BIELEFELD UNIVERSITY

Essays in International Trade, Multinational Firm Production and Economic Growth

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Declaration of Authorship

I, Phemelo Tamasiga, declare that this thesis titled, "Essays in International Trade, Multinational Firm Production and Economic Growth" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at Bielefeld University.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Abstract

Bielefeld Graduate School of Economics and Management
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Doctor of Philosophy Economics

Essays in International Trade, Multinational Firm Production and Economic Growth

by Phemelo Tamasiga

This thesis investigates questions of international trade, multinational firm production and economic growth. Firstly, Chapter 2 quantifies the total gains from openness such that firms can either export to their foreign markets or carry out horizontal FDI characterised by intra firm exports of intermediate inputs from parent firms to their subsidiaries. This is motivated by the fact that despite empirical evidence on the rising importance of intermediate good inputs in trade shares, literature on gains from trade has not adequately accounted for intermediate input share in calculating total trade gains. Intermediate input shares play an important role as they affect the trade elasticity hence welfare changes, we find that gains from trade and FDI derived in this model are higher than the current documented gains in trade literature. Chapter 3 presents a dynamic general equilibrium model of corporate taxation of heterogeneous productivity firms that enter foreign markets via exports or setting up a multinational firm subsidiary. This model is at the nexus of international trade models of firm heterogeneity and endogenous growth model without scale effects. Chapter 4 investigates international trade of exhaustible resources in a differential game over a continuous time. When exhaustible resources are concerned in questions of economics, inter-temporal trade offs between generations cannot be ignored as resources are depleting. Standard static models of international trade that argue that countries trade because of different factor endowments are not enough because resource prices change over time, and the exhaustible resource endowment is also declining over time. It is therefore natural to consider dynamic tools such as differential games to address economic questions of trade of exhaustible resources. Central to our study is the result that Hartwick's rule is broken, i.e. not all resource rents are reinvested into capital.

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Dedicated to my mother Matlhogonolo Patricia-Makgoma-Tamasiga, what was otherwise just a dream, she helped frame it into real tangible experiences, for that I am humbled and forever thankful.

Chapter 1

Introduction

1.1 Background Motivation

Central to debates and discussions forums within economic cycles is the question, how does globalization benefit economies? Many countries have developed policies that enable them to be participants in the global market, however their main concern is how international trade and foreign direct investment (hereafter, abbreviated FDI) harm or foster economic growth. Thus far, there are insurmountable volumes of literature documenting both positive and negative effects of foreign direct investment and international trade depending on the countries engaged. For example, Singh (2010) concluded that the effects of international trade on economic growth and welfare remains ambiguous in terms of both theoretical models and empirical research. In economics the subject of globalisation touches the realms of growth theory and industrial organization beyond international trade. In static models of trade and FDI there is a consensus among economists of the gains from globalisation; that is, both consumers and firms benefit from specialization and from the access to a broader variety of goods on the world market. However, a theoretical debate has ensued and it questions the advantage of globalisation through trade and FDI in dynamic set-ups. Against this background, this thesis tries to contribute to the theoretical debate surrounding the effects of trade and FDI, focusing in particular on the total gains from trade and FDI, implications for economic growth through productivity changes and the role of exhaustible resources in international trade. The following subsections; (1.1.1), (1.1.2), (1.1.3) provide brief motivations and overview of the three essays of this thesis.

1.1.1 Quantifying Gains from Trade and Foreign Direct Investment with Heterogeneous Firms

The pioneering work of Samuelson (1939) showed that there are gains from trade, consequently one of the pertinent questions in international trade literature is how best to measure these gains. The workhorse empirical tool to measure the welfare gains in international trade is the empirical gravity model of trade. There is a wide variety of theoretical models that provide the foundation for the gravity model and interpret the empirical patterns of bilateral trade. After the works of Samuelson (1939) all attempts made to understand gains from trade focused on identifying aggregate relationships through cross-country comparisons using empirical analysis. For example, Sachs and Warner (1995) argued that open economies tend to experience higher growth rates than closed economies. On the theory side, traditional models of trade, such as those based on Ricardo (1821) and Heckscher and Ohlin (1991), focus on gains from specialization by comparative advantage in settings of homogeneous firms. More recent trade models which studies the gains from trade take a different approach, they consider firms that are heterogeneous in their productivity levels and conclude that less productive firms exit the market due to stronger competition from imports, as a result resources shift to more productive firms which can then produce and sell more. The net result is an increase in average productivity. A more recent work by Arkolakis et al. (2012) has shown that both the traditional trade models with homogeneous firms and the new trade models with heterogeneous firms all yield the same measure of trade gains. Arkolakis et al. (2012)'s result is based on the assumption that only the trade elasticity and the country's expenditure share on traded goods are the two static measures necessary to measure the gains from trade. Interestingly, the gains from trade measured in Arkolakis et al. (2012) only considered inter-trade transactions. i.e, trade between non affiliated firms between countries. However, Bernard et al. (2009) carried out an empirical study that showed that a substantial volume of trade takes place within the multinational firms. This implies that there is intra-firm trade between parent firms and their subsidiaries. This provides an motivation to investigate total gains from trade taking into account both inter-firm trade and intra-firm trade.

1.1.2 Capital taxation, Heterogeneous Multinational and Trade Firms in a Model of productivity Growth

Trade theories have investigated how product variety is affected by trade flows and multinational firm production. On the other hand endogenous growth models have tackled question of how process innovation affects mass of diversified products. Amongst the first studies that combined trade and growth was the work of Rivera-Batiz and Romer (1991), their study attempted to reconcile two seemingly contradictory views; endogenous growth models with increasing returns as investigated by Romer (1990) showed that trade restrictions lead to a decline in the rate of economic growth, in contrast Grossman and Helpman (1991) showed that in some circumstances, trade restrictions could nevertheless increase economic growth. Still, the corporate taxation effects in such models are still missing even though it is apparent and clear in vast empirical studies that corporate taxes affect the specialisation pattern and location decisions of firms. Egger et al. (2010) investigated whether foreign plant ownership involves lower tax payments than domestic plant ownership. Their empirical study concluded that foreign owned firms paid lower taxes than domestic firms in high tax jurisdictions but higher taxes in lower tax jurisdictions hence the opportunity to shifts profits. Hence this paper sheds light on corporate taxation effects on heterogeneous firm productivity, economic growth and decisions of being a multinational and trade firm.

1.1.3 Impact of Exhaustible Resources on opening for International Trade: Hartwick's rule and taste for foreign goods (joint work with Anton Bondarev)

The stocks of natural resources that exist in the natural environment are both scarce and economically useful as they are used as a component of the production process or for consumption, either in their raw state or after a minimal amount of processing. These exhaustible resources are characterised by uneven distribution across countries, highly volatile prices and dominance in output and trade. Most of the exhaustible resource rich countries are typically third world poor countries under the spell of the "resource curse", that is, they fail to benefit fully from their natural resource wealth, making it difficult for governments in these countries to respond effectively to public welfare needs.

Transactions of exhaustible resource exports have been significantly on the rise and their impact on economic growth of resource rich countries have been well documented. What is peculiar about the study of international trade of exhaustible resources is that the depletion of a non-renewable resource involves an inter-temporal trade-off; if a resource is consumed today, it cannot be consumed tomorrow. These inter-temporal trade-offs pose unique challenges for policy-makers, in part because exhaustible resources are both essential to the production process and actually or potentially exhaustible. This implies that trade of exhaustible resources resources has both "static" and "dynamic" economic effects.

Static trade theory models such as Heckscher and Ohlin (1991) emphasis that a country will trade (export) a commodity which uses its abundant factor intensively or differences in factor endowments will prompt countries to export commodities for which they have comparative advantage as in the model of Ricardo (1821). This shows that endowments of natural resources may form the basis for trade in resource rich countries. In support of this reasoning, Leamer (1984) showed that the relative abundance of oil leads to net exports of crude oil and mineral abundance leads to net exports of raw materials. However the main shortcoming of trade theories is that they do not directly address this problem of exhaustibility and the inter-temporal trade-offs involved. Resource exhaustibility implies that price level also changes over time so are the levels of resources extracted, this necessitates adopting a dynamic approach that takes into account the change over time in the availability of a finite resource. This motivates the use of dynamic games where we are able to construct a continuous time model of international trade with resource exhaustibility. .

1.2 Contributions and Organisation

This thesis contains three separate chapters, Chapter 2 quantifies the gains from trade and multinational firms under a framework of heterogeneous productivities. Chapter 3 constructs a dynamic general equilibrium model of corporate taxation of firms that enter foreign markets via FDI or trade in an endogenous productivity growth framework. Finally, in Chapter 4 we study a continuous time differential game of international trade in the presence of exhaustible resources. Following I provide brief synopsis of the specific contributions of each chapter;

Chapter 2; ***Quantifying Gains from Trade and FDI with Heterogeneous Firms***
The issue of quantifying gains from trade is of long-standing importance in international trade. Gains from trade derived by Arkolakis et al. (2012) were shown to be the same in all quantitative trade models under both perfect competition models such as Armington (1969) and Eaton and Kortum (2002a) and monopolistic competition Melitz (2003). The derivation is based on the observed share of a country's trade with itself (share of expenditure on domestic goods), λ_j, and the elasticity of aggregate trade costs, ε. In this chapter we extend the result obtained by Arkolakis et al. (2012) by incorporating tradable intermediate inputs within the multinational firm. Consideration of intra firm intermediate good input is consistent with Bernard et al. (2009) as they found out that multinational firms comprise a substantial majority of U.S. trade, roughly 90% of U.S. exports and imports. We derive a closed form formulation for the total gains from international trade (both arms length trade and intra-firm trade). We find out that trade

elasticity is affected also by the share of intermediate inputs. Therefore considering intermediate good input share magnifies the gains from trade resulting in larger trade gains. We find out that, with trade elasticity of 4, when the intermediate share is 1/3 the gains from trade in the fashion of Arkolakis et al. (2012) is 3% while the gains from openness is 11% the difference between our set up (gains from trade and multinational firm intra firm trade) and the trade only gains is 8%.

Chapter 3; ***Capital Taxation, Trade and FDI in endogenous productivity Growth Model***

In this chapter we contribute to the small but growing strand of theoretical research that incorporates firm heterogeneity into models of tax policy towards mobile, multinational firms and trade. We present an extended heterogeneous firms productivity model developed by Melitz (2003) with steady-state productivity growth but without the strong scale effect. The model has an embedded frame work of innovation led productivity growth developed by Gustafsson and Segerstrom (2010). Simply put, the aim of this chapter is to provide a link between firm heterogeneity, innovation and the consequent productivity growth implications in the presence of corporate taxes. Firstly we find that corporate taxes have competing effects on the output level via decrease of the number of intermediate varieties and an increase in final out put labour supply. This implies that we get the result that for certain higher taxes we still get higher economic growth since the labour increase in final output implies increase in output in the model. This implies a higher economic growth due to its equivalence to the output growth, of course this depends on the extreme possibility that the effect through the labour channel is strong enough (dominates the effect on varieties), then higher corporate taxes may lead to higher output growth. Secondly we find that openness to trade or multinational does not affect the long run growth rate of the economy neither does it affect the share of labour allocated to R&D s_A. However trade liberalisation and openness to multinational firm production had a level effect on mass of varieties produced $M(t) \downarrow$ due to slow down in variety creation. Gustafsson and Segerstrom (2010) arrives to the same reasoning and explained that as cut-off costs decreases more firms enter export market hence a rise in the number of available varieties but this leads to more competition and lower profits, this in turn induces fewer firms to enter the market thus slowing down the variety process creation.

Chapter 4; ***Impact of exhaustible resource on opening for international trade: Hartwick's rule and taste for foreign goods***

We develop a model of world economy with two countries where one of them dubbed home sells the exhaustible resource to final producers in both countries, which compete at the final goods market. The interaction between final producers is reached via the sticky price mechanics, whereas price continuously adjusts to produce final product quantities.

We study the effect of opening up to trade on the home resource-abundant economy. It turns out that resource trade may foster structural change via substitution of resource by technology in production only if the home country is sufficiently developed and tastes of consumers are not too much biased towards foreign goods. Otherwise open trade may be detrimental for the home economy, since expenditures grow faster than resource rents and the Hartwick's rule does not hold.

Chapter 2

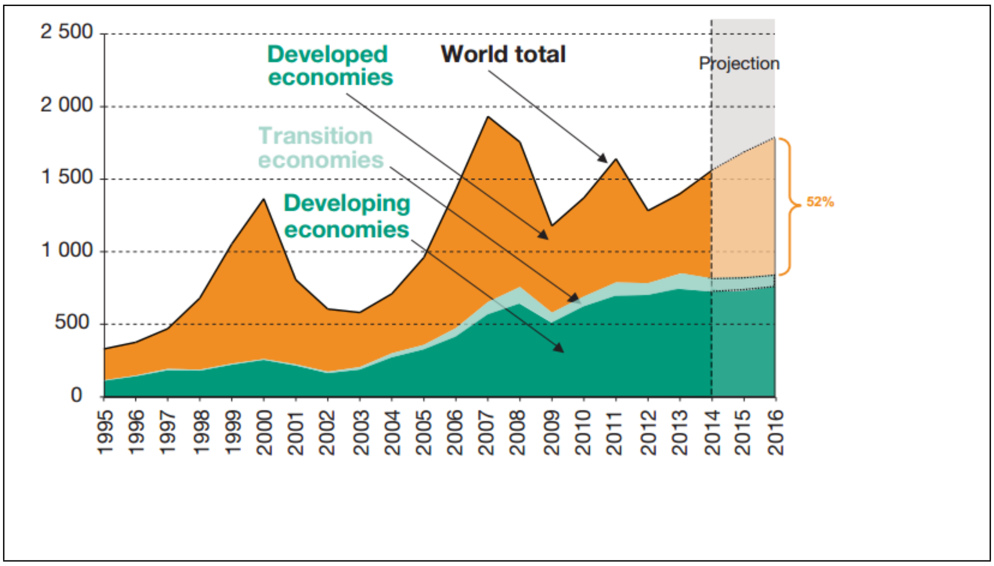
Quantifying Gains from Trade and Foreign Direct Investment with Heterogeneous Firms

2.1 Introduction

Quantifying gains from trade and foreign direct investment is a long-standing issue of economic importance. Recently Arkolakis et al. (2012) developed a static measure of gains from trade. The paper did not however address total gains from trade and FDI. According to the OECD (2013) foreign direct investment (FDI) statistics report, in the first quarter of 2013, the magnitude of global FDI inflows and outflows were 357 and 353 billion US dollars respectively (figure 2.1 shows the global trends of FDI transactions). Furthermore, UNCTAD projected that global FDI flows could rise to \$1.6 trillion in 2014, \$1.75 trillion in 2015 and \$1.85 trillion in 2016. The report asserted that the rise will be mainly driven by investments in developed economies as their economic recovery starts to take hold and spread wider. In this regard, we extend the analysis of Arkolakis et al. (2012) by quantifying total gains from both trade and FDI.

In the same frame of mind, Ramondo and Rodriguez-Clare (2013) stated that worldwide sales of multinational companies are in the order of twice that of total exports. Based on this revelation, the objective of this study is to measure gains from trade and FDI (via intra firm trade of intermediate good inputs). Where current research considers gains from FDI and trade, their approach differs from a heterogeneous productivity framework of Melitz (2003) and Helpman et al. (2004). For example, Ramondo and Rodriguez-Clare (2013) estimated gains from openness based on the framework developed by Eaton and Kortum (2002a) .

Figure 2.1: Global FDI inflows from 1995 to 2013 and projections 2014 to 2016 in Billion US$



Source:UNCTAD World Investment Report 2014

Generally gains from FDI include amongst others;

- Gains from resource transfer; Findlay (1978), Lall (1974), Loungani and Razin (2001), and Romer and Frankel (1999), asserted that FDI brings physical capital, new technology, increased competition and improved methods of conducting business. These resources are transferred into domestic firms leading to increased average productivity, increased product and process innovations;
- Employment gains; FDI has the potential to increase employment in the local economy which leads to increased spending and increased multiplier effects in the domestic economy;
- Balance of payments effects; gains from the balance of payments effects are derived from improvement in the capital account due to the inflows of new capital into the host country and improvements in the current account balance because of possible decline in imports of goods and service;
- Technology spillovers; Hymer (1976) noted that technological spillovers are derived from adopting the product, process and organizational innovations initiated by the multinational firm. It is worthwhile to mention at this point that, in our present study we only consider the change in per capita value of real income accruing to consumers as the measure of gains from FDI and trade.

In our analysis, gains from openness equals the (absolute value of) the percentage change in real income associated with moving one country from the current, observed trade and FDI equilibrium to a counter-factual equilibrium. Gains from trade were derived by Arkolakis et al. (2012) as $G_j = 1 - (\lambda_j)^{-\frac{1}{\varepsilon}}$, comprising of two static measures;(i) share of expenditure on domestic goods and (ii) an elasticity of imports with respect to variable trade costs, (trade elasticity). In this study we find that there are additional gains from FDI which is facilitated by traded intermediate inputs. Consequently total gains from trade and FDI(gains from openness) are; $\hat{W}_j = \left(\lambda_{M_F}^{\frac{1}{\gamma}} + \lambda_{E_X}\right)^{-\frac{1}{\varepsilon}}$. Where ε is the elasticity of substitution, γ is the ratio between wages in the destination country where a foreign affiliate is set up and intermediate input costs. Therefore gains from openness (Trade and FDI) are higher than gains in a trade only model as shown by Arkolakis et al. (2012).

The set up of our model is based on the model developed by Helpman et al. (2004) who extends the Melitz model by incorporating FDI and shows that firms sort into exporting and FDI based on heterogeneity on their productivity levels. They predict that only firms that are productive enough to cover the fixed cost of exporting can export. Since the fixed cost of FDI is larger than that of exporting, firms that conduct FDI must be more productive than firms that only export. However, we consider the use of intermediate goods in foreign affiliates production plants shipped from country of origin of the multinational firm. This is consistent with Bernard et al. (2009) as they found out that multinationals comprise a substantial majority of U.S. trade, roughly 90% of U.S. exports and imports. This will allow us to examine the relationship between trade and FDI. This approach is closely related to Brainard (1997), Markusen and Venables (2000), Helpman et al. (2004) who asserted that trade and FDI are complementary, i.e., a rise in trade costs leads to a replacement of exports by FDI production in destination countries. The standard production frameworks in Eaton and Kortum (2002a) and Melitz (2003) provide no relationship between the intermediate input share and the trade elasticity.

A model closely related to our work is provided by Keller and Yeaple (2009), where foreign affiliate production used intermediate inputs in production shipped from the parent firms. They found out that trade facilitates FDI and FDI boosts trade. A quantifiable multi-country general equilibrium model developed by Tintelnot (2013) showed that gains from multinational production are much smaller due to fixed costs of establishing foreign plants. The author pointed out that these gains may increase significantly if free entry is taken into account. Therefore we allow for free firm entry in our model.

With this framework, we conduct a quantitative analysis. We use data for the year 2009 for all U.S. affiliates and parent firms which are non banking. I use the BEA data set which provides aggregate exports by US parents firms to all its foreign affiliates and and aggregate imports from all US foreign affiliates to the US parents. We find out that, with trade elasticity of 4, when the intermediate share is 1/3 the gains from trade in the spirit of Arkolakis et al. (2012) is 3% while the gains from openness is 11% the difference between our set up (gains from trade and multinational firm intra firm trade) and the trade only gains is 8%.

The rest of this paper is organised as follows, Section [2.2] presents the theoretical model describing the assumptions, technology and cost structure of domestic, export and FDI firms, preferences and demand, and finally the firm's decision problem. Section [2.3], shows aggregation and equilibrium of the model. Section [2.4] shows the theoretical trade and FDI gains derived from the model, finally section [2.5] provides the quantitative relevance on the result by using data to calculate the total gains from openness. Lastly the conclusion is provided in Section [2.6].

2.2 Model

Following Helpman et al. (2004), there are N countries and $H+1$ sectors. Each country i is endowed with L_i units of labour with wage rate w_i. Consumers in each country derive utility from $H+1$ sectors; the first sector 0 produces a homogeneous good with 1 unit of labour per unit output and the H sectors produce differentiated products. We assume that the homogeneous good is freely traded with wage rate equal to 1 and it is produced under constant returns to scale which will ensures factor price equalisation as long as each and every country produces it. An exogenous fraction of income $\sum_h 1-\beta_h$ is spent on the homogeneous sector and β_h is spent on the differentiated goods sector.

2.2.1 Preferences and Demand

Preferences are a Cobb-Douglas aggregate of the homogeneous good sector and differentiated traded goods and CES across a continuum of differentiated goods in $h = 1, ..., H$ sectors.

$$U = q_0^{1-\beta_h} \prod_{h=1}^{H} \left(\int_{\omega \in \Omega_h} q_h(\omega)^{\frac{\sigma_h - 1}{\sigma_h}} d\omega \right)^{\frac{\sigma_h}{\sigma_h - 1}\beta_h} \tag{2.1}$$

The elasticity of substitution $\sigma > 1$ between varieties and within the firm is the same, hence we use the standard optimal pricing rule. We proceed in the same spirit as Helpman et al. (2004), we drop the sectoral index h in light of the view that all sectoral variables refers to a particular sector h that produces differentiated products. The consumer's problem is to maximize their utility subject to a budget constraint. This yields the following export and FDI demand functions for the differentiated products;

$$q(\omega) = \beta \frac{p(\omega)^{-\sigma}}{P_j^{1-\sigma}} Y_j \tag{2.2}$$

Where P_j is the ideal price index in country j, and $p(\omega)$ is the price charged on goods. With monopolistic competition and Dixit-Stiglitz preferences, the price charged is a constant mark-up $\frac{\sigma}{\sigma-1} = \rho$ of the costs of production, therefore a firm from country i exporting or carrying FDI in country j charges the following prices;

$$p_{ij}^{E_X}(\varphi) = \rho \frac{w_i \tau_{ij}}{\varphi} \quad \text{(price charged by exporting firm)} \tag{2.3}$$

$$p_{ij}^{M_F}(\varphi) = \rho \frac{(w_j)^{\gamma}(w_i \tau_{ij})^{1-\gamma}}{\varphi^{\gamma}} \quad \text{(price charged by a multinational firm)} \tag{2.4}$$

where, E_X, M_F refers to export and multinational firm respectively and φ is the productivity of the firm. The price index P_j of the aggregate bundle of exports and FDI is the ideal price index in country j

$$(P_j)^{1-\sigma} = \left(\int_{\varphi_{ij}^{E_X}}^{\varphi_{ij}^{M_F}} (p_{E_X})^{1-\sigma} dG(\varphi) + \int_{\varphi_{ij}^{M_F}}^{\infty} (p_{M_F})^{1-\sigma} dG(\varphi) \right) \tag{2.5}$$

The above aggregate price index can be split into price index for imported goods and price index for foreign affiliate goods as follows, (dropping all the sector sub scripts),

$$(P_j^{EX})^{1-\sigma} = N_{E_X} \left(\int_{\varphi_{ij}^{E_X}}^{\varphi_{ij}^{M_F}} (p_{ij}^{E_X})^{1-\sigma} dG(\varphi) \right) = (N_{E_X})^{\frac{1}{1-\sigma}} p_{ij}^{E_X}(\varphi_{ij}^{E_X}) \tag{2.6}$$

$$(P_j^{M_F})^{1-\sigma} = N_{M_F} \left(\int_{\varphi_{ij}^{M_F}}^{\infty} (p_{ij}^{M_F})^{1-\sigma_h} dG(\varphi) \right) = (N_{M_F})^{\frac{1}{1-\sigma}} p_{ij}^{M_F}(\varphi_{ij}^{M_F}) \tag{2.7}$$

In both cases, the price index is a multiple of the distribution of lowest prices and mass of varieties N_{E_X} and N_{M_F} for exports and FDI respectively from country i to country j. It is decreasing in average productivity cut-offs and number of varieties available, and increasing in wages, transportation costs. It then follows that the aggregate price index (2.5) can be written as the sum of equations (2.6) and (2.7) as:

$$(P_j)^{1-\sigma} = (N_{E_X})^{\frac{1}{1-\sigma}} p_{ij}^{E_X}(\varphi_{ij}^{E_X}) + (N_{M_F})^{\frac{1}{1-\sigma}} p_{ij}^{M_F}(\varphi_{ij}^{E_X}) \tag{2.8}$$

The aggregate price index is decreasing in cut-off productivities and total number of varieties provided by multinational production and imports. Since the trade of final and intermediate goods are subject to iceberg transportation costs, the aggregate price index in an increasing function of transportation costs, and wages paid to labour in foreign affiliate plant and country of origin.

