Interjurisdictional Tax Competition, Provision of Two Local Public Goods, and Environmental Policy

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by

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Abstract

This paper provides a model that integrates interjurisdictional tax competition and environmental policy. Each local government supplies two public goods—that benefit the local industry and the residents respectively—which are financed through distortionary taxation on industrial capital and pollutant emissions. In contrast to traditional theory of tax competition, we find that overprovision of local public goods may emerge in equilibrium. Since emission taxes serve to finance public spendings, the supply of public goods and the environmental quality are closely related. In the special case of a small region that cannot affect the national after-tax return to capital, we have the striking new result that in equilibrium two different regimes can occur. Either we have underprovision of public goods and an inefficiently high environmental quality, or we have overprovision of public goods and a too low environmental quality. These inefficiencies persevere as long as the federal government is not entitled to apply deliberate taxation/subsidy schemes. Correspondingly, unless regions are perfectly identical, we cannot hope to overcome the efficiency problem by symmetrical cooperative solutions.

Keywords: interjurisdictional tax competition, capital and emission taxation, efficient provision of public goods, environmental quality.

(JEL: H21, H42, H71, L51, R50)
1 Introduction

Within the last decade much work has been done on the theory of interjurisdictional tax competition. Zodrow and Mieszkowski (1986), Mintz and Tulkens (1986), Wilson (1986), (1987), and others showed that, under fairly general assumptions, interregional competition in tax rates on mobile capital leads to underprovision of local public goods. The extent of inefficiency depends, among the variety of policy tools (Hoyt (1991)), on the number of competing jurisdictions (Bucovetsky (1991)) and on a thorough specification of the policy instruments if we deal with imperfect competition (Widasin (1988)).

The establishment and imposition of environmental taxes, in particular emission taxes, that could be observed in recent years, influences capital flows through its impact on the net return to capital. Hence, the outcome of interjurisdictional competition and the provision of local public goods is determined by fiscal, industrial, and environmental policy. Therefore, a deliberate modeling of these aspects, including emission taxation, is indispensable. Nevertheless, a general treatment of this kind of interactions within a framework of appropriate, i.e., restricted, policy tools (second-best analysis) is still missing. Because of this persevered gap between political significance and theoretical treatment, we integrate interjurisdictional tax competition and environmental policy in our model.1

Our model is structured as follows. Each local government maximizes the utility of some representative resident of its jurisdiction. To achieve this, it chooses two tax rates, one on capital and one on pollutant emissions, and the supply of two local public goods that benefit local industry and residents respectively. The utility of the representative resident depends on the consumption of a private good, the prevailing environmental quality, and the provision of the residential public good. She/he receives income from her/his share of stocks of local industry and from her/his capital endowment. The local firm produces an internationally (or interregionally) traded good by the use of capital and emissions. On the output market it behaves either as a monopolistic or as a perfectly competitive firm. Thus, we allow for non-competitive behavior of the firm on the output market. By

1In economic theory there are some recent works connecting fiscal and environmental policy within an interregional setting. See for example Oates and Schwab (1988), van der Ploeg and Bovenberg (1993a), (1993b), Bovenberg and van der Ploeg (1993), and Schneider and Wellisch (1994). In these works, however, the aspects of fiscal federalism and of interjurisdictional competition only play a minor role. In particular, possibilities for a federal government to re-establish efficiency by correcting the equilibrium outcome remain disregarded and unexplored.
referring to the case of a small number of jurisdictions, non-competitive behavior of local governments is considered as well. In this case, since the supply of national capital is assumed to be fixed, the local government has some not-negligible impact on the nation-wide net return to capital.

Although we introduce an additional policy tool, the emission tax rate, the inefficiency of the public sector perseveres, because local governments take a ‘within-viewpoint’ and neglect pecuniary interjurisdictional externalities, namely the shift in tax-bases. Hence, interjurisdictional tax competition does not lead to efficiency. But the main result of the public finance literature dealing with interjurisdictional tax competition – underprovision of local public goods – is just half of the truth, if we incorporate environmental policy. Contrary to the well-known result of unambiguous underprovision of public services, we find that overprovision may also occur. The question whether under- or overprovision emerges crucially depends on local initial capital endowment and on the influence of local policy on the nation-wide equilibrium net return to capital. For the special case of a small region just two regimes may occur: The first one is characterized by underprovision of public goods, a positive tax rate on capital, and overprovision of environmental quality; whereas the second one exhibits overprovision of public goods, a negative tax rate on capital, and underprovision of environmental quality.

