{\delta_{f,ij}^n}{\omega^n} \right), \]  
(5.25)

\[ \delta_{f,ij} = \left( \frac{4b}{a} \frac{1}{\tau_{ij} w_i z_{f,i}} \right)^{\frac{1}{\eta}} \left( 1 + \frac{1}{2\eta} \right)^{\frac{1}{\eta}} \left( 1 - g_{f,ij} \right)^{\frac{1}{\eta}} \left( E_j \frac{\delta_j}{\delta_j} \right)^{\frac{1}{\eta}}, \]  
(5.26)

and

\[ g_{f,ij} = \frac{\delta_{f,ij}}{\delta_j}. \]  
(5.27)

Starting with a ceteris paribus consideration, more productive firms, i.e. firms with a higher productivity in the production and thus a lower \( z_{f,i} \), charge a lower price for each variety and export a larger scope of varieties to destinations abroad, while the markup is unaffected by the firm productivity. In the following, the interrelations between the variables are taken into account, first assuming that the overall number of varieties available in the destination country \( j \), \( \delta_j \), is constant. The elasticity of the price with respect to the transportation cost derives then as:

\[ \bar{\varepsilon}_{p,\tau} = \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} = -\frac{\eta \omega^n (1 - g_{f,ij})}{(\omega^n + \delta_{f,ij}^n) (\eta (1 - g_{f,ij}) + g_{f,ij})}, \]  
(5.28)

and the elasticity of the markup with respect to the transportation cost is given by:

\[ \bar{\varepsilon}_{\mu,\tau} = \frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{-\eta \delta_{f,ij}^n (1 - g_{f,ij}) + g_{f,ij} (\omega^n + \delta_{f,ij}^n)}{(\omega^n + \delta_{f,ij}^n) (\eta (1 - g_{f,ij}) + g_{f,ij})}. \]  
(5.29)

As can be seen, the price for each variety decreases with a decrease in the transportation cost but the price pass-through is incomplete, i.e. the elasticity of the price with respect to the transportation cost is less than one. Multi-product firms thus absorb some part of a change in the transportation cost in their markups, as the markup for each variety increases with a decrease in the transportation cost, and do not fully pass trade cost reliefs.

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67 See the Appendix F.15.1 for the details of derivation.
68 See the Appendix F.15.2 for the details of derivation.
69 See the Appendix F.15.3 for the proof.
and difficulties through to the consumers.

However, quantitative differences exist across firms\footnote{For an overview about the properties of the elasticities of the price, markup and scope: See the Table 5.1.} The transportation cost elasticity of the price is decreasing with the firm’s market dominance\footnote{See the Appendix F.15.3 for the proof.}. Thus, a firm with a higher market share has a lower pass-through of prices. Empirical evidence for this finding of the model is provided by Amiti et al. (2014). Exporters with a more dominant market positioning adjust their prices to a lesser extent to the change in the transportation cost. In this way, these firms reveal the capacity and willingness to absorb a larger part of the cost variation in their markups, as the absolute value of the elasticity of the markup with respect to the transportation cost is increasing in the firm’s market dominance\footnote{See the Appendix F.15.4 for the proof.}. The same pattern holds true for the firm’s export scope: The transportation cost elasticity of the price decreases with the firm’s export scope\footnote{See the Appendix F.15.3 for the proof.}. Firms that are larger in terms of the number of varieties shipped abroad undertake smaller price adjustments and follow a more stable and less volatile price policy. A larger part of the cost reduction is captured by these firms in form of larger markups, as the absolute value of the elasticity of the markup with respect to the transportation cost is increasing in the firm’s export scope\footnote{See the Appendix F.15.4 for the proof.}.

Quantitative differences in the firms’ price pass-through are not only in place across firms, but also within firms across their products: The transportation cost elasticity of the price is increasing in the firm’s product portfolio\footnote{See the Appendix F.15.3 for the proof.}. While for products that are closer to the firm’s core and have larger markups, firms absorb a larger part of the cost variation in those products’ markups, peripheral products that are produced with a lower productivity and higher costs react with their prices more to the change in the transportation cost. And as the absolute value of the elasticity of the markup with respect to the transportation cost is decreasing in each firm’s product portfolio\footnote{See the Appendix F.15.4 for the proof.}, a decrease in the transportation cost leads to a larger increase in the markups of the products that are closer to the firm’s core and to a smaller increase in the peripheral products’ markups.

Export prices and markups do not only vary due to the changes in the cost of transportation to the sales market abroad, but also due to the changes in the per capita income of the destination country. As it turns out, the elasticities of the price and the markup with respect to the per capita income are given by\footnote{See the Appendix F.15.5 for the details of derivation.}

$$
\vec{\varepsilon}_{p,E} = \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij} (1 - g_{f,ij}) + g_{f,ij} (\omega + \delta_{f,ij})}{(\omega + \delta_{f,ij}) (\eta (1 - g_{f,ij}) + g_{f,ij})} = \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \vec{\varepsilon}_{\mu,E}. \quad (5.30)
$$
Both, the price and the markup for each variety increase with an increase in the per capita income, and do so by the same amount, i.e. the elasticities are identical. Again, the adjustments quantitatively vary across firms: The income elasticities of the price and the markup are increasing in the firm’s market dominance. For more dominant firms, prices and markups are adjusted to a larger extent in response to a change in the per capita income. The identical pattern applies as well to larger firms: The income elasticities of the price and the markup are increasing in the firm’s export scope. Firms that export more varieties abroad undertake larger price and markup adjustments following a change in the per capita income. However, quantitative differences exist across the products within firms: The income elasticities of the price and the markup are decreasing in the firm’s product portfolio. When the per capita income changes, the prices and the markups of products that are closer to the firm’s core and in whose production the firm is more efficient in are adjusted to a larger extent than the prices and the markups of products that are further away and productional less efficient.

Besides their reactions along the prices, multi-product firms can also vary the number of products that are exported, i.e. their export scope, in response to a change in the cost of transportation and the per capita income of the destination country. From now on, the assumption of a constant overall number of varieties being available in the destination \( j \), \( \delta_j \), is suspended and instead it varies like the other variables. The elasticity of the scope with respect to the transportation cost is then given by:

\[
\varepsilon_{\delta,\tau} = \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta + 2g_{f,ij}}.
\]

As the cost of transportation decreases in the wake of a trade liberalization and exporting becomes less costly, multi-product firms expand their export scope. As it is obvious, quantitatively varying across firms, the absolute value of the elasticity of the scope with respect to the transportation cost decreases with the firm’s market dominance, due to the cannibalization effect. Firms that account for a larger sales share in the destination market and are therefore more dominant face a stronger cannibalization effect and thus reveal a smaller scope response to a change in the transportation cost; in particular, they expand their export scope less in case of a trade liberalization.

Appearing once again as a situation which is more or less rewarding for trade efforts, a variation in the destination’s per capita income leads to a structurally identical result in the firms’ extensive margin adjustment as a variation in the cost of transportation: The

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79 See the Appendix F.15.7 for the proof.
80 See the Appendix F.15.8 for the proof.
81 See the Appendix F.15.7 for the proof.
82 See the Appendix F.15.8 for the proof.
83 See the Appendix F.15.7 for the proof.
84 See the Appendix F.15.8 for the proof.
85 See the Appendix F.15.9 for the details of derivation.
86 See the Appendix F.15.10 for the proof.
elasticity of the scope with respect to the per capita income derives as

$$\varepsilon_{\delta,E} = -\varepsilon_{\delta,\tau} = \frac{d\ln \delta_{f,ij}}{d\ln E_j} = \frac{1}{\eta + 2g_{f,ij}}.$$  

An increase in the per capita income and therefore richer consumers in the destination abroad induce multi-product firms to expand their export scope, selling not only the relatively cheap core products to those consumers, but also the more expensive marginal products, for which those consumers meanwhile have some demand. However, quantitative differences exist once more across firms: The elasticity of the scope with respect to the per capita income decreases in the firm’s market dominance. Constrained by a stronger cannibalization effect, more dominant firms are less reactive in their export scope to a change in the per capita income; in particular, they expand their export scope less to an increase in the destination’s per capita income.

Replicating the above considerations of the price and the markup adjustments in response to changes in the cost of transportation and the per capita income, but now allowing for a variation in the overall number of varieties being available in the destination country, the elasticity of the price with respect to the transportation cost is then given by

$$\varepsilon_{p,\tau} = \frac{d\ln p_{f,ij}(\omega)}{d\ln \tau_{ij}} = \frac{\omega^\eta (\eta + g_{f,ij}) + g_{f,ij} \delta_{f,ij}^\eta}{(\omega^\eta + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij})},$$

and the elasticity of the markup with respect to the transportation cost derives as

$$\varepsilon_{\mu,\tau} = \frac{d\ln \mu_{f,ij}(\omega)}{d\ln \tau_{ij}} = -\frac{\eta \delta_{f,ij}^\eta + g_{f,ij} (\omega^\eta + \delta_{f,ij}^\eta)}{(\omega^\eta + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij})}. \quad (5.31)$$

As can be seen, the price for each variety decreases with a decrease in the transportation cost but the price pass-through is incomplete, i.e. the elasticity of the price with respect to the transportation cost is less than one, at the firm and product level. Empirical evidence for this finding of the model is provided by Berman et al. (2012), Chatterjee et al. (2013), Amiti et al. (2014), Li et al. (2015), Chen and Inklaar (2016), Garetto (2016) as well as Caselli et al. (2017) for exchange rates. Firms absorb some part of the cost change in their markups and do not fully pass it through to the consumers, as the markup for each variety increases with a decrease in the transportation cost. Yielding qualitatively identical results, a quantitative difference between this measure of the price pass-through, $\varepsilon_{p,\tau}$, and the one for which the overall number of varieties being available in the destination country

is hold fixed, $\bar{\varepsilon}_{p,\tau}$, quite exists. These measures’ proportion indeed is ex ante ambiguous and depends on both the firm’s market dominance and the elasticity of the marginal cost. 

$$\bar{\varepsilon}_{p,\tau} \equiv \varepsilon_{p,\tau} \iff \delta_{f,ij}^2 ((\eta - 1) g_{f,ij}^2 - \eta g_{f,ij}) \equiv \omega^9 ((\eta + 1) g_{f,ij}^2)$$

If the elasticity of the marginal cost is not too large, i.e. $\eta < 1$ and so an adjustment in the firm’s product portfolio is merely associated with a moderate cost, making firms basically more willing to undertake adjustments at their extensive margin, then $\varepsilon_{p,\tau} > \bar{\varepsilon}_{p,\tau}$, i.e. the price pass-through in case of considering a change in the overall number of varieties available in the destination country is larger than the price pass-through under the assumption of a constant $\delta_j$. In the first-mentioned case, firms thus absorb a smaller part of the cost change in their markups and pass a larger part through to the consumers. When a trade liberalization in form of a cut of the transportation cost proceeds, firms in the latter case are confronted with a stronger cannibalization effect and restrict adjustments at the extensive margin and at the intensive margin by lowering prices less, capturing more of the cost release in their markups and thus mainly generating additional profits through a better profitability on the existing product portfolio and product scale. By contrast, firms in the former case undertake adjustments at the extensive margin and lower prices more, thereby initiating larger sales per product, i.e. raising the intensive margin, therefore mainly generating additional profits through a larger extensive and intensive margin.

As before, quantitative differences in the price pass-through exist across firms and within firms across their products: The transportation cost elasticity of the price is increasing with the firm’s market dominance. Firms with a larger sales share in the destination market and that are thus more dominant in it adjust their prices more to a change in the transportation cost and absorb a smaller part of the cost change in their markups, as the absolute value of the elasticity of the markup with respect to the transportation cost is decreasing in the firm’s market dominance; in this way, these firms pass more through to the consumers. The transportation cost elasticity of the price is decreasing with the firm’s export scope. Firms that are larger in terms of the number of products exported pass through a smaller part of the change in the transportation cost to the consumers. These firms accordingly reveal a capacity or willingness to absorb a larger part of it in their markups, as the absolute value of the elasticity of the markup with respect to the transportation cost is increasing in the firm’s export scope, thereby following a more stable price policy. Finally, the transportation cost elasticity of the price is increasing in each firm’s product portfolio. While firms adjust the prices of the products that are closer to the core less and absorb a larger part of the cost variation in those products’

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93 See the Appendix F.15.16 for the details of derivation.
94 See the Appendix F.15.16 for the proof.
95 See the Appendix F.15.15 for the proof.
96 See the Appendix F.15.15 for the proof.
97 See the Appendix F.15.16 for the proof.
98 See the Appendix F.15.15 for the proof.
larger markups, peripheral products react with their prices more to the change in the transportation cost. As the absolute value of the elasticity of the markup with respect to the transportation cost is decreasing in each firm’s product portfolio\textsuperscript{[99]} a decrease in the transportation cost leads to a larger increase in the markups of the products that are closer to the firm’s core and to a smaller increase in the peripheral products’ markups.

Letting the overall number of varieties available in the destination country be variable, the elasticities of the price and the markup with respect to the per capita income are given by\textsuperscript{[100, 101]}

\[
\varepsilon_{p,E} = \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij}^{\eta} + g_{f,ij} (\omega^n + \delta_{f,ij}^{\eta})}{(\omega^n + \delta_{f,ij}^{\eta}) (\eta + 2g_{f,ij})} = \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \varepsilon_{\mu,E}.
\]

Both, the price and the markup for each variety increase with an increase in the per capita income, and do so by the same amount, i.e. the elasticities are identical. As an implication of the non-homothetic preferences, exporting firms charge higher prices for their products in richer destinations. Empirical evidence for this finding of the model is provided by \textsuperscript{[100]} Bastos and Silva (2010) for Portugal, \textsuperscript{[101]} Manova and Zhang (2012) for China, \textsuperscript{[102]} Martin (2012) for France, \textsuperscript{[103]} Simonovska (2015) for a single online retailer in the apparel sector and \textsuperscript{[104]} Görg et al. (2017) for Hungary. In general, price variations by an exporting firm within a product (product category) across heterogeneous countries may indicate differences in (i) the markups charged in the countries, as suggested by the present chapter’s theory, and/or (ii) the quality supplied to the countries. Indeed, the theoretical and empirical trade literature identifies a significant role of vertical product differentiation and product quality in shaping the patterns of international trade, also of trade between countries with different income levels\textsuperscript{[105]}, as it is this chapter’s setting. Nevertheless, by concentrating on the multi-product firms’ export scope across those heterogeneous countries, product quality represents a feature that is beyond the scope of this chapter; even so it would fit in with the present chapter’s theory, its presented narrative and the empirical evidence: With non-homothetic preferences, firms have an incentive to improve the product quality and to export higher-quality products when they face wealthier consumers with a lower marginal utility of income and a larger willingness to pay for higher quality\textsuperscript{[106]}. Another, though to the above references closely related strand of the literature observes a pattern that is typically called “pricing to market”, i.e. the price variations across countries at the

\textsuperscript{99}See the Appendix F.15.16 for the proof.
\textsuperscript{100}See the Appendix F.15.17 for the details of derivation.
\textsuperscript{101}See the Appendix F.15.18 for the details of derivation.
\textsuperscript{102}For both the respective theory and empirics: See Section 5.2.1.
\textsuperscript{103}In a recent contribution, Bastos et al. (2018) approach the topic of quality differences in exports across heterogeneous destinations by considering their inputs to the production process and in detail investigating the relationship between the destination’s income and the prices of the exporters’ inputs. They detect that exporting to richer countries leads firms to pay higher prices for inputs, which supports the presumption that selling to richer destinations is associated with an increase in the quality of the products produced and sold as well as the purchase of higher-quality inputs.
more aggregate, i.e. country, level: The prices of tradable consumption goods are higher in countries with a higher per capita income; such a finding is obtained by Alessandria and Kaboski (2011) and Bekkers et al. (2012).

Quantitative differences in the price and the markup adjustments are in place both across firms and within firms across their products: The income elasticities of the price and the markup are decreasing in the firm’s market dominance. More dominant firms adjust their prices and markups less to a change in the per capita income of the destination country. In contrast, the income elasticities of the price and the markup are increasing in the firm’s export scope. When firms export more products, they undertake larger price and markup adjustments in response to a variation in the per capita income. Finally, the income elasticities of the price and the markup are decreasing in each firm’s product portfolio. For products closer to the core, firms adjust their prices and markups in response to a change in the per capita income more, whereas the prices and the markups of peripheral products reveal smaller adjustments and a more stable policy.

<table>
<thead>
<tr>
<th>$p_{f,ij}(\omega)$</th>
<th>$\mu_{f,ij}(\omega)$</th>
<th>$\delta_{f,ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{f,ij}$</td>
<td>$\delta_{f,ij}$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$g_{f,ij}$</td>
<td>$\delta_{f,ij}$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$\tau_{ij}$</td>
<td>$\bar{\delta}_{j}$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$E_{j}$</td>
<td>$\bar{\delta}_{j}$</td>
<td>$&gt; 0$</td>
</tr>
</tbody>
</table>

Notes: The cell in row $i$ and column $j$ reports the (quantitative) change in the elasticity of the upper variable in column $j$ (at the top) with respect to the parameter in row $i$ (at the left) following an increase in the lower variable in column $j$ (at the second row), differentiated whether the number of varieties is assumed to be fixed ($\delta_{j}$) or flexible ($\bar{\delta}_{j}$) (at the second column) [*: change in the absolute value of the elasticity].