2.2.2 Technology and Cost Structure

Domestic Firm Costs

Firms in country i can enter the domestic market by paying fixed costs of entry $f_{E_i} > 0$ (measured in units of labour) which is thereafter sunk. Expectation of future positive profits is the only motivation for firms to incur these sunk costs. After observing the productivity parameter φ from the distribution $G(\varphi)$ the firm may start production and pay additional fixed costs $f_{D_i} > 0$ with marginal costs

$$C = \frac{w_i}{\varphi} \tag{2.9}$$

Depending on the level of their productivity, domestic firms have two alternatives of serving foreign markets, either through exports or setting up a foreign affiliate in the destination country[1]. Proximity concentration trade off provides a motivation for decisions to either export or FDI.

Exporting Firm Costs

If φ is sufficiently low, a firm may decide to start exporting and pays additional fixed costs for each export destination. These fixed costs are given by $f_{E_X} > 0$. The marginal cost for an exporting firm from country i to export destination j is given by

$$C_{ij}^{E_X} = \frac{w_i \tau_{ij}}{\varphi} \tag{2.10}$$

Exports are subject to melting iceberg transportation costs $\tau_{ij} > 1$, i.e., τ_{ij} units have to be shipped from country i for one unit of the good to arrive in country j. As in Melitz (2003) the marginal costs in equation (2.10) of an exporting firm increase as the transportation and labour costs increase and they decrease with a rise in firm productivity.

FDI Firm Costs

If a firm originating from country i chooses to set up a production plant in a foreign country j it incurs fixed costs of FDI defined as f_{M_F}. These fixed costs includes among others, the costs of setting up or acquiring a physical structure, marketing and information and etc. Following Irarrazabal et al. (2013) we introduce intra-firm trade into FDI to be consistent with the fact that the relationship between export and outward FDI is complementary. However our analysis does not consider any sourcing strategies for intermediate goods as in the case of Helpman (2006). A thorough illustration of technological trade off between intermediate good production in the country of origin and in the destination country is provided by Keller and Yeaple (2009).

[1] for analysis of gains from openness, we restrict our study hereafter only to exporting and FDI, see Arkolakis et al. (2012), Irarrazabal et al. (2013),

Foreign affiliates from country i in country j use intermediate inputs from country i, these intermediate goods are subject to melting iceberg transportation costs τ_{ij}. Therefore the cost function is of cobb-douglas form given by

$$C_{ij}^{M_F} = \left(\frac{1}{\varphi^{\gamma}}\right)\left(\frac{w_j}{\gamma}\right)^{\gamma}\left(\frac{w_i\tau_{ij}}{1-\gamma}\right)^{1-\gamma} \tag{2.11}$$

Where

$0 \leq \gamma < 1$, is the ratio of foreign plant labour cost to intermediate inputs used produce goods in foreign affiliates. On the right hand side of equation (2.11) the first term indicates the inverse relationship between the costs and productivity level in the same spirit as the discussion under exporting firm costs. The second term shows the wage rate in the destination country j and the third term is the cost of intermediate input used in the foreign affiliate production. Note that when $\gamma = 1$ equation (2.11) yield identical marginal costs to those in the paper of export versus FDI with heterogeneous firms by Helpman et al. (2004). From (2.11), an increase in the size of trade costs between the affiliate and country of origin results in an increase in marginal trade costs of the affiliate. Equation (2.11) derives from a cobb-douglas production function with a cost share γ for labour and $1-\gamma$ for intermediate input. This can be represented as

$$q = \left(\frac{1}{\varphi^{\gamma}}\right)\left(\frac{L}{\gamma}\right)^{\gamma}\left(\frac{Q_j^{input}}{1-\gamma}\right)^{1-\gamma} \tag{2.12}$$

Having outlined the demand for the differentiated varieties, the cost structures for export and FDI prodution, in the same spirit as Helpman et al. (2004), sales are therefore given by the following equation:

$$X_{ij_v}(\varphi_v) = \beta\left(\frac{p_{ij_v}(\varphi)}{P_j}\right)^{1-\sigma} Y_j \tag{2.13}$$

Such that $v = [E_X, M_F]$

2.2.3 Firms Decision Problem and Cut-off Costs and Productivities

A firm from country i will serve a foreign country j if the operating profits are sufficient to cover costs of entry (fixed costs). We get exporting and FDI firm profits respectively as:

$$\pi_{ij}^{E_X} = \beta P_j^{\sigma-1}\left(\frac{\rho w_i\tau_{ij}}{\varphi}\right)^{1-\sigma}\frac{Y_j}{\sigma} - f_{E_X} \tag{2.14}$$

$$\pi_{ij}^{M_F} = \beta P_j^{\sigma-1}\left(\frac{\rho w_j^{\gamma}(w_i\tau_{ij})^{1-\gamma}}{\varphi^{\gamma}}\right)^{1-\sigma}\frac{Y_j}{\sigma} - f_{M_F} \tag{2.15}$$

Firms will serve foreign markets through exports or FDI if the operating profits are greater than fixed costs. Therefore the zero profit condition for exporting and FDI firms is given by the following equations respectively:

$$\pi_{ij}^{E_X} = 0 \Longrightarrow \beta P_j^{\sigma-1}\left(\frac{\rho w_i\tau_{ij}}{\varphi}\right)^{1-\sigma}\frac{Y_j}{\sigma} = f_{E_X} \tag{2.16}$$

$$\pi_{ij}^{M_F} = 0 \Longrightarrow \beta P_j^{\sigma-1}\left(\frac{\rho w_j^{\gamma}(w_i\tau_{ij})^{1-\gamma}}{\varphi^{\gamma}}\right)^{1-\sigma}\frac{Y_j}{\sigma} = f_{M_F} \tag{2.17}$$

From the zero profit conditions of exporting and FDI firms in equations (2.16) and (2.17) we can find the cutoff productivities as follows;

Export Productivity Cut Off

We revert to the zero profit condition for exporting forms and substitute export firm costs from equation (2.10). We find that the productivity cut-off for exporting firms is given by;

$$\tilde{\varphi}_{ij}^{E_X} = \kappa\left(\frac{f_{ij}^{E_X}}{Y_j}\right)^{\frac{1}{\sigma-1}} P_j^{-1} w_i\tau_{ij} \tag{2.18}$$

where $\kappa = \left(\frac{\sigma}{\beta}\right)^{\frac{1}{\sigma-1}}\rho$. Firms with an export productivity cut-off in equation (2.18) have just break even. Therefore firms with a higher productivity than export productivity cut-off in equation (2.18) expect to make positive profits. This result is consistent with Helpman et al. (2004) and with Melitz (2003)

FDI Productivity Cut-off

This is the productivity cut-off level where the firm is indifferent between exporting and FDI. Moreover, if profits of having multinational foreign affiliates are greater than profits accrued from accessing foreign markets through exports, then FDI is the preferred choice. This is the same argument provided in Helpman et al. (2004).

$$\tilde{\varphi}_{ij}^{M_F} = \left[\kappa\left(\frac{f_{ij}^{M_F} - f_{ij}^{E_X}}{Y_j[(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1]}\right)^{\frac{1}{\sigma-1}}\frac{w_i\tau_{ij}}{P_j}\right]^{\frac{1}{\gamma}} \tag{2.19}$$

Firms with a productivities between export and FDI cut-off productivities, i.e., between (2.18) and (2.19) will serve the foreign market through exports only. Firms with a productivity cut-off given by (2.19) have just break even and make zero profits if they access the foreign market via FDI. However all firms with a productivity above the FDI productivity cut-off expect to make positive profits as they are able to cover fixed costs.

To ensure that the FDI cut-off productivity is higher than export productivity, i.e., $\varphi_{ij}^{M_F} > \varphi_{ij}^{E_X}$, we assume that $f_{M_F}\Omega_{ij}^{\gamma(\sigma-1)} > f_{E_X}\tau^{(\sigma-1)}$.

2.3 Aggregation and Equilibrium

In this section we present aggregation of cut-off productivity levels, price indices, exports and FDI sales, in country j. There is a continuum of prospective entrants that are the same ex-ante. To enter, they pay a sunk entry cost $f_{E_i} > 0$ units of labour, $w_j f_{E_X}$ in the case of exporting firms or $w_j f_{M_F}$ in the case of FDI firms. Thereafter, firms independently draw productivity levels from a common distribution from a PDF $g(\varphi) = k\underline{\varphi}^k\varphi^{-(k+1)}$ with positive support over $(0, ..., \varphi_{ii})$ and a continuous cumulative distribution $G(\varphi) = 1 - \left(\frac{\varphi}{\varphi_{ii}}\right)^k$: φ_{ii} is the minimum productivity in the productivity distribution. Productivity is assumed to be Pareto distributed and the degree of firm heterogeneity is summarized by the shape parameter $k > \sigma - 1$. This insures that the distribution of productivity draws finite variances. Since k is an inverse measure of variance, lower values of k correspond to greater firm heterogeneity (larger variance of firm productivity).

The Pareto distribution has a number of properties that make it analytically tractable hence its use on models of firm selection into export and FDI markets. From empirical evidence, it is a good approximation of the upper tail of distribution of firm sizes, this was first noted by Simon and Bonini (1958). Pareto distribution for U.S. and European firms used to predict FDI was estimated by Helpman et al. (2004). A key feature of a Pareto distributed random variable is that when truncated the random variable retains a Pareto distribution with the same shape parameter k. Therefore if entry is subject to an endogenous productivity cut-off, the distribution of the technologies that make the cut remains Pareto distributed. Another key feature of a Pareto distributed random variable is that power functions of this random variable are themselves Pareto distributed. Therefore, individual prices have a Pareto distribution, with a constant elasticity of demand, so do sales, hence firm size and variable profits are Pareto distributed.

In the same vein as Melitz (2003) we let M denote the equilibrium mass of incumbent domestic firms in each country. Firms decisions to enter into domestic industry and or into foreign markets via exports and FDI are based on comparing expected profits and costs of entering the market. The probability of entry in the home market, exporting and into FDI (conditional on successful entry) are given by the following equations respectively:

$$\theta_{iD} = 1 - G(\varphi_i^*) \tag{2.20}$$

$$\theta_{E_X} = \frac{G(\varphi_{M_F}^*) - G(\varphi_{E_X}^*)}{1 - G(\varphi_i^*)} \tag{2.21}$$

$$\theta_{M_F} = \frac{1 - G(\varphi_{M_F}^*)}{1 - G(\varphi_i^*)} \tag{2.22}$$

Using these conditional probabilities of successful entry, the mass of firms that enter foreign country via exports and FDI are given by $N_{E_X} = \theta_{E_X} N$ and $N_{M_F} = \theta_{M_F} N$, respectively. In the same spirit as Melitz (2003) we assume that the economy under study can trade with $n \geq 1$ other countries, the world is then comprised of $n + 1 \geq 2$. Total mass of varieties available to consumers in each country is given by the total mass of firms competing in the country,

$$N = N_D + nN_{E_X} + nN_{M_F} \tag{2.23}$$

Weighted productivity average

$$\widehat{\varphi} = \frac{1}{N}\left(N\widehat{\varphi}_{iD}^{\sigma-1} + nN_{E_X}\tau^{1-\sigma}\widehat{\varphi}_{E_X}^{\sigma-1} + nN_{M_F}\tau^{(1-\sigma)(1-\eta)}\widehat{\varphi}_{M_F}^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{2.24}$$

Aggregate price index as a function of weighted aggregate productivity and mass of varieties is given by $P = N^{\frac{1}{1-\sigma}}p(\widehat{\varphi})$. This price index is a decreasing as total mass of varieties increases and aggregate productivity increases. This implies that consumer welfare is given by $W = N^{\frac{1}{\sigma-1}}\rho\widehat{\varphi}$. From this welfare function, we have (a) variety gains; increase in consumer welfare with a rise in varieties, (b) productivity gains, higher weighted average productivity implies that costs of production are lower hence lower prices are charged for consumer goods.

2.3.1 Free-Entry Condition

Prior to entry, potential entrants contemplate profits they would incur if they enter and compare the expected profits to fixed costs of entry, f_{E_i}. In a stationary equilibrium as long as a firm is active it earns the same profits in each period. Ex-ante, Free entry condition implies that expectation of future profits $\overline{\pi}$ conditional on successful entry must be equal to the fixed costs of entry f_{E_i}. Net value of entry $\overline{v}(\varphi)$ at time $(t = 0)$

given the probability of dying in each period δ is

$$\overline{v}(\varphi) = E\left[\sum_{t=0}^{\infty}(1-\delta)^t\overline{\pi} - f_{E_i}\right] = \frac{1-G(\varphi^*)}{\delta}\overline{\pi} - f_{E_i} \tag{2.25}$$

According to the free entry condition, firms will enter until the net value of entry is driven to zero. Market shares of firms shrink as more firms enter the market until expected profits equal cost of entry.

Where the average revenues and profits across all domestic firms earned from both domestic, export and FDI revenues are given as:

$$\overline{r} = \pi_{iD}(\tilde{\varphi}) + n\theta_{E_X} r_{E_X}(\tilde{\varphi}) + n\theta_{M_F} r_{M_F}(\tilde{\varphi}) \tag{2.26}$$

Now taking into account the zero profit condition the above equation becomes

$$\overline{r} = \sigma(\overline{\pi} + f_{iD} + n\theta_{E_X} f_{E_X} + n\theta_{M_F} f_{M_F}) \tag{2.27}$$

$$\overline{\pi} = \pi_{iD}(\tilde{\varphi}) + n\theta_{E_X}\pi_{E_X}(\tilde{\varphi}) + n\theta_{M_F}\pi_{M_F}(\tilde{\varphi}) \tag{2.28}$$

These average Profits pin down the equilibrium mass of incumbent firms given as

$$N = \frac{R}{\overline{r}} = \frac{L}{\sigma(\overline{\pi} + f_{D_i} + n\theta_{E_X} f_{E_X} + n\theta_{M_F} f_{M_F}})\tag{2.29}$$

2.3.2 Labour Market clearing condition

The labour market clearing condition requires that total labour demand in domestic market equal labour total labour supply, i.e aggregate revenue (derived from consumers total expenditure on d differentiated goods) equal total payments to labour L. We then use the labour market clearing condition to solve for the mass of firms. The labour market clearing conditions implies that total labour used for production and the entry cost must equal total labour endowment.We get the following expression for the mass of firms:

$$N_i = \frac{L_i(\sigma-1)(\varphi/\varphi_{ii})^k}{k\sigma f_{E_i}} \tag{2.30}$$

2.3.3 Aggregation of Sales

We consider aggregation of export and FDI sales in order to determine the intensive and extensive margins.

Export sales

The approach is to multiply the probability of export conditional on successful entry by the export revenues.

$$X_{ij}^{E_X} = \frac{G(\varphi_{M_F}^*) - G(\varphi_{E_X}^*)}{1 - G(\varphi_i^*)} N_i \left(\int_{\varphi_{ij}^{E_X}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma - 1} w_i \tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{E_X}) \right) \tag{2.31}$$

Solving the integral and substituting for the export cut-off productivity yields the following decomposition for total exports into intensive and extensive margins (see appendix for derivations);

$$X_{ij}^{E_X} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{E_X}}\right)^k N_{E_X}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k - \sigma + 1}\right) f_{E_X} w_i}_{\text{intensive}} \tag{2.32}$$

At the extensive margins we have the number of firms that supply foreign market via exports and at the intensive margins we have the average sales of operating exporting firms. The effect of a change in trade costs on export sales also yields extensive and intensive margins as Chaney (2008) who has a complete exposition of the derivations);

$$\frac{dlnX_{ij}^{E_X}}{dln\tau_{ij}} = -\underbrace{(\sigma - 1)}_{\text{intensive}} - \underbrace{(k - \sigma + 1)}_{\text{extensive}} \tag{2.33}$$

The same observations as Chaney (2008) holds, i.e., intensive margins are more sensitive to change in trade barriers and the extensive margins are less sensitive. Consider a situation where the elasticity of substitution is high, this implies tough competition between firms. Lowering trade barriers will induce new and less productive firms to enter, but low productivities implies that they are at a cost disadvantage translating into smaller market share. Consequently these low productive firms have a smaller impact on aggregate trade. However if elasticity of substitution is very low, competition is low and new entrants can capture larger market resulting in larger impact on aggregate trade. Therefore, higher elasticity of substitution magnifies sensitivity of intensive margins to changes in trade costs, but sensitivity of extensive margins is less.

FDI or Multinational Sales

We proceed by multiplying the probability of entry into foreign market via FDI with revenues accrued from FDI, we get the following gravity equation

$$X_{ij}^{M_F} = \frac{1-G(\varphi_{M_F}^{*})}{1-G(\varphi_i^{*})} N_i \left(\int_{\varphi_{ij}^{M_F}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{M_F}) \right) \tag{2.34}$$

Solving the integral and substituting for the FDI productivity cut-off, we get t he following decomposition of extensive and intensive margins of FDI (see appendix);

$$X_{ij}^{M_F} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{M_F}}\right)^k N_{M_F}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1}\right) w_j (f_{M_F} - f_{E_X})}_{\text{intensive}} \tag{2.35}$$

In the same frame of mind as Irarrazabal et al. (2013), the derivation of the overall effect of a change in variable trade barriers on total affiliate sales can be decomposed into intensive and extensive margins as;

$$\frac{dlnX_{ij}^{M_F}}{dln\tau_{ij}} = -\underbrace{(1-\gamma)(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)\chi_{M_F}}_{\text{extensive}} \tag{2.36}$$

Where χ_{M_F} is defined as the elasticity of FDI cut-off to variable trade barriers.

$$\chi_{M_F} = \frac{(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)}(\gamma-1) - 1}{(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1} \tag{2.37}$$

In the case of FDI, the sensitivity of intensive margins to changes in trade barriers arises due to the intermediate inputs which are traded from country of origin of the foreign affiliate. The presence of this intra-firm trade implies that total FDI sales are inversely related to trade costs. At the extensive margins, higher trade costs leads to reduction in total FDI (multinational) sales. A higher elasticity of substitution in the presence of low trade costs induces new firms to enter foreign market via FDI provided they meet the cut-off productivity level of FDI. The higher elasticity of substitution implies fierce competition. Lower productivities of those which meet the cut-off FDI productivity implies that they are at a cost disadvantage and only capture a small portion of the market, hence the impact of these low productivity firms on FDI sales is very small. However, a decrease in elasticity of substitution implies less competition and new FDI entrant firms can capture large market size with a large impact on FDI sales.

Aggregate Sales (from all operations, i.e both exports and FDI)

Aggregate sales from all operations are therefore a sum of the total ex port(trade) and FDI (multinational) sales, given by;

$$X_{ij}^{-k} = N_i \frac{k}{k-\sigma+1}\left(\frac{Y_j}{P_j^{1-\sigma}}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij})\right)^{-k}(\sigma w_j)^{1-\frac{k}{\sigma-1}}\text{x} \\ \left(f_{E_X}^{1-\frac{k}{\sigma-1}} + (\frac{f_{M_F}-f_{E_X}}{(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)}-1})^{1-\frac{k}{\sigma-1}}\right) \tag{2.38}$$

Aggregation of Price Index

We define aggregate price, but decompose it into export price index and FDI price index (derivations in the appendix). This is to facilitate the derivation of the total welfare gains which we define in the proceeding section.

2.3.3.1 Trade only Price Index

$$P_j^{E_X} = \frac{G(\varphi_{M_F}^*)-G(\varphi_{E_X}^*)}{1-G(\varphi_i^*)}N_i\left(\int_{\varphi_{ij}^{E_X}}^{\infty}\varphi^{\sigma-1}(\frac{\sigma}{\sigma-1}w_{ij}\tau_{ij})^{1-\sigma}dG(\varphi_{ij})\right)^{\frac{1}{1-\sigma}} \tag{2.39}$$

Evaluating the integral and substituting for cut-off productivities

$$P_j^{-k} = N_{E_X}\frac{k}{k-\sigma+1}\left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}}\varphi_{min}^k\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{-k}(\sigma w_j f_{E_X}))^{1-\frac{k}{\sigma-1}} \tag{2.40}$$

FDI only Price Index

$$P_{ij}^{M_F} = \frac{1-G(\varphi_{M_F}^*)}{1-G(\varphi_i^*)}N_i\left(\int_{\varphi_{ij}^{M_F}}^{\infty}\varphi^{\sigma-1}(\frac{\sigma}{\sigma-1})^{1-\sigma}[(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)}-1](w_{ij}\tau_{ij})^{1-\sigma}dG(\varphi_{ij})\right)^{\frac{1}{1-\sigma}} \tag{2.41}$$

Evaluating the integral and substituting for cut-off productivities

$$P_j^{-k} = N_{M_F}\frac{k}{k-\sigma+1}\left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}}\varphi_{min}^k\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{-k}\left(\sigma w_j(\frac{f_{M_F}-f_{E_X}}{(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)}-1})\right)^{1-\frac{k}{\sigma-1}} \tag{2.42}$$

. Aggregate price index from all operations are therefore a sum of the total export(trade) and FDI(multinational) price indices, given by;

$$P_{ij}^{1-\sigma} = \frac{G(\varphi_{M_F}^*) - G(\varphi_{E_X}^*)}{1 - G(\varphi_i^*)} N_i \left(\int_{\varphi_{ij}^{E_X}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)$$
$$+ \frac{1 - G(\varphi_{M_F}^*)}{1 - G(\varphi_i^*)} N_i \left(\int_{\varphi_{ij}^{M_F}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)$$

Evaluating the Integral and substituting for the cut off productivities

$$\begin{aligned} P_j^{-k} =& N_i \frac{k}{k-\sigma+1} \left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}(w_i\tau_{ij})\right)^{-k} (\sigma w_j)^{1-\frac{k}{\sigma-1}} \\ \text{x} & \left(f(_{E_X})^{1-\frac{k}{\sigma-1}} + (\frac{f_{M_F} - f_{E_X}}{(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1})^{1-\frac{k}{\sigma-1}} \right) \end{aligned} \tag{2.43}$$

2.4 Welfare Analysis

We are interested in the aggregate effects of trade and FDI on the welfare measure. Welfare is given by the per capita value of real income accruing to consumers. Welfare depends on real labour income derived from export and foreign multinational affiliates in the domestic market and revenues generated by home country firms set up in foreign countries, this welfare measure is given given by:

$$W_j = \frac{w_j}{P_j} \tag{2.44}$$

This shows that the welfare depends on the wage level in country j and the aggregate price index of both trade and fdi P_j. In the following section we will derive the expenditure shares of both trade and FDI as they facilitate the computation of the welfare measure.

Expenditure Shares

Country j's welfare is expressed by Arkolakis et al. (2012) as a function of the share of expenditure that falls on domestically produced goods(which is equal to 1 minus the import penetration ratio. This share, λ_{jj} under autarky is 1 ,therefore total size of gains from openness will be equal to $1-\lambda_j$.In the considered trade models (Armington (1969) ,Eaton and Kortum (2002b), and Melitz (2003)), the change in real income associated with a change in iceberg trade costs with only trade and without FDI is computed as:

$$\hat{W} = \hat{\lambda}_{jj}^{-\frac{1}{\varepsilon}} \tag{2.45}$$

We proceed to derive change in real income associated with change in trade costs in an environment of both trade and FDI. First we determine the expenditure shares of total income of country j spent on foreign varieties, we give separate expositions of trade and FDI shares and finally the aggregate share on all foreign varies(both imports and FDI).