This result may seem to be counterintuitive at a first glance. Note, however, that if the local government has no (free) access to public funds through head taxation, local public goods must be financed either by capital or by emission taxation. Because capital is perfectly mobile, in Nash equilibrium the tax rate on capital is too low from efficiency standpoint in order to prevent capital from flowing towards other regions. The emission tax, on the other hand, is too high, i.e., it exceeds marginal damage, since it not only serves to internalize the externality but also to finance the public services. In the second regime, residents suffer to that large extent from a deterioration of the environmental quality that the efficient emission tax is such that its related tax revenues exceed the cost of the first-best provision level of public services. Even the tax revenues collected from a lower emission tax imply, besides an inefficiently low environmental quality, overprovision of local public goods. Thus, if regions are sufficiently small, we have a striking new result: Interjurisdictional tax competition either leads to

\[ ^{2}\text{To illuminate the effects of induced capital movement we do not consider real externalities. A model that also allows for pollutant transmission is given by Upmann (1995).} \]

\[ ^{3}\text{Some of the models mentioned above are included in our specification as special cases. Especially we derive a generalization of the tax formula of Zodrow and Mieszkowski (1986).} \]
underprovision of public goods and simultaneously to a too high environmental quality or we have an overprovision of public services and a too low environmental quality.

While Nash equilibrium is inefficient the question arises how a higher authority (e.g. federal regulator) can influence the equilibrium allocation to improve overall welfare. If the central government is endeavored to correct the outcome of interregional tax competition by applying a non-neutral financial interjurisdictional transfer schedule, namely to pay subsidies and penalties on local tax rates, efficiency can be re-established. More precisely, we demonstrate for the case of a small region that if in the first (second) regime capital taxation is subsidized (discouraged) and emission taxation is discouraged (subsidized) appropriately, public goods are provided efficiently and the emission tax is equal to marginal environmental damage.⁴

If the federal authority is not entitled to apply regionally differentiated subsidy/penalty schemes, we cannot hope to reach a nation-wide Pareto improvement by either enhancing or lowering all tax rates symmetrically. Even in the special case of identical regions that differ only with respect to (w.r.t.) their initial endowments, a symmetric variation of capital or emission tax rates, evaluated at the symmetric Nash equilibrium, does not imply a welfare improvement in every region. Only if all regions are perfectly identical (i.e., initial endowments are the same everywhere) such a policy measure may lead to a Pareto improvement.

This paper is structured as follows. In the next section we set up our model. Its basic ingredients are the local government, the residents, and the local industry. For the purpose of subsequent welfare analysis we also depict the dependence of factor demand on local policy tools. By means of this, in section three we portrait the optimal policy of one single local government, given the behavior of local industry and all other governments. Especially, we refer to two special cases: to the case of a small region that cannot affect the nation-wide net return to capital and to the case where one factor is fixed on short-terms. In the forth section we consider possibilities of interference of the federal government which seeks to correct the equilibrium outcome of interjurisdictional tax competition towards higher utility levels. The last section summarizes previous results.

⁴This need not be true if the federal ministry of finance has to equate exactly revenues and expenditures and lump sum transfers are not available.
2 The Model

2.1 The Local Government

Consider a nation (or union) which is made up of \( n \) regions (or states), each of them consisting of identical residents. The (representative) residents of different jurisdictions need not to be identical. Each local government is entitled to levy a proportional tax on emissions, \( \tau_e \), as well as on capital, \( \tau_h \). This imposed tax system serves the dual purpose of financing the provision of two local public goods and of affecting industrial pollutant emissions, \( E \). The latter determine environmental quality according to \( U = g(E) \), where \( g' < 0 \) and \( g'' < 0 \). Because no other tax instruments are available to the local government — in particular head taxes (lump sum transfers) are excluded — we are essentially concerned with second-best analysis. The provision of local public goods is modeled by purchases of private goods by the government. Thus, it is assumed that the public production sector is characterized by a twofold one-to-one technology.\(^5\) Denote by \( P_i \) and \( P_r \) the provision level of the industrial local public good and of the residential local public good, respectively. Hence, the (local) public budget constraint, equating expenditures and tax revenues, is given by

\[
P_i + P_r = \tau_h K + \tau_e E,
\]

where \( K \) and \( E \) represent the industrial use of capital and emissions.

2.2 The Residents

The utility level of the representative resident depends on the consumption of a private good, \( X \), the prevailing environmental quality, \( U \), and the supply of the residential local public good, \( P_r \), and is denoted by

\[
U(X, U, P_r).
\]

Local consumers take the environmental quality as well as the provision level of the local public good as given. Since local residents consume only one private good, they have no other choice than to spend their whole income on this item.

\(^5\)Note that this simplification is not essential for our results. If we model the provision of public goods by specific production processes instead of specifying them as public purchases of private goods, the results remain qualitatively unaffected.
By neglecting the labor market, resident’s income is exclusively given by non-labor income which is the sum of his/her share, $\beta \in [0,1]$, of the profits of the local firm, $\Pi$, and of his/her portion, $\theta \in [0,1]$, of the national capital stock, $K$, times the nation-wide net return rate of capital, $\rho$. Hence, the private budget constraint is given by
\[ X = \beta \Pi + \rho \theta K + \xi \] (3)

where $\xi$ denotes all income terms that do not come from the ‘home’ region. Especially, $\xi$ includes dividend income from firms of other regions and cannot be influenced by local residents neither by the local government. Therefore, both of them treat $\xi$ as exogenously given.

