How do social networks contribute to wage inequality? Insights from an agent-based analysis

Herbert Dawid Simon Gemkow
How do social networks contribute to wage inequality?
Insights from an agent-based analysis.*

H. Dawid† and S. Gemkow‡

January 2013

Abstract

Based on a closed agent-based macroeconomic simulation model (Eurace@Unibi) this paper analyzes whether the density of social networks influences via referrals the residual wage inequality in different skill groups. It is shown that an increase in network density leads to a polarization of firms and a concentration of workers with high specific skills at firms with high productivities (and wages) thereby enlarging within group wage inequality, but not between group wage inequality.

JEL: C63, J31, O33
Keywords: wage inequality, social networks, referral hiring, agent-based simulation

---

*The authors would like to thank Heike Solga and Andres Cardona for helpful comments on a previous draft. The Eurace@Unibi model has been developed in cooperation with Michael Neugart, Philipp Harting and Sander van der Hoog. Financial support of the German Science Foundation (Collaborative Research Center (SFB) 882 ”From Heterogeneities to Inequalities”) is gratefully acknowledged.

†Department of Business Administration and Economics and Center for Mathematical Economics, Bielefeld University, Germany. Email: hdawid@wiwi.uni-bielefeld.de

‡Department of Business Administration and Economics, Bielefeld University, Germany. Email: sgemkow@uni-bielefeld.de
1 Introduction

In the literature several explanations for the increasing wage inequality in the U.S. as well as in continental Europe have been given. The most prominent explanations are skill-biased-technological change (Autor and Katz (1999)), job polarization (Goos et al. (2009)), mechanical changes in the workforce composition (Lemieux (2006)), labor market institutions and episodic events (Card and DiNardo (2002)). Recent empirical literature has pointed out that much of the growth of wage inequality can however be ascribed to increasing wage dispersion among workers with the same educational attainment and/or experience (Violante (2002)). Put differently there seem to be important mechanisms independent from standard observable factors, like skills, influencing residual wage inequality. Recent empirical work suggest that residual wage inequality is strongly linked to firm heterogeneity. Faggio et al. (2010) provide evidence that the between-firm inequality of labor productivity has substantially increased over time and that the vast majority of the increase in individual wage inequality in the UK is a between-firm (rather than within-firm) phenomenon (see also Dunne et al. (2004)).

The goal of this paper is to explore the role of one such potential mechanism, namely referral hiring through social networks, on wage inequality. The importance of referral hiring, which is workers finding jobs via the help of social contacts is expressed by the claim of Montgomery (1991), that ”..it’s not what you know but who you know”. This claim is supported by Bewley (1999). He concludes in his survey of 24 studies that 30% to 60% of jobs were found by using social contacts like friends and relatives. It is well documented and widely accepted that social contacts help workers to find a job\footnote{An excellent survey over social networks and referrals can be found in Ioannides and Loury (2004).}. But the effect of finding jobs via social contacts on the wages is less clear and is discussed controversially. Pellizzari (2010) analyzes a number of European countries regarding their wage differentials between referred employees and non referred employees. He finds that wage premia and wage penalties are equally frequent across countries for employees who found their job via social contacts and attributes these differences to search strategies of firms in the recruitment process. Among others Dustmann et al. (2010), Schmutte (2010), Marmaros and Sacerdote (2002), and Simon and Warner (1992) find that referred employees receive higher wages. Lower wages for referred employees are documented in Bentolila et al. (2010), Antoninis (2006), and Pistaferri (1999).
We contribute to the discussion about wage inequality by analyzing the effects of the structure of social networks, especially their density, on wage inequality via referrals. In this paper we ask (and try to answer) the question: Does the density of the network influence the residual wage inequality in different skill groups, and the across skill wage inequality? Given that referral hiring seems to have become more frequent over the last years and social networks seem to have become more dense\footnote{For example a recent survey concerning firm recruiting activities shows that about 30\% of companies increase their investment in referral hiring and 55\% invest more in recruiting through social media (see \url{http://recruiting.jobvite.com/resources/social-recruiting-charts.php}).} answering this question might contribute to a sound understanding of the mechanisms responsible for the observed increase in wage inequality within (skill) groups.

In the present model workers have two dimensions of human capital endowments. First they have an exogenously given general skill level, which can be interpreted as formal qualification or general abilities. The general skills are equally distributed across workers and are used to introduce homophily. Workers with a certain general skill level are more likely to have friends with the same general skill level. In addition to the general skills, workers have an endogenously changing specific skill level, which can be interpreted as the productivity or abilities attained on the job. These specific skills are ex-ante not observable by firms. But they are revealed if an unemployed worker is referred by an employed friend to his employer. The employee provides the information about the specific skills of the friend to his employer. The employer prefers referred over non referred applicants if the referred applicants exceed a certain threshold of specific skills. Moreover, the employer prefers applicants with high general skills over low skilled applicants. This transmission of information in order to reduce uncertainty of firms about the productivity of workers is also used in the models of Montgomery (1991) and Dustmann et al. (2010). On the contrary, in the contributions by Calvo-Armengol and Jackson (2004, 2007) the role of social networks is that job information is just passed around by workers.

Workers increase their specific skills on the job if they work with machines with a quality such that the workers current skills are not sufficient to fully exploit the potential productivity of the machine. The speed of learning of a worker increases with his general skill level. There is a complementarity between the productivity of the capital and the specific skills of workers. The productivity of a worker-machine match is determined by the minimum of the productivity of the machine and the specific skills of the worker\footnote{Bassanini and Scarpetta (2002) and Griffith et al. (2004) provide evidence that for the adoption of a new technology adequate skills of the workforce are required.}.
productivity of the firms’ capital stocks increases over time as firms acquire newer vintages of machines. Firms determine their wage offers based on the average specific skills of employees in each skill group. Each firm posts wage offers for each general skill group, but the wage offers do not depend on whether the employee is hired through a referral or not.

