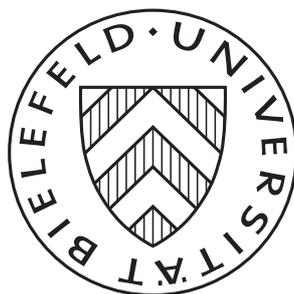


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Thoughts on Social Design

Walter Trockel and Claus-Jochen Haake



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1 Introduction

One of the fundamental problems in applications of methods and results from Mechanism Design and Implementation Theory is the effective enforcement of theoretically established equilibria by which social choice rules are implemented.

A short reflection on what is going wrong on our planet in terms of environmental, political and human disasters will result into the insight to the necessity of intelligent social design of enforceable cooperation based structures.

One of the pillars of such “social design” that had been termed *genuine implementation* in several articles by the late Leo Hurwicz is the possibility of enforcement.

In his “Fundamental theory of institutions: a lecture in honor of Leo Hurwicz”, that was presented at the North American Meeting of the Econometric Society on June 22, 2006 and published in *Review of Economic Design* (2009, vol. 13, pp. 1-2), Roger Myerson starts the abstract by writing:

“We follow Hurwicz in considering fundamental questions about social institutions.”

In section 6 of this article Myerson discusses mechanisms formalized as game forms and the way Hurwicz (2008) proposed to approach the enforcement problem via distinguishing legal games from true games.

In our present note we discuss Hurwicz’s concept of enforcement and Myerson’s proposed modification and compare them with the concept of a social system that

Walter Trockel
Bielefeld University, Bielefeld, GERMANY, e-mail: walter.trockel@uni-bielefeld.de

Claus-Jochen Haake
Paderborn University, Paderborn, GERMANY e-mail: cjhaake@wiwi.upb.de

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had been introduced by Gerard Debreu (Debreu (1952)) also known as *abstract economy* or *generalized game*.

2 Enforcement

Section 8 of Hurwicz (2008) begins with the passage:

“A need for enforcement implies the possibility of behavior that violates the rules of the game. The point is that if there were no possibility of violation, then you wouldn’t need enforcement. We refer to such strategies as illegal, as distinct from legal, of course.”

Later we find:

“To say that the legal game rules are being successfully enforced means that the outcomes of the true game ensure that illegal strategies are less attractive than legal strategies. A strong formulation of successful enforcement might require that, for every player, every illegal strategy is dominated by (that is, is less attractive than) some legal strategy. A ‘weak’ domination would require only that a player at least be no worse off by staying within the law. In fact, however, if everyone else is acting illegally, a normally law-abiding player may not find it advantageous to remain law-abiding. It seems, therefore, more reasonable to adopt a somewhat weaker concept of successful enforcement.”

Referring to this passage, (Myerson, 2009, pp. 67,68) writes:²

“Hurwicz (1998) explains that, if our legal game $G = (N, (C_i)_{i \in N}, (U_i)_{i \in N})$ is embedded in some true game H , the structural relationship must be that $H = (N, (D_i)_{i \in N}, (U_i)_{i \in N})$ has a larger strategy spaces

$$D_i \supset C_i \quad \forall i \in N$$

and has utility functions that extend those of the legal game G to the larger domain $D = \times_{j \in N} D_j$. Hurwicz (1998) then suggests that a strong formulation of successful enforcement could require that, for each player i , each illegal strategy outside C_i should be dominated by some legal strategy in C_i , so that a player’s best responses always take him into the legal game, even if others deviate.

Hurwicz (2008) remarks, however, that a normally law-abiding player might not want to remain law-abiding when others are acting illegally, and so a weaker concept of enforcement may be appropriate. Thus, I would suggest that the definition of institutional enforcement should be weakened, to say that G is enforceable in H when

$$\forall i \in N, \forall c_{-i} \in \times_{j \in N-i} C_j, \forall d_i \in D_i \setminus C_i, \exists c_i \in C_i \text{ such that } U_i(c_{-i}, c_i) > U_i(c_{-i}, d_i),$$

so that each players optimal actions are in his legal strategy set when all other actions are expected to be in their legal sets. That is, G is enforceable when its strategy sets form a curb set in H , as defined by Basu and Weibull (1991). (Curb sets are closed under rational behavior.)”