Table 5.1: Properties of the Elasticities - Quantitative Differences across the Firms and the Products

5.4 Conclusion

In a world with heterogeneous countries, this chapter examines the export product scope of multi-product firms across destinations that substantially differ in terms of their per capita income, which represents an issue so far largely ignored by the literature of international trade. Capturing several robust empirical facts, in particular demand-side drivers of trade and income, granularity of the economy, multi-product firms and their features, the analysis is employed in a non-standard model with non-homothetic preferences on the demand

\[ \text{See the Appendix F.15.19 for the proof.} \]
\[ \text{See the Appendix F.15.20 for the proof.} \]
\[ \text{See the Appendix F.15.19 for the proof.} \]
\[ \text{See the Appendix F.15.20 for the proof.} \]
\[ \text{See the Appendix F.15.19 for the proof.} \]
\[ \text{See the Appendix F.15.20 for the proof.} \]
and imperfect competition, i.e. Cournot competition, on the supply side. Multi-product firms in this setting export more varieties to countries with a higher per capita income, whereas their choice of the export scope is essentially determined by both within-firm cannibalization and income effects. By displaying this behaviour, firms expand or shorten their scope along their products ladder depending on the destination’s per capita income. The price and scope adjustments of these multi-product firms in response to changes in the income and transportation cost that are derived in the model thereby replicate several robust empirical facts.
Appendix F

F.1 Income Effect on the Price Elasticity of Demand

Given the aggregate demand

\[ x_{f,i,j}(\omega) = \frac{L_j}{b} \left( a - \lambda_j p_{f,i,j}(\omega) \right), \]

the marginal utility of income

\[ \lambda_j = \frac{1}{L_j E_j} \sum_{i=1}^I \sum_{f=1}^{m_i} \int_0^{\delta_{f,i,j}} \left( a - \frac{b}{L_j} x_{f,i,j}(\omega) \right) x_{f,i,j}(\omega) d\omega = \frac{\Delta}{L_j E_j}, \]

with \( \Delta \equiv \sum_{i=1}^I \sum_{f=1}^{m_i} \int_0^{\delta_{f,i,j}} \left( a - \frac{b}{L_j} x_{f,i,j}(\omega) \right) x_{f,i,j}(\omega) d\omega \), and the elasticity of demand with respect to the price (in its absolute value)

\[ \varepsilon_{f,i,j}(\omega) = \frac{\lambda_j p_{f,i,j}(\omega)}{a - \lambda_j p_{f,i,j}(\omega)}, \]

the effect of the per capita income on the price elasticity of demand can be derived:

\[ \frac{d\varepsilon_{f,i,j}(\omega)}{dE_j} = \frac{\partial \varepsilon_{f,i,j}(\omega)}{\partial \lambda_j} \frac{d\lambda_j}{dE_j}. \]

The per capita income thereby affects the marginal utility of income as follows:

\[ \frac{d\lambda_j}{dE_j} = \frac{\partial \lambda_j}{\partial E_j} + \sum_{i=1}^I \sum_{f=1}^{m_i} \int_0^{\delta_{f,i,j}} \frac{\partial \lambda_j}{\partial x_{f,i,j}(\omega)} \frac{dx_{f,i,j}(\omega)}{dE_j} d\omega \]

and the aggregate demand of variety \( \omega \) as follows:

\[ \frac{dx_{f,i,j}(\omega)}{dE_j} = \frac{\partial x_{f,i,j}(\omega)}{\partial \lambda_j} \frac{d\lambda_j}{dE_j} = -\frac{L_j}{b} p_{f,i,j}(\omega) \frac{d\lambda_j}{dE_j}. \]
Putting this together yields

\[
\frac{d\lambda_j}{dE_j} = \frac{\partial \lambda_j}{\partial E_j} + \sum_{i=1}^{l} \sum_{j=1}^{m} \int_0^{\delta_{f,ij}} \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} \left( -\frac{L_j}{b} p_{f,ij}(\omega) \frac{d\lambda_j}{dE_j} \right) d\omega,
\]

\[
\frac{d\lambda_j}{dE_j} = -\Delta \frac{1}{L_j E_j^2} - \sum_{i=1}^{l} \sum_{j=1}^{m} \int_0^{\delta_{f,ij}} \frac{1}{L_j E_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) - \frac{b}{L_j} x_{f,ij}(\omega) \right) \frac{L_j}{b} p_{f,ij}(\omega) d\omega \frac{d\lambda_j}{dE_j},
\]

\[
\frac{d\lambda_j}{dE_j} = \frac{\Delta}{L_j E_j^2} \left( 1 + \sum_{i=1}^{l} \sum_{j=1}^{m} \int_0^{\delta_{f,ij}} \frac{p_{f,ij}(\omega)}{b E_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \right) = -\Delta \frac{1}{L_j E_j^2}
\]

and finally

\[
\frac{d\lambda_j}{dE_j} = -\Delta \frac{1}{L_j E_j^2} \left( 1 + \sum_{i=1}^{l} \sum_{j=1}^{m} \int_0^{\delta_{f,ij}} \frac{p_{f,ij}(\omega)}{b E_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \right)^{-1} < 0,
\]

given the model assumption that \( L_j > \frac{b}{a} x_{f,ij}(\omega) \). Overall, one gets

\[
\frac{d\varepsilon_{f,ij}(\omega)}{dE_j} = \frac{\partial \varepsilon_{f,ij}(\omega)}{\partial \lambda_j} \frac{d\lambda_j}{dE_j} < 0.
\]

\[ (F.1) \]

**F.2 First-order Condition with Respect to the Quantity (FOC-Q)**

Taking the derivative of the firm’s profit function \[ \text{equation (5.8)} \] with respect to the quantity \( x_{f,ij}(\omega) \) yields the first-order condition with respect to this variable:

\[
\frac{\partial \Pi_{f,i}}{\partial x_{f,ij}(\omega)} = \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) - c_{f,ij}(\omega) + x_{f,ij}(\omega) \left( -\frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) - \frac{b}{\lambda_j L_j} \right)
\]

\[
+ \int_0^{\delta_{f,ij}} x_{f,ij}(\omega) \left( -\frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) \right) d\omega = 0,
\]

\[
\frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) + \int_0^{\delta_{f,ij}} x_{f,ij}(\omega) \left( -\frac{1}{\lambda_j^2} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) \right) d\omega \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)}
\]

\[
- x_{f,ij}(\omega) \frac{b}{\lambda_j L_j} = c_{f,ij}(\omega),
\]

\[
\frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) - x_{f,ij}(\omega) \frac{b}{\lambda_j L_j}
\]
F.3. MARGINAL UTILITY OF INCOME

Rewritten to get:

\[ + \int_0^{\delta_{f,ij}} \left( - \frac{x_{f,ij}(\omega)}{\chi_j^2} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) \right) d\omega \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} = c_{f,ij}(\omega), \]

\[ \frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) + \int_0^{\delta_{f,ij}} \left( - \frac{x_{f,ij}(\omega)}{\chi_j^2} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) \right) d\omega \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} = c_{f,ij}(\omega) \]

and

\[ \frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) - \frac{1}{\lambda_j^2} \int_0^{\delta_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} = c_{f,ij}(\omega). \]

\[ \text{(F.2)} \]

F.3 Marginal Utility of Income

Being distinctive and unique of the chapter’s model, the marginal utility of income gets influenced by the firm’s choice, i.e. the quantity produced, \( x_{f,ij}(\omega) \), and the number of products produced, \( \delta_{f,ij} \):

\[ \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} = \frac{1}{E_j L_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) - \frac{b}{L_j} x_{f,ij}(\omega) \right), \]

\[ \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} = \frac{1}{E_j L_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) \]

and

\[ \frac{\partial \lambda_j}{\partial \delta_{f,ij}} = \frac{1}{E_j L_j} \left( a - \frac{b}{L_j} x_{f,ij}(\delta_{f,ij}) \right) x_{f,ij}(\delta_{f,ij}). \]

\[ \text{(F.3)} \]

F.4 Market Dominance and FOC-Q

With the market dominance of firm \( f \) located in country \( i \) in country \( j \) given by \( g_{f,ij} \) and defined as a measure of the firm’s sales (revenue) share in the country’s market,

\[ \frac{\int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) p_{f,ij}(\omega) d\omega}{\sum_{i=1}^{I} \sum_{j=1}^{J} \int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) p_{f,ij}(\omega) d\omega} = \frac{\int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) \lambda_j p_{f,ij}(\omega) d\omega}{\sum_{i=1}^{I} \sum_{f=1}^{m_f} \int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) \lambda_j p_{f,ij}(\omega) d\omega}, \]

and

\[ \frac{\int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega}{\sum_{i=1}^{I} \sum_{f=1}^{m_f} \int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega}, \]

\[ \text{(F.5)} \]

and \textbf{equation (F.1)} as well as the expression for the marginal utility of income, given by \textbf{equation (5.5)} the first-order condition with respect to the quantity \( x_{f,ij}(\omega) \) can be rewritten to get:

\[ \frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) - \frac{1}{\lambda_j^2} \int_{0}^{\delta_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \frac{\partial \lambda_j}{\partial x_{f,ij}(\omega)} = c_{f,ij}(\omega), \]
\[
\frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) = \frac{1}{\lambda_j} \int_0^\delta x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \left( \frac{1}{E_j L_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) \right) = c_{f,ij}(\omega),
\]

\[
\frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) \left[ 1 - \frac{1}{E_j L_j} \int_0^{\delta x_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \right] = c_{f,ij}(\omega),
\]

\[
\frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) \left[ 1 - \frac{1}{E_j L_j} \sum_{i=1}^{m_f} \int_0^{\delta x_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \right] = c_{f,ij}(\omega),
\]

and

\[
\frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,ij}(\omega) \right) \left[ 1 - g_{f,ij} \right] = c_{f,ij}(\omega).
\]

(F.6)

### F.5 First-order Condition with Respect to the Scope (FOC-S)

Taking the derivative of the firm’s profit function (equation (5.8)) with respect to the scope \( \delta_{f,ij} \) yields the first-order condition with respect to this variable:

\[
\frac{\partial \Pi_{f,i}}{\partial \delta_{f,ij}} = \left( \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\delta_{f,ij}) \right) - c_{f,ij}(\delta_{f,ij}) \right) x_{f,ij}(\delta_{f,ij}) + \int_0^{\delta x_{f,ij}} -\frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) x_{f,ij}(\omega) d\omega \frac{\partial \lambda_j}{\partial \delta_{f,ij}} = 0,
\]

\[
\frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\delta_{f,ij}) \right) x_{f,ij}(\delta_{f,ij}) - \frac{1}{\lambda_j} \int_0^{\delta x_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \frac{\partial \lambda_j}{\partial \delta_{f,ij}} = 0,
\]

and

\[
\left( \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\delta_{f,ij}) \right) - c_{f,ij}(\delta_{f,ij}) \right) x_{f,ij}(\delta_{f,ij}) = \frac{1}{\lambda_j} \int_0^{\delta x_{f,ij}} x_{f,ij}(\omega) \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) d\omega \frac{\partial \lambda_j}{\partial \delta_{f,ij}}.
\]

(F.7)

### F.6 Market Dominance and FOC-S

Using the market dominance of firm \( f \) located in country \( i \) in country \( j \), given by \( g_{f,ij} \), and [equation (F.2)] as well as the expression for the marginal utility of income, given by [equation (5.5)], the first-order condition with respect to the scope \( \delta_{f,ij} \) can be rewritten to
Proven by a contradiction, the firm expands its scope until the demand for the marginal variety is driven down to zero: \( x_{f,ij}(\delta_{f,ij}) = 0 \).

First, it is assumed that the demand for the marginal variety is strictly positive: \( x_{f,ij}(\delta_{f,ij}) > 0 \). Then, the modified first-order condition with respect to scope (equation F.8)
(5.12) can be simplified to:

\[ \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,i,j}(\delta_{f,i,j}) \right) [1 - g_{f,i,j}] = c_{f,i,j}(\delta_{f,i,j}) \]

and

\[ \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,i,j}(\delta_{f,i,j}) \right) = \frac{1}{1 - g_{f,i,j}} c_{f,i,j}(\delta_{f,i,j}). \] (F.9)

Second, the first-order condition with respect to the quantity (equation (5.10)) should also be fulfilled for the marginal variety:

\[ \frac{1}{\lambda_j} \left( a - 2 \frac{b}{L_j} x_{f,i,j}(\delta_{f,i,j}) \right) [1 - g_{f,i,j}] = c_{f,i,j}(\delta_{f,i,j}), \]

which can be rearranged:

\[ \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,i,j}(\delta_{f,i,j}) \right) - \frac{b}{\lambda_j L_j} x_{f,i,j}(\delta_{f,i,j}) = \frac{1}{1 - g_{f,i,j}} c_{f,i,j}(\delta_{f,i,j}). \] (F.10)

Given the validity of both equation (F.6) and equation (F.7) it ultimately follows:

\[ \frac{b}{\lambda_j L_j} x_{f,i,j}(\delta_{f,i,j}) = 0 \]

and

\[ x_{f,i,j}(\delta_{f,i,j}) = 0, \]

which contradicts the assumption above and finishes the proof.

**F.8 Output and Price Profile**

The first-order condition with respect to the quantity (equation (5.10)) can be rearranged:

\[ a - 2 \frac{b}{L_j} x_{f,i,j}(\omega) = \frac{1}{1 - g_{f,i,j}} \lambda_j c_{f,i,j}(\omega), \]

\[ a - \frac{1}{1 - g_{f,i,j}} \lambda_j c_{f,i,j}(\omega) = 2 \frac{b}{L_j} x_{f,i,j}(\omega) \]

and

\[ x_{f,i,j}(\omega) = \frac{L_j}{2b} \left( a - \frac{1}{1 - g_{f,i,j}} \lambda_j c_{f,i,j}(\omega) \right). \]

By using equation (5.13) the optimal output profile can then be written:

\[ x_{f,i,j}(\omega) = \frac{L_j}{2b} \left( a - \frac{1}{1 - g_{f,i,j}} \frac{a}{c_{f,i,j}(\delta_{f,i,j})} [1 - g_{f,i,j}] c_{f,i,j}(\omega) \right), \]
\[ x_{f,ij}(\omega) = \frac{L_j}{2b} \left( a - a \frac{c_{f,ij}(\omega)}{c_{f,ij}(\delta_{f,ij})} \right) \]

and

\[ x_{f,ij}(\omega) = \frac{aL_j}{2b} \left( 1 - \frac{c_{f,ij}(\omega)}{c_{f,ij}(\delta_{f,ij})} \right). \]

The aggregate inverse demand is given by:

\[ p_{f,ij}(\omega) = \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right). \]

By plugging in the optimal output profile and using equation (5.13), one gets for the optimal price profile:

\[ p_{f,ij}(\omega) = \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right), \]

\[ p_{f,ij}(\omega) = \frac{1}{\lambda_j} \left( a - \frac{a}{2} \left( 1 - \frac{c_{f,ij}(\omega)}{c_{f,ij}(\delta_{f,ij})} \right) \right), \]

\[ p_{f,ij}(\omega) = \frac{1}{\lambda_j} \left( a + \frac{a}{2} c_{f,ij}(\omega) \right), \]

\[ p_{f,ij}(\omega) = \frac{c_{f,ij}(\delta_{f,ij})}{a \left[ 1 - g_{f,ij} \right]} \frac{a}{2} \left( 1 + \frac{c_{f,ij}(\omega)}{c_{f,ij}(\delta_{f,ij})} \right), \]

\[ p_{f,ij}(\omega) = \frac{1}{2 \left[ 1 - g_{f,ij} \right]} \left( c_{f,ij}(\delta_{f,ij}) + c_{f,ij}(\omega) \right) \]

and

\[ p_{f,ij}(\omega) = \frac{1}{2 \left[ 1 - g_{f,ij} \right]} \left( 1 + \frac{c_{f,ij}(\delta_{f,ij})}{c_{f,ij}(\omega)} \right) c_{f,ij}(\omega). \]

**F.9 Export Revenue**

The optimal exports of the product variety \( \omega \) by the firm to country \( j \) can be derived from equation (5.10) by rearrangement and together with the cost specification (equation (5.16)) written as:

\[ x_{f,ij}(\omega) = \frac{L_j}{2b} \left( a - \frac{1}{1 - g_{f,ij}} \lambda_j c_{f,ij}(\omega) \right) = \frac{L_j}{2b} \left( a - \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{f,i} w_i z_{f,i} \omega^n \right). \]