The simulations reported here show that in spite of the absence of explicit wage discrimination of firms between referral and non-referral hirings the average wage of workers who obtain their job through referrals is higher than that for the other workers. The ratio of referral wages and non-referral wages is positively correlated with the density of the network for all skill groups but declines with the general skill level. These patterns influence the wage inequality, which is measured via the standard deviation of overall wages, skill dependent wages, and across skill groups. Furthermore, the standard deviation of skill dependent wages is positively correlated with the network density for low general skill levels. For high general skill groups, however, the standard deviation of wages is not correlated with the network density. Moreover, the standard deviation of wages between general skill levels is not affected by the density of the social network.

The results of the simulation are based on a non-trivial mechanism. Due to the observability of the specific skills of referred applicants, firms are able to target among referred workers the best applicants with respect to their specific skills. The applicants, however, accept job offers from firms which are paying the highest wages. The firms paying the highest wages are those which have the highest productivity which feed back to the specific skills of their workforce. Hence, due to the increased transparency of the labor market in the presence of more referrals, an endogenously generated clustering of workers with high specific skills at high productive firms emerges and increases the wage inequality within groups of workers with identical general skills. The effect mainly occurs for low general skill groups because job turnover is higher for low skilled workers than for high skilled workers. The reason for the differences in turnover is that low skilled workers are dismissed more easily by firms.

Using the agent-based approach which endogenizes the evolution of the productivity of workers, the determination of wage offers and wages as well as the labor demand and considering the interplay of these factors distinguishes the present contribution from analytical search models with a network structure developed in for example Dustmann et al. (2010), Schmutte (2010), or Goel and Lang (2009).

Agent-based models have already been successfully used to shed light on numerous economic questions. Tesfatsion and Judd (2006) provides an overview of agent-based models applied in different areas of economic re-
search. In the context of labor markets and social networks agent-based models are presented for example in Gemkow and Neugart (2011), Tassier and Menczer (2001), and Pingle and Tesfatsion (2003). Furthermore, Dawid et al. (2008, 2009, 2011a), Dosi et al. (2010), and Delli Gatti et al. (2005) developed closed agent-based macroeconomic models where the contributions by Dawid et al. rely on previous versions of the model used in this paper. The innovative contribution of the present paper relative to all this work is that it is the first study focusing on the distribution of wages emerging in an agent-based macroeconomic model and considering the effects of social networks on wage inequality in different skill groups.

In the next section 2 key features of the model and the network formation algorithm are described. The parametrization and simulation set-up are explained in section 3. Afterwards in section 4 the simulation results and the mechanism driving these results are presented. Finally a summary is given in the last section 5.

2 The model - Eurace@Unibi

The analysis is conducted in a closed agent-based macroeconomic simulation model. Previous versions of the model have already been used to analyze policy measures fostering human capital endowments and the opening up of labor markets (Dawid et al. (2008, 2009, 2011a)). The complexity and the number of elaborate features of the framework is too high to present it in full detail at this point. Instead a short overview is given and the most important key features are described in order to make the results comprehensible and understandable. The labor market is explained in more detail as the analysis is focused on the role of referrals on wages. A more detailed documentation and discussion of the model can be found in Dawid et al. (2011b).

2.1 Overview

The economy is populated by different types of agents. The main actors are households, consumption good producers (firm) and one investment good producer (igfirm). The central unit of time is a day. Five working days are one week, four weeks are one month and twelve months are one year.

The agents make decisions following rules and the choice of the decision rules in the Eurace@Unibi model is based on a systematic attempt to incorporate rules that resemble empirically observable behavior documented in the relevant literature. Concerning households, this means that for ex-
ample empirically identified saving rules are used and purchasing choices are
described using models from the Marketing literature with strong empirical
support. With respect to firm behavior we follow the ‘Management Science
Approach’, which aims at implementing relatively simple decision rules that
match standard procedures of real world firms as described in the correspond-
ing management literature. A more extensive discussion of the Management
Science approach can be found in Dawid and Harting (2012). Decisions or
actions of agents can be event driven or time driven. Time driven decisions
are made on a daily, weekly, monthly, or yearly base depending on the agent
type and the context of the decision. The real side of the economy, i.e.
the consumption goods market, the investment goods market, and the labor
market are modeled in much detail with elaborate decision rules and agent
interactions. The model also incorporates a rudimentary financial and credit
market in order to close the model.

As usual households have a dual role as consumers and workers. A worker
$w$ has two dimensions of human capital endowments namely an exogenously
given general skill level $b_{gen}^w$ and an endogenously increasing specific skill
level $b_{w,t}$. General skills can be interpreted as formal qualification or general
embodied abilities while specific skills are experiences or abilities obtained on-
the-job reflecting the productivity of each worker. There exist five general
skill levels
described by different values of $b_{gen}^w$, i.e. $b_{gen}^w \in \{1, 2, 3, 4, 5\}$. $b_{gen}^w = 1$ is the lowest general skill level and $b_{gen}^w = 5$ the highest. General
skill levels determine how fast workers can acquire specific skills by on the job learning. General skills are observable by firms in the hiring process,
while specific skills are not. They become observable to employers during
the production process.