Strictly speaking, the relation between Myerson’s definition of enforceability and the curb sets of Basu and Weibull (1991) is more subtle. In contrast to their definition that is based on mixed strategy profiles interpreted as players’ beliefs about

² An earlier version of Hurwicz’s Nobel prize lecture (Hurwicz, 2008) appeared in 1998 as working paper at the University of Minnesota.

which pure strategies in compact strategy sets of the other players will be played and that thereby makes use of expected utilities there are no “expectations” in the less specific framework or in the definitions used by Hurwicz (2008) and Myerson (2009). The sentence “so that each player’s optimal actions are in his legal strategy set when all others’ actions are expected to be in their legal sets” used by Myerson adequately describes the situation in the model of Basu and Weibull (1991), but it is meaningless in his context and should be modified as follows in order to fit the pure strategy framework.

The argument in favor of a weaker definition of enforceability used by Hurwicz (2008) and followed by Myerson by his suggestion of the modified definition above does not completely convince us.

A designer’s attitude that accepts deviation from law-abiding behavior of an individual provided the others act illegally, would not be consistent with the central goal of a mechanism to regulate activities in a society by restricting the sets of acceptable actions via unattractive payoffs. Such a weak enforcement fails, for instance, in cases of coordination of the players on a more advantageous illegal equilibrium. Only the trust of everybody in punishment of each single individual that deviates from legal behavior creates confidence in the effectiveness of the mechanism.

A weakness that we feel to be common to either of the above definitions of enforcement is the fact that it is totally determined on the mechanism side rather than also depending on the individuals’ preferences as reflecting the respective society’s values, customs and norms.

We shall deal with this aspect in the next section.

3 Social systems

Debreu (1952) introduced the notion of a social system in the first paragraph of his seminal article “A Social Equilibrium Existence Theorem” and used this notion again on page 397 of his Nobel Memorial Lecture on “Economic Theory in the Mathematical Mode” (Debreu (1984)). The fact that in this purely mathematical analysis, references to Economics and Game Theory are made only in four footnotes may have been causal for the now prevailing habit in the related literature of denoting a social system as a generalized game or an abstract economy (e.g., Shafer and Sonnenschein (1975), Tian (1990), Tian and Zhou (1992)). In particular in the Operations Research literature the problem of existence of equilibria in social systems has become popular under the name *generalized Nash equilibrium problem (GNEP)*. Facchinei and Kanzow (2010) write:

“As we already mentioned, many researchers from different fields worked on the GNEP, and this explains why this problem has a number of different names in the literature including pseudo-game, social equilibrium problem, equilibrium programming, coupled constraint equilibrium problem, and abstract economy. We will stick to the term generalized Nash equilibrium problem that seems the favorite one by OR researchers in recent years.”

They do not cite Debreu (1952) in their references but at least refer to Arrow and Debreu (1954), where the Equilibrium Existence Theorem of Debreu (1952) was first applied in Economics.

In sociology, where generalized games recently have gained some popularity, their origins in Debreu (1952) seem to have remained widely unnoticed (cf. Burns and Roszkowska (2005)).

In this note we want to stress the relevance that Debreu's modeling of a social system may have in an agenda of social and institutional design as propagated in the work of the late Leo Hurwicz. Accordingly, we will introduce Debreu's social system adding at the same time a way of interpreting it under the aspects of genuine implementation and enforcement as discussed in our first two sections.