For the price of the variety, one therefore gets:

\[ p_{f,ij}(\omega) = \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) = \frac{1}{\lambda_j} \left( a - \frac{b}{L_j} \frac{L_j}{2b} \left( a - \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{f,i} w_i z_{f,i} \omega^n \right) \right) \]

and

\[ p_{f,ij}(\omega) = \frac{1}{\lambda_j} \left( a + \frac{1}{2} \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{f,i} w_i z_{f,i} \omega^n \right) = \frac{1}{2 \lambda_j} \left( a + \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{f,i} w_i z_{f,i} \omega^n \right). \]
The firm’s revenue from the exports of the product variety \( \omega \) to country \( j \) is then given by:

\[
p_{f,ij}(\omega)x_{f,ij}(\omega) = \frac{1}{2\lambda_j} \left( a + \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{ij} w_i z_{f,i} \omega^q \right) L_j \left( a - \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{ij} w_i z_{f,i} \omega^q \right)
\]

and

\[
p_{f,ij}(\omega)x_{f,ij}(\omega) = \frac{L_j}{4b\lambda_j} \left( a^2 - \left( \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{ij} w_i z_{f,i} \omega^q \right)^2 \right) \omega^q.
\]

By summing it up over the firm’s export scope and using equation (5.13) for the firm’s optimal scope, one gets the firm’s revenue from exports to country \( j \):

\[
R_{f,ij} = \int_0^{\delta_{f,ij}} p_{f,ij}(\omega)x_{f,ij}(\omega)d\omega = \int_0^{\delta_{f,ij}} \frac{L_j}{4b\lambda_j} \left( a^2 - \left( \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{ij} w_i z_{f,i} \omega^q \right)^2 \right) \omega^q d\omega,
\]

\[
R_{f,ij} = \frac{L_j}{4b\lambda_j} \left[ a^2 - \left( \frac{1}{2\eta + 1} \left( \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{ij} w_i z_{f,i} \omega^q \right)^2 \omega^{2\eta+1} \right) \right]_0^{\delta_{f,ij}},
\]

\[
R_{f,ij} = \frac{L_j}{4b\lambda_j} \left( a^2 \delta_{f,ij} - \left( \frac{1}{2\eta + 1} \left( \frac{1}{1 - g_{f,ij}} \lambda_j \tau_{ij} w_i z_{f,i} \omega^q \right)^2 \delta_{f,ij}^{2\eta+1} \right) \right),
\]

\[
R_{f,ij} = \frac{L_j}{4b\lambda_j} \left( a^2 \delta_{f,ij} - \left( \frac{1}{2\eta + 1} \left( \frac{a}{\delta_{f,ij}} \right)^2 \delta_{f,ij}^{2\eta+1} \right) \right) = \frac{L_j}{4b\lambda_j} \left( a^2 \delta_{f,ij} - \frac{a^2}{2\eta + 1} \delta_{f,ij} \right),
\]

\[
R_{f,ij} = \frac{a^2 L_j}{4b\lambda_j} \left( \delta_{f,ij} - \frac{1}{2\eta + 1} \delta_{f,ij} \right) = \frac{a^2 L_j}{2b\lambda_j} \frac{\eta}{2\eta + 1} \delta_{f,ij}.
\]

and

\[
R_{f,ij} = \frac{a^2 L_j}{2b} \frac{\eta}{(2\eta + 1)} \frac{\tau_{ij} w_i z_{f,i} \delta_{f,ij}}{a [1 - g_{f,ij}]} \delta_{f,ij} = \frac{a}{2b} \frac{\eta}{(2\eta + 1)} \frac{L_j \tau_{ij} w_i z_{f,i} \delta_{f,ij}}{[1 - g_{f,ij}]} \delta_{f,ij}^{2\eta+1},
\]

(F.11)

### F.10 Export Product Scope

The firm’s optimal export scope is implicitly given by:

\[
c_{f,ij}(\delta_{f,ij}) = \frac{a}{\lambda_j} [1 - g_{f,ij}],
\]
such that

\[
\lambda_j = \frac{1}{E_j L_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \int_0^{\delta_{f,ij}} \left( a - \frac{b}{L_j} x_{f,ij}(\omega) \right) x_{f,ij}(\omega) d\omega.
\]

By plugging in the optimal quantity from the modified first-order condition (equation (5.10)), the integrand gets the form:

\[
(a - \frac{b}{L_j} x_{f,ij}(\omega)) x_{f,ij}(\omega) = \left( a - \frac{b}{L_j} \frac{L_j}{2b} \left( a - \frac{\lambda_j}{1 - g_{f,ij}} c_{f,ij}(\omega) \right) \right) \frac{L_j}{2b} \left( a - \frac{\lambda_j}{1 - g_{f,ij}} c_{f,ij}(\omega) \right),
\]

whereas the marginal utility of income can be written as:

\[
F.10. \text{EXPORT PRODUCT SCOPE}
\]

By plugging in the optimal quantity from the modified first-order condition (equation (5.10)), the integrand gets the form:

\[
(a - \frac{b}{L_j} x_{f,ij}(\omega)) x_{f,ij}(\omega) = \left( a - \frac{1}{2} \left( a - \frac{\lambda_j}{1 - g_{f,ij}} c_{f,ij}(\omega) \right) \right) \frac{L_j}{2b} \left( a - \frac{\lambda_j}{1 - g_{f,ij}} c_{f,ij}(\omega) \right),
\]

such that

\[
\lambda_j = \frac{1}{E_j L_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \int_0^{\delta_{f,ij}} \frac{L_j}{4b} \left( a^2 - \left( \frac{\lambda_j}{1 - g_{f,ij}} \tau_{ij} w_i z_{f,i} \omega^\eta \right)^2 \right) d\omega,
\]

\[
\lambda_j = \frac{1}{4b E_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \int_0^{\delta_{f,ij}} \left( a^2 - \left( \frac{\lambda_j}{1 - g_{f,ij}} \tau_{ij} w_i z_{f,i} \omega^\eta \right)^2 \right) \omega^{2\eta+1} d\omega,
\]

\[
\lambda_j = \frac{1}{4b E_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \left[ a^2 \omega - \frac{1}{2\eta+1} \right] \delta_{f,ij}
\]

and

\[
\lambda_j = \frac{1}{4b E_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \left( a^2 \delta_{f,ij} - \frac{1}{2\eta+1} \right) \delta_{f,ij}.
\]

With the expression for \( \lambda_j \) from equation (5.13), the marginal utility of income finally gets:

\[
\lambda_j = \frac{1}{4b E_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \left( a^2 \delta_{f,ij} - \frac{1}{2\eta+1} \right) \delta_{f,ij}.
\]
\[
\lambda_j = \frac{1}{4bE_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \left( a^2 \delta_{f,ij} - \frac{1}{2\eta + 1} \left( \frac{a}{\delta_{f,ij}} \right)^2 \delta_{f,ij}^{2\eta+1} \right),
\]
\[
\lambda_j = \frac{1}{4bE_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \left( a^2 \delta_{f,ij} - \frac{a^2}{2\eta + 1} \delta_{f,ij} \right),
\]
\[
\lambda_j = \frac{a^2}{4bE_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \left( \delta_{f,ij} \left( 1 - \frac{1}{2\eta + 1} \right) \right) = \frac{a^2}{4bE_j} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \frac{2\eta}{2\eta + 1} \delta_{f,ij}
\]

and
\[
\lambda_j = \frac{a^2}{4bE_j} \frac{2\eta}{2\eta + 1} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \frac{\delta_{f,ij}}{2\eta + 1} = \frac{a^2}{4bE_j} \frac{2\eta}{2\eta + 1} \delta_{j},
\]

where \(\delta_j = \sum_{i=1}^{I} \sum_{f=1}^{m_i} \delta_{f,ij}\) denotes the number of varieties available in country \(j\). For the firm’s optimal scope, one therefore derives
\[
c_{f,ij}(\delta_{f,ij}) = \frac{a}{\lambda_j} \left[ 1 - g_{f,ij} \right],
\]
\[
\tau_{ij}w_i z_{f,i} \delta^{\eta}_{f,ij} = a \left[ 1 - g_{f,ij} \right] \frac{4bE_j}{a^2} \frac{2\eta + 1}{2\eta} \frac{1}{\delta_j},
\]
\[
\tau_{ij}w_i z_{f,i} \delta^{\eta}_{j,ij} = 4bE_j \left( 1 + \frac{1}{2\eta} \right) \left[ 1 - g_{f,ij} \right] \frac{1}{\delta_j},
\]
\[
\delta^{\eta}_{f,ij} = \frac{4b}{a \tau_{ij}w_i z_{f,i}} \left( 1 + \frac{1}{2\eta} \right) \left[ 1 - g_{f,ij} \right] \frac{E_j}{\delta_j}
\]

and
\[
\delta_{f,ij} = \left( 4b \frac{1}{a \tau_{ij}w_i z_{f,i}} \left( 1 + \frac{1}{2\eta} \right) \left[ 1 - g_{f,ij} \right] \frac{E_j}{\delta_j} \right)^{-\frac{\eta}{2}}. \tag{F.12}
\]

### F.11 Market Dominance

The market dominance of firm \(f\) located in country \(i\) in country \(j\) can be written as the ratio of the firm’s export scope to the destination and the overall number of varieties available in country \(j\):
\[
g_{f,ij} = \frac{R_{f,ij}}{\sum_{i=1}^{I} \sum_{f=1}^{m_i} R_{f,ij}} = \frac{a^2 L_j}{2b\lambda_j} \frac{\eta}{(2\eta + 1)} \delta_{f,ij}
\]

and
\[
g_{f,ij} = \frac{a^2 L_j}{2b\lambda_j} \frac{\eta}{(2\eta + 1)} \sum_{i=1}^{I} \sum_{f=1}^{m_i} \delta_{f,ij} = \frac{\delta_{f,ij}}{\sum_{i=1}^{I} \sum_{f=1}^{m_i} \delta_{f,ij}} = \frac{\delta_{f,ij}}{\delta_j}.
\]
F.12 Relationship between the Market Dominance and the Scope

For the relationship between a firm’s market dominance and its export scope, one gets:

\[
\frac{d\delta_{f,ij}}{dg_{f,ij}} = \frac{1}{\eta + 1} \left[ \frac{4b}{a} \frac{1}{\tau_{ij} w_{i}z_{f,i}} \left[ 1 + \frac{1}{2\eta} \right] \left[ 1 - g_{f,ij} \right] g_{f,ij} E_j \right]^{\frac{1}{\eta + 1}} \frac{4b}{a} \frac{1}{\tau_{ij} w_{i}z_{f,i}} \left[ 1 + \frac{1}{2\eta} \right] E_j \times \left( 1 - g_{f,ij} - g_{f,ij} \right) \leq 0
\]

and

\[
\frac{d\delta_{f,ij}}{dg_{f,ij}} \leq 0 \iff 1 - 2g_{f,ij} \leq 0 \iff 0.5 \leq g_{f,ij}.
\]  

(F.13)

F.13 Export Costs and Export Profits

By plugging in the optimal quantity from the modified first-order condition (equation (5.10)) and by using equation (5.13) for the optimal scope, the firm’s costs from exporting to country \(j\) are:

\[
C_{f,ij} = \int_{0}^{\delta_{f,ij}} c_{f,i}(\omega) x_{f,i}(\omega) d\omega = \int_{0}^{\delta_{f,ij}} \tau_{ij} w_{i}z_{f,i} \omega^{n} x_{f,i}(\omega) d\omega,
\]

\[
C_{f,ij} = \int_{0}^{\delta_{f,ij}} \tau_{ij} w_{i}z_{f,i} \omega^{n} \frac{L_j}{2b} \left( a - \frac{1}{1 - g_{f,ij}} \lambda_{j} c_{f,i}(\omega) \right) d\omega,
\]

\[
C_{f,ij} = \int_{0}^{\delta_{f,ij}} \tau_{ij} w_{i}z_{f,i} \omega^{n} \frac{L_j}{2b} \left( a - \frac{1}{1 - g_{f,ij}} \lambda_{j} \tau_{ij} w_{i}z_{f,i} \omega^{n} \right) d\omega,
\]

\[
C_{f,ij} = \frac{L_j}{2b} \left[ \frac{1}{\eta + 1} a \tau_{ij} w_{i}z_{f,i} \delta_{f,i}^{\eta+1} - \frac{1}{2n + 1 [1 - g_{f,ij}]} \lambda_{j} \left( \tau_{ij} w_{i}z_{f,i} \right)^{2n+1} \right] \delta_{f,ij}^{\eta+1},
\]

\[
C_{f,ij} = \frac{L_j}{2b} \left( \frac{1}{\eta + 1} a \tau_{ij} w_{i}z_{f,i} \delta_{f,i}^{\eta+1} - \frac{1}{2n + 1 [1 - g_{f,ij}]} \lambda_{j} \left( \tau_{ij} w_{i}z_{f,i} \right)^{2n+1} \right) \left( \tau_{ij} w_{i}z_{f,i} \right)^{2n+1},
\]

\[
C_{f,ij} = \frac{L_j}{2b} \left( \frac{1}{\eta + 1} a \tau_{ij} w_{i}z_{f,i} \delta_{f,i}^{\eta+1} - \frac{1}{2n + 1 [1 - g_{f,ij}]} \lambda_{j} \left( \tau_{ij} w_{i}z_{f,i} \right)^{2n+1} \right) \left( \tau_{ij} w_{i}z_{f,i} \right)^{2n+1},
\]

\[
C_{f,ij} = \frac{L_j}{2b} \left( \frac{1}{\eta + 1} a \tau_{ij} w_{i}z_{f,i} \delta_{f,i}^{\eta+1} - \frac{1}{2n + 1 [1 - g_{f,ij}]} \lambda_{j} \left( \tau_{ij} w_{i}z_{f,i} \right)^{2n+1} \right) \left( \tau_{ij} w_{i}z_{f,i} \right)^{2n+1},
\]

and

\[
C_{f,ij} = \frac{L_j}{2b} \frac{\eta}{(\eta + 1)(2\eta + 1)} \delta_{f,i}^{\eta+1},
\]  

(F.14)
The export profits of the firm from serving country \( j \) are then given by:

\[
\Pi_{f,ij} = R_{f,ij} - C_{f,ij} = \frac{L_j}{2b} \frac{\eta}{(2\eta + 1)} \left( \frac{1}{1 - g_{f,ij}} \right)^{\eta+1} \delta_{f,ij}^{\eta+1} - \frac{L_j}{2b} a \tau_{ij} w_{i} z_{f,i} \frac{\eta}{(\eta + 1)(2\eta + 1)} \delta_{f,ij}^{\eta+1},
\]

\[
\Pi_{f,ij} = \frac{L_j}{2b} a \tau_{ij} w_{i} z_{f,i} \delta_{f,ij}^{\eta+1} \left( \frac{\eta}{(2\eta + 1)} \frac{1}{1 - g_{f,ij}} \right) - \frac{\eta}{(\eta + 1)(2\eta + 1)} \delta_{f,ij}^{\eta+1},
\]

\[
\Pi_{f,ij} = \frac{L_j}{2b} a \tau_{ij} w_{i} z_{f,i} \delta_{f,ij}^{\eta+1} \left( \frac{\eta}{(\eta + 1)(2\eta + 1)} \right) - \frac{\eta(1 - g_{f,ij})}{(\eta + 1)(2\eta + 1)(1 - g_{f,ij})},
\]

\[
\Pi_{f,ij} = \frac{L_j}{2b} a \tau_{ij} w_{i} z_{f,i} \delta_{f,ij}^{\eta+1} \left( \frac{\eta}{(\eta + 1)(2\eta + 1)(1 - g_{f,ij})} \right) - \frac{\eta(1 - g_{f,ij})}{(\eta + 1)(2\eta + 1)(1 - g_{f,ij})},
\]

and

\[
\Pi_{f,ij} = \frac{L_j}{2b} a \tau_{ij} w_{i} z_{f,i} \delta_{f,ij}^{\eta+1} \eta \left( \frac{\eta + g_{f,ij}}{(\eta + 1)(2\eta + 1)} \right) - \frac{\eta(1 - g_{f,ij})}{(\eta + 1)(2\eta + 1)(1 - g_{f,ij})},
\]

(F.15)

### F.14 Export Ratio

The sales ratio of two varieties \( \omega \) and \( \omega' \) of firm \( f \) in country \( j \) is given by:

\[
\frac{r_{f,ij}(\omega)}{r_{f,ij}(\omega')} = \frac{L_j}{4b\lambda_j} \left( a^2 - \left( \frac{1}{1-g_{f,ij}} \right) \lambda_j \tau_{ij} w_{i} z_{f,i} (\omega')^{\eta} \right) = a^2 - \left( \frac{1}{1-g_{f,ij}} \right) \lambda_j \tau_{ij} w_{i} z_{f,i} (\omega')^{\eta},
\]