2.2 Consumption goods market

Consumption goods are produced by the firms once in a month on their acti-
vation day. The activation days of firms are asynchronously distributed over
the month. The consumption goods are homogenous regarding quality, but
horizontally differentiated and heterogeneous in prices. Each firm conducts
a detailed production planning containing the calculation of the vertically
differentiated input factors capital and labor. Planned production quanti-
ties and prices are determined by using estimated residual demand curves
obtained from simulated purchase surveys on a test market. Each firm sends
a query to a representative sample of households containing different prices

\footnote{Although the choice of five general skill levels has been inspired by the five levels used in the International Adult Literacy Survey (IALS) the general skill groups used in the model should not be seen as a representation of the five IALS levels.}
for its good once in a year. Households answer this query by sending their purchasing decision for each queried price. These answers are based on the same decision rules households employ in their actual purchasing decisions. Based on the answers and the development of the economy firms form demand expectations and conduct a detailed cost and profit analysis. They set planned productions quantities and prices in order to maximize the expected discounted profits over their planning horizon taking into account the estimated marginal costs and the elasticity of demand. Firms store the produced goods in a mall. Each household visits the mall once a week but not all households on the same day of the week. They spend their consumption budget in order to purchase consumption goods. Since the consumption goods are homogenous regarding the quality but heterogeneous in prices less expensive goods are more likely to be chosen. The decision which good to buy is described using a logit-choice model with strong empirical foundation in the Marketing literature (see e.g. Malhotra (1984)). In several parts of the Eurace@Unibi model choices of decision makers are described by logit models. These models are well suited to capture decisions where individuals try to maximize some objective function which depends on observable and unobservable variables.

The production technology of firm \( i \) is represented by a linear-limitational production function. This Leontief production function implies a fixed proportion of the input factors capital and labor. It is assumed that one unit of capital is used by one worker. Furthermore, it is assumed that employees with the highest general skills use the most productive machines in order to exploit the productivity of the machines as much as possible. Thus, machines and workers are deployed in descending order regarding their productivity. The effective productivity of the capital-worker combination is the minimum of the productivity of the capital and the specific skills of the worker \( \min[A^v, b_{w,t}] \), where \( A^v \) is the quality of the used vintage \( v \) and \( b_{w,t} \) the specific skills of worker \( w \) in period \( t \). Since the size of the capital stock for a vintage might not necessarily be an integer, it might happen that a worker cannot spend his full working time on the same vintage. In that case the remaining fraction of the working time of this worker is allocated to next lower vintage.

As mentioned above, workers embody an exogenously given general skill level and an endogenously changing level of specific skills. The specific skills are increasing over time during the production process. Workers are learning on-the-job when using the currently employed machines (see e.g. Argote and Epple (1990)). It is assumed that workers with higher general skills \( b_{w}^{gen} \) learn faster than workers with lower general skills. The condition for the learning of workers is that the productivity of the machine used by a worker \( w \) is
higher than his specific skills \( A^v > b_{w,t} \). In the opposite case the specific skills of the worker remain unchanged. Formally the updating of specific skills of a worker \( w \) employed by firm \( i \) can be written as

\[
b_{w,t} = b_{w,t-1} + \sum_{v \in V: s_{w,t} > 0} s_{w,v,t} \cdot \chi(b_{w}^{\text{gen}}) \cdot \max[0, A^v - b_{w,t}],
\]

where \( s_{w,v,t} \) denotes the fraction of time the worker spends with machine of vintage \( v \) in period \( t \). The function \( \chi(b_{w}^{\text{gen}}) \) increases with general skills \( b_{w}^{\text{gen}} \) and \( 0 < \chi(b_{w}^{\text{gen}}) < 1 \).

### 2.3 Investment goods market:

Investment goods\(^5\) are produced by one investment goods producer. The investment goods producer offers different vintages of the investment good with different qualities on every day. The supply of each vintage is infinite. In order to simplify the model at this point it is assumed that the investment goods producer is able to produce without any input factors and any costs. The introduction time of new vintages with improved quality follows an exogenous stochastic process, where the quality of a machine determines the maximal productivity that can be reached with the machine in case it is used by workers with sufficiently high specific skills. The investment goods producer expands the set of offered vintages as soon as a new vintage becomes available. Prices for the vintages are determined by the investment goods producer using a combination of a cost based approach and a value based approach. In order to close the model the revenues of the investment goods producer are paid out as dividends to the households. Consumption goods producers use the investment goods as an input factor in their production. They have to choose between the offered vintages if they want to expand their capital stock or to replace the depreciated fraction. In order to make a vintage choice decision firms estimate the costs and expected future benefits of the different vintages, which depend on the distribution of skills in their workforce. A consumption good producer has to choose between the different vintages \( v \in \{1, ..., V_t\} \) offered at different productivities \( A^v \) and different prices \( p^v \) by the investment goods producer where \( V_t \) denotes the index of the most recent vintage at time \( t \). The benefit of an offered vintage is the additional output it generates. The costs are the price \( p^v \). In order to determine the benefit of a vintage \( v \) firm \( i \) takes into consideration the complementarity between the productivity of the vintage \( A^v \) and the average specific skills of its workforce \( B_{i,t} \). The effective productivity of a vintage

---

\(^5\)Investment goods can also be denoted as machines or capital.
v is estimated by min[A^v, B_{i,t}]. If A^v > B_{i,t} firm i cannot fully exploit the technology of the vintage since it is constraint by the specific skills of the workforce. Since the specific skills of the workforce are generally increasing over time firm i estimates the development of the specific skills for the next S months. The potential effective productivity of a vintage v is then given by min[A^v, \tilde{B}_{i,t+s}] for each month s ≤ S, where \tilde{B}_{i,t+s} denotes the expected future specific skills of the firm’s workforce. The sum over the discounted potential effective productivities

\[ B^v = \sum_{s=0}^{S} \left( \frac{1}{1+\rho} \right)^s \min[A^v, \tilde{B}_{i,t+s}] \]

with discount rate \( \rho > 0 \) is used as a proxy for the benefit of vintage v.