Trade Share (Income spent on Imports)

We present here an expression of total fraction of income of country j spent on goods from country i (trade shares) as a function of wages and labour allocated in each country and parameters(see appendix for derivations)

$$\lambda_{ij}^{E_X} = \frac{(L_i/f_{iE})\varphi_{min_i}^{k} w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{ij})^{-k} f_{ij_{E_X}}^{1-\frac{k}{\sigma-1}}}{\sum\limits_{v}(L_{vj}/f_{vE})\varphi_{min_v}^{k} w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{vj})^{-k} f_{vj}^{1-\frac{k}{\sigma-1}}} \tag{2.46}$$

FDI Expenditure Share

This is the total income spent on goods supplied by multinationals in the host country (see appendix for derivations)

$$\lambda_{ij}^{M_F} = \frac{(L_i/f_{iE})\varphi_{min_i}^{k} w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{ij})^{-k}(f_{M_F}-f_{E_X})^{1-\frac{k}{\sigma-1}}(\Omega_{vj}\tau_{ij})^{\gamma(\sigma-1)}-1)^{1-\frac{k}{\sigma-1}}}{\sum\limits_{v}(L_{vj}/f_{vE})\varphi_{min_v}^{k} w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{vj})^{-k}(f_{M_F}-f_{E_X})_{vj}^{1-\frac{k}{\sigma-1}}(\Omega_{vj}\tau_{vj})^{\gamma(\sigma-1)}-1)^{1-\frac{k}{\sigma-1}}} \tag{2.47}$$

In both the trade and FDI share, the exponent on the wages differs from the exponent on trade costs due to the assumption that fixed exporting costs are incurred in terms of labour in the source country. The aggregate expenditure share on all foreign varieties is given by adding the last two expressions for trade (2.46) and FDI (2.47) expenditures. These expenditure share expressions are then used to write the generalised welfare formula in equation (2.44) as a function of expenditure shares and elasticities. Under Pareto assumption knowing a country's domestic share of trade and FDI and shape parameter of the productivity distribution k is sufficient to determine welfare gains from openness. To achieve this, we first write the export price index in equation (40) and FDI price index in equation (42) as a function of country j's share of trade(FDI) with itself, its export(FDI) wage, the export(FDI) labour allocation and parameters.

Welfare derived from Trade

$$W_j = \left(\frac{w_j}{P_j}\right) = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-k} \sigma^{-\frac{k}{\sigma-1}} \frac{\sigma-1}{k-\sigma+1} \varphi_{min_j}^k f_{EX}^{1-\frac{k}{\sigma-1}} f_E\right]^{\frac{1}{k}} (L_j)^{\frac{1}{\sigma-1}} (\lambda_j)^{-\frac{1}{k}} \tag{2.48}$$

Gains from trade are given by,(the domestic trade shares in the closed economy are fixed at $\lambda^{\text{closed}} = 1$):

$$\hat{W}_j = \frac{W_j^{open}}{W_j^{closed}} = \hat{\lambda}_j^{-\frac{1}{k}} \tag{2.49}$$

5In a similar manner a change in welfare can be expressed as a function of domestic expenditure on FDI as

Total Welfare(Gains from Openness)

$$\begin{aligned} W_j^* &= \left(\frac{w_j}{P_j}\right) = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-k} \sigma^{-\frac{k}{\sigma-1}} \frac{\sigma-1}{k-\sigma+1} \varphi_{min_j}^k \right. \\ & \left. \left((f_{M_F} - f_{E_X})^{1-\frac{k}{\sigma-1}} [\Omega j^{\gamma(\sigma-1)} - 1]^{1-\frac{k}{\sigma-1}} + f_{EX}^{1-\frac{k}{\sigma-1}} \right) f_{jE}^{-1} \right]^{\frac{1}{k}} \\ & (L_j)^{\frac{1}{\sigma-1}} \left(\lambda_{M_F}^{\frac{1}{\gamma}} + \lambda_{E_X} \right)^{-\frac{1}{k}} \end{aligned}$$

Country j's gains from openness is given by;

$$\hat{W}_j = \left(\lambda_{M_F}^{\frac{1}{\gamma}} + \lambda_{E_X} \right)^{-\frac{1}{k}} \tag{2.50}$$

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2.5 Quantitative Relevance: Theory to Data

As a final step in this chapter, we examine the quantitative relevance of our results, we compare the welfare properties of the trade only set up as in Arkolakis et al. (2012) and intra-firm trade within the multinationals firms. We choose standard values for the heterogeneous firm model's parameters based on central estimates from the existing empirical literature. Our starting point is to specify values for the intermediate input share $(1-\gamma)$ and elasticity of substitution σ and the shape parameter of the Pareto distribution k and the share of expenditure on domestically produced goods λ_j. We limit this quantitative analysis to the U.S. as the data on USA parent firms and their foreign affiliates are readily and publicly available on the BEA (Bureau of Economic Analysis) website.

We quickly point out that to obtain estimates for the shape parameter and trade elasticity parameters we would need data on iceberg trade costs and tariffs. In the cases of the Armington (1969), Krugman (1980) and Eaton and Kortum (2002a)) models, one would estimate a gravity equation under the assumption $\varepsilon = k$, in the Melitz (2003) case, we have $\varepsilon \neq k$. This implies that an estimation exercise of the gravity equation will have to be carried out. At the objective of this paper is not concerned with the estimation of trade elasticities or other model parameters, we acknowledge that this is not a trivial exercise. The goal of this paper is to shed light on the total gains from trade and FDI in order to enable comparison with the quantitative exercise of Arkolakis et al. (2012).

The share of domestic expenditure under autarky would be equal to one, this implies that the total size of the gains from trade, defined as the absolute value of the percentage change in real income as we move from the observed equilibrium to autarky, is $1-\lambda^{\frac{-1}{\varepsilon}}$ for trade only gains; $1-\lambda^{\frac{-1}{\varepsilon(1-\gamma)}}$ for FDI (intra-firm trade) only gains and. In Arkolakis et al. (2012), the calculations of the import penetration ratios are based on the 2006 edition of the OECD Input-Output Database: as imports over gross output (rather than GDP), so that they can be interpreted as a share of (gross) total expenditures allocated to imports. There is a difference in these two approaches; using gross outputs includes intermediate imports while using GDP would not include imported intermediates. We use OECD Stan database for gross outputs.

Using the BEA data base we extract intra firm exports of intermediate inputs from U.S. parent firms to their affiliates abroad (U.S. Exports of Goods Shipped by U.S. Parents to Foreign Affiliates) for the year 2009. The BEA estimates cover the universe of non-bank U.S. affiliates of foreign companies. From this data set we are able to distinguish both export level of US parent to its affiliates in the destination countries and import level from affiliates to the Parent. We adopt the BEA definition of U.S. direct investment abroad as *Ownership or control, directly or indirectly, by one U.S. person, or entity, of 10 percent or more of the voting securities of an incorporated foreign business enterprise or an equivalent interest in an unincorporated foreign business enterprise.* A limitation of this quantitative analysis is data restrictions due on affiliate intra-firm trade, the BEA data set only focuses on U.S. affiliate and parent firms data. An inoculation to this data challenge is to use imports by U.S. parents from their foreign affiliates as an approximation for affiliates intra-firm trade activities in the U.S.

For arm's length export data we use the OECD STAN Bilateral Trade Database (BTDIxE) which provides values of imports and exports of goods. BTDIxE was designed to extend the BTD database which provided bilateral trade in goods by industry only. We then proceed to deduct the intra firm trade values from the total trade values obtained in order to determine the arm's length trade values. We then assign the shape parameter of the Pareto distribution. Anderson and Van Wincoop (2004) offer a review studies that offer gravity-based estimates for the trade elasticity all within the range of -5 and -10. Simonovska and Waugh (2014) estimate a trade elasticity of 4.10 or 4.27 depending on the data used. Any of these values would lead to quantitatively similar results. Concerning the share if intermediate goods used, we take the value $\frac{1}{3}$ which is the most widely used in trade literature.

In the table below we compare the gains from arm's length trade, intra firm trade and total gains for the U.S. Here the gains from openness is the total gains from trade and FDI. The last column is the percentage point difference between the gains from openness and the gains from trade in the fashion of Arkolakis et al. (2012)

Gains from Openness in the U.S. for the year 2009				
Parameters	Gains (Arkolakis et al. (2012))	FDI (Intra Firm Trade) Gains	Gains from Openness	% point Increase
$k = 4$, $\gamma = 1/3$	3%	11%	14%	11%
$k = 4$, $\gamma = 1/2$	3%	12%	15%	12%
$k = 10$, $\gamma = 1/3$	1%	5 %	6,00%	4%
$k = 10$, $\gamma = 1/2$	1%	5,1%	6,10%	5%

The results in this table indicate that the total gains from openness rise with the share of intermediate goods used in the multinational affiliates. With trade elasticity of 4, when the intermediate share is 1/3 the gains from trade in the spirit of Arkolakis et al. (2012) is 3% while the gains from openness is 11% the difference between our set up (gains from trade and multinational firm intra firm trade) and the trade only gains in the set up by Arkolakis et al. (2012) is 8%. When we increased the trade elasticity to 10 keeping the intermediate good shares constant at 1/3 the differences in the gains from openness and trade only gains was 3%. In the second case where the intermediate input share is 1/2, the difference was given by is 9% when the elasticity of trade is 10. At both trade elasticities for the same input shares we observe higher gains from openness than in the set up with only trade gains as in Arkolakis et al. (2012). This shows that intra-firm trade of intermediate inputs within the multinational firm is an important source of trade gains that cannot be ignored.

2.6 Conclusion

In this paper, we have revisited the welfare gains from trade for new trade models in the presence of multinational firm production. Provided there are positive marginal cost savings and positive profits from multinational firm production, we get additional gains from FDI. Therefore omission of FDI via intra-firm trade if intermediate good inputs underestimates gains from openness relative to autarky. We anticipate that the bias is largest in economies characterized by high intra-firm trade between parent firms and their foreign affiliates, mostly developed countries.

The multinational affiliates used intermediate inputs provided via intra firm trade, this gives us insights about the interaction of trade and multinational firms. Whenever $\gamma = 1$, there is no intra-firm trade of intermediate goods hence trade and FDI are substitutes as in the case of Helpman et al. (2004)in which FDI was favoured whenever trade costs are high(tariff-jumping). However when $\gamma = 0$, an increase in trade barriers leads to costly intra-firm firm trade of affiliates hence, affiliate profits diminish. Therefore for low values of γ, accessing foreign markets via FDI is most preferred, for high values of γ firm will access foreign market via exports.

We carried out a quantitative analysis to verify the analytical results we obtained in what could otherwise be thought of as the decomposition of the Arkolakis et al. (2012) measure of trade taking into account intra-firm trade of intermediate inputs within the multinational firm. We find ou that the total gains from openness rise with the share of intermediate goods used in the multinational affiliates. With trade elasticity of 4, the difference between our set up (gains from trade and multinational intra firm trade) and the trade only gains in the set up by Arkolakis et al. (2012) is 8%. When we increased the trade elasticity to 10 keeping the intermediate good shares constant at 1/3 the differences in the gains from openness and trade only gains was 3%. We conclude to state that multinational intra firm trade is an important channel of trade gains, and interaction of the trade elasticity and the intermediate good input share in our set up accounts for this increase in the trade gains as compared to the result of Arkolakis et al. (2012)

In this framework there is a positive correlation between the trade elasticity parameter and the intermediate good input share, hence gains from intra firm trade(FDI) are affected by both γ and ε. For higher productivity dispersions, the percentage point differences are higher between the gains from openness in our study and the gains from trade as calculated in the fashion of Arkolakis et al. (2012) and for higher intermediate good inputs, and for the same productivity dispersion the percentage differences in the gains are lower for low intermediate good input. It is important to note that the trade elasticity parameter used in this study was average for all sectors. As argued by Ossa (2015), average sectoral parameter underestimate total gains from trade, therefore an interesting direction of future research will be to consider different sectoral elasticities and different input shares by sector. Of course this depends on the availability of disaggregated data per sector per destination country of intra-firm trade.

Chapter 3

Capital Taxation, Heterogeneous Multinational and Trade Firms in a Model of Endogenous productivity Growth

3.1 Introduction

Fiscal policy affects the location decisions of firms and plays an important role in stimulating innovation led growth through its effects on investment in research and development (R&D) activities. Countries face a dilemma in determining tax policies to maximize the benefits of trade and FDI such as productivity growth, and stable funding for investment. As an example, emerging market and developing economies often implement tax holidays or tax exemptions in special economic zones to attract more FDI and promote trade. However, these incentives erode tax bases, most notably of the corporate income tax. A vast amount of economics literature confirms that various types of investment activity or the location of corporate investments and profits are responsive to differences in countries tax regimes.

It is well documented that statutory corporation tax rates have been falling across the G7 economies. This decline shows some evidence of convergence to main rates of between 30% and 40%. Econometric evidence points out that the main reasons of declining corporate taxes is the international tax competition for mobile capital and firms, particularly in Europe. In a panel regression of the impact of tax structures on economic growth, Arnold (2008) used a sample of firm-level data for 21 OECD countries over 1996-2004 to find out whether firms incurring higher corporate tax rates shows any lower levels of total factor productivity and investment compared to firms facing lower corporate tax rates. Further studies on the impact of corporate taxation on investments is captured on an empirical investigation by Schwellnus and Arnold (2008) who asserted that; firstly high corporate taxes may reduce incentives for productivity-enhancing innovations by reducing their post-tax returns; secondly, high corporate taxes may reduce incentives for risk taken by firms with negative consequences for productivity; thirdly high corporate taxes may reduce incentives to invest in physical capital by increasing the user cost of capital.

Arnold et al. (2011) provide strong evidence that firms in less profitable industries are affected differently to those in more profitable ones, they are unable to assess whether the overall effect of corporate taxation on productivity growth is positive, negative or zero. Up until that paper, the productivity effects of taxation on firm behaviour had usually been inferred from their effects on indirect channels such as R&D and capital investment. According to Bernard et al. (2007) firms - even in a narrowly defined sector - differ vastly in their size and productivity. This empirically observed heterogeneity has become a core element of recent theoretical and empirical research in various fields of economics and international trade literature based on the seminal theoretical contribution by Melitz (2003). It is therefore very clear that, the heterogeneity of firms is also relevant to the design of corporate tax policies across countries. The existing theoretical literature on international corporate taxation has largely been confined to settings where all firms are identical. Even though there are several findings in the literature on the relationship between firm size, growth and innovation, however none have investigated how these are affected by corporate taxes. Against this background the focus of this chapter is to study the link between heterogeneous multinational and trade firms, innovation and the consequent productivity growth implications in the presence of corporate taxes. The eminent importance of such a study is further highlighted by Chu et al. (2015), who stated that it would be interesting to study the relationships among innovation, growth, and taxes using an R&D-based endogenous growth model proposed by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1998), as will be the approach in this chapter.

There are several strands of literature at the nexus of trade and public finance that derive optimal tax policies in models with heterogeneous firms. Pflüger and Südekum (2013) analyse optimal equilibrium subsidies to market entry in an open economy model of policy competition. Davies and Eckel (2010) analyse tax rate competition for internationally mobile, heterogeneous firms, whereas Krautheim and Schmidt-Eisenlohr (2011) derive Nash equilibrium tax rates when the location of firms is fixed but profits can be shifted between countries. These papers, however, focus on tax rate competition and do not endogenise the simultaneous determination of productivity growth led by innovation activities, corporate taxation and investment decisions. In addition these papers only pay attention to static steady states and gives no intuition on dynamic analysis of the effects of corporate taxation on productivity growth as they take productivity to be exogenously given.

We present an extended version of the model of Helpman et al. (2004) with firm heterogeneity and trade with steady-state productivity growth driven by innovation in the spirit of Gustafsson and Segerstrom (2010). They set up a model of heterogeneous firm productivity embedded in an R&D led endogenous growth model without scale effects. Previous works in new trade theory and growth has featured scale effects which are proved to be at odds with empirics for example, Baldwin and Robert-Nicoud (2008). In the model developed by Baldwin and Robert-Nicoud (2008), the steady-state productivity growth rate is an increasing function of population size, which implies that larger economies should grow faster. This strong scale effect property is present in all early R&D-driven endogenous growth models, including Grossman and Helpman (1991), but it is clearly at odds with empirical evidence. As Jones (1995) has pointed out, there have been no upward trends in the productivity growth rates of the United States, France, Germany or Japan since 1950 in spite of substantial increases in population size and R&D employment. In response to this critique, a variety of R&D-driven growth models have been developed that do not have the strong scale effect property, including Jones (1995), Segerstrom (1998) and Howitt (1999).

Based on the model in this study, we find out that high corporate taxes may reduce incentives for productivity-enhancing innovations by reducing their post-tax returns. Moreover we find out that higher corporate taxes raise the interest rate and lowers the fraction of workers in the innovation sector. There are however competing effects of taxation on the output. The rest of this paper is organised as follows, section [3.2] presents the setup of the theoretical model and the assumptions used, section [3.3] presents the solution of the model and the results in the form of propositions, finally section [3.4] presents the main conclusions of the paper.

3.2 Theoretical model of trade and FDI with firms heterogeneity, innovation-led growth and corporate taxation

3.2.1 Model Setup and Assumptions

We set up a heterogeneous productivity model of trade and FDI in the same spirit as Melitz (2003) and Helpman et al. (2004). The production structure in the economy is set up in a similar manner to Jones (1995). The endogenous growth approach without scale effects is in the spirit of Gustafsson and Segerstrom (2010), i.e the growth rate along the balanced growth path is not an increasing function of population size. Jones (1995) argued that the presence of strong scale effects is at odds with time series data suggesting that Gustafsson and Segerstrom (2010) is superior from empirical point of view.

House-holds and Labour Supply

The economy of each country comprises of a fixed measure of households $L \equiv L(0)$ with an infinite horizon. Initially each household has one member, but the household size grows at an exogenously given exponential rate n, hence at every point in time there is $L(t) = L(0)e^{nt}$ individuals. Each individual inelastically supplies one unit of labour (there is no dis-utility from work) meaning that labour/leisure trade off does not enter the preferences. Labour is employed either in the production of final good $L_Y(t)$, or in innovation $L_I(t)$. We assume perfect mobility of labour across sectors in the economy but not across countries such that wages are equal in the R&D and final good sector. The reward for labour is the wages w_t. Preferences are identical across individuals and households. Consumer preferences are similar in both countries and household maximize infinite lifetime utility, taking the growing number of household members into account, the inter-temporal utility of the household is defined by;

$$U = \int_t^{\infty} e^{-(\rho-n)t}\frac{C_{it}^{1-\frac{1}{\zeta}}-1}{1-\frac{1}{\zeta}}dt \tag{3.1}$$

where ρ is the rate of time preference, ζ denotes the inverse of the elasticity of inter-temporal substitution, C_{it} is the consumption of the final good. Household income comprises of labour income, interest payments on assets of the household and profits from firms in the intermediate goods sector. These incomes sums up to the out produced Y. The income is used for consumption and investment undertaken by firms in the intermediate good sector. Therefore household maximises infinite time horizon utility (3.1) subject to an inter-temporal budget constraint

$$\dot{v(t)} = w(t) + ((1-\tau_v)r(t) - n)v_t - c(t) + T \tag{3.2}$$

such that w denotes the wage rate, r denotes the interest rate(pre-tax return on assets), v denotes the assets, n is the growth rate of population, T denotes transfers from the government, τ_v denotes the tax rate for asset income. Whereby the following transversality condition holds

$$\lim_{t \leftarrow \infty} v_t exp \left\{ - \int_0^t ((1 - \tau_v) r_s - n) ds \right\} \geq 0 \tag{3.3}$$

The above equation is the usual or standard household's no Ponzi game condition, that is fulfilled for a time path on which assets grow at the same rate as consumption, necessary conditions are also sufficient.

Government

The government in our economy receives tax revenues from capital income taxes. In this model we abstract from neither government spending nor public provision of services, i.e., government spending neither enhances welfare nor raises production possibilities on the economy. The reason for that assumption is that we neglect any distortions resulting from variations in government spending. Inclusion of public capital in the production function would have consequences on the productivity growth, however we want to explain productivity growth effects of innovation without a parallel mechanism that affects productivity growth of the factors used. The Ricardian equivalence holds; that is, government taxation does not affect household decisions. We also assume that the government rebates all the taxation revenues immediately to the households, hence it is not important to monitor the asset position of the government. Therefore we make an underlying simplifying assumption that government collects taxes and returns them as lump sum to households. The per period budget constraint of the government is given by:

$$\tau_v r(t) v(t) = T(t) \tag{3.4}$$

Innovation

New varieties are introduced by R&D and innovation i.e., firms first employ researchers to develop a new variety. Firms generate knowledge by carrying out innovations, it takes $b_I(t)$ units of labour to generate one unit of knowledge. Assuming identical production functions in producing intermediate varieties in the same spirit as Baldwin and Robert-Nicoud (2008) an entrant is required to hire $b_I F_I$ workers and pay the associated wage bill $wb_I F_I$. The fixed costs of innovation F_I measures the strength of barriers to entry. Expected value of the firm is equal to total innovation cost. Free entry condition implies that when there are positive profits, more firms enter into research and development of blue prints for intermediate goods thereby driving down the value of innovation until profits are set to zero. The basic set up of the innovation is adopted from Gustafsson and Segerstrom (2010).

Recall that it takes b_I units of labour to generate one unit of knowledge. Then b_I is treated as a parameter by firms but it can change over time due to knowledge spillovers.

$$b_I(t) = \frac{L(t)^\kappa}{(M_D(t) + \lambda M_F(t))^\phi L_I(t)^{\theta-1}} \tag{3.5}$$

where M_D and M_F are the number of varieties produced in the home and the foreign countries. Symmetry implies that $M_D(t) = M_F(t) = M_t$, where M_t is the total number of varieties produced in each economy. Therefore we can re write (3.5) as:

$$b_I(t) = \frac{L(t)^\kappa}{(1+\lambda)^\phi M_t^\phi L_I(t)^{\theta-1}} \tag{3.6}$$

where $L(t)$ is the total labour endowment in the economy, $L_I(t)$ is the labour devoted to production of new blue prints in the innovation process $\lambda \in [0,1]$ is the intensity of international knowledge spillovers. $\lambda = 0$ corresponds to no international spillovers and $\lambda = 1$ corresponds to perfect international spillovers. The parameter $\phi < 1$ is an R&D parameter that measures the strength of inter-temporal knowledge spillovers, if $\phi = 0$ then there are no inter-temporal knowledge spillovers, if $\phi > 0$ researchers experience standing on the shoulders of giants effect and benefit from past innovations and become more productive in creating new knowledge as the stock of knowledge increases. For the case $\phi < 0$ researchers become less productive in creating new knowledge, they face the fishing out of ideas effect.

Intermediate Goods Sector

We consider intermediate good producers with heterogeneous productivities, this is a point of departure from endogenous growth literature which usually assumes homogeneous technologies between producers in the intermediate goods sector. There is a large pool of potential entrants into this sector and each potential entrant faces sunk costs of F. After obtaining a blue print, each firm draws firm specific marginal costs a_i from a stationary Pareto distribution

$$G_i(a) = Pr(a < a) = \int_0^a g(a)da = \left(\frac{a}{a_{i0}}\right)^{k_i}, \quad a \in [o, a_{i0}] \tag{3.7}$$

where the shape parameter is denoted by k and the scale of the distribution is denoted a_{i0}. This distribution relates firm productivity and firm size. When the shape parameter is lower firms are more heterogeneous,i.e there exists an inverse relationship between heterogeneity and the shape parameter k. For the mean and variance of the distribution to be finite we assume that $k > 0$. There is only one chance for an intermediate firm to draw the unit cost parameter and it remains the same there after.