By using equation (5.13) for the firm's optimal scope, it can be rewritten:

\[
\frac{r_{f,ij}(\omega)}{r_{f,ij}(\omega')} = \frac{a^2 - \left( \frac{a}{1-g_{f,ij}} \right) \lambda_j \tau_{ij} w_{i} z_{f,i} (\omega')^{\eta}}{a^2 - \left( \frac{a}{1-g_{f,ij}} \right) \lambda_j \tau_{ij} w_{i} z_{f,i} (\omega')^{\eta}} = a^2 - \left( \frac{a}{1-g_{f,ij}} \right) \lambda_j \tau_{ij} w_{i} z_{f,i} (\omega')^{\eta}.
\]

(F.16)

For two varieties \( \omega \) and \( \omega' \) with \( \omega < \omega' \), the sales ratio is decreasing in the firm’s scope:

\[
\frac{\partial}{\partial \delta_{f,ij}} \left( \frac{r_{f,ij}(\omega)}{r_{f,ij}(\omega')} \right) = \frac{2\eta \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta-1} \omega^{2\eta} - 1}{1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta}} \left( 1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta} \right) 2\eta \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta-1} \omega^2.
\]

\[
\left( 1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta} \right)^2,
\]

\[
\frac{\partial}{\partial \delta_{f,ij}} \left( \frac{r_{f,ij}(\omega)}{r_{f,ij}(\omega')} \right) = \frac{2\eta \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta-1} \omega^{2\eta} - 1}{1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta}} \left( 1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta} \right) 2\eta \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta-1} \omega^2.
\]

\[
\left( 1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta} \right)^2,
\]
\[
\frac{\partial}{\partial \delta_{f,ij}} \left( \frac{r_{f,ij}(\omega)}{\delta_{f,ij}(\omega)} \right) = \frac{2\eta \delta_{f,ij}^{2\eta - 1} \left( \omega^{2\eta} \left( 1 - \left( \frac{\omega'}{\delta_{f,ij}} \right)^{2\eta} \right) - \left( 1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta} \right) \right)}{\left( 1 - \left( \frac{\omega'}{\delta_{f,ij}} \right)^{2\eta} \right)^2}.
\]

\[
\frac{\partial}{\partial \delta_{f,ij}} \left( \frac{r_{f,ij}(\omega)}{\delta_{f,ij}(\omega)} \right) \leq 0 \iff \omega^{2\eta} \left( 1 - \left( \frac{\omega'}{\delta_{f,ij}} \right)^{2\eta} \right) - \left( 1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta} \right) \left( \omega' \right)^{2\eta} \leq 0
\]

\[
\iff \omega^{2\eta} \left( 1 - \left( \frac{\omega'}{\delta_{f,ij}} \right)^{2\eta} \right) \leq \left( 1 - \left( \frac{\omega}{\delta_{f,ij}} \right)^{2\eta} \right) \left( \omega' \right)^{2\eta}
\]

\[
\iff \omega^{2\eta} \leq \left( \frac{\omega'}{\delta_{f,ij}} \right)^{2\eta}
\]

\[
\iff \omega \leq \omega'.
\]

F.15 Comparative Statics

F.15.1 Price Elasticity with Respect to the Transportation Cost (PETC)

Taking the logarithm of equation (5.26) yields

\[
\ln \delta_{f,ij} = \frac{1}{\eta} \left( \ln (4b) - \ln (a\tau_{ij}w_{i}z_{f,i}) \right) + \frac{1}{\eta} \ln \left( 1 + \frac{1}{2\eta} \right) + \frac{1}{\eta} \ln (1 - g_{f,ij}) + \frac{1}{\eta} \ln (E_j - \ln \delta_j)
\]

and, by taking the total derivative while assuming a constant \( \delta_j \), the elasticity of the scope with respect to the transportation cost is given by

\[
\frac{d}{d \ln \tau_{ij}} \ln \delta_{f,ij} = \frac{\partial}{\partial \ln \tau_{ij}} \ln \delta_{f,ij} + \frac{\partial}{\partial \ln g_{f,ij}} \ln g_{f,ij} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}}.
\]

Taking the logarithm of equation (5.27) yields

\[
\ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j
\]

and, by taking the total derivative while assuming a constant \( \delta_j \), the elasticity of the market dominance with respect to the transportation cost is given by

\[
\frac{d}{d \ln \tau_{ij}} \ln g_{f,ij} = \frac{\partial}{\partial \ln \delta_{f,ij}} \ln \delta_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}}
\]
Taking the logarithm of equation (5.24) yields
\[ \ln p_{f,i,j}(\omega) = \ln (\tau_{ij} w_i z_{f,i}) - \ln (2 (1 - g_{f,i,j})) + \ln (\omega^n + \delta_{f,i,j}^n) \]
and, by taking the total derivative, the elasticity of the price with respect to the trans-
portation cost is given by
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} = \frac{\partial \ln p_{f,i,j}(\omega)}{\partial \ln \tau_{ij}} + \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} + \frac{\partial \ln p_{f,i,j}(\omega)}{\partial \ln g_{f,i,j}} \frac{d \ln g_{f,i,j}}{d \ln \tau_{ij}}.
\]
and
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} = 1 + \frac{g_{f,i,j}}{\omega^n + \delta_{f,i,j}^n} \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} + \frac{1}{1 - g_{f,i,j}} \frac{d \ln g_{f,i,j}}{d \ln \tau_{ij}}.
\]
By using equation (F.18), the last equation can be reduced to
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} = 1 + \left( \frac{\eta \delta_{f,i,j}^n}{\omega^n + \delta_{f,i,j}^n} + \frac{g_{f,i,j}}{1 - g_{f,i,j}} \right) \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} \tag{F.19}
\]
and equation (F.17) can also be reduced to
\[
\frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,i,j}}{\eta (1 - g_{f,i,j})} \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}},
\]
\[
\frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} \left( 1 + \frac{g_{f,i,j}}{\eta (1 - g_{f,i,j})} \right) = \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} \left( \frac{\eta (1 - g_{f,i,j}) + g_{f,i,j}}{\eta (1 - g_{f,i,j})} \right) = -\frac{1}{\eta}
\]
and
\[
\frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} = -\frac{1}{\eta} \frac{\eta (1 - g_{f,i,j})}{\eta (1 - g_{f,i,j}) + g_{f,i,j}} = -\frac{1}{\eta} \frac{1 - g_{f,i,j}}{\eta (1 - g_{f,i,j}) + g_{f,i,j}}. \tag{F.20}
\]
Finally, plugging equation (F.20) into equation (F.19) provides the price elasticity with
respect to the transportation cost:
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} = 1 + \left( \eta \delta_{f,i,j}^n (1 - g_{f,i,j}) + g_{f,i,j} \left( \omega^n + \delta_{f,i,j}^n \right) \right) \left( -\frac{1 - g_{f,i,j}}{\eta (1 - g_{f,i,j}) + g_{f,i,j}} \right);
\]
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} = 1 - \frac{\eta \delta_{f,i,j}^n (1 - g_{f,i,j}) + g_{f,i,j} \left( \omega^n + \delta_{f,i,j}^n \right)}{\left( \omega^n + \delta_{f,i,j}^n \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)};
\]
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} = 1 - \frac{\eta \delta_{f,i,j}^n - \eta g_{f,i,j} \delta_{f,i,j}^n g_{f,i,j} + g_{f,i,j} \omega^n + g_{f,i,j} \delta_{f,i,j}^n}{\left( \omega^n + \delta_{f,i,j}^n \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)};\]
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} = \frac{\left( \omega^n + \delta_{f,i,j}^n \right) \left( \eta - \eta g_{f,i,j} + g_{f,i,j} \right) - \eta \delta_{f,i,j}^n + \eta g_{f,i,j} \omega^n - g_{f,i,j} \delta_{f,i,j}^n}{\left( \omega^n + \delta_{f,i,j}^n \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)};
\]
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} = \frac{\eta \omega^n + \eta \delta_{f,i,j}^n - \eta g_{f,i,j} \omega^n - \eta g_{f,i,j} \delta_{f,i,j}^n + g_{f,i,j} \omega^n + g_{f,i,j} \delta_{f,i,j}^n}{\left( \omega^n + \delta_{f,i,j}^n \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)}.
\]
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\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{\eta \omega^n - \eta g_{f,ij} \omega^n}{(\omega^n + \delta^n_{f,ij}) (\eta - \eta g_{f,ij} + g_{f,ij})},
\]

and

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{\eta \omega^n (1 - g_{f,ij})}{(\omega^n + \delta^n_{f,ij}) (\eta (1 - g_{f,ij}) + g_{f,ij})}.
\]  

(F.21)

F.15.2 Markup Elasticity with Respect to the Transportation Cost (METC)

Taking the logarithm of equation (5.26) yields

\[
\ln \delta_{f,ij} = \frac{1}{\eta} (\ln (4b) - \ln (a \tau_{ij} w_i z_{f,i})) + \frac{1}{\eta} \ln \left(1 + \frac{1}{2 \eta}\right) + \frac{1}{\eta} \ln (1 - g_{f,ij}) + \frac{1}{\eta} (\ln E_j - \ln \delta_j)
\]

and, by taking the total derivative while assuming a constant \(\delta_j\), the elasticity of the scope with respect to the transportation cost is given by

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln \delta_{f,ij}}{\partial \ln \tau_{ij}} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}}.
\]  

(F.22)

Taking the logarithm of equation (5.27) yields

\[
\ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j
\]

and, by taking the total derivative while assuming a constant \(\delta_j\), the elasticity of the market dominance with respect to the transportation cost is given by

\[
\frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}}.
\]  

(F.23)

Taking the logarithm of equation (5.25) yields

\[
\ln \mu_{f,ij}(\omega) = \ln (1) - \ln (2 (1 - g_{f,ij})) + \ln \left(1 + \frac{\delta^n_{f,ij}}{\omega^n}\right)
\]

and, by taking the total derivative, the elasticity of the markup with respect to the transportation cost is given by

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln \tau_{ij}} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}}
\]

and

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = \left(1 + \frac{\delta^n_{f,ij}}{\omega^n}\right)^{-1} \eta \frac{\delta^n_{f,ij}}{\omega^n} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}}.
\]  

(F.24)
Plugging equation (F.23) into equation (F.22) yields

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,ij}}{\eta(1 - g_{f,ij})} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} \left(1 + \frac{g_{f,ij}}{\eta(1 - g_{f,ij})}\right) = \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} \left(\frac{\eta(1 - g_{f,ij}) + g_{f,ij}}{\eta(1 - g_{f,ij})}\right) = -\frac{1}{\eta}
\]

and

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta} \frac{\eta(1 - g_{f,ij})}{\eta(1 - g_{f,ij}) + g_{f,ij}} = -\frac{1 - g_{f,ij}}{\eta(1 - g_{f,ij}) + g_{f,ij}}.
\]

Plugging equation (F.23) and equation (F.25) into equation (F.24) provides the elasticity of the markup with respect to the transportation cost

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = \left(1 + \frac{\delta_{f,ij}}{\omega}\right)^{-1} \frac{\eta \delta_{f,ij}}{\omega} \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}},
\]

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = \left(\frac{\omega + \delta_{f,ij}}{\omega(1 - g_{f,ij}) + g_{f,ij}} + \frac{g_{f,ij}}{1 - g_{f,ij}}\right) \left(\frac{-1 - g_{f,ij}}{\eta(1 - g_{f,ij}) + g_{f,ij}}\right),
\]

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = \left(\frac{\eta \delta_{f,ij} (1 - g_{f,ij}) + g_{f,ij} (\omega + \delta_{f,ij})}{(\omega + \delta_{f,ij})(1 - g_{f,ij})}\right) \left(\frac{-1 - g_{f,ij}}{\eta(1 - g_{f,ij}) + g_{f,ij}}\right),
\]

and

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = -\frac{\eta \delta_{f,ij} (1 - g_{f,ij}) + g_{f,ij} (\omega + \delta_{f,ij})}{(\omega + \delta_{f,ij})(\eta(1 - g_{f,ij}) + g_{f,ij})}.
\]

F.15.3 Properties of the PETC

The price elasticity with respect to the transportation cost is less than one:

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} < 1 \Leftrightarrow \eta \omega (1 - g_{f,ij}) \frac{1 - g_{f,ij}}{(\omega + \delta_{f,ij})(\eta(1 - g_{f,ij}) + g_{f,ij})} < 1
\]

\[
\Leftrightarrow \eta \omega (1 - g_{f,ij}) < (\omega + \delta_{f,ij})(\eta(1 - g_{f,ij}) + g_{f,ij})
\]

\[
\Leftrightarrow \eta \omega - \eta g_{f,ij} \omega < \omega + \delta_{f,ij}(\eta(1 - g_{f,ij}) + g_{f,ij})
\]

\[
\Leftrightarrow \eta \omega - \eta g_{f,ij} \omega + \eta(1 - g_{f,ij}) < \omega + \delta_{f,ij}(\eta(1 - g_{f,ij}) + g_{f,ij})
\]

\[
0 < \eta(1 - g_{f,ij}) + \delta_{f,ij}(1 - g_{f,ij}) + g_{f,ij} \delta_{f,ij} = g_{f,ij}\omega + \eta \delta_{f,ij}(1 - g_{f,ij}) + g_{f,ij} \delta_{f,ij}.
\]

The transportation cost elasticity of the price is decreasing with the firm’s market dominance:

\[
\partial \left(\frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}}\right) = -\eta \omega (\omega + \delta_{f,ij})(\eta(1 - g_{f,ij}) + g_{f,ij}) - \eta \omega (1 - g_{f,ij})(\omega + \delta_{f,ij})(-\eta + 1)
\]

\[
\left((\omega + \delta_{f,ij})(\eta(1 - g_{f,ij}) + g_{f,ij})\right)^2,
\]
\[
\frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = -\eta \omega^n \left( \omega^n + \delta_{f,ij}^n \right) \left[ \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \left( 1 - \eta \right) \right],
\]

\[
\frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = -\eta \omega^n \left( \omega^n + \delta_{f,ij}^n \right) \left[ \eta - \eta g_{f,ij} + g_{f,ij} + 1 - \eta - g_{f,ij} + \eta g_{f,ij} \right]
\]

and

\[
\frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = -\eta \omega^n \left( \omega^n + \delta_{f,ij}^n \right) \frac{\left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)}{\left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)^2} < 0.
\]

The transportation cost elasticity of the price is decreasing with the firm’s export scope:

\[
\frac{\partial}{\partial f_{ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = -\eta \omega^n \left( 1 - g_{f,ij} \right) \eta \delta_{f,ij}^{n-1} \left( \eta \left( 1 - f_{ij} \right) + f_{ij} \right)
\]

and

\[
\frac{\partial}{\partial f_{ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = -\eta^2 \omega^n \delta_{f,ij} \left( 1 - g_{f,ij} \right) \left( \eta \left( 1 - f_{ij} \right) + f_{ij} \right) \frac{\left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)}{\left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)^2} < 0.
\]

The transportation cost elasticity of the price is increasing in each firm’s product portfolio:

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = \eta^2 \omega^{n-1} \left( 1 - g_{f,ij} \right) \left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - f_{ij} \right) + f_{ij} \right)
\]

\[
- \eta \omega^n \left( 1 - g_{f,ij} \right) \eta \omega^{n-1} \left( \eta \left( 1 - f_{ij} \right) + f_{ij} \right) \frac{\left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)}{\left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)^2},
\]

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = \left( 1 - g_{f,ij} \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right) \left[ \eta^2 \omega^{n-1} \left( \omega^n + \delta_{f,ij}^n \right) - \eta^2 \omega^{2n-1} \right]
\]

and

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = \eta^2 \omega^{n-1} \left( 1 - g_{f,ij} \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right) \left[ \omega^n + \delta_{f,ij}^n - \omega^n \right]
\]

and

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = \eta^2 \omega^{n-1} \delta_{f,ij} \left( 1 - g_{f,ij} \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right) \frac{\left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)}{\left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)^2} > 0.
\]