The probability Prob_{i,t}^v for choosing vintage v is determined by a logit model using the ratio of the benefit and the costs of a vintage \( r^v = \frac{B^v}{p^v} \) and a parameter \( \gamma^{Vint} \) for the sensitivity of choice

\[ \text{Prob}_{i,t}^v = \frac{\exp(\gamma^{Vint} \cdot r^v)}{\sum_{v=1}^{V} \exp(\gamma^{Vint} \cdot r^v)}. \]

The higher \( \gamma^{Vint} \) the higher is the probability to choose the vintage with the highest value of \( r^v \).

2.4 Labor market and referral hiring

Labor supply

Unemployed workers are searching for jobs. An unemployed worker searches actively on average on two randomly chosen days in a month. He takes the wage offers of a randomly chosen set of firms posting vacancies into consideration and compares them with his reservation wage. In case that his reservation wage is lower than the wage offer he sends one application to the corresponding firm containing his general skill level. The maximum number of overall non referral applications per month is exogenously given. On the remaining days of the month he is not actively searching. But he might receive one or more referrals from his employed friends for one or more firms. In that case he sends one application to each referred firm indicating that he received a referral, if the wage offer is higher than his reservation wage. In that sense a referral from a friend activates an otherwise non active unemployed on that day. It is also possible that he receives referrals on days when he is actively searching. In that case he sends (referral) applications
to referred firms and applications to non-referred firms.

**Labor demand**

The labor demand is determined by consumption good firms in their production planning. In case of an expansion of the production quantity firm $i$ posts vacancies containing wage offers $w_{i,t,g}$ for each general skill level $g$ in period (month) $t$ on its activation day. The wage offer has two constituent parts. The first part is the market driven base wage $w_{i,t}^{\text{base}}$. The base wage is paid per unit of specific skill. If the firm cannot fill its vacancies it increases the base wage to attract more workers. The second part is related to the expected productivity of a worker. Firms build this expectation based on the average specific skills in each general skill group inside the firm $\bar{b}_{i,t,g}$. For each of the general skill groups the firm $i$ offers different wages. The wage offer of firm $i$ for an applicant with general skill $g$ is given by

$$w_{i,t,g} = \bar{b}_{i,t,g} \cdot w_{i,t}^{\text{base}}$$

In addition, the firms inform their current employees about the vacancies and the corresponding wage offers. The employees can pass this information to their unemployed friends by making a referral. It is assumed that the specific skills of a referred applicant can be observed by the firm due to information the firm receives from the employee referring the worker. In that sense a referral has to be distinguished from purely passing the job information. Firms consider a referral only if the revealed specific skills of the applicant are higher than the economy wide average specific skill of level workers with this general skill. Hence, firms are using the referral hiring in order to get highly productive workers. An employee makes only one referral per month and only if the specific skills of his unemployed friend exceeds this specific skill requirement.\(^6\) If the employee has more than one unemployed friend fulfilling the specific skill requirement he chooses randomly among those friends. Since the wage offer is the same for referred and non-referred applicants firms expect that on average the ratio between the wage offer and the specific skill is lower in the case of a referred applicant. Put differently, on average they get higher specific skills for the same costs. Hence, in the hiring process firms prefer referred applicants over non referred applicants. Inside both groups they also prefer applicants with higher general skills over applicants with lower general skills because high general skill workers will improve their specific skills faster.

\(^6\)Beside the fact that in this model firms try to get employees with high specific skills via the referrals, [Saloner (1985)](10) argued that employees tend to refer more productive workers. Referring a low productive friend could be disreputable.
Preferring referred over non referred applicants due to the reduction of uncertainty is in line with [Montgomery (1991)]. If the firms plan to decrease the workforce employees with low general skills are dismissed first, because they have generally a lower speed of learning. At the end of each month wages of employees are increased by the monthly productivity growth rate. This can be interpreted as a reduced form representation of collective wage bargaining.

Matching algorithm

Consumption good firms decide once a month on their activation day whether to post vacancies or not. The matching between job seekers and vacancies works in the following way:

Step 1:

Firms post vacancies including wage offers and inform their workforce about the vacancies and wage offers. Each employee refers one unemployed friend if he has an unemployed friend whose specific skills exceed the economy wide average specific skills in his general skill group. If the employee does not have such a friend he does not give a referral and if he has more than one such friend he chooses randomly.

Step 2:

Each job seeker checks the posted vacancies on average on two days in a month. Furthermore, he might receive referrals which activate him on days when he is not searching. From the list of vacancies and/or referrals he extracts those postings which fit in terms of his reservation wage. He sends applications to randomly chosen non referred firms and (referral) applications to referred firms.

Step 3:

If the number of applicants is smaller or equal to the number of vacancies the firms send job offers to every applicant. If the number of applicants is higher than the number of vacancies firms send job offers to as many applicants as they have vacancies to fill. Firms prefer referred applicants over non referred applicants when sending job offers. Inside both groups they prefer applicants with higher general skills over applicants with lower general skills.

Step 4:
Each unemployed ranks the incoming job offers according to the offered wages. The ranking depends solely on the offered wages and not on referrals. The highest ranked job offer is accepted by each unemployed. The other job offers are ignored.

**Step 5:**

Firms adjust their vacancy list for filled jobs and their labor force for new employees.

**Step 6:**

If an unemployed $k$ did not find a job he reduces his reservation wage by an exogenously given fraction $\phi \in [0, 1]$, that is $w_{k,t+1}^R = (1 - \phi)w_{k,t}^R$. The lower bound for the reservation wage are the unemployment benefits that $k$ receives from the government. If an unemployed $k$ found a job his new reservation wage is set to his actual wage, $w_{k,t}^R = w_{k,i,t}$. If the number of unfilled vacancies exceeds a threshold value $V$ the firm increases the base wage offer by an exogenously given fraction $\upsilon \in [0, 1]$, that is $w_{i,t+1}^{base} = (1 + \upsilon)w_{i,t}^{base}$. Go to step 1.