Essentially potential intermediate producers decide on devotion of resources to develop intermediate goods. Then the firms draws its costs and decide whether to supply the domestic market or foreign market via trade and FDI. Consider a firm that develops a new variety at time t, let a_D be the unit labour requirement that is incurred by a firm that is indifferent between incurring the fixed costs $b_I F_D$ of serving only the domestic market and immediately shutting down production process. Let a_{E_X} be the marginal costs that a firm incurs if it decided to serve the foreign market via exports with additional fixed costs of $b_I F_{E_X}$ and, let a_{E_M} be the marginal cost that the firm incur when it decides to access the foreign market via FDI with additional fixed costs of $b_I F_{E_M}$

To guarantee non negative average net profits for MNEs, the per period fixed costs $F_c it$, $c \in \{d, x, j\}$ satisfies the following condition:

$$\frac{\Omega_j}{\Omega_i} F_{iit} < \tau^{\epsilon} F_{xit} < F_{jit} \quad \forall i \in \{i, j\}, j \neq i \tag{3.8}$$

An intermediate firm has monopoly over production of its variety, that is, it is advantageous for each firm to produce a distinct variety and charge a monopoly price instead of entering into direct competition with other firms by manufacturing an identical variety. As production does not feature economies of scope (fixed costs has to be paid per variety), firms will not produce more than one variety. Assuming that there ia free entry into consumption good production the result is a well known monopolistic competition market structure.

Final Goods Sector

The final out put Y is produced using labour and a continuum of intermediate goods. This production function is analogous to that of Romer (1990);

$$Y_t = A L_{Y_t}^{1-\Phi} \int_{\omega \in \Omega} x_t(\omega)^{\Phi} d\omega, \quad 0 \leq \Phi < 1, \tag{3.9}$$

where L_{Y_t} denotes labour employed in the final good production. $A > 0$ denotes a measure of exogenous aggregate productivity which is non R&D driven. The set of available intermediate goods is denoted Ω and $x_t(\omega)$ is the quantity of intermediate variety ω. The elasticity of substitution between intermediate goods is given by $\epsilon = \frac{1}{1-\Phi}$. The final good is treated as the numeraire and sold at unit price, i.e $P_{Y_t} = 1$, for purposes of this analysis we assume that the final good is non tradable.

3.3 Solution of the model

In this section I present the solution of the model, I describe equilibrium properties that hold. We first define the decentralized equilibrium is a sequence of allocations $\{c_t, K, Y, L, L_Y L_I, x, G\}_{t=0}^{\infty}$ and prices $\{p_{it}, w_t, r_t\}_{t=0}^{\infty}$ and policy $\{\tau_v\}$ such that for all t;

1. households maximize their lifetime utility, i.e., c_t, v_t solves the household problem
2. competitive final goods firms choose $\{x_{it}\}_0^N\, and L_Y$ to maximize profits taking prices as given.
3. L_{I_t} solves the research and development firm problem taking V, w as given
4. K_t satisfies $\int_0^{N_t} x_{it}di = K_t$
5. $Y_t = \left(L_Y^{1-\Phi} \int_0^{N_t} x_{it}^{\Phi}\right)$
6. w_t clears the labour market; $L_I(t) + L_Y(t) = L(t)$
7. r_t clears the capital markets
8. $L_t = L_0 e^{nt}$
9. the government budget constraint is balanced i.e., $\tau_v r(t)v(t) = T$
10. Monopolistic intermediate good firms maximize their profits by choosing $\{p_{it}\}_{i=0}^{N_{it}}$

3.3.1 Household optimization problem

Inter-temporal maximisation of lifetime utility (3.1) by a representative house old subject to the budget constraint (3.2) yields the standard Euler equation which shows consumption growth rate as the depending on the difference between the interest rate r and the rate of time preference ρ.

$$\frac{\dot{c_{it}}}{c_{it}} = \zeta((1-\tau_v)r - \rho) \tag{3.10}$$

The households of all countries have equal access to an international financial market, leading to a common interest rate across countries, and common motions for the evolution of household expenditure defined by equation (3.10) above. Output growth is driven by growth of inputs and technology.

$$g_Y = n + g \tag{3.11}$$

This implies that the growth rate of output and capital in steady state does not depend on tax rates. This is one of the distinguishing features with endogenous growth models of Romer and Frankel (1999), Grossman and Helpman (1991) which all indicates that strong scale effects so that growth rate may be influenced by tax rates. Since capital grows at the same rate as consumption and output, capital income taxes do not affect balanced growth rate of consumption. The growth rate of consumption is given by;

$$\begin{aligned} g_c &= \frac{\dot{c}_t}{c_t} = \zeta((1-\tau_v)r_t - \rho) \\ &\rightarrow g_Y - n \\ r^* &= \frac{\frac{g}{\zeta} + \rho}{1-\tau_v} \end{aligned} \tag{3.12}$$

This equation shows that the steady state interest rate depends on the tax rate applied to capital income.Higher capital income taxes raises the interest rate. Higher interest rates lower the present value of future profits.

3.3.2 Intermediate good sector Profit maximization

A firm with cost $a(\omega)$ from country i maximizes its operating profits by choosing its optimal price p_{ci} for $c \in \{D, E_X, E_M\}$ to serve the domestic and foreign markets via export and multinational firms respectively. The firm problem is given by;

$$\max_{p_{ci}(\omega)} (p_{ci}(\omega) - \tau^z a(\omega)) x_{ci}(p_(ci)) \tag{3.13}$$

where $z \in \{0, 1\}$ such that $z = 0$ in the case of domestic and multinational firms and $z = 1$ in the case of exporting firms with iceberg trade costs of τ. The optimal prices for country i firm to serve the domestic market, export and set up horizontal MNE is thus given by the following respectively;

$$p_{Di} = \frac{a(\omega)}{\Phi} \qquad p_{E_{Xi}} = \frac{\tau a(\omega)}{\Phi} \qquad p_{E_{Mi}} = \frac{a(\omega)}{\Phi}$$

The above prices are constant mark-ups over the marginal costs. Now substituting the prices into the profit equation given by (3.13) together with the demand given by (C.1) yield optimal profit;

$$\pi_{ci} = \left(\frac{1-\Phi}{\Phi}\right) (\tau^z a(\omega))^{\frac{-\Phi}{1-\Phi}} A_i^{\frac{1}{1-\Phi}} \Phi^{\frac{2}{1-\Phi}} L_d \tag{3.14}$$

3.3.3 Final Good Profit Maximization

We assume a perfectly competitive final goods market, therefore marginal products are rewards for the factors used in production of the final good. From the first order conditions we determine the (factor prices) wages and the price of the intermediate goods;

$$\frac{\partial Y_t}{\partial L_t} = w_t = 1 - \Phi L_{Y_t}^{1-\Phi} \int_{\omega \in \Omega_t} x_t(\omega)^{\Phi} d\omega = \frac{(1-\Phi)Y_t}{L_{Y_t}}, \quad 0 < \Phi < 1 \tag{3.15}$$

$$\frac{\partial dY_t}{\partial dX_t} = p(\omega) = \Phi L_{Y_t}^{1-\Phi} (x_t(\omega))^{\Phi - 1} \rightarrow \Phi Y_t = K_t P_t \tag{3.16}$$

where the wages are denoted by w_t and the price is denoted by $p(\omega)$ for the good ω. Since we consider monopolistic competition in the intermediate goods sector, the price in (3.16) is not affected by price settings of other intermediate good producers. We have that $K_t = \int_{\omega \in \Omega} x(\omega) d(\omega)$ and $P_{ij} = \left(\int_{\omega \in \Omega} p_{ij}^{1-\epsilon} d\omega \right)^{\frac{1}{1-\epsilon}}$ is the price index of the economy.

Summing over the differentiated input varieties we obtain the aggregate price index associated with the intermediate sector:

$$P_{it} = \left[\sum_{i=H,F} \int_{\omega} p(\omega)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{3.17}$$

Note that the price of final good is taken as the numeraire this implies that:

$$\left(\frac{w}{1-\Phi} \right)^{1-\Phi} \left(\frac{P}{\Phi} \right)^{\Phi} = P_Y = 1 \tag{3.18}$$

Equation (3.15) implies that the wages can be written as:

$$w_t = A^{-\frac{1}{1-\Phi}} (1-\Phi) \Phi^{\frac{\Phi}{1-\Phi}} P_t^{\frac{-\Phi}{1-\Phi}} \tag{3.19}$$

subsequently Equations (3.15), (3.19) implies that:

$$Y_t = A^{-\frac{1}{1-\Phi}} \Phi^{\frac{\Phi}{1-\Phi}} P(t)^{\frac{-\Phi}{1-\Phi}} L_Y(t) \tag{3.20}$$

Final good producers decide on the optimal amount of the intermediate inputs, taking prices of the intermediate goods as given. Solving the maximisation problem yields home country's demand for variety ω;

$$x(\omega) = \frac{p(\omega)^{-\epsilon}}{P^{1-\epsilon}}\Phi Y_t \quad \text{with} \quad P = \left(\int_{\omega\in\Omega} p(\omega)^{1-\epsilon} d(\omega)\right)^{-\frac{1}{1-\epsilon}} \tag{3.21}$$

Where $p(\omega)$ is the price of variety ω, taking into account the distribution function of productivity levels of the firms we can re-write the price index as $P = \left(\int_{\varphi}^{\infty} p(\varphi)^{1-\epsilon} dG(\varphi)\right)^{\frac{1}{1-\epsilon}}$.

3.3.4 Entry and Exit Decisions

Cut-off Productivity Levels

First it is important to note that in contrast to the model of Melitz (2003) who assumes an exogenous death rate for all firms to enable steady state transitions in response to trade, we employ the notion of innovation to facilitate transition between steady states. Assume that entrant's entry costs are financed by issuing equities. The return on these equities is given by the usual arbitrage condition. Denoting $\pi_j(a)$ the instantaneous profits of the firm j

$$V_{jt}(a) = \int_t^{\infty} e^{-\bar{r}(s-t)} \pi_{jt}(a) ds \tag{3.22}$$

where $\bar{r} = \int_t^s r(\nu)$ denotes the cumulative interest rate up to the time $s \geq t$, $V_{jt}(a)$ is the market value of firm j. Differentiating (3.22) with respect to time t, the dividend payments and capital gains have to equal to common return on either asset.

$$\pi_{jt}(a) + \dot{V}_{jt}(a) = r_t V_{jt}(a) \tag{3.23}$$

In an environment with taxes on capital income, the above equation implies that capital gains from value of intellectual property are taxed, this gives the following equation;

$$(1-\tau_v)r_t = (1-\tau_v)\frac{\pi_{it}}{V_{jt}} + \frac{\dot{V}_{jt}(a)}{V_{jt}(a)} \tag{3.24}$$

Solving for $V_{jt}(a)$ gives

$$V_{jt}(a) = \frac{(1-\tau_v)\pi_{jt}(a_{jt})}{(1-\tau_v)r_t - \frac{\dot{V}_{jt}(a)}{V_{jt}(a)}}, \quad \text{where} \quad j = D, E_X, E_M \tag{3.25}$$

Equating the value of the firm to the costs we obtain;

$$V_{jt}(a) = wb_I(t)F_j \quad \text{such that} \quad j = D, E_X, E_M \tag{3.26}$$

Substituting the profit function given by (3.14) gives the entry condition into the domestic market, as follows

$$\frac{(1-\tau_v)A^{\sigma}(1-\Phi)L_{Y_t}\Phi^{2\sigma-1}a_D^{1-\sigma}}{(1-\tau_v)r-\left(\frac{\dot{b_I(t)}}{b_I(t)}+\frac{\dot{w}}{w}\right)} = wb_I(t)F_D \tag{3.27}$$

For firms accessing the foreign market via trade the value function is as follows:

$$\frac{(1-\tau_v)A^{\sigma}(1-\Phi)\varrho L_{Y_t}\Phi^{2\sigma-1}a_{E_X}^{1-\sigma}}{(1-\tau_v)r-\left(\frac{\dot{b_I(t)}}{b_I(t)}+\frac{\dot{w}}{w}\right)} = wb_I(t)F_{E_X} \tag{3.28}$$

where $\varrho \equiv \tau^{1-\sigma}$, with ($\varrho = 0$ describing the case of autarky and $\varrho = 1$ describing free trade)

The observation that not all firms export implies that the ($a_D > a_{E_X}$) and the costs associated with foreign market entry are higher than domestic. Therefore we get the following:

$$\frac{a^*_{Dt}}{a^*_{E_X}} = \left(\frac{F_{E_X}}{F_D\varrho}\right)^{\frac{1}{\sigma-1}} \tag{3.29}$$

Finally for the firm entering via FDI we have:

$$\frac{(1-\tau_v)A^{\sigma}(1-\Phi)L_{Y_t}\Phi^{2\sigma-1}a_{E_M}^{1-\sigma}}{(1-\tau_v)r-\left(\frac{\dot{\vartheta}}{\vartheta}+\frac{\dot{w}}{w}\right)} = \frac{w\vartheta_{I_t}(F_{E_M}-F_{E_X})}{1-\varrho} \tag{3.30}$$

In a similar fashion dividing (3.30) by (3.28) gives the ratio of FDI cut-off costs to that of exporting firms.

$$\frac{a^*_{E_M}}{a^*_{E_X}} = \left(\frac{(F_{E_M}-F_{E_X})\varrho}{F_{E_X}(1-\varrho)}\right)^{\frac{1}{\sigma-1}} \tag{3.31}$$

Cut-off costs can be determined by using the zero profits conditions obtained from equations (3.27), (3.28), (3.30). The threshold values of a_D, a_{E_X}, a_{E_M} can be determined by finding the marginal firm that pays fixed costs to enter and make zero profits. The marginal costs of domestic firms is higher than that of firms serving foreign markets through exports and FDI and has the lowest fixed costs among all the operating firms, this implies that $a_{D_t} < a_{E_{X_t}} < a_{E_{M_t}}$. This translates to the notion that all firms with marginal costs less than domestic threshold ($a < a_{D_t}$) will serve the domestic market. Moreover, the most productive firms have an opportunity to sell to foreign markets via FDI followed by exports. Tariff jumping motive of setting up a multinational subsidiary in the foreign country implies that there no ice berg trade costs incurred by the multinational firm, however the trade off is incurring larger fixed costs of setting up a subsidiary plant. A firm chooses to set up a foreign subsidiary rather than simply exporting when the gains from avoiding trade costs outweigh the costs of maintaining capacity in foreign markets.

Free Entry Condition

As in Melitz (2003), in addition to the cut-off conditions the free entry condition has to hold in equilibrium. While the monopolistic competition features monopolistic pricing, market entry is not restricted. The free entry condition implies that firms can develop a new variety and by free entry condition the ex-ante expected profit of developing a new variety must equal the cost of innovation. Without this free entry condition, infinitely many firms would enter the market if the expected value of market entry was higher than the costs, on the other hand if the expected value of market entry was lower than the cost, there would be no incentive for any firm to develop new varieties. The expected market entry is given by the sum of expected pay-offs from the domestic and the foreign markets (via trade and FDI) net of fixed costs of entry into the markets, this must equal the cost of developing a new variety.

$$\int_0^{a_D} \Big[V_D(a) - wb_I F_D\Big] dG(a) + \int_{a_{E_M}}^{a_{E_X}} \Big[V_{E_X}(a) - wb_I F_{E_X}\Big] dG(a) + \tag{3.32}$$

$$\int_0^{a_{E_M}} \Big[V_{E_M}(a) - wb_I F_{E_M}\Big] dG(a) = wb_I(t)(t)F_I(t) \tag{3.33}$$

Equation (3.33) simply states that the expected value of net profits in the local and foreign markets via trade and FDI must be equal to the cost of variety creation. This condition can also be expressed ex post conditional on the probability of successful entry as follows;

$$\int_0^{a_D} \Big[V_D(a) - wb_I F_D\Big] \frac{g_i(a)}{G_i(a_{Dt})} M_{Dt} da + \int_{a_{E_M}}^{a_{E_X}} \Big[V_{E_X}(a) - wb_I F_{E_X}\Big] \frac{g_j(a)}{G_j(a_{E_X}) - G_j(a_{E_M})} M_{E_X} da + \tag{3.34}$$

$$\int_0^{a_{E_M}} \Big[V_{E_M}(a) - wb_I F_{E_M}\Big] \frac{g_j(a)}{G_j(a_{E_X})} M_{E_M} da = wb_I(t)F_I(t) \tag{3.35}$$

where $\frac{g_i(a)}{G_i(a_{Dt})}$ is the density function conditional on firms survival, $\frac{g_j(a)}{G_j(a_{E_X})-G_j(a_{E_M})}$ is the density function conditional on the distribution of foreign exporters and $\frac{g_j(a)}{G_j(a_{E_X})}$ is the density function conditional on the distribution of firms accessing the foreign market via FDI. The conditional distributions of the intermediate firms are governed by the cut-off costs. The number of firms are given by the product of the density functions conditional on distribution (conditional probabilities) of exporter firms and FDI firms multiplied by the total number of firms available in the domestic country.

In the same vein as Melitz (2003), we proceed by calculating the probabilities of MNEs and exporters conditional on the surviving firms in the domestic market respectively as follows

$$\beta_D = G(a_i) \quad \textit{probability of being in the domestic market in country } i \tag{3.36}$$

$$\beta_{E_X} = \frac{G(a_{E_X}) - G(a_{E_M})}{G(a_i)} \quad \textit{probability of being an exporter conditional on surviving firms} \tag{3.37}$$

$$\beta_{E_M} = \frac{G(a_{E_M})}{G(a_i)} \quad \textit{probability of being a multinational firm conditional on surviving firms} \tag{3.38}$$

Using these conditional probabilities of successful entry, the mass of firms that enter foreign country via exports and FDI are given by

$$M_{E_X} = \beta_{E_X} M_D \quad \text{mass of exporting firms} \tag{3.39}$$

$$M_{E_M} = \beta_{E_M} M_D \quad \text{mass of firms entering via} \tag{3.40}$$

This implies that the total mass of varieties are given by

$$M = M_D + \beta_{E_X} M_D + \beta_{E_M} M_D \tag{3.41}$$

Substituting equation (3.27), (3.28), (3.30) into the free entry condition given by equation (3.35) we get the following

$$\int_0^{a_D} \left(\frac{(1-\tau_v) A^\sigma (1-\Phi) L_{Y_t} \Phi^{2\sigma-1} a_D^{1-\sigma}}{(1-\tau_v) r - \left(\frac{\dot{b_I}}{b_I} + \frac{\dot{w}}{w} \right)} - w b_I(t) F_D \right) dG(a) + \tag{3.42}$$

$$\int_{a_{E_M}}^{a_{E_X}} \left(\frac{(1-\tau_v) A^\sigma (1-\Phi) \varrho L_{Y_t} \Phi^{2\sigma-1} a_{E_X}^{1-\sigma}}{(1-\tau_v) r - \left(\frac{\dot{b_I}}{b_I} + \frac{\dot{w}}{w} \right)} - w b_I(t) F_{E_X} \right) dG(a)$$
$$+ \int_0^{a_{E_M}} \left(\frac{(1-\tau_v) A^\sigma (1-\Phi) L_{Y_t} \Phi^{2\sigma-1} a_{E_M}^{1-\sigma}}{(1-\tau_v) r - \left(\frac{\dot{b_I}}{b_I} + \frac{\dot{w}}{w} \right)} - \frac{w b_I(t) (F_{E_M} - F_{E_X})}{1-\varrho} \right) dG(a) = b_I(t) F_{(I)} \tag{3.43}$$

This simplifies to;

$$\frac{(1-\tau_v)(1-\Phi) L_Y}{((1-\tau_v) r - (\frac{\dot{b_I}}{b_I} + \frac{\dot{w}}{w}))} = M_D \bar{F} \tag{3.44}$$

where

$$\bar{F} \equiv F_I \frac{1}{G(a_{Dt})} + F_D + F_{E_X}\left(\frac{G(a_{E_X}(t)) - G(a_{E_M}(t))}{G(a_{D_t})}\right) + F_{E_M}\left(\frac{G(a_{E_M}(t))}{G(a_{D_t})}\right)$$
$$\bar{F}(t) = F_I\left(\frac{\bar{a}}{a_D(t)}\right)^k + F_D + F_{E_X}\left(\frac{a_{E_X}(t)}{a_D(t)}\right)^k + F_{E_M}\left(\frac{a_{E_M}(t)}{a_D(t)}\right)^k \tag{3.45}$$

$$\Delta_t \equiv \int_o^{a_{D_t}} a^{1-\sigma}\frac{g_a(a)}{G_a(a_D)}da + \varrho\int_{a_{E_M}}^{a_{E_X}} a^{1-\sigma}\frac{g_a(a)}{G_a(a_D)}da + \int_0^{a_{E_M}} a^{1-\sigma}\frac{g_a(a)}{G_a(a_D)}da \tag{3.46}$$

As in Gustafsson and Segerstrom (2010), we define Δ_t as the average marginal cost weighted by the expost distribution of successful entry that considers the variable trade costs. $\bar{F}_t$ is the expected cost of innovation in units of knowledge

The flow of varieties is determined by the labour devoted to R&D divided by the labour units required for successful innovation.

$$\dot{M}(t) = \frac{L_I(t)}{b_I(t)\bar{F}} \tag{3.47}$$

Where $L_{it} = \sum l_{it}$ is the sum of all the R&D carried out by firms in the economy. The denominator contains the expected costs of new variety and time (t). Equation (3.47) implies that the steady state rate of innovation is therefore given by;

$$g \equiv \frac{\dot{M(t)}}{M(t)} = \frac{L_I(t)}{Fb_I(t)M_D} = \frac{L_I^{\theta}(t)(1+\lambda)^{\phi}}{\bar{F}M_t^{1-\phi}L(t)^{\kappa}} \tag{3.48}$$

Aggregation

Using the average productivity equation (3.46) and the domestic and foreign profit maximizing prices into the the price index we can then find a closed form solution to the aggregate price index. Following Gustafsson and Segerstrom (2010) in steady state the CES price index in equation (3.17) can be written as;

$$\begin{aligned}P(t)^{1-\epsilon} &= M_D\int_0^{a_D} p_D(a_D)^{1-\epsilon}\frac{g(a)da}{G(a_D(t))} + M_{E_X}(t)\int_{a_{E_M}}^{a_E} p_{E_X}(a_E)^{1-\epsilon}\frac{g_a(a)}{G_a(a_D)}da + \\ &M_{E_M}\int_0^{a_F} p_{E_M}(a_F)^{1-\epsilon}\frac{g_a(a)}{G_a(a_D)}da \\ &= \left(\frac{\epsilon}{\epsilon-1}\right)^{1-\epsilon} M(t)\Delta(t)^{1-\epsilon} \\ &= \Phi^{1-\epsilon}M(t)\Delta(t)^{1-\epsilon}\end{aligned} \tag{3.49}$$

This equation shows that the price index is affected my the mass of varieties M, elasticity of substitution and the average productivity. Higher levels of varieties drives down the price index and the higher elasticity of substitution also pins down the price index. Output is given by equation (3.9), taking into account the cut-off productivity, output can be aggregated as:

$$\begin{aligned} Y_{it} =& A_i L_i \Bigg(\int_0^{a_D} X_D(a)^{\Phi} \frac{g_i(a)}{G_i(a_{iit})} M_D da + \int_{a_E}^{a_F} X_E(a)^{\Phi} \frac{g(a)}{G(a_E) - G(a_F)} M_E da + \\ & \int_0^{a_F} X(a)^{\Phi} \frac{g(a)}{G(a_F)} M_F \Bigg) \end{aligned} \tag{3.50}$$

$$Y_{it} = A^{\frac{1}{1-\Phi}} \Phi^{\frac{2\Phi}{1-\Phi}} L_Y M_{it} \Delta^{1-\epsilon} \tag{3.51}$$

where we recall that $\epsilon = \frac{1}{1-\Phi}$ is the elasticity of substitution between the intermediate goods and $M_{it} = M_D(t) + M_{E_X}(t) + M_{E_M}(t)$ is the total number of intermediates. And $X_{it}(\tilde{it})$ representing the intermediate goods produced by each firm i is produced according the following costs