**F.15.4 Properties of the METC**

The absolute value of the elasticity of the markup with respect to the transportation cost is increasing in the firm’s market dominance:

\[
\frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln m_{f,ij}(\omega)}{d \ln \tau_{ij}} \right) = \frac{\left( -\eta \delta_{f,ij}^n + \omega^n + \delta_{f,ij}^n \right) \left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)}{\left( \omega^n + \delta_{f,ij}^n \right) \left( \eta \left( 1 - g_{f,ij} \right) + g_{f,ij} \right)^2}.
\]
\[
\frac{\partial \left( \frac{d \ln \mu_{f,i,j}(\omega)}{d \ln \tau_{f,i,j}} \right)}{\partial g_{f,i,j}} = \frac{\left( \omega^n + \delta^n_{f,i,j} \right) \left( (-\eta^n \delta^n_{f,i,j} + \omega^n + \delta^n_{f,i,j}) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right) \right)}{\left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)^2} - \frac{\left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta \delta^n_{f,i,j} \left( 1 - g_{f,i,j} \right) + g_{f,i,j} \right)}{\left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)^2} \left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)^2 \left( \eta \delta^n_{f,i,j} \left( 1 - g_{f,i,j} \right) + g_{f,i,j} \right) \omega^n \left( 1 - g_{f,i,j} \right) + g_{f,i,j} \omega^n + g_{f,i,j} \omega^n \right) \\
+ \frac{\left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta \delta^n_{f,i,j} - \eta g_{f,i,j} \delta^n_{f,i,j} + g_{f,i,j} \delta^n_{f,i,j} - \eta g_{f,i,j} \delta^n_{f,i,j} - \eta g_{f,i,j} \delta^n_{f,i,j} - g_{f,i,j} \omega^n - g_{f,i,j} \delta^n_{f,i,j} \right)}{\left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)^2} + \frac{\left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta g_{f,i,j} - \eta^n g_{f,i,j} \delta^n_{f,i,j} + g_{f,i,j} g_{f,i,j} \omega^n + g_{f,i,j} \delta^n_{f,i,j} \right)}{\left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)^2} \left( \eta \delta^n_{f,i,j} \left( 1 - g_{f,i,j} \right) + g_{f,i,j} \right) \omega^n \left( 1 - g_{f,i,j} \right) + g_{f,i,j} \omega^n + g_{f,i,j} \omega^n \right) \\
\text{and} \quad \frac{\partial \left( \frac{d \ln \mu_{f,i,j}(\omega)}{d \ln \tau_{f,i,j}} \right)}{\partial g_{f,i,j}} = \frac{\eta \omega^n \left( \omega^n + \delta^n_{f,i,j} \right)}{\left( \omega^n + \delta^n_{f,i,j} \right) \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right)^2} > 0.
\]

The absolute value of the elasticity of the markup with respect to the transportation cost is increasing in the firm’s export scope:
and
\[
\frac{\partial \left( \frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} \right)}{\partial \delta f_{i,j}} = \frac{(\eta (1 - g_{f,i,j}) + g_{f,i,j}) (1 - g_{f,i,j}) \eta^2 \delta_{f,i,j}^{-1} \omega^n}{(\omega^n + \delta_{f,i,j}^n) (\eta (1 - g_{f,i,j}) + g_{f,i,j})^2} > 0.
\]

The absolute value of the elasticity of the markup with respect to the transportation cost is decreasing in each firm’s product portfolio:
\[
\frac{\partial \left( \frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} \right)}{\partial \omega} = \frac{g_{f,i,j} \eta \omega^{n-1} (\omega^n + \delta_{f,i,j}^n) (\eta (1 - g_{f,i,j}) + g_{f,i,j})}{(\omega^n + \delta_{f,i,j}^n) (\eta (1 - g_{f,i,j}) + g_{f,i,j})^2} - \frac{\left( \eta \delta_{f,i,j}^n (1 - g_{f,i,j}) + g_{f,i,j} (\omega^n + \delta_{f,i,j}^n) \right) (\eta (1 - g_{f,i,j}) + g_{f,i,j}) \eta \omega^{n-1}}{(\omega^n + \delta_{f,i,j}^n) (\eta (1 - g_{f,i,j}) + g_{f,i,j})^2}.
\]

\[
\frac{\partial \left( \frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} \right)}{\partial \omega} = \frac{(\eta (1 - g_{f,i,j}) + g_{f,i,j}) (\eta \delta_{f,i,j}^n \omega^{n-1} (1 - g_{f,i,j}) + g_{f,i,j} \eta \omega^{n-1} (\omega^n + \delta_{f,i,j}^n))}{(\omega^n + \delta_{f,i,j}^n) (\eta (1 - g_{f,i,j}) + g_{f,i,j})^2}.
\]

and
\[
\frac{\partial \left( \frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} \right)}{\partial \omega} = -\frac{(\eta (1 - g_{f,i,j}) + g_{f,i,j}) \eta^2 \delta_{f,i,j}^n \omega^{n-1} (1 - g_{f,i,j})}{(\omega^n + \delta_{f,i,j}^n) (\eta (1 - g_{f,i,j}) + g_{f,i,j})^2} < 0.
\]

\[\textbf{F.15. Comparative Statics} \]

**F.15.5 Price Elasticity with Respect to the Per Capita Income (PEPCI)**

Taking the logarithm of equation \((5.24)\) yields
\[
\ln p_{f,i,j}(\omega) = \ln (\tau_{ij} w_i z_{f,i}) - \ln (2 [1 - g_{f,i,j}]) + \ln (\omega^n + \delta_{f,i,j}^n)
\]
and, by taking the total derivative, the elasticity of the price with respect to the per capita income is given by
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln E_j} = \frac{\partial \ln p_{f,i,j}(\omega)}{\partial \ln E_j} + \frac{\partial \ln p_{f,i,j}(\omega)}{\partial \ln \delta_{f,i,j}} \frac{d \ln \delta_{f,i,j}}{d \ln E_j} + \frac{\partial \ln p_{f,i,j}(\omega)}{\partial \ln g_{f,i,j}} \frac{d \ln g_{f,i,j}}{d \ln E_j}.
\]

And
\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,i,j}^n \ln \delta_{f,i,j}}{\omega^n + \delta_{f,i,j}^n} \frac{d \ln \delta_{f,i,j}}{d \ln E_j} + \frac{g_{f,i,j}}{1 - g_{f,i,j}} \frac{d \ln g_{f,i,j}}{d \ln E_j}.
\]

Taking the logarithm of equation \((5.26)\) yields
\[
\ln \delta_{f,i,j} = \frac{1}{\eta} (\ln (4b) - \ln (a \tau_{ij} w_i z_{f,i})) + \frac{1}{\eta} \ln \left( 1 + \frac{1}{2\eta} \right) + \frac{1}{\eta} \ln (1 - g_{f,i,j}) + \frac{1}{\eta} (\ln E_j - \ln \delta_j).
\]
and, by taking the total derivative while assuming a constant \( \delta_j \), the elasticity of the scope with respect to the per capita income is given by

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{\partial \ln \delta_{f,ij}}{\partial \ln E_j} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \frac{d \ln g_{f,ij}}{d \ln E_j}.
\] (F.28)

Taking the logarithm of equation (5.27) yields

\[
\ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j
\]

and, by taking the total derivative while assuming a constant \( \delta_j \), the elasticity of the market dominance with respect to the per capita income is given by

\[
\frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{\partial \ln g_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{d \ln \delta_{f,ij}}{d \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j} = 1 - \frac{g_{f,ij}}{\eta (1 - g_{f,ij}) + g_{f,ij}}.
\] (F.29)

By using equation (F.29), equation (F.28) can be written as

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij}) + g_{f,ij}}.
\] (F.30)

Finally, by plugging equation (F.29) and equation (F.30) into equation (F.27), the price elasticity with respect to the per capita income derives as

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij}^\eta}{\omega^n + \delta_{f,ij}^\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j},
\]

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \left( \frac{\eta \delta_{f,ij}^\eta (1 - g_{f,ij}) + g_{f,ij} (\omega^n + \delta_{f,ij}^\eta)}{\omega^n + \delta_{f,ij}^\eta (1 - g_{f,ij})} \right) \frac{d \ln \delta_{f,ij}}{d \ln E_j},
\]

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \left( \frac{\eta \delta_{f,ij}^\eta (1 - g_{f,ij}) + g_{f,ij} (\omega^n + \delta_{f,ij}^\eta)}{\omega^n + \delta_{f,ij}^\eta (1 - g_{f,ij})} \right) \left( \frac{1}{\eta (1 - g_{f,ij}) + g_{f,ij}} \right)
\]

and

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij}^\eta (1 - g_{f,ij}) + g_{f,ij} (\omega^n + \delta_{f,ij}^\eta)}{\omega^n + \delta_{f,ij}^\eta (\eta (1 - g_{f,ij}) + g_{f,ij})}.
\] (F.31)
F.15.6 Markup Elasticity with Respect to the Per Capita Income (MEPCI)

Taking the logarithm of equation (5.26) yields
\[ \ln \delta_{f,ij} = \frac{1}{\eta} \left( \ln (4b) - \ln (a\tau_{ij}w_{iz}f_{ji}) \right) + \frac{1}{\eta} \ln \left( 1 + \frac{1}{2\eta} \right) + \frac{1}{\eta} \ln (1 - g_{f,ij}) + \frac{1}{\eta} \ln (E_j - \ln \delta_j) \]

and, by taking the total derivative while assuming a constant \( \delta_j \), the elasticity of the scope with respect to the per capita income is given by
\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{\partial \ln \delta_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j} - \frac{1}{\eta} \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j}.
\]  
(F.32)

Taking the logarithm of equation (5.27) yields
\[ \ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j \]

and, by taking the total derivative while assuming a constant \( \delta_j \), the elasticity of the market dominance with respect to the per capita income is given by
\[
\frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\partial \ln g_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j}.
\]  
(F.33)

Taking the logarithm of equation (5.25) yields
\[ \ln \mu_{f,ij}(\omega) = \ln (1) - \ln (2 [1 - g_{f,ij}]) + \ln \left( 1 + \frac{\delta_{f,ij}^\eta}{\omega^\eta} \right) \]

and, by taking the total derivative, the elasticity of the markup with respect to the per capita income is given by
\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln E_j} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j}.
\]

and
\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \left( 1 + \frac{\delta_{f,ij}^\eta}{\omega^\eta} \right)^{-1} \frac{\delta_{f,ij}^\eta}{\omega^\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j}.
\]  
(F.34)

Plugging equation (F.33) into equation (F.32) yields
\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \frac{d \ln \delta_{f,ij}}{d \ln E_j},
\]
\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} \left( 1 + \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \right) = \frac{1}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j} \left( \frac{\eta (1 - g_{f,ij}) + g_{f,ij}}{\eta (1 - g_{f,ij})} \right) = \frac{1}{\eta}. \]
and

\[ \frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{1}{\eta} \frac{\eta (1 - g_{f,ij})}{\eta (1 - g_{f,ij}) + g_{f,ij}} = \frac{1 - g_{f,ij}}{\eta (1 - g_{f,ij}) + g_{f,ij}}, \]  

(F.35)

Plugging equation (F.33) and equation (F.35) into equation (F.34) provides the elasticity of the markup with respect to the per capita income

\[ \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \left( 1 + \frac{\delta_{f,ij}^\eta}{\omega^\eta} \right)^{-1} \frac{\eta \delta_{f,ij}^\eta}{\omega^\eta} + \frac{g_{f,ij}}{1 - g_{f,ij}} \]  

\[ \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \left( \frac{\omega^\eta}{\omega^\eta + \delta_{f,ij}^\eta} \right) \eta \delta_{f,ij}^\eta + \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{1 - g_{f,ij}}{\eta (1 - g_{f,ij}) + g_{f,ij}}, \]

\[ \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \left( \frac{\eta \delta_{f,ij}^\eta (1 - g_{f,ij}) + g_{f,ij} (\omega^\eta + \delta_{f,ij}^\eta)}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij})} \right) \frac{1 - g_{f,ij}}{\eta (1 - g_{f,ij}) + g_{f,ij}}, \]

and

\[ \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij}^\eta (1 - g_{f,ij}) + g_{f,ij} (\omega^\eta + \delta_{f,ij}^\eta)}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij})} \frac{1 - g_{f,ij}}{\eta (1 - g_{f,ij}) + g_{f,ij}}. \]  

(F.36)

F.15.7 Properties of the PEPCI

The income elasticity of the price is increasing in the firm’s market dominance:

\[ \frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{-\eta \delta_{f,ij}^\eta + \omega^\eta + \delta_{f,ij}^\eta}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij}) + g_{f,ij}} \]  

\[ \frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{-\eta \delta_{f,ij}^\eta + \omega^\eta + \delta_{f,ij}^\eta}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij}) + g_{f,ij}} \frac{-\eta + 1}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij}) + g_{f,ij}} \]  

\[ \frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{-\eta \delta_{f,ij}^\eta + \omega^\eta + \delta_{f,ij}^\eta}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij}) + g_{f,ij}} \frac{-\eta + 1}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij}) + g_{f,ij}} \]  

\[ \frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{-\eta \delta_{f,ij}^\eta + \omega^\eta + \delta_{f,ij}^\eta}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij}) + g_{f,ij}} \frac{-\eta + 1}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij}) + g_{f,ij}} \]  

\[ -\eta \delta_{f,ij}^\eta + \omega^\eta + \delta_{f,ij}^\eta - \eta g_{f,ij} \delta_{f,ij}^\eta + \omega^\eta - \eta g_{f,ij} \delta_{f,ij}^\eta + \omega^\eta + \eta \delta_{f,ij}^\eta - \eta g_{f,ij} \delta_{f,ij}^\eta + g_{f,ij} \delta_{f,ij}^\eta \]  

\[ = \frac{-\eta \delta_{f,ij}^\eta + \omega^\eta + \delta_{f,ij}^\eta}{(\omega^\eta + \delta_{f,ij}^\eta) (1 - g_{f,ij}) + g_{f,ij}} \times \]
\[
\eta \delta_{f,i}^j - \eta g_{f,i,j} \delta_{f,i}^j + g_{f,i,j} \omega^\eta + g_{f,i,j} \delta_{f,i}^j - \eta^2 \delta_{f,i}^j + \eta^2 g_{f,i,j} \delta_{f,i}^j - \eta g_{f,i,j} \omega^\eta + \eta g_{f,i,j} \delta_{f,i}^j \left( (\omega^\eta + \delta_{f,i}^j) (\eta (1 - g_{f,i,j}) + g_{f,i,j}) \right)^2
\]

and
\[
\frac{\partial}{\partial g_{f,i,j}} \left( \frac{d \ln P_{f,i,j}(\omega)}{d \ln E_j} \right) = \frac{\left( \omega^\eta + \delta_{f,i}^j \right) \eta \omega^\eta}{\left( (\omega^\eta + \delta_{f,i}^j) (\eta (1 - g_{f,i,j}) + g_{f,i,j}) \right)^2} > 0.
\]

The income elasticity of the price is increasing in the firm’s export scope:
\[
\frac{\partial}{\partial \delta_{f,i}^j} \left( \frac{d \ln P_{f,i,j}(\omega)}{d \ln E_j} \right) = \eta \delta_{f,i}^j \left( \eta (1 - g_{f,i,j}) + g_{f,i,j} \right) \frac{\eta \omega^\eta + \eta \delta_{f,i}^j - \eta g_{f,i,j} \omega^\eta - \eta g_{f,i,j} \delta_{f,i}^j + g_{f,i,j} \omega^\eta + g_{f,i,j} \delta_{f,i}^j \left( (\omega^\eta + \delta_{f,i}^j) (\eta (1 - g_{f,i,j}) + g_{f,i,j}) \right)^2}{\left( (\omega^\eta + \delta_{f,i}^j) (\eta (1 - g_{f,i,j}) + g_{f,i,j}) \right)^2}
\]

and
\[
\frac{\partial}{\partial \delta_{f,i}^j} \left( \frac{d \ln P_{f,i,j}(\omega)}{d \ln E_j} \right) = \frac{\eta \omega^\eta - \eta g_{f,i,j} \omega^\eta}{\left( (\omega^\eta + \delta_{f,i}^j) (\eta (1 - g_{f,i,j}) + g_{f,i,j}) \right)^2} > 0.
\]

The income elasticity of the price is decreasing in each firm’s product portfolio:
\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln P_{f,i,j}(\omega)}{d \ln E_j} \right) = \eta g_{f,i,j} \omega^\eta - \eta g_{f,i,j} \omega^\eta - \eta g_{f,i,j} \delta_{f,i}^j \left( (\omega^\eta + \delta_{f,i}^j) (\eta (1 - g_{f,i,j}) + g_{f,i,j}) \right)^2
\]

and
\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln P_{f,i,j}(\omega)}{d \ln E_j} \right) = \frac{\eta \omega^\eta - \eta g_{f,i,j} \omega^\eta}{\left( (\omega^\eta + \delta_{f,i}^j) (\eta (1 - g_{f,i,j}) + g_{f,i,j}) \right)^2} > 0.
\]
\[-\eta \omega^{-1} (\eta (1 - g_{f,ij}) + g_{f,ij}) \eta \delta_{f,ij}^\gamma - \eta g_{f,ij} \delta_{f,ij}^\gamma + g_{f,ij} \omega^n + g_{f,ij} \delta_{f,ij}^\gamma \left((\omega^n + \delta_{f,ij}^\gamma) (\eta (1 - g_{f,ij}) + g_{f,ij})\right)^2,\]

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = \eta \omega^{-1} (\eta (1 - g_{f,ij}) + g_{f,ij}) \left(\frac{\eta g_{f,ij} \delta_{f,ij}^\gamma - \eta \delta_{f,ij}^\gamma}{(\omega^n + \delta_{f,ij}^\gamma) (\eta (1 - g_{f,ij}) + g_{f,ij})^2}\right),
\]

and

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = -\eta \omega^{-1} (\eta (1 - g_{f,ij}) + g_{f,ij}) \left(\frac{\eta \delta_{f,ij}^\gamma (1 - g_{f,ij})}{(\omega^n + \delta_{f,ij}^\gamma) (\eta (1 - g_{f,ij}) + g_{f,ij})^2}\right) < 0.
\]

**F.15.8 Properties of the MEPCI**

Since the markup elasticity MEPCI equals the price elasticity PEPCI, i.e.