This algorithm runs on each day and is aborted after two iterations. Referrals are only made in the first iteration as well as the update of the base wage offer. It happens that not all firms can fill their vacancies and not all unemployed find a job. This leads to labor market frictions and rationing of firms regarding their labor demand.

### 3 Parametrization and set-up of experiment

Table 1 summarizes the general setup in terms of numbers and types of the most important agents in the framework. The intention of the analysis to get a qualitative understanding of the role of referrals on wage inequality in the presence of changing number of friends and homophily. Hence, no attempt has been made to match the skill distribution with empirical data from a particular economy, but the general skill levels are assumed to be distributed equally across the households. Each of the five general skill groups contains 320 households. The idea is to avoid effects which are the result of different general skill group sizes. Table 2 presents the initialization values of the key variables. The technological frontier $A^V_0$ is the productivity of the latest
Table 1: General set up

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households</td>
<td>1600</td>
</tr>
<tr>
<td>Consumption goods producers</td>
<td>80</td>
</tr>
<tr>
<td>Investment good producers</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Initialization

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technological Frontier: $A^0$</td>
<td>1.7</td>
</tr>
<tr>
<td>Capital Stock: $K_{i,0}$</td>
<td>19.0</td>
</tr>
<tr>
<td>Productivity Capital Stock: $A_{i,0}$</td>
<td>1.5</td>
</tr>
<tr>
<td>Specific Skill Level: $b_{w,t}$</td>
<td>1.5</td>
</tr>
<tr>
<td>Initial Base Wage Offer: $w_{i,0}^{base}$</td>
<td>1</td>
</tr>
</tbody>
</table>

vintage type offered by the investment goods producer in $t = 0$. The technological progress is driven by an exogenous stochastic process and is on average 2% per year. The initial capital stock of each firm $K_{i,0}$ is 19.0 and consists of one vintage type. Taken into account the one to one correspondence of workers and machines each firm wants to hire 19 workers at the beginning. Having 80 firms and 1600 households this would lead to an unemployment rate of 5%. The productivity of the initial capital stock is $A^0 = 1.5$, where it is assumed that initially all firms use a vintage $v_0 < V_0$ whose productivity is slightly below the technological frontier. The same holds for the specific skills $b_{w,t}$ of workers. Consequently, at the beginning workers are not able to learn since there is no gap between the technology and the specific skills. But if the first firms decide to increase their capital stock or to replace the depreciated capital stock with machines having a higher productivity, i.e. machines of the latest vintage types, a gap between technology and specific skills for some workers is generated. At that point the learning regarding the specific skills starts to take place. All households are starting with the same specific skill level 1.5 and hence, the initial wage offers $w_{i,t,g}$ and therefore the wages are also 1.5. Taken together the points mentioned above it should be highlighted that wage and specific skill distributions emerge endogenously as a result of the dynamic interplay of production decisions of firms and labor market interactions. We do not present the full calibration of the model.
The full set of used parameter values is motivated by empirical observations and chosen such that a set of empirical stylized facts on different levels of aggregation are reproduced by the model [Dawid et al. 2011b].

In the analysis four types of social networks are compared. These different networks were generated by a stochastic algorithm, which allows to produce networks with given degree distribution and inbreeding homophily index\(^7\) (see Gemkow 2011) for details. The networks considered in the following analysis differ with respect to their degree distribution and therefore with respect to the average number of friends and the density. The degree distribution of all four networks follows a power-law distribution, where the probability for a household to have \(k\) links is given by

\[ p_k = C k^{-\alpha}. \]

The constant \(C\) normalizes the degree distribution such that \(\sum_{k_{\min}}^{k_{\max}} p_k = 1\). According to Newman (2010) the exponent \(\alpha\) of the power-law distribution lies in the range of \(2 \leq \alpha \leq 3\) and is set to 2.5 for all four networks. Empirical evidence suggests that in many social networks there exists a \(k_{\min}\) such that the power-law properties hold for \(k \geq k_{\min}\), whereas for \(k < k_{\min}\) we have \(p_k < p_{k_{\min}}\). To keep things simple it is assumed here that \(p_k = 0\) for \(k < k_{\min}\), which means that \(k_{\min}\) is the lower bound of links per household. The upper bound for the number of links per household is given by \(k_{\max}\). The technical maximum number of links per household is given by the number of total household (1600), but it does not seems reasonable to assume that one household might know all other households. In the baseline network 1 the lower and the upper limits are set to \(k_{\min} = 1\) and \(k_{\max} = 20\). The four different networks are generated by multiplying both limits with a parameter \(M\) with \(M \in [1, 2, 4, 8]\) and are calibrated in such a way that the inbreeding homophily index \(IH\) is always \(\approx 0.48\). This value lies within the range identified for empirical data in Currarini et al. (2009). Table 3 summarizes the four different networks. For each simulation for the four considered cases a stochastically generated network with the given properties was generated.

\(^7\)The inbreeding homophily index \(IH_g\) for one general skill group \(g\) used by Currarini et al. (2009) is given by \(IH_g = \frac{H_g - n_g^2}{1 - n_g}\) where \(n_g\) is the fraction of household with general skill \(g\) and \(H_g = s_g + d_g\) is the homophily index which measures the fraction of links to households with the same general skill level. \(s_g\) \((d_g)\) denotes the number of links to households with the same (different) general skill level. The inbreeding homophily index is than given by \(IH = \sum_{g=1}^{5} n_g \cdot IH_g\).