In the same spirit we recalculate the wages from equation(3.15) given by:

$$w = (1-\Phi) A^{\epsilon} \Phi^{2(\epsilon-1)} M \Delta^{1-\epsilon} L_Y \tag{3.52}$$

This equation implies that the wage rate grows at the same rate with (M) the number of varieties ($\frac{\dot{w}}{w} = \frac{\dot{M_D}}{M_D} = g$) and From average cut-off costs we can then re-write (3.50) as follows:

$$Y_{it} = A^{\frac{1}{1-\Phi}} \Phi^{\frac{2\Phi}{1-\Phi}} L_Y M_{it} \Delta_{it}^{1-\epsilon} \tag{3.53}$$

In the same fashion we can rewrite the intermediate goods using the price index 3.5

$$X(\Delta) = A^{\frac{1}{1-\Phi}} \Phi^{\frac{2}{1-\Phi}} L_Y \Delta^{\frac{-1}{1-\Phi}} \tag{3.54}$$

This denotes the intermediate goods produced by an average intermediate goods firm that serve a country i and the total cost for intermediate goods is represented by:

$$\begin{aligned} I_{it} &= \tilde{a_{it}} M_{it} X_i(\Delta_{it}) \\ &= \tilde{a_{it}} A \frac{1}{1-\epsilon} \Phi^{\frac{2}{1-\sigma}} L_Y M_{it} \Delta_{it}{}^{\frac{-1}{1-\epsilon}} \\ &= \Phi^2 Y_{it} \end{aligned} \tag{3.55}$$

Final Goods sector equilibrium

Income can be used to generate intermediate goods or for consumption purposes;

$$Y = I + C \tag{3.56}$$

where C is the total consumption and I is investment into production of intermediate goods. We have already derived investment into intermediate goods given by equation (3.55). This implies that consumption is therefore given by: $C = (1 - \Phi^2)Y$

Labour Market Equilibrium

In this model labour is used in the innovation and in production of final output. The labour market in this model is perfectly competitive. Aggregate labour in this economy is the sum of labour allocated to the final good production and L_Y and labour allocated to the R&D sector L_I.

$$L = L_Y + L_I \tag{3.57}$$

From equation 54 this is given by:

$$L_I = gFb_I(t)M_D \tag{3.58}$$

Recall that

$$g \equiv \frac{\dot{M}_t}{M_t} = \frac{L_I}{Fb_I(t)M_D} = \frac{L_I(t)(1+\lambda)^{\phi}}{\bar{F}M_t^{1-\phi}} \tag{3.59}$$

This implies that

$$\dot{M}_t = \frac{L_I}{Fb_I(t)} = \frac{L_I(t)(1+\lambda)^{\phi}}{\bar{F}M^{-\phi}} \tag{3.60}$$

Hence the growth rate of innovations in steady state is given by:

$$\frac{\dot{b_I}}{b_I} = \kappa\frac{\dot{L}}{L} - \phi\frac{\dot{M}}{M} - (\theta - 1)\frac{\dot{L_I}}{L_I} \tag{3.61}$$

Taking into account that in steady state the growth rate of innovations is equal to the growth rate of the population we can re-write the above equation (3.61) of innovation growth rate as follows (where $\frac{\dot{L}}{L} = n = \frac{\dot{L_I}}{L_I}$);

$$\frac{\dot{b_I}}{b_I} = (\kappa - \theta + 1)n - \phi g \tag{3.62}$$

3.3.5 Steady State Equilibrium

Steady State Cut-offs

In steady state the cut-offs for the domestic, export and multinational firm are respectively given by:

$$\begin{aligned}\frac{(1-\tau_v)\Phi a_D^{\epsilon-1}L_Y}{(1-\tau_v)r+(\theta-1)g-(\kappa-\theta+1)n} &= M\Delta^{\sigma-1}b_I F_D\\ \frac{(1-\tau_v)\Phi a_X^{\epsilon-1}\rho L_Y}{(1-\tau_v)r+(\theta-1)g-(\kappa-\theta+1)n} &= M\Delta^{\sigma-1}b_I F_X\\ \frac{(1-\tau_v)\Phi a_I^{\epsilon-1}L_Y}{(1-\tau_v)r+(\theta-1)g-(\kappa-\theta+1)n} &= \frac{M\Delta^{\sigma-1}b_I(F_{E_M}-F_{E_X})}{(1-\rho)}\end{aligned} \tag{3.63}$$

Therefore substituting the steady state given by (3.63) into the free entry condition (3.44), we obtain;

$$\frac{(1-\tau_v)\Phi L_Y}{(1-\tau_v)r+(\theta-1)g-(\kappa-\theta+1)} = M_D b_I F \tag{3.64}$$

We now proceed to use the above equation (3.64) to solve for the steady state value of labour allocated to the final good production and we obtain;

$$L_Y = \frac{1}{\Phi}\frac{Fb_I M_D}{(1-\tau_v)}[(1-\tau_v)r+(\phi-1)g+(\theta-\kappa-1)n] \tag{3.65}$$

And the total Labour in this economy is given by:

$$L = L_Y + L_I = \frac{Fb_I M_D}{(1-\tau_v)\Phi}\left\{r(1-\tau_v)+(\Phi(1-\tau_v)+\phi-1)g+(\theta-\kappa-l)n\right\} \tag{3.66}$$

The steady state share of labour allocated to the R&D sector is given by:

$$s_A = \frac{L_I}{L} = \frac{1}{1+\frac{1}{\Phi(1-\tau_v)}\left[\frac{\rho+n(\theta-\kappa-1)}{g}+(1/\zeta+\phi-1)\right]} \tag{3.67}$$

Where s_A is the share of labour allocated to innovation activities in the model.

Proposition 3.1 (**Labour allocated to innovation** (L_I) **is decreasing in taxation**).

From this equation (3.67) it is clear that higher capital taxes raised the interest rate but reduces the share of labour used in innovation sector, more over the share of intermediate inputs in production declines. This implies that higher tax rates leads to a decline in number of varieties along a BGP. This is shown by deriving the number of varieties along the BGP M_D;

$$M_{Dt}^{*} = \left[\frac{L^{\theta-\kappa}s_A^{\theta}}{g^{*}}\frac{(1+\lambda)^{\phi}}{F}\right]^{\frac{1}{1-\phi}} \tag{3.68}$$

In this model the only source of productivity growth is the expansion of intermediate inputs. Therefore it is worthwhile to note that trade and multinational production only affects the intermediate variety by changing the fixed costs and knowledge spillovers and the capital tax which in turn affects the labour allocated to production of intermediate goods.

Proposition 3.2 (**The growth rate**).

Expansion of intermediate goods is the only source of productivity growth, therefore, output growth rate and aggregate consumption are all equal to the rate of productivity growth given by

$$g^{*} = \frac{1}{1/\zeta + \Phi(1-\tau_v) + \phi - 1}\left(\Phi(1-\tau_v)\left[\frac{g^{\theta-1}(1+\lambda)^{\phi}L^{\theta-\kappa}}{FM_D^{1-\phi}}\right]^{\frac{1}{\theta}} - \rho - (\theta-\kappa-1)n\right) \tag{3.69}$$

According to (3.69) it is clear that there are two implications, firstly public policy changes that affect trade liberalization has no effect on the growth rate, this result is also obtain by Segerstrom (1998). This is due to the fact that the iceberg trade costs τ does not appear in equation (3.69). Secondly the taxation rate affects the steady state growth and the magnitude of the fixed costs incurred in accessing foreign markets either via trade or FDI.

Proposition 3.3 (**Impact of capital taxation on growth rate are non linear: we deduce that in a high tax economy with high productivity firms, the sensitivity of economic growth to an increase in taxes is magnified, whereas it is dampened in the case of low tax economy with low productivity firms**).

A close inspection of equation (3.69) shows that the impact of low capital e taxes on long run growth rate is low or minimal, however increasing taxes leads to rise in the marginal effect of taxation on growth. In our current model the non linearity arises from the heterogeneous firm productivities. In a low capital tax economy, increase in taxes results in exit of low productivity firms. The reasoning is as follows, low productivity is a cost disadvantage, any cost disadvantage translates into large losses of market share. These less productive firms can capture only a small market share. The impact of those new entrants on aggregate growth rate is small. In the case of high productivity firms, in a high tax economy, an increase in taxes leads to a large decline in growth rate.

Comparing the above result to Segerstrom (1998) recall the definition of the growth rate

$$g \equiv \frac{\dot{M}_t}{M_t} = \frac{L_I}{Fb_I(t)M_D} = \frac{L_I(t)(1+\lambda)^{\phi}}{\bar{F}M_t^{1-\phi}} \tag{3.70}$$

log differentiating with respect to time and setting the growth rate equal to zero in steady state along a balanced growth path we obtain:

$$g* = \frac{(\theta-\kappa)n}{1-\phi} \tag{3.71}$$

That is, the growth rate is dependent on population growth rate and on the level of technological spillovers. There are some restriction, i.e $\theta > \kappa$ and $\phi < 1$. This growth rate only shows that rate of innovation depends only on exogenous variables. It follows that the steady state share of labour devoted to R&D is obtained by substituting the growth rate along a balanced growth path (3.71) in equation (3.67) to obtain

$$s_A^* = \frac{1}{1+\frac{1}{\Phi(1-\tau_v)}\left[\frac{(\rho-n)(1-\phi)}{(\theta-\kappa)n}+1/\zeta\right]} \tag{3.72}$$

It is clear that a higher capital taxation τ_v decreases the amount of labour devoted to innovation activities as output (final good) production becomes more labour intensive. Moreover higher the elasticity of substitution and higher share of intermediate good inputs increases the share of labour in innovation. However the steady state s_A^* in (3.72) shows that neither export fixed costs or FDI fixed costs affect the share of labour devoted to innovation.

Proposition 3.4 (**Growth rate of output and consumption are equal to growth rate of productivity** $g_Y = g = g_c$).

On the growth rate of output, first we turn to the price index and real consumption in steady state. Log differentiating the price index in equation (3.49)with respect to time and recalling that Δ is time invariant gives the steady state growth rate of the price index

$$g_P = \frac{1}{1-\epsilon}g = \frac{(\theta-\kappa)n}{(1-\epsilon)(1-\phi)} < 0 \tag{3.73}$$

Recalling that $\epsilon > 1$,this implies that along a balanced growth path the price index is falling.

Recall that ;

$$\begin{aligned} Y =& A^{\frac{1}{1-\Phi}}\Phi^{\frac{2}{1-\Phi}}LM\Delta^{\frac{-1}{1-\Phi}} \\ y =& A^{\frac{1}{1-\Phi}}\Phi^{\frac{2}{1-\Phi}}(1-s_A)M\Delta^{\frac{-1}{1-\Phi}} \end{aligned} \tag{3.74}$$

More over we derived that $C_t = (1-\Phi^2)Y_t \rightarrow c_t = (1-\Phi^2)y_t$. Bearing in mind that in steady state all variables are constant except for the number of intermediates M_D, the growth rate of consumption is equal to the growth rate of output. In this model creation of intermediates drives the productivity growth, if innovation activities stop, then both output and consumption growth also stops.

Moreover we observe from (3.74) that the growth rate does not depend on tax rates. This is s distinguishing feature of semi endogenous growth models. In contrast, first generation growth models such as that Romer (1990), Aghion and Howitt (1992) all have scale effects such that the growth rate of output may be influenced by tax instruments.

Proposition 3.5 (**Effect of corporate taxation on output**).

From (3.74), we deduce that there are competing effects of the corporate tax on the out put; an increase in financial taxes lowers N but increases the fraction of workers producing the final out put, i.e $(1-s_A)$ rises as the the corporate tax increases. If the effect through the labour channel is strong enough (dominates the effect on N), then higher corporate taxes may lead to higher output growth. This result of positive impact of corporate taxes on out put growth was also a result in Uhlig and Yanagawa (1996). In the same vein Chen et al (2016) argued that in the case of tax shifting from labour income to capital income tax, raising capital income taxation increases the steady state equilibrium growth rate.

Proposition 3.6 (**Endogenously determined variables F, a_D,a_{E_M},a_{E_M} are constant for all t, they are solely dependent on exogenous parameters**).

We already pointed out that the productivities are distributed Pareto, therefore it is convenient to define that

$$\beta \equiv \frac{k}{\epsilon - 1} > 1 \tag{3.75}$$

We therefore define the weighted average productivity as;

$$\begin{aligned}\Delta^{1-\epsilon} &= \int_0^{a_D} a^{1-\epsilon}\frac{g_i(a)}{G_i(a_D)}da + \rho\int_{a_{E_M}}^{a_{E_X}} a^{1-\epsilon}\frac{g_j(a)}{G(a_{E_X})-G(a_{E_M})}da + \int_0^{a_{E_M}} a^{1-\epsilon}\frac{g_j(a)}{G_j(a_{E_M})}da \\ &= \frac{\beta}{\beta-1}a_D^{1-\epsilon}(1+\Omega_{E_X}+\Omega_{E_M})\end{aligned} \tag{3.76}$$

where $\Omega_{E_X} = \rho^{\beta}\left(\frac{F_{E_X}}{F_D}\right)^{\beta}$ and $\Omega_{E_M} = (1-\rho)^{\beta}\left(\frac{F_{E_M}-F_{E_X}}{F_D}\right)^{1-\beta}$ The equilibrium values for the cut-offs are given as;

$$\begin{aligned} a_{D^*} &= \bar{a}\left[\frac{F_I(\beta-1)}{F_D(1+\Omega_{E_x}+\Omega_{E_M})}\right]^{\frac{1}{k}} \\ a_{E_X^*} &= \bar{a}\left[\frac{F_I(\beta-1)(\Omega_{E_X}+\Omega_{E_M})}{F_{E_X}(1+\Omega_{E_x}+\Omega_{E_M})}\right]^{\frac{1}{k}} \\ a_{E_M^*} &= \bar{a}\left[\frac{F_I(\beta-1)(\Omega_{E_X}+\Omega_{E_M})}{(F_{E_M}-F_{E_X})(1+\Omega_{E_x}+\Omega_{E_M})}\right]^{\frac{1}{k}} \end{aligned} \tag{3.77}$$

The analysis of the cutoff productivities above shows that $\bar{F}$, a_D^*, a_{E_X}, a_{E_M} are time invariant , and solely dependent on exogenous parameters, this implies that the weighted equilibrium is also time invariant and also a constant.

3.3.6 Steady State Effects of Openness (Trade Liberalisation and FDI)

In this section we proceed to examine the steady state impact (long run effects) trade liberalization an and FDI which can occur either through a fall in variable trade costs or fixed costs of foreign market entry. The degree of openness can be measured by changes in (Ω_{E_X}) and Ω_{E_M}. That is effects of oneness can be measured by changes in the iceberg transportation costs τ and the fixed costs of reign market entry via trade or FDI F_{E_X} and F_{E_M}.

Recall that $\Omega_{E_X} \equiv \rho^{\beta}\left(\frac{F_{E_X}}{F_I}\right)^{1-\beta}$ where, $\rho = \tau^{1-\epsilon}$, and $\Omega_{E_M} \equiv (1-\rho)^{\beta}\left(\frac{F_{E_M}-F_{E_X}}{F_I}\right)^{1-\beta}$

In order to study the effects of openness on Growth separate them by first looking at effects of Trade Liberalisation and thereafter effects of Attracting MNEs.

Effects on Trade Liberalisation

In the same spirit as Gustafsson-Segerstrom (2010), Figure 1 shows that trade liberalisation implies that Ω_{E_X} increases due to a decrease in ice berg trade costs $\tau \downarrow$ or a decrease in the fixed costs of entering foreign markets through trade F_{E_X}. when governments change corporate tax rates, by virtue of firms heterogeneity in the composition of their capital stock, investments financing and involvement in foreign markets, they induce heterogeneous effects across firms. The partial derivatives of Ω_{E_M} with respect to ρ and F_{E_X} are given as;

$$\frac{\partial \Omega_{E_X}}{\partial \rho} = \beta\frac{\Omega_{E_X}}{\rho} > 0 \qquad \frac{\partial \Omega_{E_X}}{\partial F_{E_X}} = (1-\beta)\frac{\Omega_{E_X}}{F_{E_X}} \tag{3.78}$$

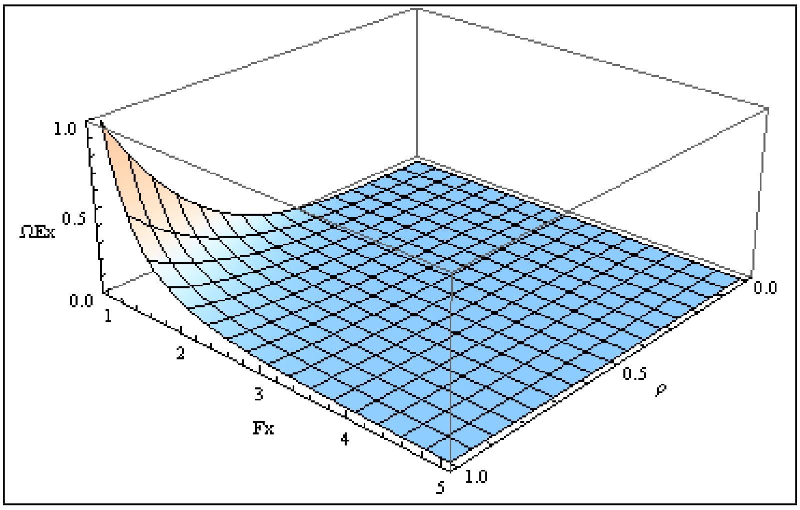

FIGURE 3.1: Degree of Trade openness as a function of ρ and Fixed costs of export, taking k=4 and $\epsilon = 2$

Figure 1 clearly depicts that if the infinitely increasing the iceberg trade costs ($\tau \to \infty, \Leftrightarrow \rho \to 0$), or increasing the relative fixed costs of export firms to domestic firms labelled as Fx, this implies that the trade openness ($\Omega_{Ex} \to 0$) and the economy is closed due to prohibitively high costs of doing trade.

Effects on Multinational Firms

We consider the fact that part of the MNEs fixed costs are policy related. Therefore the MNEs fixed costs are treated as a policy variable for the number of firms that enter the foreign markets via setting up subsidiary plants.

$$\frac{\partial \Omega_{E_M}}{\partial \rho} < 0 \qquad \frac{\partial \Omega_{E_M}}{\partial F_{E_M} - F_{E_X}} < 0 \tag{3.79}$$

Figure 2 clearly depicts that if iceberg trade costs are infinitely decreasing ($\tau \to 0, \Leftrightarrow \rho \to \infty$), or increasing the relative fixed costs of FDI fixed costs to domestic fixed costs labelled as Fm, this implies that openness to multinational firms decreases as shown by the negative derivative in the first part of equation (3.79).

In terms of the impact of trade costs on openness to trade and openness to multinational firms we derived two opposite effects $\frac{\partial \Omega_{E_X}}{\partial \rho} => 0$ and $\frac{\partial \Omega_{E_M}}{\partial \rho} < 0$. Then we must address which of the two is the dominationg force. To address this conundrum, we study the find the sign of the first derivative of total openness i.e

$$\frac{\partial (\Omega_{E_X} + \Omega_{E_M})}{\partial \rho} > 0 \tag{3.80}$$

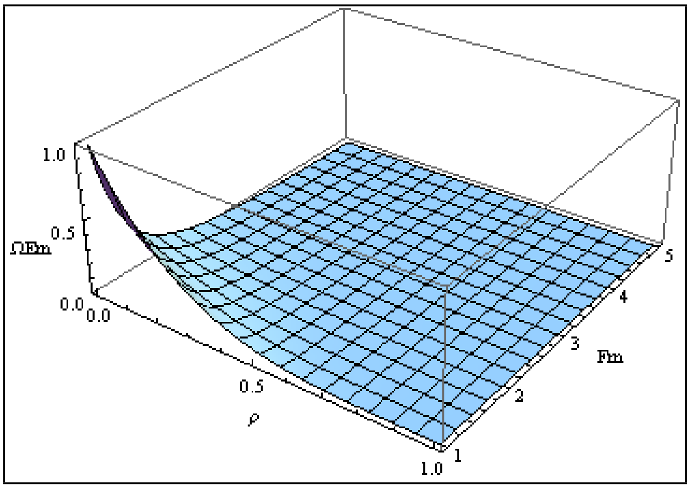

FIGURE 3.2: Degree of Openness to Multinational Firm as a function of ρ and Fixed costs of export and Fixed costs of setting up a multinational firm, taking k=4 and $\epsilon = 2$

The above equation shows that the openness to trade is the dominating effect as the derivative is positive

Effect on Cutoff Levels

We already proved that the cutoff unit labour requirements do not change over time, i.e, they are constant. In a trade only model without multinational firms, Trade liberalisation (Ω_{E_X}) increases the minimum productivity required to enter into the domestic market $a_d \uparrow$ and lowers the productivity to start exporting ($a_{E_X} \uparrow$). This result is analogous to Gustafsson-Segerstrom (2010); Chaney (2008) and Melitz (2003). This implies that when trade barriers are decreased, exporting becomes cheaper and firms with lower productivities can afford to enter the export market, this implies that the cutoff for entering export market ($a_{E_x} \uparrow$) increases.

$$\frac{\partial a_D}{\partial \Omega_D} = -\frac{1}{k}\frac{a_{E_X}}{1+\Omega_{E_X}} < 0, \qquad \frac{\partial a_{E_X}}{\partial \Omega_{E_X}}|_{F_{E_X}}, \text{const} \quad = \frac{a_{E_X}}{k\Omega_{E_X}(1-\Omega_{E_X})} \tag{3.81}$$

Next we study how this result compare to our present set up with multinational firms. This implies that we need to study the impact of openness on total weighted productivity which is a composite of both export and multinational productivity. This leads to the following proposition,

Proposition 3.7 (**The effect of a Openness on the average weighted productivity.**).

At the aggregate level, trade liberalisation leads to overall productivity increases. This of course depends on fulfilling the endogenous sorting criteria, i.e., $\frac{F_{E_X}}{F_{E_M}} < \tau^{1-\epsilon}$. We establish this proposition by taking the first derivative of the weighted productivity Δ. Moreover we establish that a decrease in the fixed costs of entering foreign market via FDI increased average weighted productivity.

$$\frac{\partial \Delta}{\partial \rho} > 0 \quad (\frac{\partial \Delta}{\partial \tau} < 0), \qquad \frac{\partial \Delta}{\partial F_{EX}} < 0, \qquad \frac{\partial \Delta}{\partial F_{E_M}} < 0 \tag{3.82}$$

We can interpret the above equation as follows, trade liberalisation makes exporting to be cheaper and induces firms that did not initially export but closer to the export cut-off productivity to start exporting, this increases competition since the number of imported varieties lowers domestic market shares in every market. Furthermore a decline in the both the export fixed costs and FDI fixed cots increases the average weighted productivity. Reducing the FDI costs paves a way for the most productive export firms next to the FDI productivity threshold to engage in FDI which increases competition . This induces tougher selection and increases productivity.

Proposition 3.8 (**The effect of a Marginal Increase in τ_v in the growth rate of the economy is negative.**).

Arithmetic computation gives

$$\frac{dg}{d\tau_v} < 0 \tag{3.83}$$

Although the sign of the first derivative is negative, it is key to state that a rise in the tax corporate taxes depresses growth for lower values of elasticity of substitution (lower level of intermediate goods) and but accelerate growth for higher values of elasticity of substitution, this translates into inverse relationship between market power and tax corporate taxes.