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j},
\]

their properties are identical:

\[
\frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) > 0,
\]

\[
\frac{\partial}{\partial \delta_{f,ij}} \left( \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{\partial}{\partial \delta_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) > 0
\]

and

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) < 0.
\]

**F.15.9 Scope Elasticity with Respect to the Transportation Cost (SETC’)**

Taking the logarithm of equation (5.26) yields

\[
\ln \delta_{f,ij} = \frac{1}{\eta} (\ln (4b) - \ln (a \tau_{ij} w_i z_{f,ij})) + \frac{1}{\eta} \ln \left(1 + \frac{1}{2 \eta}\right) + \frac{1}{\eta} \ln (1 - g_{f,ij}) + \frac{1}{\eta} (\ln E_j - \ln \delta_j)
\]

and, by taking the total derivative, the elasticity of the scope with respect to the transportation cost is given by

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln \delta_{f,ij}}{\partial \ln \tau_{ij}} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln \tau_{ij}}
\]

and

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln \tau_{ij}}.
\]
Taking the logarithm of equation (5.27) yields
\[ \ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j \]

and, by taking the total derivative, the elasticity of the market dominance with respect to the transportation cost is given by
\[
\frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln \tau_{ij}}
\]

and
\[
\frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} - \frac{d \ln \delta_j}{d \ln \tau_{ij}}. \tag{F.38}
\]

Taking the logarithm of the general relation
\[ \delta_j = \sum_i \sum_f \delta_{f,ij} \]

yields
\[ \ln \delta_j = \ln \left( \sum_i \sum_f \delta_{f,ij} \right) \]

and, by taking the total derivative, the elasticity of the overall number of varieties available in country \( j \) with respect to the transportation cost is given by
\[
\frac{d \ln \delta_j}{d \ln \tau_{ij}} = \frac{\partial \ln \delta_j}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{\delta_{f,ij}}{\delta_j} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = g_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}}. \tag{F.39}
\]

Plugging equation (F.38) and equation (F.39) into equation (F.37) provides the elasticity of the scope with respect to the transportation cost
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \left( \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} - \frac{d \ln \delta_j}{d \ln \tau_{ij}} \right) - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln \tau_{ij}},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \left( \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} - g_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} \right) - \frac{1}{\eta} \frac{g_{f,ij}}{d \ln \tau_{ij}},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} - \frac{g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta} - 2 \frac{g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} \left( 1 + 2 \frac{g_{f,ij}}{\eta} \right) = -\frac{1}{\eta},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} \left( \frac{\eta + 2 g_{f,ij}}{\eta} \right) = -\frac{1}{\eta}.
and
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta + 2g_{f,ij}}.
\] (F.40)

**F.15.10 Properties of the SETC′**

The absolute value of the elasticity of the scope with respect to the transportation cost is decreasing in the firm’s market dominance:

\[
\frac{\partial}{\partial g_{f,ij}} \left( \left| \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} \right| \right) = -\frac{2}{(\eta + 2g_{f,ij})^2} = -\frac{2}{(\eta + 2g_{f,ij})^2} < 0.
\]

**F.15.11 Scope Elasticity with Respect to the Per Capita Income (SEPCI′)**

Taking the logarithm of equation (5.26) yields

\[
\ln \delta_{f,ij} = \frac{1}{\eta} \left( \ln (4b) - \ln (a\tau_{ij}w_{i}z_{f,i}) \right) + \frac{1}{\eta} \ln \left( 1 + \frac{1}{2\eta} \right) + \frac{1}{\eta} \ln (1 - g_{f,ij}) + \frac{1}{\eta} (\ln E_j - \ln \delta_j)
\]

and, by taking the total derivative, the elasticity of the scope with respect to the per capita income is given by

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{\partial \ln \delta_{f,ij}}{\partial \ln E_j} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln E_j}
\]

and

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \frac{d \ln g_{f,ij}}{d \ln E_j} - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln E_j}.
\] (F.41)

Taking the logarithm of equation (5.27) yields

\[
\ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j
\]

and, by taking the total derivative, the elasticity of the market dominance with respect to the per capita income is given by

\[
\frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln E_j}
\]

and

\[
\frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{d \ln \delta_{f,ij}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j}.
\] (F.42)

Taking the logarithm of the general relation

\[
\delta_j = \sum_i \sum_f \delta_{f,ij}
\]
yields
\[
\ln \delta_j = \ln \left( \sum_i \sum_f \delta_{f,ij} \right)
\]
and, by taking the total derivative, the elasticity of the overall number of varieties available in country \( j \) with respect to the per capita income is given by
\[
\frac{d \ln \delta_j}{d \ln E_j} = \frac{\partial \ln \delta_j}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} = g_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln E_j}. 
\] (F.43)

Plugging equation (F.42) and equation (F.43) into equation (F.41) provides the elasticity of the scope with respect to the per capita income
\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{1}{\eta} - \frac{2 g_{f,ij}}{\eta} \left( \frac{d \ln \delta_{f,ij}}{d \ln E_j} \right)^2 - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln E_j},
\]
and
\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{1}{\eta} + 2 g_{f,ij}.
\] (F.44)

**F.15.12 Properties of the SEPCI’**

The elasticity of the scope with respect to the per capita income is decreasing in the firm’s market dominance:
\[
\frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln \delta_{f,ij}}{d \ln E_j} \right) = \frac{-2}{(\eta + 2 g_{f,ij})^2} < 0.
\]

**F.15.13 Price Elasticity with Respect to the Transportation Cost (PETC’)**

Taking the logarithm of equation (5.24) yields
\[
\ln p_{f,ij}(\omega) = \ln (\tau_{ij} w_i z_{f,i}) - \ln (2 [1 - g_{f,ij}]) + \ln (\omega^{\eta} + \delta_{f,ij}^{\eta})
\]
and, by taking the total derivative, the elasticity of the price with respect to the transportation cost is given by
\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{\partial \ln p_{f,ij}(\omega)}{\partial \ln \tau_{ij}} + \frac{\partial \ln p_{f,ij}(\omega)}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln p_{f,ij}(\omega)}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln p_{f,ij}(\omega)}{\partial \ln \delta_f} \frac{d \ln \delta_f}{d \ln \tau_{ij}}.
\]

and
\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} = 1 + \frac{\eta \delta_{f,ij}^\eta}{\omega^{\eta} + \delta_{f,ij}^\eta} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}}.
\] (F.45)

Taking the logarithm of equation (5.27) yields
\[
\ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j
\]

and, by taking the total derivative, the elasticity of the market dominance with respect to the transportation cost is given by
\[
\frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_f} \frac{d \ln \delta_f}{d \ln \tau_{ij}}
\]

and
\[
\frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} - \frac{d \ln \delta_j}{d \ln \tau_{ij}}.
\] (F.46)

Taking the logarithm of the general relation
\[
\delta_j = \sum_i \sum_f \delta_{f,ij}
\]

yields
\[
\ln \delta_j = \ln \left( \sum_i \sum_f \delta_{f,ij} \right)
\]

and, by taking the total derivative, the elasticity of the overall number of varieties available in country j with respect to the transportation cost is given by
\[
\frac{d \ln \delta_j}{d \ln \tau_{ij}} = \frac{\partial \ln \delta_j}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{\delta_{f,ij}}{\delta_j} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{g_{f,ij}}{d \ln \tau_{ij}}.
\] (F.47)

Taking the logarithm of equation (5.26) yields
\[
\ln \delta_{f,ij} = \frac{1}{\eta} \left( \ln (4b) - \ln (a \tau_{ij} w_{i,z_{f,i}}) \right) + \frac{1}{\eta} \ln \left( 1 + \frac{1}{2\eta} \right) + \frac{1}{\eta} \ln (1 - g_{f,ij}) + \frac{1}{\eta} \ln (E_j - \ln \delta_j)
\]

and, by taking the total derivative, the elasticity of the scope with respect to the transportation cost is given by
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln \delta_{f,ij}}{\partial \ln \tau_{ij}} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln \tau_{ij}}
\]
Plugging equation (F.46) and equation (F.47) into equation (F.45) yields

\[
\frac{\text{d} \ln p_{f,ij}(\omega)}{\text{d} \ln \tau_{ij}} = 1 + \frac{\eta \delta_{f,ij}^\eta}{\omega^n + \delta_{f,ij}^\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} + \frac{g_{f,ij}}{1 - g_{f,ij}} \left( \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} - \frac{\text{d} \ln \delta_j}{\eta \text{d} \ln \tau_{ij}} \right),
\]

and

\[
\frac{\text{d} \ln p_{f,ij}(\omega)}{\text{d} \ln \tau_{ij}} = 1 + \frac{\eta \delta_{f,ij}^\eta}{\omega^n + \delta_{f,ij}^\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} + \frac{g_{f,ij}}{1 - g_{f,ij}} \left( \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} - \frac{\text{d} \ln \delta_j}{\eta \text{d} \ln \tau_{ij}} \right),
\]

Plugging equation (F.46) and equation (F.47) into equation (F.49) yields

\[
\frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \left( \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} - \frac{\text{d} \ln \delta_j}{\eta \text{d} \ln \tau_{ij}} \right) - \frac{1}{\eta} \frac{\text{d} \ln \delta_j}{\text{d} \ln \tau_{ij}},
\]

\[
\frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} = -\frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \left( 1 - g_{f,ij} \right) \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} - \frac{1}{\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}},
\]

\[
\frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} = -\frac{1}{\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} - \frac{g_{f,ij}}{\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} - \frac{g_{f,ij}}{\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}},
\]

and

\[
\frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} = -\frac{1}{\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} - \frac{g_{f,ij}}{\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}} - \frac{g_{f,ij}}{\eta} \frac{\text{d} \ln \delta_{f,ij}}{\text{d} \ln \tau_{ij}},
\]

Plugging equation (F.50) into equation (F.49) provides the elasticity of the price with respect to the transportation cost

\[
\frac{\text{d} \ln p_{f,ij}(\omega)}{\text{d} \ln \tau_{ij}} = 1 + \frac{\eta \delta_{f,ij}^\eta + g_{f,ij} (\omega^n + \delta_{f,ij}^\eta)}{\omega^n + \delta_{f,ij}^\eta} \left( -\frac{1}{\eta + 2g_{f,ij}} \right),
\]

\[
\frac{\text{d} \ln p_{f,ij}(\omega)}{\text{d} \ln \tau_{ij}} = 1 - \frac{\eta \delta_{f,ij}^\eta + g_{f,ij} (\omega^n + \delta_{f,ij}^\eta)}{(\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij})},
\]
\[ \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{\omega^n + \delta_{f,ij}^n}{(\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij})} (\eta + 2g_{f,ij}) - \eta \delta_{f,ij} - g_{f,ij} (\omega^n + \delta_{f,ij}^n), \]

\[ \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{\eta \omega^n + \eta \delta_{f,ij}^n + 2g_{f,ij}\omega^n + 2g_{f,ij}\delta_{f,ij}^n - \eta \delta_{f,ij}^n - g_{f,ij}\omega^n - g_{f,ij}\delta_{f,ij}^n}{(\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij})}, \]

and

\[ \frac{d \ln p_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{\eta \omega^n + g_{f,ij}\omega^n + g_{f,ij}\delta_{f,ij}^n}{(\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij})} = \frac{\omega^n (\eta + g_{f,ij}) + g_{f,ij}\delta_{f,ij}^n}{(\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij})}. \quad (F.51) \]

**F.15.14 Markup Elasticity with Respect to the Transportation Cost (METC')**

Taking the logarithm of equation (5.25) yields

\[ \ln \mu_{f,ij}(\omega) = \ln (1) - \ln (2 (1 - g_{f,ij})) + \ln \left( 1 + \frac{\delta_{f,ij}^n}{\omega^n} \right) \]

and, by taking the total derivative, the elasticity of the markup with respect to the transportation cost is given by

\[ \frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln \tau_{ij}} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln \tau_{ij}} \]

and

\[ \frac{d \ln \mu_{f,ij}(\omega)}{d \ln \tau_{ij}} = \left( 1 + \frac{\delta_{f,ij}^n}{\omega^n} \right)^{-1} \frac{\eta \delta_{f,ij}^n}{\omega^n} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}}. \quad (F.52) \]

Taking the logarithm of equation (5.27) yields

\[ \ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j \]

and, by taking the total derivative, the elasticity of the market dominance with respect to the transportation cost is given by

\[ \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln \tau_{ij}} \]

and

\[ \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} = \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} - \frac{d \ln \delta_j}{d \ln \tau_{ij}}. \quad (F.53) \]

Taking the logarithm of the general relation

\[ \delta_j = \sum_i \sum_f \delta_{f,ij} \]
yields
\[ \ln \delta_j = \ln \left( \sum_i \sum_f \delta_{f,ij} \right) \]
and, by taking the total derivative, the elasticity of the overall number of varieties available in country \( j \) with respect to the transportation cost is given by
\[
\frac{d \ln \delta_j}{d \ln \tau_{ij}} = \frac{\partial \ln \delta_j}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln \tau_{ij}} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = g_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}}.
\] (F.54)

Taking the logarithm of equation (5.26) yields
\[
\ln \delta_{f,ij} = \frac{1}{\eta} \left( \ln (4b) - \ln (a \tau_{ij} w_i z_{f,i}) \right) + \frac{1}{\eta} \ln \left( 1 + \frac{1}{2\eta} \right) + \frac{1}{\eta} \ln (1 - g_{f,ij}) + \frac{1}{\eta} (\ln E_j - \ln \delta_j)
\]
and, by taking the total derivative, the elasticity of the scope with respect to the transportation cost is given by
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{\partial \ln \delta_{f,ij}}{\partial \ln \tau_{ij}} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln \tau_{ij}} + \frac{\partial \ln \delta_j}{\partial \ln \tau_{ij}} \frac{d \ln \delta_j}{d \ln \tau_{ij}}
\]
and
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln \tau_{ij}}.
\] (F.55)

Plugging equation (F.53) and equation (F.54) into equation (F.55) yields
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \frac{d \ln g_{f,ij}}{d \ln \tau_{ij}} - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln \tau_{ij}},
\]
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} (1 - g_{f,ij}) \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} - \frac{1}{\eta} g_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}},
\]
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{1}{\eta} - \frac{g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} - \frac{g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}},
\]
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = \frac{1}{\eta} - \frac{2 g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}},
\]
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} \left( 1 + 2 \frac{g_{f,ij}}{\eta} \right) = -\frac{1}{\eta},
\]
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} \left( \eta + 2 g_{f,ij} \right) = -\frac{1}{\eta}
\]
and
\[
\frac{d \ln \delta_{f,ij}}{d \ln \tau_{ij}} = -\frac{1}{\eta + 2 g_{f,ij}}.
\] (F.56)
Plugging equation (F.53) and equation (F.54) into equation (F.52) yields

\[
\frac{d \ln \mu_{f,i,j}(\omega)}{d \ln \tau_{ij}} = \left(1 + \frac{\delta^\eta_{f,i,j}}{\omega^\eta} \right)^{-1} \eta \frac{\delta^\eta_{f,i,j}}{\omega^\eta} \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} + \frac{g_{f,i,j}}{1 - g_{f,i,j}} \left( \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} - \frac{d \ln \delta_j}{d \ln \tau_{ij}} \right),
\]

\[
\frac{d \ln \mu_{f,i,j}(\omega)}{d \ln \tau_{ij}} = \left(1 + \frac{\delta^\eta_{f,i,j}}{\omega^\eta} \right)^{-1} \frac{g_{f,i,j}}{1 - g_{f,i,j}} \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} + \frac{g_{f,i,j}}{1 - g_{f,i,j}} \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}},
\]

\[
\frac{d \ln \mu_{f,i,j}(\omega)}{d \ln \tau_{ij}} = \left(1 + \frac{\delta^\eta_{f,i,j}}{\omega^\eta} \right)^{-1} \eta \frac{\delta^\eta_{f,i,j}}{\omega^\eta} \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} + g_{f,i,j} \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}} = \left( \frac{\omega^\eta}{\omega^\eta + \delta^\eta_{f,i,j} + g_{f,i,j}} \right) \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}},
\]

and

\[
\frac{d \ln \mu_{f,i,j}(\omega)}{d \ln \tau_{ij}} = \frac{\eta \frac{\delta^\eta_{f,i,j} + g_{f,i,j} (\omega^\eta + \delta^\eta_{f,i,j})}{\omega^\eta + \delta^\eta_{f,i,j}}}{d \ln \tau_{ij}} = \frac{d \ln \delta_{f,i,j}}{d \ln \tau_{ij}}.
\]