In Debreu (1952) a social system S is defined as

$$S = (N, (D_i)_{i \in N}, (U_i)_{i \in N}, (\beta_i)_{i \in N})$$

The correspondence β_i from $D_{-i} := \times_{j \in N-i} D_j$ to D_i associates with each action profile $d_{-i} \in D_{-i}$ the set of all those actions of individual i in D_i that are unanimously declared legal by the individuals $j \in N-i$ via the joint choice of the action profile d_{-i} . The correspondence β from D to D is defined by $\beta(d) := \times_{i \in N} \beta_i(d_{-i})$.

This definition does not exclude the possibility that the mechanism declares additional actions or action profiles illegal. An action d_i is illegal per se, if it is not contained in $\beta_i(d_{-i})$ for any $d_{-i} \in D_{-i}$. We will not include this possibility, however, in our present considerations.

Debreu distinguished between *actions* and *choices* of agents $i \in N$ from the sets D_i and $\beta_i(d_{-i})$, respectively. We interpret here his actions as corresponding to strategies in Hurwicz's true game and choices as strategies in legal action profiles: An action profile is legal if it is a choice profile, i.e. if each agent's action in the profile is confirmed by the actions of all others in the profile.

Notice that in contrast to Hurwicz we do not have a legal game whose extension is the true game! Our set of legal action profiles need not have a product structure. So legality of an action is always contingent on all other agent's actions.

To illustrate this point, consider the scenario, in which a committee has to take an ethically difficult decision. Each member can take the actions to "join" or "stay away" from the committee meeting. The decision can only be taken, if half of the members choose "join". As the decision has to be taken, illegal action profiles are only those, in which more than one half of the individuals choose to stay away. Phrased differently, a single individual choosing "stay away" need not mean that the action profile itself is illegal, although everyone choosing to stay away should certainly be called an illegal action profile.

Any product of subsets of the players' action sets, however, that builds a set of legal action profiles would define a legal subgame game of the true game.

The similarity to curb sets is obvious but neither is such a legal game necessarily a curb set nor vice versa. The difference lies in the fact that legality in our context has nothing to do with rationality. The latter will enter when we start considering

equilibria of legal games, an intention behind Hurwicz's idea of *genuine implementation*.

A detailed study of these relations could contribute a solid edifice of enforcement and social mechanism design.

References

- Arrow KJ, Debreu G (1954) Existence of an equilibrium for a competitive economy. *Econometrica: Journal of the Econometric Society* pp 265–290
- Basu K, Weibull JW (1991) Strategy subsets closed under rational behavior. *Economics Letters* 36(2):141–146
- Burns TR, Roszkowska E (2005) Generalized game theory: Assumptions, principles, and elaborations grounded in social theory. *Studies in Logic, Grammar and Rhetoric* 8(21):7–40
- Debreu G (1952) A social equilibrium existence theorem. *Proceedings of the National Academy of Sciences* 38(10):886–893
- Debreu G (1984) Economic theory in the mathematical mode. *The Scandinavian Journal of Economics* 86(4):393–410
- Facchinei F, Kanzow C (2010) Generalized nash equilibrium problems. *Annals of Operations Research* 175(1):177–211
- Hurwicz L (1994) Economic design, adjustment processes, mechanisms, and institutions. *Economic Design* 1(1):1–14
- Hurwicz L (2008) But who will guard the guardians? *The American Economic Review* 98(3):577–585
- Myerson RB (2009) Fundamental theory of institutions: a lecture in honor of leo hurwicz. *Review of Economic Design* 13(1-2):59
- Shafer W, Sonnenschein H (1975) Equilibrium in abstract economies without ordered preferences. *Journal of Mathematical Economics* 2(3):345–348
- Tian G (1990) Equilibrium in abstract economies with a non-compact infinite dimensional strategy space, an infinite number of agents and without ordered preferences. *Economics Letters* 33(3):203–206
- Tian G, Zhou J (1992) The maximum theorem and the existence of nash equilibrium of (generalized) games without lower semicontinuities. *Journal of Mathematical Analysis and Applications* 166(2):351–364