3.4 Conclusion

Differences that persist between multinational firms, such as their size, investments and innovation levels are usually the mechanisms established in trade literature as some of the determinants of multinational firms location decisions. In this paper we contribute to literature on heterogeneous firm productivity and taxation by investigating the corporate taxation effects on multinational and trade firms. We build a model that modifies Romer (1990), Jones (1995) and Gustafsson and Segerstrom (2010) by incorporating what we term corporate taxes (tax on financial income). We then carry out an analysis of the economy whose growth is driven by innovation activities. We then explore how corporate taxation affects such an economy in the presence of firms that operate in the domestic market and foreign market either via international trade or via foreign direct investment.

Firstly we find that corporate taxes have competing effects on the output level via decrease of the number of intermediate varieties and an increase in final out put labour supply. This implies that we get the result for certain higher taxes we still get higher economic growth since the labour increase in final output implies increase in output in the model. This implies a higher economic growth due to its equivalence to the output growth. Secondly we find that openness to trade or multinational does not affect the long run growth rate of the economy neither does it affect the share of labour allocated to R&D s_A. However trade liberalisation and openness to multinational firm production had a level effect on mass of varieties produced $M(t) \downarrow$ due to slow down in variety creation. Gustafsson and Segerstrom (2010) arrives to the same reasoning and explained that as a_{E_X} increases more firms enter export market hence a rise in the number of available varieties but this leads to more competition and lower profits, this in turn induces fewer firms to enter the market thus slowing down the variety process creation. On the other hand openness to trade and multinational firm production diverts resources from product innovation as more units of knowledge are needed to cover the fixed costs of market entry hence lower productivity growth. This is a very important result in the dynamic set up as compared to the result in the static set up model of Helpmann (2004) which pointed out that trade liberalisation and openness to multinational firm production are always growth enhancing. To avoid monotony and repetition, we indicate that a paper by Gustafsson and Segerstrom (2010) quantifies the overall effect even though their setup is only restricted to trade.

Chapter 4

Impact of exhaustible resource on opening for international trade: Hartwick's rule and taste for foreign goods

The problem of exhaustible resources traded by resource abundant countries and the effects on economic growth is of long standing importance in economics. van der Ploeg (2012) points out that natural resource rents worldwide now exceed \$4 trillion per annum, amounting to some 7 percent of global GDP, and Ruta and Venables (2012) claims that (non-renewable) resources account for 20% of world trade. Hence understanding the growth effects of natural resource revenue in an open economy cannot be overemphasised.

Static trade theory models following Ricardo, like one of Heckscher and Ohlin (1991) emphasis that a country will trade (export) a commodity which uses its abundant factor intensively or differences in factor endowments will prompt countries to export commodities for which they have comparative advantage. This shows that endowments of natural resources may form the basis for trade in resource abundant countries. The main shortcoming of trade theories is that they not directly address this problem of exhaustibility and the inter-temporal trade-offs involved. Due to resource exhaustibility, prices change over time so are the levels of resources extracted, this necessitates adopting a dynamic approach that takes into account the change over time in the availability of a finite resource. Recently some dynamic arguments have been established for example by Bajona and Kehoe (2010), but still effects of exploitation of natural resources in the open economy are missing. In support of this view, World Trade Organisation report (2010) stated that the traditional model of trade like of Heckscher and Ohlin (1991), does not directly address the inter-temporal trade-offs involved in resource exhaustibility.

Therefore understanding the impact of trade on exhaustible resources requires adopting a dynamic approach that takes into account how finite resource stock varies over time. Exhaustibility of finite resources requires analysis of how the resource stock dynamic impacts production and prices and therefore the application of dynamic games framework seems quiet natural here. In light of this view and motivation, the objective of this paper is to analyse the impact of exhaustible resources on opening up for international trade. We construct a dynamical theoretical framework of international trade with two countries and three sectors (final good producers, intermediate good and resource sector). In our study, exhaustible resource is used in production of final output like in model of Sethi (1979) with resource monopoly optimizing over the price of the resource and not its extraction rate, like in seminal paper of Hotelling (1931). We determine price evolution of the finite resource and final output when a small country opens to trade. Moreover, we explore the extent to which exhaustible resource constraints can be overcome by substitution and technological change.

The findings of our investigation are that when a resource abundant country opens up to trade, demand for intermediate goods rises boosting technical change. This result is obtained from direct comparison of evolution of intermediate goods demand under both regimes (autarky and trade). Secondly trade opening leads to an increase in rate of capital accumulation derived from non renewable resource revenue in the open economy regime. Moreover opening of the domestic market leads to the switch in the mix of factors being used for production, as the resource price increases overtime due to exhaustibility, final good producers substitute it with intermediates in the long run boosting technical change and total factor productivity due to new varieties of intermediates goods being produced.

Our novel result is breakage of Hartwick Rule in a dynamic frame work of exhaustible resource trade. In principle Hartwick rule shows that when resource rents are reinvested in physical capital, consumption levels could be sustained. However in this paper we find out that in the case of a trading resource abundant country, not all resource rents are re-invested into the capital accumulation. This stems from consumer's bias for foreign products, that is, the more home consumers like foreign products, the more likely expenditures will increase higher, than resource revenues. Therefore opening up to trade may benefit home country only if it is not too much backwards and there is some support of home produced products.

This paper contributes to several strands of related literatures studying optimality of resource extraction and effects on economic growth. The first strand is the Ramsey type growth models, like those of Stiglitz (1974), Dasgupta and Heal (1974) and Garg and Sweeney (1978). The second strand is the endogenous growth models like of Schou (1996), Aghion and Howitt (1998), Scholz and Ziemes (1999), Barbier (1999), Grimaud and Rouge (2003). Unlike in our analysis, both strands of literature do not consider impact of trade on resource price changes and extraction paths, final output price evolution or capital accumulation via trade. According to Todo (2003) developing countries usually rely on foreign direct investment(FDI) as a major source of technological development. We suggest that another channel which received less attention in models of economic growth is international trade, through which there is improved technical change (through the switch of the mix of intermediate good inputs and exhaustible resources) and increased resource rents. The third strand is the literature on what is termed the "resource curse" as established by Sachs and Warner (1995) , which depicts a negative relationship between countries natural-resource abundance and dependence and their economic growth. In our paper we show that opening to trade may benefit the resource abundant country only if it is not too much backwards and there is some support of home produced goods.

While most studies of endogenous growth with exhaustible resources focus on an isolated economy without trade and or FDI, a few studies related to our present work involving natural resources and trade have been carried out. A model most closely related to our work is presented by Gaitan and Roe (2012). Though they consider an infinite time horizon of trade with exhaustible resources, the key focus of their study is distinct from the objective of our paper. Their main aim was to show plausible conditions under which the interdependence of countries through trade can give rise to the resource curse. In their study they do not explicitly account for resource price determination in autarky and in trade as we have demonstrated in this paper by applying the open loop strategies. Moreover they abstract from intermediate goods driving technical change, but they compare capital accumulation under closed and open economy. Finally we account for breakage of Hartwick's rule in when a resource abundant country is opened to trade, which is not the aim of their exposition. A more recent study is by Yenokyan et al. (2014) who investigated economic growth with trade in factors of production. They concluded that trade in goods can raise growth rates of trading partners without any technology transfer or international transfer. Our study departs from their approach by considering technical change induced by intermediate goods used in final output production.

The rest of the paper is organised as follows, section [4.1] presents the model, section [4.2] contains the solution (with technical steps in Appendices), in section [4.3] we obtain main results and propositions of the paper, in section [4.4] we provide the conclusion and discuss possible extensions of the idea of this paper.

4.1 Model

Consider the world economy with two countries, home, h and foreign, f. Home country possesses an endowment of exhaustible resource which is used in production both in home and foreign countries. The economies of both countries are otherwise symmetrical: there are representative households consuming final product being made in both countries, final goods producers, intermediate goods sectors and resource sector. Intermediate goods are produced in the Romer-fashion but we assume constant variety of available intermediaries technologies in the basic set-up. We describe each sector of the home economy in turn since it is our main focus. Derivations for foreign economy are symmetric and can be easily made.

4.1.1 Households

Consumption is represented by the representative household who supplies L units of labour. As it is already standard (see for example Peretto and Connolly (2007)), we assume that the population is constant and equal in both countries of mass $L_{h,f} = 1$. We also assume that wage is equal across countries and is a numeraire such that $w = 1$. The only source of income for the households is the resource rent plus interest payments on savings. All of the income may be either consumed or saved: i.e a representative household owns the resource stock with associated revenue stream $p_R R$. Letting $r_h = r_f = r$ in the same spirit as Gaitan and Roe (2012) such that interest parity condition holds, the flow budget constraint of the representative household at home is:

$$\dot{A}_h = rA_h + p_R R + 1 - pE_h. \tag{4.1}$$

Where E_h is the expenditures in the home country, E_f is expenditures in the foreign country, p_R is the resource price, R is the amount of exhaustible resource, A_h and A_f are the household asset holdings in the home and foreign country respectively. In the foreign country the budget constraint is the same but without resource rent:

$$\dot{A}_f = rA_f + 1 - pE_f. \tag{4.2}$$

The only capital in this economy are financial assets of households which are not depreciated. This assumption is not crucial and cannot influence any results of the paper except for the initial conditions on capital (assets) accumulation.

The household maximizes lifetime utility over an infinite time horizon subject to the inter-temporal budget constraint and the usual No-Ponzi-game condition. We assume a constant discount rate and CES preferences preferences over goods provided by the home and foreign country. Thus the household maximizes

$$\max_{C_i} \int_0^{\infty} \mathrm{e}^{-\rho t} U(C_i) dt,\ i \in \{h, f\}; \tag{4.3}$$

where C_i is a consumption at country i resulting from the combination of home produced final good and foreign traded good:

$$C_i = \left[\varphi^{\frac{1}{\chi}} (C_{i,i})^{\frac{\chi-1}{\chi}} + (1-\varphi)^{\frac{1}{\chi}} (C_{-i,i})^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}} \tag{4.4}$$

and utility is logarithmic:

$$U(C_i) = \ln C_i \tag{4.5}$$

Note that χ denotes the inter temporal elasticity of substitution between home and foreign consumption goods, φ and $(1-\varphi)$ denote preferences over home and foreign goods respectively, where φ is the home bias; $\varphi \in [0,1]$. A consumer also derives utility from consuming imported goods, the more the bias is, the less a representative household prefers imported goods.

4.1.2 Final goods sector

In this sector there exist home and foreign producers. They use resource being extracted in the home country and a range of intermediate inputs (country-specific) in production technology:

$$Y_i = \int_0^{N_i} Q_{ij}^{\eta} dj R_i^{\gamma},\ i \in \{h, f\}, j \in [0; N_i], \tag{4.6}$$
$$0 < \eta < 1,\ 0 < \gamma < 1.$$

Where The sum of η and γ are in principle total factor productivity. labour being used inelastically with $L = 1$.

The instantaneous profit function for final producer in country i is given by:

$$\pi_i = pY_i - \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj - p_R R_i - 1,\ \ i \in \{h, f\}. \tag{4.7}$$

where last term appears due to labour costs.

We assume that the resource is traded by the home country to both home and foreign firms and the price p_R is the same for both final sector producers at home and abroad, but varieties of intermediaries are in general different and non-tradable across countries.

The equilibrium price at time t is related to industry output by means of a linear inverse demand function:

$$p^{\infty}(t) = (a - Y_h(t) - Y_f(t)). \tag{4.8}$$

We assume that the actual market price, $p(t)$, at time t is not equal to its equilibrium level $p^{\infty}(t)$, but moves towards it following a first-order adjustment process, i.e

$$\dot{p(t)} = s\left[a - p - Y_h(t) - Y_f(t)\right], \tag{4.9}$$

The speed of adjustment is captured by parameter s, with $0 < s \leq \infty$.

The problem of final producers thus involves instantaneous profit maximization with respect to the demand for intermediate products (capital) and inter-temporal optimization with respect to the resource demand.

4.1.3 Resource sector

In our simple model, the resource sector is a monopoly, owned by the households. The behaviour is thus described by a standard resource-extraction model. The profits are given to consumers, who own this sector. In the same spirit as Dasgupta and Heal (1974), and Hartwick (1990) the firm extracts resources without any costs and offers them to the firms of the final-goods sector for a price p_R per unit, not differentiating across countries. Letting $S(t)$ denote the resource stock at time t and assuming no storage we have resource constraint

$$S_0 \geq \int_0^{\infty} R(t)dt. \tag{4.10}$$

The deterministic evolution of $S(t)$ is described by total extraction equal to the demand from both countries:

$$S(0) = S_0;\, \dot{S} = -R = -R_h^D - R_f^D \tag{4.11}$$

where, R_h^D and R_f^D are the exhaustible resource demand for the home and foreign country respectively. Since storage is not possible, the quantity supplied to the market at any time is equal to the total quantity extracted. The extracted resource is completely homogeneous across all firms, hence there is no product differentiation in the resource sector. Since we assume costless extraction, discounted profit to be maximized is given by

$$\max_{p_R} \int_o^\infty p_R R e^{-\int_0^t r(\tau)d(\tau)} dt \tag{4.12}$$

where r denotes the interest rate.

4.1.4 Intermediate producers

The intermediate goods sector consists of a variety N_i of products. We assume the range of existing intermediates to be constant, $\dot{N}_{h,f} = 0$ in both countries to simplify the analysis. Every variety is owned by a single firm, which is interpreted as R&D firm in standard dynamic case. All such monopolists have homogeneous technology of producing variety at hand. The production function for monopolist j in country i is given by

$$Q_{ij} = K_{ij}, \int_0^{N_i} Q_{ij} dj = V_i; j \in [0; N_i] \tag{4.13}$$

thus the total assets stock is used for production of intermediaries (capital).

In each country, each firm j in the intermediate good sector maximizes the present discounted value of profits flow:

$$V_{ij} = \max_{p_{Q_{ij}}} \int_t^\infty e^{-r(s)} [p_{Q_{ij}} Q_{ij} - r_i K_{ij}] ds \tag{4.14}$$

and establishes the equilibrium price for each individual intermediate product given the demand from the final producers sector.

4.2 Solution

4.2.1 Households

The maximization problem of the household, (4.3) subject to (4.1) or (4.2) yields demand for homogeneous final product. Households solve the problem in two steps,

1. **They decide how to allocate expenditures between imports and domestically produced goods.**

   Let E_i be the aggregate consumption expenditure in country $i = h, f$. The instantaneous expenditure constraint is given be

$$E_h = p_h C_{hh} + p_f C_{fh} \tag{4.15}$$

$$E_f = p_f C_{ff} + p_h C_{hf} \tag{4.16}$$

No trade friction implies that the law of one price holds across countries,

$$p_h = p_f = p \tag{4.17}$$

thus prices of final goods sold at home and foreign countries are equal.

2. **Agents choose the time profile of expenditures by maximising present value utility.**

$$\max_C \int_0^\infty \mathrm{e}^{-\rho t} \ln\left[\frac{E_i}{\varphi^\chi p^{1-\chi} + (1-\varphi)^\chi p^{1-\chi}}\right] dt; \tag{4.18}$$

subject to (4.1) for the home country and to (4.2) for the foreign country.

Optimality conditions give the standard Euler equation in which at each moment in time the rate of growth of expenditures on consumption is equal to the difference between the instantaneous interest rate and the rate of time preference.

$$\frac{\dot{E}_i}{E_i} = r_i - \rho,\ i \in \{h, f\}. \tag{4.19}$$

Assuming equal interest rates across countries, $r_h = r_f$ yields equal growth rates of expenditures.

4.2.2 Final producers

Final producers are facing the consumers demand, C_h, C_f and optimize over demand for resource, R_i^D and demand for intermediate products, Q_{ij}^D. The total resource demand is dynamic and has to be chosen by both firms, but demand for intermediaries is obtained via maximization of the instantaneous profit functions.

We solve this problem in two steps.

1. **Demand for intermediate products.**

Perfect competition and profit maximization implies that marginal products are equalised to factor prices. The demand for the jth variety of intermediate input in country i can be written in the well known Dixit-Stiglitz form:

$$Q_{ij}^D : \eta Q_{ij}^{\eta-1} R_i^\gamma - p_{Q_{ij}} = 0; \tag{4.20}$$

$$Q_{ij}^D = \left(\frac{\eta}{p_{Q_{ij}}} R_i^\gamma\right)^{\frac{1}{1-\eta}} = \eta Y_i \frac{p_{ij}^\epsilon}{\int_0^{N_i} p_{ik,k\neq j}^{1-\epsilon} dk} \tag{4.21}$$

Note that $\epsilon = \frac{1}{1-\eta}$ is the elasticity of substitution between varieties . The demand (4.21) is the downward sloping for each intermediate input j and is known to the monopolistic intermediate producer.

2. **Demand for the exhaustible resource.**

   The only dynamic choice variable for both foreign and home final producers is the resource demand. The dynamic problem for both firms is thus:

$$\max_{R_i^D} \int_0^\infty \mathrm{e}^{-rt} \left(pY_i - \int_0^N p_{Q_{ij}} Q_{ij} di - p_R R_i^D - 1 \right) dt, \tag{4.22}$$

s.t.

$$\dot{p} = s\left[a - p - Y_h - Y_f\right] \tag{4.23}$$

   where Q_{ij} is given by (4.21) and output is given by (4.6) for the home firm and the same for the foreign firm.

In non cooperative games there are basically two strategies that can be applied; open loop strategies and feedback strategies. Open loop equilibrium means that strategies implemented by the players must be functions of time alone where as feedback (closed loop) strategies are functions of time and the state. Closed-loop solutions to differential games are mainly considered in the special case with linear dynamics and quadratic costs. In this paper we employ the open-loop equilibrium concept we derive resource demand through application of the Maximum Principle for both countries. The system of equations is highly non-linear, therefore open loop strategies as opposed to closed loop will enable us to obtain analytically tractable solution. Closed loop strategies will be analytically challenging and require numerically approaches such as collocation methods of value function estimations. Details can be found in the Appendix C.2. The optimal demand for the resource is

$$R_i^D = \frac{\gamma}{1-\eta}^{1-\frac{\gamma}{1-\eta}} \left(\frac{(p-\lambda_i)-\eta}{p_R} \right)^{1-\frac{\gamma}{1-\eta}} N_i^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{\eta}{p_Q} \right)^{\frac{\eta}{1-\eta}} \right)^{1-\frac{\gamma}{1-\eta}} \tag{4.24}$$

a function of final product's price p and the shadow costs of production λ_i. Provided the resource demand is a differentiable function, we can reformulate the problem to get rid of shadow costs in the system. For that we use F.O.C. (C.16) to express λ_i through resource demand and price and then make use of (C.21) to get the differential equation on resource demand as a function of resource price, giving (C.24) in Appendix C.2.

$$\dot{R}_i = \left(\frac{\dot{p_R}}{p_R} - (1+r) \right) \frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R} \left(a - Y_{-i} - (2+r)p\right) \frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}}. \tag{4.25}$$

From equation (4.24) is trivial to deduce that an increase in the resource price p_R leads to a decline in the resource demand. Increasing varieties of intermediate inputs N would imply that a monopolist charges a lower price p_Q which gives rise to an increase in resource price. The net effect is a decrease in non-renewable resource demand. As the substitution parameter η increases (more intermediates inputs substituting resource inputs) p_R rises hence less resources are demanded by final good producers confirming the above assertion.

4.2.3 Resource extraction

In this section we solve for the price and the quantity demanded of the intermediate good. Profit maximization problem in the resource sector becomes :

$$\max_{p_R} \int_o^\infty p_R(t)R(t)e^{-\int_0^t r(\tau)d\tau}dt, \tag{4.26}$$

$$s.t.$$

$$\int_o^\infty R(t) \leq S(0) \tag{4.27}$$

$$\dot{S} = -R \tag{4.28}$$

with $R = R_h^D + R_f^D$. The application of Maximum Principle (details in Appendix C.3) yields constant price growth rate to ensure infinite time of total extraction: Thus the growth of resource price is proportional to the interest rate:

$$\frac{\dot{p}_R}{p_R} = r\frac{\eta + \gamma - 1}{\gamma}. \tag{4.29}$$

This is consistent with Hotelling (1931) rule that the optimal extraction path of an exhaustible resource is one along which the exhaustible resource price increases at the rate of interest times the ratio of total productivity and intermediaries (which are substitutes to the resource). This is a no-arbitrage condition equalizing the returns between the exhaustible resource and other assets.

4.2.4 Intermediate goods producers

The producer maximizes profit function:

$$\max_{p_{Q_{ij}}} \pi_{Q_{ij}}(Q_{ij}) = p_Q Q_{ij} - rK_{ij} \tag{4.30}$$

$$s.t.$$

$$Q_{ij} = K_{ij}; j \in [0 : N_i]. \tag{4.31}$$

Since every intermediate producer is a monopolist in his/her own product, the firm optimizes over the price with the demand given by (4.21). From this the price for every intermediary may be defined and turns to be equal across intermediaries (since there is no technical change and homogeneous technologies for all of them).

$$r = \eta^2 \left(\frac{R_i^{\gamma}}{Q^{1-\eta}} \right) \tag{4.32}$$

Substituting for Q_D we get the optimal pricing rule of a constant mark up over the marginal cost

$$p_{Qij} = p_{Qi} = \frac{r}{\eta} \tag{4.33}$$

Due to product symmetry, prices in all sectors are identical, The only parameters affecting prices are r and η. We assume interest parity such that $r_i = r_{-i}$, which implies the equal growth rates given by the Euler equation (4.19). Low values of η allow monopolist to charge a higher mark up and earn higher revenues and profits. η captures intermediate input share in output, low values of η presume less demand for intermediate inputs.

We can substitute for p_Q and rewrite Q^D as:

$$Q_{ij}^D = \left(\frac{\eta}{p_Q} R_i^{\gamma} \right)^{\frac{1}{1-\eta}} = \left(\frac{\eta^2}{r} R_i^{\gamma} \right)^{\frac{1}{1-\eta}} = \eta^{2\epsilon} r^{-\epsilon} R_i^{\epsilon\gamma}, \tag{4.34}$$

where $\epsilon = \frac{1}{1-\eta}$. Now substituting for the pricing rule in the profit function we get

$$\begin{aligned} \pi_i &= \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj - rK_i = \eta^{2\epsilon - 1} r^{1-\epsilon} R_i^{\epsilon\gamma} - \eta^{2\epsilon -} r^{1-\epsilon} R_i^{\epsilon\gamma} \\ &= (1-\eta) \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj = \frac{1-\eta}{\eta} r \int_0^{N_i} Q_{ij} dj \end{aligned} \tag{4.35}$$

That is, monopolist profits at every instance is $(1-\eta)$ share of revenue. Thus with $\eta > 1$ the R&D sector is making losses and economy collapses.

4.2.5 Equilibrium, market clearing

consists of:

1. Final goods market clearing $C_h + C_f = Y_h + Y_f$;
2. Capital market clearing for every country $K_i = \int_0^{N_i} Q_{ij} dj$;
3. Intermediaries market clearing for every country $Q_{ij} = Q_{ij}^D$;
4. Resource market clearing, $R = R_h^D + R_f^D$.