(F.57)

Plugging equation (F.56) into equation (F.57) provides the elasticity of the markup with respect to the transportation cost

\[
\frac{d \ln \mu_{f,i,j}(\omega)}{d \ln \tau_{ij}} = \frac{\eta \frac{\delta^\eta_{f,i,j} + g_{f,i,j} (\omega^\eta + \delta^\eta_{f,i,j})}{\omega^\eta + \delta^\eta_{f,i,j}}}{d \ln \tau_{ij}} = \frac{\eta}{\omega^\eta + \delta^\eta_{f,i,j} + g_{f,i,j}}.
\]

(F.58)

F.15.15 Properties of the PETC'

The price elasticity with respect to the transportation cost is less than one:

\[
\frac{d \ln p_{f,i,j}(\omega)}{d \ln \tau_{ij}} < 1 \quad \Leftrightarrow \quad \frac{\omega^\eta (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j}}{\omega^\eta + \delta^\eta_{f,i,j} (\eta + 2g_{f,i,j})} < 1
\]

\[
\Leftrightarrow \quad \omega^\eta (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j} < (\omega^\eta + \delta^\eta_{f,i,j}) (\eta + 2g_{f,i,j})
\]

\[
\Leftrightarrow \quad \eta \omega^\eta + g_{f,i,j} \omega^\eta + g_{f,i,j} \delta^\eta_{f,i,j} < \eta \omega^\eta + 2g_{f,i,j} \omega^\eta + \eta \delta^\eta_{f,i,j} + 2g_{f,i,j} \delta^\eta_{f,i,j}
\]

\[
\Leftrightarrow \quad 0 < g_{f,i,j} \omega^\eta + \eta \delta^\eta_{f,i,j} + g_{f,i,j} \delta^\eta_{f,i,j}.
\]

For the proportion of the PETC and the PETC', one gets

\[
\varepsilon_{p,\tau} \equiv \frac{\eta \omega^\eta (1 - g_{f,i,j})}{\omega^\eta + \delta^\eta_{f,i,j} (\eta (1 - g_{f,i,j}) + g_{f,i,j})} \Leftrightarrow \frac{\eta \omega^\eta (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j}}{\omega^\eta + \delta^\eta_{f,i,j} (\eta + 2g_{f,i,j})}
\]

\[
\Leftrightarrow \quad \eta \omega^\eta (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j} \Leftrightarrow \frac{\eta \omega^\eta (1 - g_{f,i,j})}{\eta (1 - g_{f,i,j}) + g_{f,i,j}} \Leftrightarrow \frac{\omega^\eta (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j}}{\eta + 2g_{f,i,j}}
\]

\[
\Leftrightarrow \quad \eta \omega^\eta (1 - g_{f,i,j}) (\eta + 2g_{f,i,j}) \Leftrightarrow \frac{\omega^\eta (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j}}{\eta (1 - g_{f,i,j}) + g_{f,i,j}}
\]

\[
\Leftrightarrow \quad (\eta \omega^\eta - \eta g_{f,i,j} \omega^\eta) (\eta + 2g_{f,i,j}) \Leftrightarrow \frac{\omega^\eta (\eta + g_{f,i,j} \omega^\eta + g_{f,i,j} \delta^\eta_{f,i,j}) (\eta - \eta g_{f,i,j} + g_{f,i,j})}{\eta (1 - g_{f,i,j}) + g_{f,i,j}}
\]
\[ \Leftrightarrow \eta^2 \omega^n + 2 \eta g_{f,i,j} \omega^n - \eta^2 g_{f,i,j} \omega^n - 2 \eta g_{f,i,j}^2 \omega^n + \eta^2 g_{f,i,j}^2 \omega^n + \eta g_{f,i,j} \omega^n + \eta g_{f,i,j}^2 \omega^n - \eta^2 g_{f,i,j}^2 \omega^n \\
+ g_{f,i,j}^2 \omega^n + \eta g_{f,i,j} \delta^\eta_{f,i,j} - \eta g_{f,i,j}^2 \delta^\eta_{f,i,j} + g_{f,i,j}^2 \delta^\eta_{f,i,j} \]

\[ \Leftrightarrow -2 \eta g_{f,i,j}^2 \omega^n \leq \eta g_{f,i,j}^2 \omega^n + g_{f,i,j}^2 \omega^n + \eta g_{f,i,j} \delta^\eta_{f,i,j} - \eta g_{f,i,j}^2 \delta^\eta_{f,i,j} + g_{f,i,j}^2 \delta^\eta_{f,i,j} \]

\[ \Leftrightarrow \eta g_{f,i,j} \delta^\eta_{f,i,j} - \eta g_{f,i,j} \delta^\eta_{f,i,j} - g_{f,i,j}^2 \delta^\eta_{f,i,j} \leq \eta g_{f,i,j}^2 \omega^n - \eta^2 g_{f,i,j} \omega^n + 2 \eta g_{f,i,j}^2 \omega^n \]

\[ \Leftrightarrow \delta^\eta_{f,i,j} (\eta g_{f,i,j} - \eta g_{f,i,j} - g_{f,i,j}^2) \leq \eta g_{f,i,j}^2 \omega^n + g_{f,i,j}^2 \omega^n \]

\[ \Leftrightarrow \delta^\eta_{f,i,j} (\eta g_{f,i,j} - \eta g_{f,i,j} - g_{f,i,j}^2) \leq \omega^n (\eta g_{f,i,j} + g_{f,i,j}^2) \]

\[ \Leftrightarrow \delta^\eta_{f,i,j} ((\eta - 1) g_{f,i,j} - \eta g_{f,i,j}) \leq \omega^n ((\eta + 1) g_{f,i,j}) \]

The transportation cost elasticity of the price is increasing with the firm’s market dominance:

\[ \frac{\partial}{\partial g_{f,i,j}} \left( \frac{\ln p_{f,i,j}(\omega)}{\ln \tau_{i,j}} \right) = \frac{\left( \omega^n + \delta^\eta_{f,i,j} \right) (\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}) - \left( \omega^n (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j} \right) 2 (\omega^n + \delta^\eta_{f,i,j})}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} \]

\[ \frac{\partial}{\partial g_{f,i,j}} \left( \frac{\ln p_{f,i,j}(\omega)}{\ln \tau_{i,j}} \right) = \frac{\left( \omega^n + \delta^\eta_{f,i,j} \right) (\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}) - 2 (\omega^n (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j})}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} \]

\[ \frac{\partial}{\partial g_{f,i,j}} \left( \frac{\ln p_{f,i,j}(\omega)}{\ln \tau_{i,j}} \right) = \frac{\left( \omega^n + \delta^\eta_{f,i,j} \right) (\eta \omega^n + \eta \delta^\eta_{f,i,j} + 2 g_{f,i,j} \omega^n + 2 g_{f,i,j} \delta^\eta_{f,i,j} - 2 \eta \omega^n - 2 g_{f,i,j} \omega^n - 2 g_{f,i,j} \delta^\eta_{f,i,j})}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} \]

and

\[ \frac{\partial}{\partial g_{f,i,j}} \left( \frac{\ln p_{f,i,j}(\omega)}{\ln \tau_{i,j}} \right) = \frac{\left( \omega^n + \delta^\eta_{f,i,j} \right) (-\eta \omega^n + \eta \delta^\eta_{f,i,j})}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} = \frac{\eta \left( \omega^n + \delta^\eta_{f,i,j} \right) (\delta^\eta_{f,i,j} - \omega^n)}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} > 0. \]

The transportation cost elasticity of the price is decreasing with the firm’s export scope:

\[ \frac{\partial}{\partial \delta_{f,i,j}} \left( \frac{\ln p_{f,i,j}(\omega)}{\ln \tau_{i,j}} \right) = \frac{\eta \delta^\eta_{f,i,j}^{-1} g_{f,i,j} (\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}) - \left( \omega^n (\eta + g_{f,i,j}) + g_{f,i,j} \delta^\eta_{f,i,j} \right) (\eta + 2 g_{f,i,j}) \eta \delta^\eta_{f,i,j}^{-1}}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} \]

\[ \frac{\partial}{\partial \delta_{f,i,j}} \left( \frac{\ln p_{f,i,j}(\omega)}{\ln \tau_{i,j}} \right) = \frac{\eta \delta^\eta_{f,i,j}^{-1} (\eta + 2 g_{f,i,j}) (g_{f,i,j} (\omega^n + \delta^\eta_{f,i,j}) - \omega^n (\eta + g_{f,i,j}) - g_{f,i,j} \delta^\eta_{f,i,j})}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} \]

\[ \frac{\partial}{\partial \delta_{f,i,j}} \left( \frac{\ln p_{f,i,j}(\omega)}{\ln \tau_{i,j}} \right) = \frac{\eta \delta^\eta_{f,i,j}^{-1} (\eta + 2 g_{f,i,j}) (g_{f,i,j} \omega^n + g_{f,i,j} \delta^\eta_{f,i,j} - \omega^n - g_{f,i,j} \omega^n - g_{f,i,j} \delta^\eta_{f,i,j})}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} \]

and

\[ \frac{\partial}{\partial \delta_{f,i,j}} \left( \frac{\ln p_{f,i,j}(\omega)}{\ln \tau_{i,j}} \right) = - \frac{\eta \delta^\eta_{f,i,j}^{-1} (\eta + 2 g_{f,i,j}) \eta \omega^n}{((\omega^n + \delta^\eta_{f,i,j}) (\eta + 2 g_{f,i,j}))^2} < 0. \]
The transportation cost elasticity of the price is increasing in each firm’s product portfolio:

\[
\frac{\partial \left( \frac{\ln p_{f,ij}(\omega)}{\ln \tau_{ij}} \right)}{\partial \omega} = \eta \omega^{n-1} (\eta + g_{f,ij}) (\omega^n + \delta_{f,ij}^n) (\eta + 2g_{f,ij}) \\
\frac{(\omega^n + \delta_{f,ij}^n) (\eta + 2g_{f,ij})^2}{},
\]

\[
\frac{\partial \left( \frac{\ln p_{f,ij}(\omega)}{\ln \tau_{ij}} \right)}{\partial \tau_{ij}} = -\frac{(\omega^n (\eta + g_{f,ij}) + g_{f,ij} \delta_{f,ij}^n) (\eta + 2g_{f,ij}) \eta \omega^{n-1}}{((\omega^n + \delta_{f,ij}^n) (\eta + 2g_{f,ij})^2),}
\]

\[
\frac{\partial \left( \frac{\ln p_{f,ij}(\omega)}{\ln \tau_{ij}} \right)}{\partial g_{f,ij}} = \eta \omega^{n-1} (\eta + 2g_{f,ij}) \frac{(\eta \omega^n + \eta \delta_{f,ij}^n + g_{f,ij} \omega^n + g_{f,ij} \delta_{f,ij}^n - \eta \omega^n - g_{f,ij} \omega^n - g_{f,ij} \delta_{f,ij}^n)(\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij})^2}{,}
\]

and

\[
\frac{\partial \left( \frac{\ln p_{f,ij}(\omega)}{\ln \tau_{ij}} \right)}{\partial g_{f,ij}} = \eta \omega^{n-1} (\eta + 2g_{f,ij}) \eta \delta_{f,ij}^n \frac{(\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij})^2}{,} > 0.
\]

F.15.16 Properties of the METC'

The absolute value of the elasticity of the markup with respect to the transportation cost is decreasing in the firm’s market dominance:

\[
\frac{\partial \left| \frac{\ln \mu_{f,ij}(\omega)}{\ln \tau_{ij}} \right|}{\partial g_{f,ij}} = \frac{(\omega^n + \delta_{f,ij}^n)(\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij}) - (\eta \delta_{f,ij}^n + g_{f,ij} \omega^n + \delta_{f,ij}^n) 2 (\omega^n + \delta_{f,ij}^n)}{((\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij}))^2},
\]

\[
\frac{\partial \left| \frac{\ln \mu_{f,ij}(\omega)}{\ln \tau_{ij}} \right|}{\partial g_{f,ij}} = \frac{(\omega^n + \delta_{f,ij}^n)(\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij}) - 2 (\eta \delta_{f,ij}^n + g_{f,ij} \omega^n + \delta_{f,ij}^n)}{((\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij}))^2},
\]

\[
\frac{\partial \left| \frac{\ln \mu_{f,ij}(\omega)}{\ln \tau_{ij}} \right|}{\partial g_{f,ij}} = \frac{(\omega^n + \delta_{f,ij}^n)(\eta \omega^n + 2g_{f,ij} \omega^n + \eta \delta_{f,ij}^n + 2g_{f,ij} \delta_{f,ij}^n - 2 \eta \delta_{f,ij}^n - 2g_{f,ij} \omega^n - 2g_{f,ij} \delta_{f,ij}^n)}{((\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij}))^2},
\]

\[
\frac{\partial \left| \frac{\ln \mu_{f,ij}(\omega)}{\ln \tau_{ij}} \right|}{\partial g_{f,ij}} = \frac{(\omega^n + \delta_{f,ij}^n)(\eta \omega^n - \eta \delta_{f,ij}^n)}{((\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij}))^2} = \frac{(\omega^n + \delta_{f,ij}^n) \eta (\delta_{f,ij}^n - \omega^n)}{((\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij}))^2},
\]

and

\[
\frac{\partial \left| \frac{\ln \mu_{f,ij}(\omega)}{\ln \tau_{ij}} \right|}{\partial g_{f,ij}} = \frac{- \eta (\delta_{f,ij}^n - \omega^n)}{((\omega^n + \delta_{f,ij}^n)(\eta + 2g_{f,ij}))^2} < 0.
\]
The absolute value of the elasticity of the markup with respect to the transportation cost is increasing in the firm’s export scope:

\[
\frac{\partial \left( \frac{d\ln \mu_{f,ij}(\omega)}{d\ln \tau_{ij}} \right)}{\partial \delta_{f,ij}} = \frac{(\eta^2 \delta_{f,ij}^{\eta-1} + \eta g_{f,ij} \delta_{f,ij}^{\eta-1}) (\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij})}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2} - \frac{(\eta \delta_{f,ij}^\eta + g_{f,ij} (\omega^n + \delta_{f,ij}^\eta)) \eta (\eta + 2g_{f,ij}) \delta_{f,ij}^{\eta-1}}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2},
\]

and

\[
\frac{\partial \left( \frac{d\ln \mu_{f,ij}(\omega)}{d\ln \tau_{ij}} \right)}{\partial \delta_{f,ij}} = \frac{(\eta + 2g_{f,ij}) (\eta^2 \delta_{f,ij}^{\eta-1} \omega^n + \eta^2 \delta_{f,ij}^{\eta-1} \omega^n + \eta g_{f,ij} \delta_{f,ij}^{\eta-1} \omega^n + \eta g_{f,ij} \delta_{f,ij}^{\eta-1} \delta_{f,ij}^\eta)}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2} - \frac{(\eta + 2g_{f,ij}) (\eta^2 \delta_{f,ij}^{\eta-1} \delta_{f,ij}^\eta + \eta g_{f,ij} \omega^n \delta_{f,ij}^{\eta-1} + \eta g_{f,ij} \delta_{f,ij}^{\eta-1})}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2}.
\]

The absolute value of the elasticity of the markup with respect to the transportation cost is decreasing in each firm’s product portfolio:

\[
\frac{\partial \left( \frac{d\ln \mu_{f,ij}(\omega)}{d\ln \tau_{ij}} \right)}{\partial \omega} = \frac{\eta g_{f,ij} \omega^{n-1} (\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij})}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2} - \frac{(\eta \delta_{f,ij}^\eta + g_{f,ij} (\omega^n + \delta_{f,ij}^\eta)) \eta (\eta + 2g_{f,ij}) \omega^{n-1}}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2},
\]

and

\[
\frac{\partial \left( \frac{d\ln \mu_{f,ij}(\omega)}{d\ln \tau_{ij}} \right)}{\partial \omega} = \frac{(\eta + 2g_{f,ij}) (\eta g_{f,ij} \omega^{n-1} (\omega^n + \delta_{f,ij}^\eta) - \eta \omega^{n-1} (\eta \delta_{f,ij}^\eta + g_{f,ij} (\omega^n + \delta_{f,ij}^\eta)))}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2},
\]

\[
\frac{\partial \left( \frac{d\ln \mu_{f,ij}(\omega)}{d\ln \tau_{ij}} \right)}{\partial \omega} = \frac{(\eta + 2g_{f,ij}) (\eta g_{f,ij} \omega^{n-1} \omega^n + \eta g_{f,ij} \omega^{n-1} \delta_{f,ij}^\eta)}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2} - \frac{(\eta + 2g_{f,ij}) (\eta^2 \omega^{n-1} \delta_{f,ij}^\eta + \eta \omega^{n-1} g_{f,ij} \omega^n + \eta \omega^{n-1} g_{f,ij} \delta_{f,ij}^\eta)}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2}.
\]

and

\[
\frac{\partial \left( \frac{d\ln \mu_{f,ij}(\omega)}{d\ln \tau_{ij}} \right)}{\partial \omega} = -\frac{(\eta + 2g_{f,ij}) \eta^2 \omega^{n-1} \delta_{f,ij}^\eta}{((\omega^n + \delta_{f,ij}^\eta) (\eta + 2g_{f,ij}))^2} < 0.
\]
F.15.17 Price Elasticity with Respect to the Per Capita Income (PEPCI')