\(^8\)See Newman (2010) for a summary of the characteristics of different social networks such as actor, telephone call, email, or sexual contact networks and for further references. Lewis et al. (2008) show the power-law behavior of the degree distribution for Facebook data.
Table 3: Networks

<table>
<thead>
<tr>
<th>Properties / Networks</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{\text{min}} )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>( k_{\text{max}} )</td>
<td>20</td>
<td>40</td>
<td>80</td>
<td>160</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Average number of links</td>
<td>( \approx 2 )</td>
<td>( \approx 4 )</td>
<td>( \approx 9 )</td>
<td>( \approx 18 )</td>
</tr>
</tbody>
</table>

The networks always stay constant over time. For each of the five considered networks 40 batch-runs over 10000 periods (days, 500 months, 42 years) were conducted. The qualitative statements made in the following section are backed up by statistical significance test w.r.t. this data. The results in the next section are mainly displayed using box plots that represent the distribution across the 20 batch-runs for each network. Each of the 40 data points per box plot represents the average of the last 20 monthly observations of a single run.

4 Results and Mechanism

The main research question addressed in this paper is whether the increase in density of social networks and the increase in referral hiring are a potential mechanism responsible for the observable increase of wage inequality within groups with the same educational attainment and/or experience. In order to address this issue we compare the within group wage inequality as well as the between group inequality for different densities of the social network. Within group wage inequality is measured by the standard deviation of wages of employed individuals within a certain general skill group divided by the average wage of workers in this group. To measure the between group inequality we calculate the average wage in each skill group and determine the standard deviation of these values normalized by the average wage in the population.

The discussion of our results is organized in a way that we first state the main insights concerning the effects of network density on wage inequality in terms of three observations. Afterwards we highlight and discuss the mechanisms driving these observations. It should be pointed out that the aim of the analysis is to gain a qualitative understanding of the mechanism driving the results. Therefore, the focus lies rather on identifying correlations and patterns in the variables of interest when varying the network density rather than on quantitative statements and comparisons of the size of different ef-
fects that are identified. Having discussed the effect of increased network density on wage inequality we then briefly explain the effect on the wage level of such a change in the social network.

Figure 1 shows the within group wage inequality for all five skill groups for increasing network density. It can be easily seen that the standard deviation of wages is positively correlated with the network density. But the correlation decreases with higher general skills. For general skill groups 3 and 4 no systematic effect of an increase in network density on the wage inequality can be seen while for skill group 5 the correlation is slightly negative. These points are summarized in our first main observation.

**Observation 1:** *Increasing the density of the social network significantly increases the within group wage inequality for the low general skill groups.*

Figure 2 presents for each of the four social networks in panels (a)-(d) the dynamics of the standard deviation of wages for skill groups 1-5. It becomes obvious that in addition to observation 1 the standard deviation of wages decreases with the general skill level for a given density. Concerning the dynamics of the standard deviation of wages, we observe an almost stationary pattern.

The overall wage inequality in the population results from the combination of within group and across group variation (as well as from composition effects if the fractions of employees from the different skill groups change). As mentioned in the introduction factors like skill biased technological change have been identified as main drivers of the increasing wage inequality between skill groups. The focus here is on within group inequality, but it is nevertheless important to understand whether an increase in the density of the social network also has implication on between group inequality. As can be seen from Figure 3 the density has no systematic effect in this respect. Obviously a key assumption here is that the expected number of links is independent from the general skill level of the worker. Exploring the effects of heterogeneity between skill groups in this respect would certainly be interesting and empirically relevant but is beyond the scope of the present analysis. The following observation highlights that an increase in network density should indeed be mainly seen as a trigger for an increase of within group rather than between group wage inequality.

**Observation 2:** *Increasing the density of the social network does not in-*.  

---

9We applied Wilcoxon rank tests to confirm the statistical significance of our different observations. The results are available upon request from the authors.
Figure 1: Standard deviation of wages per general skill level.
Figure 2: Standard deviation of wages for skill groups 1-5 per network. Skill 1 solid, skill 2 dashed, skill 3 dotted, skill 4 dashed-dotted, skill 5 long-dashed
We now return to the wage inequality within a skill group. In the presence of job referrals the wages earned by members of this group can be separated into two classes: the wages earned by workers who got their job through referrals and those who went through the regular labor market. It should be noted that an assumption of our model is that each firm pays identical wages to all its incoming employees with a certain general skill level no matter whether the employee was hired through a referral or not. This might suggest that there should be no systematic difference between the two classes of wages mentioned above. However, as can be clearly seen from panels (a) and (b) of Figure 4 wages of workers who obtained their jobs through referrals are significantly higher than those of the other workers. The mechanism responsible for this fact will be discussed below. An implication of this observation is that the wage inequality within a skill group can be decomposed into the inequality within referral wages, the inequality within non-referral wages and the gap between referral and non-referral wages. The lower panels of Figure 4 indicate that not only the gap between referral and non-referral wages increase the between group wage inequality.

\[\text{Between Standard Deviation Wage}\]

\[\text{Figure 3: Standard Deviation of Wages between Skill Levels}\]

\[\text{creases the between group wage inequality.}\]

\[\text{For reasons of clarity and simplicity we depict in this and all the following figures only the highest and the lowest skill group. The qualitative patterns for the three intermediate skill groups are fully in accordance with the ones shown in the figures.}\]
wages increases as the density of the social network goes up, but that for low skill groups also the inequality within the group of referral wages goes up significantly. Although for the highest skill group we also observe an increase of the referral/non-referral wage ratio and of the standard deviation of referral wages if the network density goes up, the size of both effects is much weaker than for the low skill group. For the group of non-referral wages (corresponding figure is not shown here) no positive effect of increased network density on wage inequality can be observed. Summarizing, we get:

**Observation 3:** Increasing the density of the social network increases for each skill group the wage gap between workers who obtained jobs with respectively without referrals. Furthermore, it increases for low skill groups the wage inequality within the group of workers who obtained their job by referrals.