Now observe that the resource price growth rate is constant, (4.29). Thus the resource demand dynamics (4.25) are transformed into

$$\forall i \in \{h; f\} : \dot{R}_i = \left(r\frac{\gamma+\eta-1}{\gamma} - (1+r) \right) R_i - \\ \frac{1}{p_R} \left(a - N_{-i} \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} R_{-i}^{\frac{\gamma}{1-\eta}} - (2+r)p \right) \frac{\gamma}{\gamma+\eta-1} N_i \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} R_i^{\frac{\gamma}{1-\eta}} \tag{4.36}$$

and substituting for output and resource price solution (which is a straightforward monotonic solution for the ODE (4.29)) one gets the triple of equations, which describes the evolution of the world economy:

$$\dot{R}_h = \left(r\frac{\gamma+\eta-1}{\gamma} - (1+r) \right) R_h - \\ \frac{1}{p_0} \mathrm{e}^{-r\frac{\eta+\gamma-1}{\gamma}t} \left(a - N_f \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} R_f^{\frac{\gamma}{1-\eta}} - (2+r)p \right) \frac{\gamma}{\gamma+\eta-1} N_h \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} R_h^{\frac{\gamma}{1-\eta}}; \tag{4.37}$$

$$\dot{R}_f = \left(r\frac{\gamma+\eta-1}{\gamma} - (1+r) \right) R_f - \\ \frac{1}{p_0} \mathrm{e}^{-r\frac{\eta+\gamma-1}{\gamma}t} \left(a - N_h \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} R_h^{\frac{\gamma}{1-\eta}} - (2+r)p \right) \frac{\gamma}{\gamma+\eta-1} N_f \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} R_f^{\frac{\gamma}{1-\eta}}; \tag{4.38}$$

$$\dot{p} = \left(a - p - R_h^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} N_h - R_f^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} N_f \right). \tag{4.39}$$

where p_0 denotes the initial price for the resource.

The evolution of the world economy with two countries and open trade for exhaustible resource and final product is thus given by the triple (4.37), (4.38), (4.39).

It is clear that under autarky the resource demand from the foreign country is zero and the system is reduced to:

$$\dot{R}_h^A = \left(r\frac{\gamma+\eta-1}{\gamma} - (1+r) \right) R_h - \frac{1}{p_0} \mathrm{e}^{-r\frac{\eta+\gamma-1}{\gamma}t} \left(a - (2+r)p \right) \frac{\gamma}{\gamma+\eta-1} N_h \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} R_h^{\frac{\gamma}{1-\eta}}; \tag{4.40}$$

$$\dot{p}^A = \left(a - p - R_h^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} N_h \right). \tag{4.41}$$

where superscript A denotes autarky regime. If the trade for resource is absent as well as for final product the dynamics of the home economy is characterized by the couple (4.40), (4.41).

4.3 Results

We analyse here the comparative dynamical behaviour of the system under open trade of the resource and under autarky, when the resource is traded only within country as well as the final product. It is relatively straightforward to see from the price dynamics, (4.39), (4.41) that opening the national markets may either decrease or increase prices of the final product and subsequently change the total demand for the resource. This in turn characterises the usage of intermediaries (capital and technology).

We start by defining the steady states of both systems.

Proposition 4.1 (Steady states existence).
Steady states for both systems (4.39), (4.37), (4.38) *and* (4.41), (4.40) *exist only if*

$$\gamma + \eta > 1 \tag{4.42}$$

otherwise resource demand has increasing growth rates until the full exhaustion of the resource.

Indeed observe that only under this condition the exponent in resource demand equations have negative power yielding contraction in time. Non-autonomous ODE systems may have steady states only if their non-autonomous part is a contraction. Hence the result.

This means prices and resource demand may be stabilized in the system if economy (both domestic and abroad) has high enough overall productivity. In this case the accumulation of capital can substitute for resource in production and thus demand for resource will stabilize at some level as well as prices. If instead, productivity is relatively low, there is not enough additional savings to replace resources in production and rising demand for goods (because of higher income of households) would lead to resource demand increase and thus price increases up to the point when the resource is fully exhausted. At this time no production will take place, since we assumed Cobb-Douglas production technology.

4.3.1 Autarky

The steady state for the autarky is defined by stable price for final product and stable demand for the resource.

The implicitly defined steady state level of resource demand under autarky results from substituting (4.44) into (4.43) and observing that the demand is stabilized only when

1. Economy is productive, e. g. $\gamma + \eta > 1$;
2. The fluctuations of demand stop, e. g. $e^{-r\frac{\eta+\gamma-1}{\gamma}t} \to 1$.

If the total factor productivity is lower than 1, $\gamma + \eta < 1$ demand cannot be stabilized., since the non-autonomous part does not vanish at infinity, $e^{-r\frac{\eta+\gamma-1}{\gamma}t} \to \infty$ and the system explodes. If conditions above hold, there exist three different steady state levels of resource demand: one of them trivially sets demand to zero, another one is close to zero and the last one is non-trivially set at some positive level. These follow from the algebraic properties of the polynomial (4.43), which has up to 3 real roots.

$$\left(r\frac{\gamma+\eta-1}{\gamma} - (1+r)\right) R_h^A - \frac{1}{p_0}\left(a - (2+r)p^A\right)\frac{\gamma}{\gamma+\eta-1} N_h^A \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} (R_h^A)^{\frac{\gamma}{1-\eta}} = 0, \tag{4.43}$$

$$\bar{p}^A = a - (R_h^A)^{\frac{\gamma}{1-\eta}} \left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}} N_h^A. \tag{4.44}$$

4.3.2 Opened trade

The steady state for the system under trade is defined similarly from the system (4.37),(4.38),(4.39) under the condition of productive economy and thus vanishing exponent in resource demand equations:

$$\left(r\frac{\gamma+\eta-1}{\gamma}-(1+r)\right)R_h^O-$$
$$\frac{1}{p_0}\left(a-N_f\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}(R_f^O)^{\frac{\gamma}{1-\eta}}-(2+r)p^O\right)\frac{\gamma}{\gamma+\eta-1}N_h^O\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}(R_h^O)^{\frac{\gamma}{1-\eta}}=0 \tag{4.45}$$

$$\left(r\frac{\gamma+\eta-1}{\gamma}-(1+r)\right)R_f^O-$$
$$\frac{1}{p_0}\left(a-N_h^O\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}(R_h^O)^{\frac{\gamma}{1-\eta}}-(2+r)p^O\right)\frac{\gamma}{\gamma+\eta-1}N_f^O\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}(R_f^O)^{\frac{\gamma}{1-\eta}}=0 \tag{4.46}$$

$$\bar{p}^O=a-(R_h^O)^{\frac{\gamma}{1-\eta}}\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}N_h^O-(R_f^O)^{\frac{\gamma}{1-\eta}}\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}N_f^O. \tag{4.47}$$

with superscript O denoting the open trade regime.

As it can be seen, the same is true for the opened system. The difference comes from the boost in resource demand and lower price for the final product experienced by home final producers. At the same time under the open trade revenues of resource monopoly are growing, increasing the households income. This income is then used for faster capital accumulation and higher production of existing varieties.

For simplicity of further analysis we assume that varieties which are produced at home and abroad are fixed and are not changed as a result of opening trade, that is $N_h^A = N_h^O = N_h$, $N_f^A = N_f^O = N_f$ thus only the level of production of each variety , Q_{ij} may change. We start with the direct comparison of two steady states for the home economy.

Compare first steady state price levels in the opened and closed economy. The difference is given by

$$\bar{p}^A-\bar{p}^O=\left(\frac{\eta^2}{r}\right)^{\frac{\eta}{1-\eta}}\left((R_f^O)^{\frac{\gamma}{1-\eta}}N_f+(R_h^O)^{\frac{\gamma}{1-\eta}}N_h-(R_h^A)^{\frac{\gamma}{1-\eta}}N_h\right) \tag{4.48}$$

which is the difference between home output under open trade and autarky plus foreign output. This difference can be negative (thus final product price increases) only if the home output fall more due to foreign competition then the total foreign output. This could be true only if the home economy is relatively large comparing to the rest of the world (foreign) and representative households preferences are biased towards foreign final product. In this case the resulting reduction of home output is so strong, that it is not compensated by foreign output and final price increases instead of decreasing due to increased competition.

Proposition 4.2 (Steady states prices comparison).
The steady state price decreases after opening up trade, $\bar{p}^A - \bar{p}^O > 0$ if:

1. *The home final output at least does not decrease under trade,*

$$(R_h^O)^{\frac{\gamma}{1-\eta}} - (R_h^A)^{\frac{\gamma}{1-\eta}} \geq 0 \tag{4.49}$$

2. *The home output decreases to a lesser extent than the foreign output increases:*

$$(R_h^O)^{\frac{\gamma}{1-\eta}} - (R_h^A)^{\frac{\gamma}{1-\eta}} < 0,\ N_h|(R_h^O)^{\frac{\gamma}{1-\eta}} - (R_h^A)^{\frac{\gamma}{1-\eta}}| < (R_f^O)^{\frac{\gamma}{1-\eta}} N_f \tag{4.50}$$

otherwise the final product price increases, $\bar{p}^A - \bar{p}^O < 0$

This is a straightforward conclusion which does not need formal proof. Since our main goal is to study how the opening trade would affect the developing country, we assume $N_f/N_h > 1$ meaning the rest of the world is more developed. This increases the likelihood of final price to decrease as a result of opening trade. From this observation we deduce a corollary:

Corollary 4.3. *The higher is the technological distance between home and foreign country, N_f/N_h the more final price reduction have to be expected from opening trade.*

Now we analyse the changes in the resource rent, which governs the capital accumulation at home.

Proposition 4.4 (Resource rent).

1. *The price of the resource does not change from opening up the trade as long as $r_f = r_h$ and production technologies are the same, $\eta_f = \eta_h, \gamma_h = \gamma_h$:*

$$p_R^O = p_R^A \tag{4.51}$$

2. *The resource rent under open trade (as long as resource is available) is higher than under autarky by the factor of*

$$p_R R^O - p_R R^A = p_R \left(R_f^O + (R_h^O - R_h^A)\right) > 0. \tag{4.52}$$

Proof is made by:

1. Checking the resource price movement equation which is independent of country-specific parameters, (4.29);

2. Done by checking resource demand (C.26) with zero and non-zero foreign demand and observing from proposition 1 that home demand changes is negative.

Thus the resource rent grows from opening trade as long as the final price drops (the home country is technologically significantly backward) and technology is not super-productive $\eta < 1$. Resource rent may also grow if the final price grows but technology is very productive, since the relationship in Proposition 1 is reversed.

Observe that the quantity $(R_h)^{\frac{\gamma}{1-\eta}}$ governs the home demand for each of the intermediaries expressed in terms of resource demand. As long as $\eta < 1$ intermediaries are complements with the resource and increase in resource demand leads to the increase in intermediaries usage too. However if $\eta > 1$ which means more productive technologies, the increase in resource demand will actually decrease intermediaries usage, making them substitutes. This of course holds only as long as prices both for intermediaries and the resource have fixed schedules, that is, $r_h = r_f$. Otherwise the home economy would adapt its interest rate to the world one, making intermediaries cheaper.

From the (4.21) it follows, that under open trade the demand for intermediate products as a substitute for resource is higher, since the resource demand is lower. This result is in tandem with a widely held view in literature that introducing new intermediate goods leads to an increase in total factor productivity(TFP) and causes growth. Thus the opening of the domestic market leads to the switch in the mix of factors being used for production, boosting technical change. This last is reflected in higher levels of Q_{hj}, since the price ratio between the resource and intermediaries becomes more favourable for the latter.

Proposition 4.5 (**Structural change under open trade**).

When the resource abundant country opens the trade, then:
For $\eta < 1$*:*

1. *If* $r_h > r_f$ *the demand for intermediate products increases, boosting technical change. through cheaper capital;*

2. *If* $r_h < r_f$ *the opposite effect occurs, making intermediaries relatively more expensive and the economy more resource oriented;*

3. If $r_h = r_f$ intermediaries usage may decrease at home, following the resource demand.

It is interesting to note that it is not always good for technical change in the home country to open up trade. The effect would depend on the relative price of capital at home and abroad and on the productivity of technology. If capital at home is scarce which is often the case for developing countries, structural change may be ignited by the change in relative price of technology and resource based factors. However if the technology is more productive, the effect would be reversed, since additional revenue will increase resource expenditures and not the intermediaries demand. However this can be the case only for $\eta > 1$, which means collapse of the home economy without technical change. Thus we restrict ourselves only to the case of $\eta < 1$.

This may be demonstrated by direct comparison of evolution of intermediate demand (4.21) under both regimes. The ordering would follow for the case of $R_h^A > R_h^O$ in the steady state.

Proposition 4.6 (Hartwick's rule in the economy under open trade).

Due to Proposition 1 Hartwick's rule does not always hold in the opened economy: not all resource rents are re-invested into the capital accumulation.

Proof follows from the fact that consumption expenditures may increase after opening trade, $E_h^O \geq E_h^A$, both under decrease and increase of final price, depending on the taste for foreign goods , $1 - \varphi$ parameter. Then it is possible to find such a configuration of parameters space, $\varphi, \rho, \eta, \gamma, r$ that simultaneously

$$E_h^O > E_h^A,\, p_R R^O \leq p_R R^A \tag{4.53}$$

hold. This is sufficient for Hartwick's rule to fail.

Observe that we do not discuss here how *generic* is the situation of Hartwick's rule breakage. This would require much more detailed simulations and parametric analysis. However some raw estimations point to the key role of elasticity φ. The more home consumers like foreign products, the more likely expenditures will increase higher, than resource revenues. This is indeed the case for many developing countries: taste for foreign products is high, and demand for resource is low since the rest of the world is more technologically advanced. Thus we claim that according to our analysis, the open trade may benefit home country only if it is not too much backwards and there is some support of home produced products.

4.4 Conclusion

In this paper we have studied the consequences of opening up to trade in a resource abundant economy, with particular attention on technical change, resource rents, capital accumulation. The dynamic game, two country approach adopted in this paper allows us to study trade effects and exhaustibility of non renewable resources in a unified framework.

Our approach is different from endogenous growth models that study a closed economy and do not take into account opening to trade. We summarise the main results of the paper as follows; Firstly opening to trade leads to a switch in the mix of factors of production. That is, opening to trade leads to more extraction of exhaustible resources to supply both the home and foreign market, as resources get exhausted the resource price rises and intermediate goods substitute for exhaustible resources in production and thus demand for resource will stabilize at some level as well as prices. In the case of relatively low productivity, there is not enough additional savings to replace resources in production and rising demand for goods. This in-turn leads to resource demand increase and thus price increases up to the point when the resource is fully exhausted.

Secondly, resource rents grow from opening trade as long as the final price drops (this occurs when the home country is technologically significantly backward) and technology is not super-productive $\eta < 1$. Resource rent may also grow if the final price grows but technology is very productive. More over, As long as long as the parameter governing technology (productivity) $\eta < 1$ intermediates are complements with the resource and increase in resource demand leads to the increase in intermediaries usage too. However if $\eta > 1$ which means more productive technologies, the increase in resource demand will actually decrease intermediaries usage, making them substitutes. This gives the fourth conclusion; under open trade the demand for intermediate products as a substitute for resource is higher, since the resource demand is lower.

A final important result is in relation to Hartwick rule, in this study we were able to show that not all resource rents are re-invested into the capital accumulation as claimed by Hartwick rule. This stems from consumer's bias for foreign products, that is, the more home consumers like foreign products, the more likely expenditures will increase higher, than resource revenues. Therefore opening up to trade may benefit home country only if it is not too much backwards and there is some support of home produced products.

From the model and the results above, we infer policy implications and recommendations for the developing exhaustible resource country. Firstly opening up to trade may boost home direct investments through faster capital accumulation and will benefit consumers without harming domestic industry. The higher is the current resource stock, the more beneficial such a policy may be. We argue that larger initial exhaustible resource stocks leads to increased capital stock in the long run for the resource abundant country since they receive trade proceeds of the exhaustible resources to the foreign countries. Larger revenues may in turn be used as FDI replacement to stimulate consumption, this is in the same spirit as Asheim (1986) who asserted that resource-rich economy can allow itself to use revenues arising from resource exports to finance additional consumption.

Lastly investing in R&D in extraction technology might offset the depletion of today's resources, this is a possible extension to our model. Stuermer and Schwerhoff (2013) found out in their study that resource stock may be increased through R&D investment in extraction technology, they argue that even if non-renewable resource use and production increase exponentially, resource prices might stay constant in the long term. In the same spirit, WTO (2010) reported that allowing for technological change in the extractive sector can effectively increase the supply of resources by contributing to new discoveries and allowing extraction of stocks that could not be reached before.

The are several extensions that could be investigated using our framework. The model allows for immediate extension of results on arbitrary n of resource-importing countries. The extension of the number of exporting countries is non-trivial, since if one allows for strategic interactions between them, the cartel collision behaviour may be observed.

It is also interesting to study the effects of price discrimination for the resource monopoly, as it is frequently the case: home producers enjoy lower prices than foreign ones. In our set-up such a discrimination may indeed boost home country revenues and thus capital accumulation, but the decrease in foreign output would drive prices for the final product higher and decrease the utility of households. Thus the price discrimination should be treated with caution while might be implemented if the difference in home and foreign technologies, represented by the level of available varieties is big enough.

Moreover, the process of technical change need not to be restricted to the growth in the level of usage of existing intermediaries. One could also assume dynamic character of N_h, N_f in the spirit of Peretto and Connolly (2007), Belyakov et al. (2011) or Bondarev (2012). However this extension should require rather challenging analytical elaboration.

The limited capacity of the resource seems to play no role in the current model, because the time of extraction is not optimized upon by the resource monopolist. However this time of full exploitation should play an important role in the decision of how much resource to export to foreign country and at which price. Moreover the market structure of the resource should be paid more attention. The existence of oligopolistic structure is common for resource abundant economies and is a matter of ongoing debates. This structure may be modelled by the means of usual dynamic oligopoly with the application of the same Differential games apparatus.

Appendix A

Gains from Trade and FDI with Heterogenous Firms

A.1 Aggregation Price Index

A.1.1 Trade Only Price Index

$$P_j^{E_X} = \frac{G(\varphi_{M_F}^*) - G(\varphi_{E_X}^*)}{1 - G(\varphi_i^*)} N_i \left(\int_{\varphi_{ij}^{E_X}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij} \tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)^{\frac{1}{1-\sigma}} \tag{A.1}$$

Under assumption of Common Pareto distribution this becomes

$$P_j^{E_X} = N_{E_X} \left(\int_{\varphi_{ij}^{M_F}}^{\varphi_{ij}^{E_X}} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij} \tau_{ij})^{1-\sigma} k \varphi_{min}^k \varphi^{k+1} d(\varphi_{ij}))^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{A.2}$$

Evaluating the integral and substituting for cutoff productivities

$$P_j^{-k} = N_{ij} \frac{k}{k-\sigma+1} \left(\frac{1}{Y_j} \right)^{1-\frac{k}{\sigma-1}} \varphi_{min}^k \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij} \right)^{-k} \left(\sigma w_j f_{ij}^{E_X}) \right)^{1-\frac{k}{\sigma-1}} \tag{A.3}$$

.

A.1.2 FDI only Price Index

$$P_{ij}^{M_F} = \frac{1-G(\varphi_{M_F}^{*})}{1-G(\varphi_i^{*})} N_i \left(\int_{\varphi_{ij}^{M_F}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)^{\frac{1}{1-\sigma}} \tag{A.4}$$

Under assumption of Common Pareto distribution this becomes

$$P_{ij}^{M_F} = N_{M_F} \left(\int_{\varphi_{ij}^{M_F}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} [(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1] k \varphi_{min}^{k} \varphi^{k+1} d(\varphi_{ij}))^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{A.5}$$

Evaluating the integral and substituting for cutoff productivities

$$P_j^{-k} = N_{ij} \frac{k}{k-\sigma+1} \left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}} \varphi_{min}^{k} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{-k} \left(\sigma w_j (\frac{f_{M_F} - f_{E_X}}{(\Omega\tau_{ij})^{\gamma(\sigma-1)} - 1})\right)^{1-\frac{k}{\sigma-1}} \tag{A.6}$$

.

A.1.3 Aggregate price index of all foreign varieties

For the Aggregate price index of all foreign varieties supplied via exports and FDI we start with the following integral

$$\begin{aligned} P_{ij}^{1-\sigma} &= \frac{G(\varphi_{M_F}^{*}) - G(\varphi_{E_X}^{*})}{1-G(\varphi_i^{*})} N_i \left(\int_{\varphi_{ij}^{E_X}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right) \\ &+ \frac{1-G(\varphi_{M_F}^{*})}{1-G(\varphi_i^{*})} N_i \left(\int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right) \end{aligned}$$

$$P_{ij}^{1-\sigma} = N_{E_X}\left(\int_{\varphi_{ij}^{E_X}}^{\infty} \varphi^{\sigma-1}(\frac{\sigma}{\sigma-1}w_{ij}\tau_{ij})^{1-\sigma}k\varphi_{min}^{k}\varphi^{k+1}d(\varphi_{ij})\right)$$
$$+N_{M_F}\left(\int_{\varphi_{ij}^{M_F}}^{\infty} \varphi^{\sigma-1}(\frac{\sigma}{\sigma-1})^{1-\sigma}[(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)}-1](w_{ij}\tau_{ij})^{1-\sigma}k\varphi_{min}^{k}\varphi^{k+1}d(\varphi_{ij})\right)$$

Evaluating the Integral and substituting for the cut off productivities

$$P_j^{-k} = N_{ij}\frac{k}{k-\sigma+1}\left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}(w_i\tau_{ij})\right)^{-k}(\sigma w_j)^{1-\frac{k}{\sigma-1}}\left(f(_{ij}^{E_X})^{1-\frac{k}{\sigma-1}}+(\frac{f_{M_F}-f_{E_X}}{(\Omega\tau_{ij})^{\gamma(\sigma-1)}-1})^{1-\frac{k}{\sigma-}} \right. \tag{A.7}$$

A.2 Aggregation of Sales

A.2.1 Export sales

$$X_{ij}^{E_X} = \frac{G(\varphi_{M_F}^{*})-G(\varphi_{E_X}^{*})}{1-G(\varphi_i^{*})}N_{E_X}\left(\int_{\varphi_{ij}^{E_X}}^{\infty}\varphi^{\sigma-1}(\frac{\sigma}{\sigma-1}w_{ij}\tau_{ij})^{1-\sigma}\beta\frac{Y_j}{P_j^{1-\sigma}}dG(\varphi_{ij}^{E_X})\right) \tag{A.8}$$

Under the assumption of a common Pareto distribution for all countries, this becomes:

$$X_{ij}^{E_X} = N_{E_X}\left(\int_{\varphi_{ij}^{E_X}}^{\infty}\varphi^{\sigma-1}(\frac{\sigma}{\sigma-1}w_{ij}\tau_{ij})^{1-\sigma}\beta\frac{Y_j}{P_j^{1-\sigma}}k\varphi_{min}^{k}\varphi^{k+1}d(\varphi_{ij}^{E_X})\right) \tag{A.9}$$

Evaluating the integrals we get

$$X_{ij}^{E_X} = \underbrace{\left(\frac{k}{k-\sigma+1}\varphi_{min}^{\sigma-1}\right)M_{ij}^{E_X}}_{\text{supply capacity}}\underbrace{\left(\frac{\varphi_{min}}{\varphi_{E_X}}\right)^{-k-\sigma+1}\frac{Y_j}{P_j^{1-\sigma}}(\frac{\sigma}{1-\sigma}w_{ij}\tau_{ij})^{1-\sigma}}_{\text{market capacity}} \tag{A.10}$$

Using the export productivity cut-off the last equation above can be re-written as:

$$X_{ij}^{E_X} = N_{E_X} \frac{k}{k-\sigma+1} \left(\frac{Y_j}{P_j^{1-\sigma}} \right)^{\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij} \right)^{-k} \left(\sigma w_j f_{ij}^{E_X}) \right)^{1-\frac{k}{\sigma-1}} \tag{A.11}$$

Equivalently this can be re-written as:

$$X_{ij}^{E_X} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{E_X}} \right)^k N_{E_X}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1} \right) f_{E_X} w_j}_{\text{intensive}} \tag{A.12}$$

The effect of trade costs on export sales yields the extensive and intensive margins as

$$\frac{dlnX_{ij}^{E_X}}{dln\tau_{ij}} = -\underbrace{(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)}_{\text{extensive}} \tag{A.13}$$