Taking the logarithm of equation (5.24) yields

\[
\ln p_{f,ij}(\omega) = \ln (\tau_{ij} w_i z_{f,i}) - \ln (2 [1 - g_{f,ij}]) + \ln \left( \omega^n + \delta^n_{f,ij} \right)
\]

and, by taking the total derivative, the elasticity of the price with respect to the per capita income is given by

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\partial \ln p_{f,ij}(\omega)}{\partial \ln E_j} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln E_j} + \frac{\partial \ln g_{f,ij}}{\partial \ln E_j} + \frac{\partial \ln p_{f,ij}(\omega)}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\partial \ln p_{f,ij}(\omega)}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j}
\]

and

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta^n_{f,ij}}{\omega^n + \delta^n_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j}. \quad (F.59)
\]

Taking the logarithm of equation (5.27) yields

\[
\ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j
\]

and, by taking the total derivative, the elasticity of the market dominance with respect to the per capita income is given by

\[
\frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln E_j}
\]

and

\[
\frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{d \ln \delta_{f,ij}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j}. \quad (F.60)
\]

Taking the logarithm of the general relation

\[
\delta_j = \sum_i \sum_f \delta_{f,ij}
\]

yields

\[
\ln \delta_j = \ln \left( \sum_i \sum_f \delta_{f,ij} \right)
\]

and, by taking the total derivative, the elasticity of the overall number of varieties available in country \( j \) with respect to the per capita income is given by

\[
\frac{d \ln \delta_j}{d \ln E_j} = \frac{\partial \ln \delta_j}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\delta_{f,ij}}{\delta_j} \frac{d \ln \delta_{f,ij}}{d \ln E_j} = g_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln E_j} \quad (F.61)
\]
Taking the logarithm of equation (5.26) yields

\[
\ln \delta_{f,ij} = \frac{1}{\eta} \left( \ln (4b) - \ln (a \tau_{ij} w_i z_{f,ij}) \right) + \frac{1}{\eta} \ln \left( 1 + \frac{1}{2 \eta} \right) + \frac{1}{\eta} \ln (1 - g_{f,ij}) + \frac{1}{\eta} (\ln E_j - \ln \delta_j)
\]

and, by taking the total derivative, the elasticity of the scope with respect to the per capita income is given by

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{\partial \ln \delta_{f,ij}}{\partial \ln E_j} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j} + \frac{\partial \ln \delta_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln E_j},
\]

and

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = 1 - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \left( \frac{d \ln \delta_{f,ij}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j} \right) - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln E_j}.
\]

Plugging equation (F.60) and equation (F.61) into equation (F.62) yields

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = 1 - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \left( \frac{d \ln \delta_{f,ij}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j} \right) - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln E_j},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = 1 - \frac{g_{f,ij}}{\eta (1 - g_{f,ij})} \left( \frac{d \ln \delta_{f,ij}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j} \right) - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln E_j},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = 1 - \frac{2 g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = 1 - \frac{2 g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = 1 - \frac{2 g_{f,ij}}{\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j},
\]

\[
\frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{1}{\eta + 2 g_{f,ij}}.
\]

Plugging equation (F.60), equation (F.61) and equation (F.63) into equation (F.59) provides the elasticity of the price with respect to the per capita income

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij}^\eta}{\omega^n + \delta_{f,ij}^\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{g_{f,ij}}{1 - g_{f,ij}} \left( \frac{d \ln \delta_{f,ij}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j} \right),
\]

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij}^\eta}{\omega^n + \delta_{f,ij}^\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{g_{f,ij}}{1 - g_{f,ij}} \left( \frac{d \ln \delta_{f,ij}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j} \right),
\]

\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij}^\eta + g_{f,ij} \left( \omega^n + \delta_{f,ij}^\eta \right)}{\omega^n + \delta_{f,ij}^\eta} \frac{d \ln \delta_{f,ij}}{d \ln E_j} = \frac{\eta \delta_{f,ij}^\eta + g_{f,ij} \left( \omega^n + \delta_{f,ij}^\eta \right)}{\omega^n + \delta_{f,ij}^\eta} \frac{1}{\eta + 2 g_{f,ij}}.
\]
and
\[
\frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta^\eta_{f,ij} + g_{f,ij} (\omega^n + \delta^\eta_{f,ij})}{(\omega^n + \delta^\eta_{f,ij}) (\eta + 2g_{f,ij})}. \tag{F.64}
\]

**F.15.18 Markup Elasticity with Respect to the Per Capita Income (MEPCI')**

Taking the logarithm of equation (5.25) yields
\[
\ln \mu_{f,ij}(\omega) = \ln (1) - \ln (2 (1 - g_{f,ij})) + \ln \left(1 + \frac{\delta^\eta_{f,ij}}{\omega^n}\right)
\]
and, by taking the total derivative, the elasticity of the markup with respect to the per capita income is given by
\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln E_j} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j} + \frac{\partial \ln \mu_{f,ij}(\omega)}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln E_j}
\]
and
\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \left(1 + \frac{\delta^\eta_{f,ij}}{\omega^n}\right)^{-1} \eta \delta^\eta_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{g_{f,ij}}{1 - g_{f,ij}} \frac{d \ln g_{f,ij}}{d \ln E_j}. \tag{F.65}
\]

Taking the logarithm of equation (5.27) yields
\[
\ln g_{f,ij} = \ln \delta_{f,ij} - \ln \delta_j
\]
and, by taking the total derivative, the elasticity of the market dominance with respect to the per capita income is given by
\[
\frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + \frac{\partial \ln g_{f,ij}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln E_j}
\]
and
\[
\frac{d \ln g_{f,ij}}{d \ln E_j} = \frac{d \ln \delta_{f,ij}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j}. \tag{F.66}
\]

Taking the logarithm of the general relation
\[
\delta_j = \sum_i \sum_f \delta_{f,ij}
\]
yields
\[
\ln \delta_j = \ln \left(\sum_i \sum_f \delta_{f,ij}\right)
\]
and, by taking the total derivative, the elasticity of the overall number of varieties available in country $j$ with respect to the per capita income is given by

$$\frac{d \ln \delta_j}{d \ln E_j} = \frac{\partial \ln \delta_j}{\partial \ln \delta_{f,i,j}} \frac{d \ln \delta_{f,i,j}}{d \ln E_j} = \frac{\delta_{f,i,j}}{\delta_j} \frac{d \ln \delta_{f,i,j}}{d \ln E_j} = g_{f,i,j} \frac{d \ln \delta_{f,i,j}}{d \ln E_j}. \tag{F.67}$$

Taking the logarithm of \textbf{equation (5.26)} yields

$$\ln \delta_{f,i,j} = \frac{1}{\eta} (\ln (4b) - \ln (a \tau_{ij} w_i z_{f,i})) + \frac{1}{\eta} \ln \left(1 + \frac{1}{2\eta}\right) + \frac{1}{\eta} \ln (1 - g_{f,i,j}) + \frac{1}{\eta} (\ln E_j - \ln \delta_j)$$

and, by taking the total derivative, the elasticity of the scope with respect to the per capita income is given by

$$\frac{d \ln \delta_{f,i,j}}{d \ln E_j} = \frac{\partial \ln \delta_{f,i,j}}{\partial \ln E_j} + \frac{\partial \ln \delta_{f,i,j}}{\partial \ln g_{f,i,j}} \frac{d \ln g_{f,i,j}}{d \ln E_j} + \frac{\partial \ln \delta_{f,i,j}}{\partial \ln \delta_j} \frac{d \ln \delta_j}{d \ln E_j}$$

and

$$\frac{d \ln \delta_{f,i,j}}{d \ln E_j} = \frac{1}{\eta} - \frac{g_{f,i,j}}{\eta (1 - g_{f,i,j})} \frac{d \ln \delta_{f,i,j}}{d \ln E_j} - \frac{1}{\eta} \frac{d \ln \delta_j}{d \ln E_j}. \tag{F.68}$$

Plugging \textbf{equation (F.66)} and \textbf{equation (F.67)} into \textbf{equation (F.68)} yields

$$\frac{d \ln \delta_{f,i,j}}{d \ln E_j} = \frac{1}{\eta} - \frac{g_{f,i,j}}{\eta (1 - g_{f,i,j})} \left(1 - g_{f,i,j}\right) \frac{d \ln \delta_{f,i,j}}{d \ln E_j} - \frac{1}{\eta} g_{f,i,j} \frac{d \ln \delta_{f,i,j}}{d \ln E_j},$$

$$\frac{d \ln \delta_{f,i,j}}{d \ln E_j} = \frac{1}{\eta} - \frac{g_{f,i,j}}{\eta (1 - g_{f,i,j})} \left(1 - g_{f,i,j}\right) \frac{d \ln \delta_{f,i,j}}{d \ln E_j} - g_{f,i,j} \frac{d \ln \delta_{f,i,j}}{d \ln E_j},$$

$$\frac{d \ln \delta_{f,i,j}}{d \ln E_j} = \frac{1}{\eta} - 2 \frac{g_{f,i,j} d \ln \delta_{f,i,j}}{\eta d \ln E_j},$$

$$\frac{d \ln \delta_{f,i,j}}{d \ln E_j} \left(1 + \frac{2 g_{f,i,j}}{\eta}\right) = \frac{1}{\eta},$$

$$\frac{d \ln \delta_{f,i,j}}{d \ln E_j} \left(\frac{\eta + 2 g_{f,i,j}}{\eta}\right) = \frac{1}{\eta}$$

and

$$\frac{d \ln \delta_{f,i,j}}{d \ln E_j} = \frac{1}{\eta + 2 g_{f,i,j}}. \tag{F.69}$$

Plugging \textbf{equation (F.66)} and \textbf{equation (F.67)} into \textbf{equation (F.65)} yields

$$\frac{d \ln \mu_{f,i,j}(\omega)}{d \ln E_j} = \left(1 + \frac{\delta_{f,i,j}^\eta}{\omega'^\eta}\right)^{-1} \eta \delta_{f,i,j} d \ln \delta_{f,i,j} + \frac{g_{f,i,j}}{(1 - g_{f,i,j})} \left(\frac{d \ln \delta_{f,i,j}}{d \ln E_j} - \frac{d \ln \delta_j}{d \ln E_j}\right),$$

$$\frac{d \ln \mu_{f,i,j}(\omega)}{d \ln E_j} = \left(1 + \frac{\delta_{f,i,j}^\eta}{\omega'^\eta}\right)^{-1} \eta \delta_{f,i,j} d \ln \delta_{f,i,j} + \frac{g_{f,i,j}}{(1 - g_{f,i,j})} \left(1 - g_{f,i,j}\right) \frac{d \ln \delta_{f,i,j}}{d \ln E_j},$$
\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \left(1 + \frac{\delta_{f,ij}}{\omega}\right)^{-1} \eta \delta_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln E_j} + g_{f,ij} \frac{d \ln \delta_{f,ij}}{d \ln E_j} = \left(\frac{\eta \delta_{f,ij}}{\omega + \delta_{f,ij}} + g_{f,ij}\right) \frac{d \ln \delta_{f,ij}}{d \ln E_j}
\]

and
\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij} + g_{f,ij} (\omega + \delta_{f,ij})}{\omega + \delta_{f,ij}} \frac{d \ln \delta_{f,ij}}{d \ln E_j}.
\] 

Plugging equation (F.69) into equation (F.70) provides the elasticity of the markup with respect to the per capita income

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij} + g_{f,ij} (\omega + \delta_{f,ij})}{\omega + \delta_{f,ij}} \frac{1}{\eta + 2g_{f,ij}}
\]

and
\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \frac{\eta \delta_{f,ij} + g_{f,ij} (\omega + \delta_{f,ij})}{(\omega + \delta_{f,ij}) (\eta + 2g_{f,ij})}.
\]

**F.15.19 Properties of the PEPCI′**

The income elasticity of the price is decreasing in the firm’s market dominance:

\[
\frac{\partial \left(\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j}\right)}{\partial g_{f,ij}} = \frac{(\omega + \delta_{f,ij}) (\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}) - (\eta \delta_{f,ij} + g_{f,ij} (\omega + \delta_{f,ij})) 2 (\omega + \delta_{f,ij})}{((\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2},
\]

\[
\frac{\partial \left(\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j}\right)}{\partial g_{f,ij}} = \frac{(\omega + \delta_{f,ij}) ((\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}) - 2 (\eta \delta_{f,ij} + g_{f,ij} (\omega + \delta_{f,ij})))}{((\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2},
\]

\[
\frac{\partial \left(\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j}\right)}{\partial g_{f,ij}} = \frac{(\omega + \delta_{f,ij}) (\eta \omega - \eta \delta_{f,ij} + 2g_{f,ij} \omega + 2g_{f,ij} \delta_{f,ij} - 2\eta \delta_{f,ij} - 2g_{f,ij} \omega - 2g_{f,ij} \delta_{f,ij})}{((\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2},
\]

\[
\frac{\partial \left(\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j}\right)}{\partial g_{f,ij}} = \frac{(\omega + \delta_{f,ij}) ((\omega + \delta_{f,ij}) (\eta - \delta_{f,ij}))}{((\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2} = \frac{(\omega + \delta_{f,ij}) (\eta - \delta_{f,ij})}{((\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2},
\]

and

\[
\frac{\partial \left(\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j}\right)}{\partial g_{f,ij}} = -\frac{(\omega + \delta_{f,ij}) (\eta (\delta_{f,ij} - \omega))}{((\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2} = -\frac{\eta (\delta_{f,ij} - \omega^2)}{(\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2 < 0.
\]

The income elasticity of the price is increasing in the firm’s export scope:

\[
\frac{\partial \left(\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j}\right)}{\partial \delta_{f,ij}} = \frac{(\eta \omega + \omega \delta_{f,ij})}{((\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2} = \frac{\eta (\delta_{f,ij} - \omega^2)}{(\omega + \delta_{f,ij}) (\eta + 2g_{f,ij}))^2},
\]
\[
\frac{\partial}{\partial \delta_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = \eta \delta_{f,ij}^{\eta-1} \left( \eta + 2 g_{f,ij} \right) \left( (\eta + g_{f,ij}) (\omega^n + \delta_{f,ij}^n) - \eta \delta_{f,ij}^n - g_{f,ij} (\omega^n + \delta_{f,ij}^n) \right) \frac{1}{\left( (\omega^n + \delta_{f,ij}^n) (\eta + 2 g_{f,ij}) \right)^2},
\]

\[
\frac{\partial}{\partial \delta_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = \eta \delta_{f,ij}^{\eta-1} \left( \eta + 2 g_{f,ij} \right) \left( (\eta + g_{f,ij}) (\omega^n + \delta_{f,ij}^n) - \eta \delta_{f,ij}^n - g_{f,ij} (\omega^n + \delta_{f,ij}^n) \right) \frac{1}{\left( (\omega^n + \delta_{f,ij}^n) (\eta + 2 g_{f,ij}) \right)^2},
\]

and

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = \eta \delta_{f,ij}^{\eta-1} \left( \eta + 2 g_{f,ij} \right) \left( (\eta + g_{f,ij}) (\omega^n + \delta_{f,ij}^n) - \eta \delta_{f,ij}^n - g_{f,ij} (\omega^n + \delta_{f,ij}^n) \right) \frac{1}{\left( (\omega^n + \delta_{f,ij}^n) (\eta + 2 g_{f,ij}) \right)^2}.
\]

The income elasticity of the price is decreasing in each firm’s product portfolio:

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = \eta \delta_{f,ij}^{\eta-1} \left( \eta + 2 g_{f,ij} \right) \left( (\eta + g_{f,ij}) (\omega^n + \delta_{f,ij}^n) - \eta \delta_{f,ij}^n - g_{f,ij} (\omega^n + \delta_{f,ij}^n) \right) \frac{1}{\left( (\omega^n + \delta_{f,ij}^n) (\eta + 2 g_{f,ij}) \right)^2} < 0.
\]

F.15.20 Properties of the MEPCI’

Since the markup elasticity MEPCI’ equals the price elasticity PEPCI’, i.e.

\[
\frac{d \ln \mu_{f,ij}(\omega)}{d \ln E_j} = \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j},
\]

their properties are identical:

\[
\frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{\partial}{\partial g_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) < 0,
\]

\[
\frac{\partial}{\partial \delta_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{\partial}{\partial \delta_{f,ij}} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) > 0,
\]

and

\[
\frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) = \frac{\partial}{\partial \omega} \left( \frac{d \ln p_{f,ij}(\omega)}{d \ln E_j} \right) < 0.
\]
Bibliography


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