Observations 1 and 3 indicate that an increase in network density increases wage inequality within the skill groups because of an increases in the gap between referral and non-referral wages and an increase in inequality within referral wages. Furthermore, the effect is more pronounced for low skill groups.

We now turn to the exploration of the mechanisms responsible for these observations. The kind of influence an increased network density has on the wages of workers with low general skills is far from obvious. Since there is no wage discrimination between referred and non-referred workers with identical general skills within a firm the increased wage inequality must result from the wage dispersion between firms and a sorting effect resulting in the systematic overrepresentation of workers hired through referrals in firms that pay relatively high wages. The firms paying above average wages are those with above average productivity. Since firm productivity depends on the quality of the firm’s capital stock as well as the level of specific skills of its employees we depict in Figure 5 the correlation between the specific skills in a firm per skill group and its fraction of referral hirings as well as the correlation between the quality of the capital stock of a firm and its fraction of referral hirings. Both correlations are significantly positive indicating that indeed firms with a high productivity for a certain skill group have a larger fraction of employees of that group hired through referrals. These firms are able to attract the workers with the largest specific skills, which explains the observed gap between referral and non-referral wages. The correlation is much stronger for the specific skills in a firm, which is due to the fact that in most cases the skills of workers are the limiting factor rather than the quality of the employed machine which means that actual productivity is
Figure 4: Ratio between referral and non-referral wages for general skill levels 1 (a) and 5 (b). Standard deviation of referral wages for general skill groups 1 (c) and 5 (d).
mainly determined by specific skills in a firm. The mechanism responsible for this observation is quite clear. When hiring through referrals firms are able to observe the specific skills of workers, which means that they can target the workers with above average specific skills. All firms try to do this, but the most productive firms are most attractive for workers since they offer the highest wages. Therefore, the most advanced firms have a larger probability of filling their vacancies with above average workers using referrals and this process reinforces the relative productivity advantage of these firms.

Due to the complementarity between specific skills and the quality of the capital stock, also the relative technological advantage of these firms is reinforced, but this effect seems to be relatively small. An increase of the frequency of referralhirings due to an increased network density therefore should lead to an increase in the heterogeneity of firms with respect to the specific skills of their workers. Figure 6 shows the standard deviation of average specific skills of the workforce across firms and confirms this conclusion for the considered low general skill group. For workers with general skill 5 no such systematic effect on the standard deviation of specific skills arises. On the one hand this is due to the much lower frequency of job turnovers of workers of this skill group, which stems from the fact that firms are more reluctant to dismiss workers with high general skills in case they reduce their work force. Hence, the described clustering of high general skill workers at certain firms can only occur much slower as can be seen in Figure 7. Figure 7 presents the Herfindahl Index of employees for skill levels 1 and 5. The Herfindahl Index is used to measure the concentration of employees with a certain general skill level across firms. If workers in all general skill groups were equally distributed across firms the Herfindahl indices would read $1/80 = 0.0125$. Hence, Figure 7 shows that workers of all skill groups are concentrated within the set of firms and the concentration is significantly higher for low general skill groups. Furthermore, the Herfindahl index is positively correlated with the network density for all skill groups. On the other hand, the effect of allocation patterns of workers to firms on the specific skill dynamics is much lower for workers in the highest skills groups. These workers are always exposed to the highest vintages available in the firms, which vary relatively little across firms. For low skill groups the vintage they can work with depends crucially on the distribution of general skill groups within the workforce of the firm and firms vary substantially in this respect. Figure 8 shows that firms workforce compositions tend to concentrate only

---

11The Herfindahl Index for a given general skill group in period $t$ is defined as $HH_g = \sum_{i=1}^F x_{i,t,g}^2$, where $F$ denotes the number of consumption good producers and $x_{i,t,g}$ the fraction of employees with general skills $g$ working in firm $i$. 

22
Figure 5: Correlation between the fraction of referred employees with a particular general skill level in firms and the specific skills of employees in this general skill group: general skill levels 1 (a) and 5 (b). Correlation between the productivity of the used capital stock in firms and the fraction of referred employees in their workforce.
Figure 6: Standard Deviation of specific skills between firms for general skill levels 1 (a) and 5 (b)

Figure 7: Herfindahl index of employees for general skill levels 1 (a) and 5 (b)
on a few general skill groups and deviate from the composition of the economy wide workforce.\footnote{If workers would be matched to firms randomly the composition of the workforce of firms regarding the general skills should be nearly equal to the economy wide composition of general skills of employed households. The following measure, which is used in Figure 8, would be 0 if the workforce composition of all firms would be equal to the economy wide composition.}

Putting this together with Figure 7 implies that the gap in terms of on the job learning between firms is much more pronounced for workers in low skill groups because low skill workers get access to more productive machines in firms where the workforce is concentrated in low skill levels.

Given the higher dismissal rate and the larger heterogeneity of specific skills there is a relatively large fraction of low skilled unemployed with relatively high specific skills and therefore a high reservation wage. These unemployed refuse referrals from friends working in firms with a low concentration of low skilled employees and consequently low wage offers. They wait until they find a firm which is paying a high wage for their low skill group, which means that this firm has a high fraction of (referred) employees in that low

\begin{equation}
Concentration_t = \frac{1}{F} \sum_{i=1}^{F} \sum_{g=1}^{G} (r_{i,t,g} - r_{t,g})^2
\end{equation}

where $F$ is the number of firms and $G$ is the number of general skill levels. $r_{t,g} = e_{t,g}/e_t$ is the fraction of all employees $e_{t,g}$ with general skill level $g$ of all employees $e_t$ in the economy. The same ratios are calculated for the employees $e_{i,t,g}$ in each firm $i$ that is $r_{i,t,g} = e_{i,t,g}/e_{i,t}$. 