A.2.2 FDI or Multinational Sales

$$X_{ij}^{M_F} = \frac{1-G(\varphi_{M_F}^*)}{1-G(\varphi_i^*)} N_{M_F} \left(\int_{\varphi_{ij}^{M_F}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{M_F}) \right) \tag{A.14}$$

Under the assumption of a common Pareto distribution for all countries, this becomes:

$$X_{ij}^{M_F} = N_{M_F} \left(\int_{\varphi_{ij}^{M_F}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} k \varphi_{min}^k \varphi^{k+1} d(\varphi_{ij}^{M_F}) \right) \tag{A.15}$$

Evaluating the integrals we get

$$X_{ij}^{M_F} = \underbrace{\left(\frac{k}{k-\sigma+1} \varphi_{min}^{\sigma-1} \right) N_{M_F}}_{\text{supply capacity}} \underbrace{\left(\frac{\varphi_{min}}{\varphi_{M_F}} \right)^{-k-\sigma+1} \frac{Y_j}{P_j^{1-\sigma}} [(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1](\frac{\sigma}{1-\sigma} w_{ij}\tau_{ij})^{1-\sigma}}_{\text{market capacity}} \tag{A.16}$$

Using the FDI productivity cut-off the last equation above can be re-written as:

$$X_{ij}^{M_F} = N_{ij}^{M_F} \frac{k}{k-\sigma+1} \left(\frac{Y_j}{P_j^{1-\sigma}} \right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}) \right)^{-k} \left(\sigma w_j (\frac{f_{M_F} - f_{E_X}}{(\Omega \tau_{ij})^{\gamma(\sigma-1)} - 1}) \right)^{1-\frac{k}{\sigma-1}} \tag{A.17}$$

In the same spirit as the trade case(export) this can be re-written as;

$$X_{ij}^{M_F} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{fdi}} \right)^k N_{ij}^{M_F}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1} \right) w_j (f_{M_F} - f_{E_X})}_{\text{intensive}} \tag{A.18}$$

Following Irarrazabal et al. (2013)The overall effect of an increase in variable trade barriers on total affiliate sales can be decomposed into intensive and extensive margin as ;

$$\frac{dlnX_{ij}^{M_F}}{dln\tau_{ij}} = -\underbrace{(1-\gamma)(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)\chi_{M_F}}_{\text{extensive}} \tag{A.19}$$

Where χ_{fdi} is defines as the elasticity of FDI cutoff to variable trade barriers.

$$\chi_{M_F} = \frac{(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)}(\gamma-1) - 1}{(\Omega_{ij}\tau_{ij})^{\gamma(\sigma-1)} - 1} \tag{A.20}$$

A.2.3 Aggregate Sales(from all operations, i.e both exports and FDI)

$$X_{ij}^{-k} = N_{ij} \frac{k}{k-\sigma+1} \left(\frac{Y_j}{P_j^{1-\sigma}} \right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}) \right)^{-k} (\sigma w_j)^{1-\frac{k}{\sigma-1}} \left(f_{E_X}^{1-\frac{k}{\sigma-1}} + (\frac{f_{M_F} - f_{E_X}}{(\Omega \tau_{ij})^{\gamma(\sigma-1)} - 1})^{1-\frac{k}{\sigma-1}} \right) \tag{A.21}$$

A.3 Expenditure shares

A.3.1 Trade Share(Income spent on Imports

$$\lambda_{ij}^{E_X} = \frac{X_{ij}^{E_X}}{\sum\limits_v X_{vj}} = \frac{\left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*} \right)^k N_i w_i f_{ij_{E_X}} \frac{\sigma k}{k-\sigma+1}}{\sum\limits_v \left(\frac{\varphi_{ii}^*}{\varphi_{vj}^*} \right)^k N_i w_i f_{vj_{E_X}} \frac{\sigma k}{k-\sigma+1}} \tag{A.22}$$

Using the solution for the equilibrium mass of firms derived from the labour market clearing condition and free entry condition we get

$$\lambda_{ij}^{E_X} = \frac{(L_i/f_{iE})\varphi_{min_i}^k \left(\varphi_{ij}^*\right)^{-k} w_i f_{ij_{E_X}}}{\sum_v (L_{vj}/f_{vE})\varphi_{min_v}^k \left(\varphi_{vj}^*\right)^{-k} w_v f_{vj_{E_X}}} \tag{A.23}$$

Using the export cutoff productivity

$$\lambda_{ij}^{E_X} = \frac{(L_i/f_{iE})\varphi_{min_i}^k w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)} (\tau_{ij})^{-k} f_{ij_{E_X}}^{1-\frac{k}{\sigma-1}}}{\sum_v (L_{vj}/f_{vE})\varphi_{min_v}^k w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)} (\tau_{vj})^{-k} f_{vj_{E_X}}^{1-\frac{k}{\sigma-1}}} \tag{A.24}$$

A.3.2 FDI Expenditure Share

$$\lambda_{ij}^{M_F} = \frac{X_{ij}^{M_F}}{\sum_v X_{vj}} = \frac{\left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^k N_i w_i (f_{ij_{M_F}} - f_{ij_{E_X}}) \frac{\sigma k}{k-\sigma+1}}{\sum_v \left(\frac{\varphi_{ii}^*}{\varphi_{vj}^*}\right)^k N_i w_i (f_{vj_{M_F}} - f_{vj_{E_X}}) \frac{\sigma k}{k-\sigma+1}} \tag{A.25}$$

Using the solution for the equilibrium mass of firms derived from the labour market clearing condition and free entry condition we get

$$\lambda_{ij}^{M_F} = \frac{(L_i/f_{iE})\varphi_{min_i}^k \left(\varphi_{ij}^*\right)^{-k} w_i (f_{ij_{M_F}} - f_{ij_{E_X}})}{\sum_v (L_{vj}/f_{vE})\varphi_{min_v}^k \left(\varphi_{vj}^*\right)^{-k} w_v (f_{vj_{M_F}} - f_{vj_{E_X}})} \tag{A.26}$$

Using the export FDI productivity

$$\lambda_{ij}^{M_F} = \frac{(L_i/f_{iE})\varphi_{min_i}^k w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)} (\tau_{ij})^{-k} f_{ij_{E_X}}^{1-\frac{k}{\sigma-1}} (\Omega_{vj}\tau_{ij})^{\eta(\sigma-1)} - 1)^{1-\frac{k}{\sigma-1}}}{\sum_v (L_{vj}/f_{vE})\varphi_{min_v}^k w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)} (\tau_{vj})^{-k} f_{vj_{ex}}^{1-\frac{k}{\sigma-1}} (\Omega_{vj}\tau_{vj})^{\eta(\sigma-1)} - 1)^{1-\frac{k}{\sigma-1}}} \tag{A.27}$$

A.3.3 Aggregate Expenditure share

The aggregate expenditure share on all foreign varieties is given by adding the last two expressions for trade and FDI expenditure shares

Appendix B

Capital Taxation, Heterogeneous Multinational and Trade Firms in a Dynamic Model of Endogenous productivity Growth

B.1 Profits of the home, export and FDI firms

- **Domestic firm maximisation problem**

$$\max_{p_D(\omega)(\varphi)} p_{D_t}(\omega)x_{D_t}(\omega) - a(\omega)x_{D_t}(\omega) \tag{B.1}$$

This yields price as a constant markup over the marginal costs

$$p_D = \frac{\sigma}{\sigma - 1}a(\omega) \tag{B.2}$$

and the revenue of this domestic firm are given by

$$R_D(\omega) = A^\sigma L_{Y_t}\Phi^{2\sigma-1}a(\omega^{1-\sigma}) \tag{B.3}$$

This implies that the domestic firm profits are given by

$$\pi_D(\omega) = A^\sigma(1 - \Pi)L_{Y_t}\Phi^{2\sigma-1}a(\omega^{1-\sigma}) \tag{B.4}$$

- **Trade firm maximisation problem**

$$\max_{p_{Ex}(\omega)(\varphi)} p_{E_x}(\omega)x_{Ex_t}(\omega) - a(\omega)x_{Ex_t}(\omega) \tag{B.5}$$

This yields price as a constant mark-up over the marginal costs including the iceberg costs τ

$$p_D = \frac{\sigma}{\sigma - 1}a(\omega)\tau \tag{B.6}$$

and the revenue of this exporting firm are given by

$$R_{E_x}(\omega) = A^{\sigma}L_{Y_t}\Phi^{2\sigma-1}\tau^{1-\sigma}a(\omega^{1-\sigma}) \tag{B.7}$$

This implies that the exporting firm profits are given by

$$\pi_{E_x}(\omega) = A^{\sigma}(1 - \Phi)L_{Y_t}\Phi^{2\sigma-1}\tau^{1-\sigma}a(\omega^{1-\sigma}) \tag{B.8}$$

- **FDI firm maximisation problem**

$$\max_{p_{Em}(\omega)(\varphi)} p_{E_m}(\omega)x_{Em_t}(\omega) - a(\omega)x_{Em_t}(\omega) \tag{B.9}$$

The multinational firm price is also a constant markup over costs, and in the same fashion as for the domestic and trade firm we specify the price, revenue and profits as follows.

$$p_{E_m} = \frac{\sigma}{\sigma - 1}a(\omega) \tag{B.10}$$

and the revenue of this domestic firm are given by

$$R_{E_m}(\omega) = A^{\sigma}L_{Y_t}\Phi^{2\sigma-1}a(\omega^{1-\sigma}) \tag{B.11}$$

This implies that the domestic firm profits are given by

$$\pi_{E_m}(\omega) = A^{\sigma}(1 - \Phi)L_{Y_t}\Phi^{2\sigma-1}a(\omega^{1-\sigma}) \tag{B.12}$$

Appendix C

Impact of exhaustible resource on opening for international trade: Hartwick's rule and taste for foreign goods

C.1 Derivation of the Euler Equation

First derive the Indirect Utility
Lets rewrite the utility $U(C)$ as

$$U(C_{i,i}, C_{-i,i}) = \ln\left(\varphi C_{ii}^{\rho} + (1-\varphi)C_{-i,i}^{\rho}\right)^{\frac{1}{\rho}},\ i \in \{h, f\} \tag{C.1}$$

The consumers' budget constrained choice problem has the Lagrangian

$$\mathcal{L} = U(C_{ii}, C_{-ii}) - \lambda\left(E_i - pC_{ii} - pC_{-ii}\right) \tag{C.2}$$

The demand functions then satisfy the familiar tangency condition:

$$\frac{MRS_{ii}}{MRS_{-ii}} = \frac{\varphi C_{ii}^{\rho-1}}{(1-\varphi)C_{-ii}^{\rho-1}} = \frac{p_{ii}}{p_{-ii}} = 1 \tag{C.3}$$

Solving the equation for C_{-ii} in terms of C_{ii} we get:

$$C_{-ii} = C_{ii}\left(\frac{(1-\varphi)}{\varphi}\right)^{\chi} \tag{C.4}$$

Remember that $\chi = \frac{1}{1-\rho}$, then substituting C_{-ii} into the expenditures we get

$$pC_{ii} + pC_{-ii} = pC_{ii} + pC_{ii}\left(\frac{(1-\varphi)}{\varphi}\right)^{\chi} = E_i \tag{C.5}$$

$$C_{ii} = \frac{E_i}{p + p\left(\frac{(1-\varphi)}{\varphi}\right)^{\chi}} = \frac{(p/\varphi)^{-\chi}}{\varphi^{\chi}p^{1-\chi} + (1-\varphi)^{\chi}p^{1-\chi}}E_i \tag{C.6}$$

By symmetry the demand function for the foreign goods consumed at home is given by

$$C_{-ii} = \frac{(p/(1-\varphi))^{-\chi}}{\varphi^{\chi}p^{1-\chi} + (1-\varphi)^{\chi}p^{1-\chi}}E_i \tag{C.7}$$

The corresponding indirect utility function is therefore given by

$$\overline{\mathcal{U}} = \frac{E_i}{\varphi^{\chi}p^{1-\chi} + (1-\varphi)^{\chi}p^{1-\chi}} \tag{C.8}$$

$$\max_{C} \int_0^{\infty} \mathrm{e}^{-\rho t} \ln\left[\frac{E_i}{\varphi^{\chi}p^{1-\chi} + (1-\varphi)^{\chi}p^{1-\chi}}\right] dt; \tag{C.9}$$

subject to (4.1) for the home country and to (4.2) for the foreign country. Optimality conditions give the standard Euler equation in which at each moment in time the rate of growth of expenditures on consumption is equal to the difference between the instantaneous interest rate and the rate of time preference.

$$\frac{\dot{E}_i}{E_i} = r_i - \rho,\ i \in \{h, f\}. \tag{C.10}$$

Assuming equal interest rates across countries, $r_h = r_f$ yields equal growth rates of expenditures.

C.2 Derivations for final producers

The (current value) Hamiltonian functions for final producer in country i is:

$$\mathcal{H}_i = pY_i - \int_0^{N_i} p_{Q_{ij}} Q_{ij} dj - p_R R_i - \lambda_i \left(s\left[a - p - Y_i(t) - Y_{-i}(t)\right]\right), \tag{C.11}$$

substituting in it for Y_i from (4.6) and normalizing $s = 1$ for simplicity

$$\mathcal{H}_i = p\left[R_i^{\gamma}\int_0^{N_i} Q_{ij}^{\eta}dj\right] - \int_0^{N_i} p_{Q_{ij}}Q_{ij}dj - p_R R_i + \lambda_i\Bigg[a - p - \left(R_i^{\gamma}\int_0^{N_i} Q_{ij}^{\eta}dj\right) - \left(R_{-i}^{\gamma}\int_0^{N_{-i}} Q_{-ij}^{\eta}dj\right)\Bigg] \tag{C.12}$$

and then for Q_{ij} from (4.21) one may write down explicitly Hamiltonian as a function of resource for both firms :

$$\mathcal{H}_i = p\left[R_i^{\gamma}\int_0^{N_i}\left(\left[\frac{\eta}{p_Q}R_i^{\gamma}\right]^{\frac{1}{1-\eta}}\right)^{\eta}dj\right] - \int_0^{N_i} p_{Q_{ij}}\left(\left[\frac{\eta}{p_Q}R_i^{\gamma}\right]^{\frac{1}{1-\eta}}\right)dj - p_R R_i + \lambda_i\left[a - p - \left[R_i^{\gamma}\int_0^{N_i}\left(\left[\frac{\eta}{p_Q}R_i^{\gamma}\right]^{\frac{1}{1-\eta}}\right)^{\eta}dj\right] - \left[R_{-i}^{\gamma}\int_0^{N_{-i}}\left(\left[\frac{\eta}{p_Q}R_{-i}^{\gamma}\right]^{\frac{1}{1-\eta}}\right)^{\eta}dj\right]\right]. \tag{C.13}$$

Assuming similar prices for all the intermediates, $p_{Q_{ij}} = p_Q$ we can integrate over and obtain

$$\mathcal{H}_i = pR_i^{\frac{\gamma}{1-\eta}}\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}N_i - R_i^{\frac{\gamma}{1-\eta}}\left(\frac{\eta^{\frac{1}{\eta}}}{p_Q}\right)^{\frac{\eta}{1-\eta}}N_i - p_R R_i + \lambda_i\Bigg[a - p - R_i^{\frac{\gamma}{1-\eta}}\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}N_i - \tag{C.14}$$

$$R_{-i}^{\frac{\gamma}{1-\eta}}\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}N_{-i}\Bigg]. \tag{C.15}$$

Taking F.O.C for resource demand:

$$\frac{\partial\mathcal{H}_i}{\partial R_i} = \frac{\gamma}{1-\eta}pR_i^{\frac{\gamma}{1-\eta}-1}N_i\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} - \frac{\gamma}{1-\eta}R_i^{\frac{\gamma}{1-\eta}-1}\left(\frac{\eta^{\frac{1}{\eta}}}{p_Q}\right)^{\frac{\eta}{1-\eta}}N_i - p_R + \lambda_i\left[-\frac{\gamma}{1-\eta}R_i^{\frac{\gamma}{1-\eta}-1}N_i\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right] \tag{C.16}$$

Equating the derivative to zero we get resource demand as a function of prices:

$$p_R = \frac{\gamma}{1-\eta}R^{\frac{\gamma}{1-\eta}-1}N_i\left[p\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} - \lambda_i\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} - \left(\frac{\eta^{\frac{1}{\eta}}}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right],$$

$$p_R = \frac{\gamma}{1-\eta}R^{\frac{\gamma}{1-\eta}-1}N_i\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\left[(p-\lambda_i)-\eta\right] \tag{C.17}$$

$$R_i^D = \frac{\gamma}{1-\eta}^{1-\frac{\gamma}{1-\eta}}\left(\frac{(p-\lambda_i)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}}N_i^{1-\frac{\gamma}{1-\eta}}\left(\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} \tag{C.18}$$

which is (4.24). Expressing λ_i from (4.24) through resource demands yields

$$\lambda_i = p - \frac{p_R}{\frac{\gamma}{1-\eta}R_i^{\frac{\gamma}{1-\eta}-1}N_i\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}} - \eta \tag{C.19}$$

giving

$$\dot{\lambda}_i = \dot{p} - \frac{\dot{p}_R \frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} - p_R \frac{\gamma}{1-\eta}\left(\frac{\gamma}{1-\eta}-1\right) R_i^{\frac{\gamma}{1-\eta}-2} \dot{R}_i N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}}{\left(\frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^2} \tag{C.20}$$

Costate equations are given by:

$$\dot{\lambda}_i = r\lambda_i - \frac{\partial \mathcal{H}_i}{\partial p} = (r+1)\lambda_i - R_i^{\gamma \frac{1}{1-\eta}} \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} N_i \tag{C.21}$$

Given resource demand, (4.24), the evolution of shadow costs is defined by prices and itself:

$$\dot{\lambda}_i = (r+1)\lambda_i - \frac{\gamma}{1-\eta}^{1-\frac{\gamma}{1-\eta}} \left(\frac{(p-\lambda_i)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_i^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} N_i, \tag{C.22}$$

and the same for λ_{-i}. At the same time we may make use of (C.19) and (C.20) to express dynamics of resource demand for each country from (C.21):

$$\dot{\lambda}_i = r\lambda_i - \frac{\partial \mathcal{H}_i}{\partial p} = (r+1)\lambda_i - R_i^{\gamma\frac{1}{1-\eta}} \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} N_i =$$
$$(r+1)\left(p - \frac{p_R}{\frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}} - \eta\right) - R_i^{\gamma\frac{1}{1-\eta}} \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} N_i,$$
$$\left(R_i^{\gamma\frac{1}{1-\eta}} \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} N_i = Y_i,\ \frac{\gamma}{1-\eta} R_i^{\frac{\gamma}{1-\eta}-1} N_i \left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}} = \frac{\partial Y_i}{\partial R_i}\right), \tag{C.23}$$
$$\dot{p} - \frac{\dot{p_R}\frac{\partial Y_i}{\partial R_i} - p_R \frac{\partial^2 Y_i}{\partial^2 R_i}\dot{R}_i}{\left(\frac{\partial Y_i}{\partial R_i}\right)^2} = (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) - Y_i,$$
$$\left(\dot{p} - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\left(\frac{\partial Y_i}{\partial R_i}\right)^2 = \dot{p_R}\frac{\partial Y_i}{\partial R_i} - p_R\frac{\partial^2 Y_i}{\partial^2 R_i}\dot{R}_i,$$
$$\dot{R}_i = \frac{\dot{p_R}\frac{\partial Y_i}{\partial R_i} - \left(\dot{p} - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{p_R\frac{\partial^2 Y_i}{\partial^2 R_i}},$$
$$\dot{R}_i = \frac{\dot{p_R}}{p_R}\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left(\dot{p} - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}},$$
$$\dot{R}_i = \frac{\dot{p_R}}{p_R}\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left((a - p - Y_i - Y_{-i}) - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}}$$
$$\dot{R}_i = \frac{\dot{p_R}}{p_R}\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left((a - p - Y_i - Y_{-i}) - (r+1)\left(p - \frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right) + Y_i\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}} =$$
$$\frac{\dot{p_R}}{p_R}\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left(a - Y_{-i} - (2+r)p + (1+r)\frac{p_R}{\frac{\partial Y_i}{\partial R_i}}\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}} =$$
$$\left(\frac{\dot{p_R}}{p_R} - (1+r)\right)\frac{\frac{\partial Y_i}{\partial R_i}}{\frac{\partial^2 Y_i}{\partial^2 R_i}} - \frac{1}{p_R}\left(a - Y_{-i} - (2+r)p\right)\frac{\left(\frac{\partial Y_i}{\partial R_i}\right)^2}{\frac{\partial^2 Y_i}{\partial^2 R_i}}. \tag{C.24}$$

Yielding the resource demand dynamics as a function of itself, final price and outputs of both producers (which are in turn also functions of resource demand).

Evolution of the price itself is:

$$\dot{p} = \left(a - p - R_i^{\gamma}\int_0^{N_i} Q_{ij}^{\eta}dj - R_{-i}^{\gamma}\int_0^{N_{-i}} Q_{-ij}^{\eta}dj\right) \tag{C.25}$$

C.3 Resource price determination

The total demand for resource is

$$R = \left(\frac{\gamma}{1-\eta}\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{(p-\lambda_h)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_h^{1-\frac{\gamma}{1-\eta}} + \left(\frac{(p-\lambda_f)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_f^{1-\frac{\gamma}{1-\eta}}\right) \tag{C.26}$$

The optimization for resource monopolist is carried out with respect to price of the resource, since demand is given and no market segmentation is assumed. Current value Hamiltonian is:

$$\mathcal{H}^R = p_R R - \lambda_R R \tag{C.27}$$

with R given by (C.26). Differentiation w. r. t. p_R yields price proportional to shadow costs of extraction:

$$\frac{\partial \mathcal{H}^R}{\partial p_R} = R + p_R \frac{\partial R}{\partial p_R} - \lambda_R \frac{\partial R}{\partial p_R} = 0, \tag{C.28}$$

with derivative of resource demand proportional to the demand itself

$$\begin{aligned}
&\frac{\partial R}{\partial p_R} = \\
&-(1-\frac{\gamma}{1-\eta})\frac{1}{p_R}\left(\frac{\gamma}{1-\eta}\left(\frac{\eta}{p_Q}\right)^{\frac{\eta}{1-\eta}}\right)^{1-\frac{\gamma}{1-\eta}} \left(\left(\frac{(p-\lambda_h)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_h^{1-\frac{\gamma}{1-\eta}} + \right.\\
&\left.\left(\frac{(p-\lambda_f)-\eta}{p_R}\right)^{1-\frac{\gamma}{1-\eta}} N_f^{1-\frac{\gamma}{1-\eta}}\right) = \\
&-(1-\frac{\gamma}{1-\eta})\frac{1}{p_R}R,
\end{aligned}$$

we define the price of the resource:

$$-(1-\frac{\gamma}{1-\eta})R(1-\lambda_R/p_R) + R = 0, \tag{C.29}$$

$$\frac{\lambda_R}{p_R} \overset{R\neq 0}{=} \frac{\gamma}{\eta+\gamma-1}. \tag{C.30}$$

The dynamics of the co-state is:

$$\dot{\lambda}_R = r\lambda_R - \frac{\partial \mathcal{H}^R}{\partial S} = r\lambda_R. \tag{C.31}$$

transversality condition is given by

$$\lim_{t\to\infty} p_R(t)R(t)e^{-\int_0^t r(\tau)d\tau} = 0 \tag{C.32}$$

Thus the growth of resource price is proportional to the interest rate:

$$\dot{p_R} = \frac{\eta+\gamma-1}{\gamma}\dot{\lambda_R} = r\frac{\eta+\gamma-1}{\gamma}\lambda_r \to \frac{\dot{p}_R}{p_R} = r\frac{\eta+\gamma-1}{\gamma}. \tag{C.33}$$

giving (4.29).

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