---

Figure 8: Deviation of firms workforce composition from the economy wide composition of general skill of employed households.
skill group thereby reinforcing the concentration pattern explained above. Figure 9 displays that in network 1 panel (a) and 4 (b) low skilled workers refuse a higher percentage of referrals than high skilled workers.

From the perspective of the workers the described mechanism induces a path dependency in the sense that workers who happen to acquire high specific skills early on have higher chances to be hired by technologically more advanced firms, which due to the learning by doing effects reinforces their relative specific skill advantage with respect to their peers. This effect becomes stronger the more transparent the market becomes in a sense that specific skills of workers are observable for potential employees. Hence, increasing the density of the social network leads to the emergence of larger heterogeneity among workers of identical general skills with respect to their specific skills and hence to larger wage inequality also within the group of referral wages, as stated in Observation 3.

Having explored in some detail the mechanisms responsible for Observations 1 and 3 we now briefly deal with the effect of an increase of network density on the wage level. Figure 10 clearly shows that the wage level of all skill groups decreases if the expected number of links per worker goes up. To understand this effect, it has to be realized that wages for a certain general skill group in a given firm are determined as the product of the base wage offer of that firm and the estimated average productivity, as measured by the average specific skills of workers with that general skills in the firm. The base wage offer captures the tightness of the labor market as firms increase their
base wage offer whenever their fraction of vacancies they are unable to fill exceeds a certain level. Changes in the wage level therefore might result from changes in the labor market tightness as well as from changes in the average specific skills of workers. Considering these two components of the wage separately clearly shows that the decrease in wages is due to a decrease in the base wage offer of firms rather than to systematic changes in the workers specific skills (corresponding boxplots are not reported here). Indeed Figure 11, showing the average fraction of unfilled vacancies of all firms, clearly indicates that an increase in the density of the social network leads to a decrease in the labor market friction and hence to reduced tightness on the labor market. Accordingly, firms increase their base wage offers with lower frequency and wage levels go down. The observation that a more dense social network leads to a higher number of successful employer-employee matches and hence to lower labor market frictions is quite intuitive. Increasing the number of links per employee makes it more likely that an employed worker knows a suitable searching worker if an opening in that firm appears. Therefore, the number of referrals goes up and, since these referrals generate potential employer-employee matches in addition to the regular labor market interactions, the probability that the vacancy can be filled increases.
Observation 4: Increasing the density of the social network decreases the wage level for all skill groups. This effect is due to decreased labor market frictions inducing lower incentives of firms to increase base wage offers.

The decreased market frictions associated with higher network densities do not only influence the wage level, but also affects the heterogeneity of base wage offers in a firm. Reduced frictions imply that firms increase the base wage offers for incoming employees less frequently, which means that the heterogeneity of base wages paid to different employees, who entered the firm at different points in time, goes down. This induced reduction in heterogeneity of base wages within each firm negatively affects the overall wage inequality and also the residual wage inequality for each skill groups. For the low skill groups this effect is clearly dominated by the mechanisms described above, which lead to an increase in heterogeneity of specific skills of workers and of the productivity-related wage component across firms. For the highest general skill group 5, where these mechanisms have only a very minor effect, as discussed above, the induced reduction in base wage heterogeneity dominates and this leads to the negative dependence on residual wage inequality on network density for skill group 5, as observed in Figure 1(e).

5 Conclusions

The starting point of this paper are two empirical observations. First, residual wage inequality within groups of workers with the same educational at-
tainment and/or experience has gone up in recent years. Second, the involve-
ment of households in social networks as well as the hiring of firms through
social networks is increasing. Using an agent-based closed macroeconomic
model we have shown that these two observations might be connected. We
have highlighted a mechanism indicating how the increase in the density
of the social network can contribute to increasing residual wage inequality
even in the absence of any explicit wage discrimination of employers between
referral and non-referral hirings. The main message is that the increased
transparency on the labor market, implied by a more intensive use of refer-
rals, induces a stronger clustering of workers with high specific skills (and
therefore high productivity) at the most productive firms. This leads to an
increased heterogeneity of firms with respect to productivity and of workers
with respect to specific skills implying the increased wage inequality. Fur-
thermore, the increased transparency reduces labor market frictions, thereby
reducing the fraction of unfilled vacancies and the wage level. Clearly, other
factors might also contribute to the observed increase in residual wage in-
everty, and it is beyond the scope of this paper to quantify which fraction
of the observed effect is due to the mechanisms described here. Considering
this issue might be the starting point of future empirical work.

Since the mechanism identified in this paper relies on the interplay of the
evolution of productivity of firms, the skills of workers and of the match-
ing between firms and workers, a model capturing the heterogeneity of firms
and workers as well as linking the dynamics of goods and labor markets is
necessary to identify this pattern. This study therefore demonstrates that
agent-based models, which combine these features, are useful tools to study
issues of (income) inequality and to explore mechanisms contributing to the
emergence of inequality. So far agent-based models have hardly been em-
ploved in this respect and it seems that the potential of these models in the
area of inequality research could be exploited much more intensively.

Although the model incorporates elaborate features regarding the social
network and the referral hiring there are several important possible exten-
sions. One could allow for systematic heterogeneity of workers with respect
to the number of their links and study in how far a systematic relationship
between the heterogeneity of workers with respect to the number of links
and wage inequality emerges. Furthermore, the mechanism identified here
suggest that increasing the homophily in the social network should further
contribute to the emergence of wage inequality. A systematic exploration of
this issue should deepen our understanding of the relationship between the
structure of social networks and the wage distribution. Finally, it should be
examined in how far the mechanism described here is affected by the speed
of technological change captured in the model by the speed of movement of
the technological frontier (i.e. the productivity of the most recent capital vintage). All this is left for future research.

References