### 2.3 The Local Industry

The local firm produces some output good by the use of capital, $K$, and emissions, $E$, subject to the provision of the local industrial public good, $P_t$, (e.g. infrastructure), such that $Q = F(K, E; P_t)$. Let $\rho$ denote the net price of capital; then its the after tax (or consumer) price is given by $p_k := \rho + \tau_k$, whereas the price of emissions is equal to its tax rate, $p_e := \tau_e$. The firm’s output is sold on an international market\(^7\) where the firm may have market power. For ease of tractability, we will consider the two polar cases: either the firm has monopoly power or behaves as a competitive firm.

Note that this good, produced and sold by the local firm, is different from that one consumed by local residents. Therefore, the welfare of the local residents does not directly depend on firm’s output decisions.\(^8\)

\(^6\)The portion $1 - \beta$ of the profits of the local firm belongs either to residents of other regions or to foreigners. If, however, we consider the whole nation we assume that all stocks of national firms are held by domestic consumers. Since we exclude foreigners as capital owners of national firms, we rule out the possibility of providing public goods at the expense of abroad consumers.

\(^7\)Alternatively, we may assume that the firm produces some capital good that is sold on either a national or international market. The crucial fact is that no local consumer (and thus no national consumer) does demand this item; instead, it is either demanded by abroad consumers of by firms of other branches.

\(^8\)We may imagine that the consumer good is produced either in some other region or abroad. If we wish to consider the case where the local firm produces the consumer good we have to multiply the consumption of the private good in (3) by the appropriate price, $p(Q)$. In this case, private welfare depends directly, through related consumer surplus, on local firm’s output. For a more detailed analysis of this case see UPMANN (1995).
The firm is faced with an inverse downward sloping demand curve \( p \) which is not 'too convex' in the sense that
\[
p'' \leq -\frac{2p'}{Q} \quad \forall Q \in \mathbb{R}^+. \tag{4}
\]
Then its profit is given by
\[
\pi(K, E; p_k, \tau_e, P_i) = p(Q)F(K, E; P_i) - (\rho + \tau_k)K - \tau_e E, \tag{5}
\]
where \( F \) is concave in all variables\(^9\) with \( F_{ke} \leq 0.\(^{10}\)

Condition (4) ensures that the revenue \( \hat{R}(Q) = p(Q)Q \) is concave in \( Q \) which together with the concavity of \( F \) implies that revenue \( R(K, E; P_i) := \hat{R}(F(K, E; P_i)) \) is also concave in all its arguments. The firm maximizes its profit w.r.t. \( K \) and \( E \) yielding the first order conditions (f.o.c.s),
\[
R_k = (p + p'Q)F_k = (\rho + \tau_k), \tag{6}
\]
\[
R_e = (p + p'Q)F_e = \tau_e. \tag{7}
\]
In conjunction with \( Q = F(K, E; P_i) \) this implicitly defines the (unconditional) factor demands for capital, \( K(p_k, \tau_e, P_i) \), and emissions, \( E(p_k, \tau_e, P_i) \). Substituting the factor demand into (5) gives the reduced profit function which depends on factor prices and the supply of industrial public services only,
\[
\Pi(p_k, \tau_e, P_i) := p(F(K(\cdot), E(\cdot)))F(K(\cdot), E(\cdot)) - (\rho + \tau_k)K(\cdot) - \tau_e E(\cdot). \tag{8}
\]

### 2.4 Comparative Statics of Factor Demands

First we investigate firm’s behavior on variations of the tax rates, \( \tau_k \) and \( \tau_e \). To see how the firm reacts to a change of the tax rates, differentiate (6) and (7) totally for a fixed supply level of industrial public services. This procedure yields, in matrix notation,
\[
\begin{bmatrix}
   dK \\
   dE
\end{bmatrix} = \frac{1}{\det(hess(R))} \begin{bmatrix}
   R_{ee} & -R_{ke} \\
   -R_{ke} & R_{kk}
\end{bmatrix} \begin{bmatrix}
   dp_k \\
   d\tau_e
\end{bmatrix}, \tag{9}
\]
\(^9\)Subindices of functions denote partial derivatives unless stated otherwise.
\(^{10}\)We can relax this assumption by allowing for \( F_{ke} > 0 \). If this cross derivative is sufficiently bounded, our main results are unaffected. In a forthcoming paper we present a generalization of our model by explicitly giving lower and upper bounds for \( F_{ke} \).
where \( \text{det}(\text{hess}(R)) \) denotes the determinant of the Hessian of \( R(K, E; \bar{P}_i) \). Because of the concavity of \( R \) we know that \( \text{hess}(R) \) is negative semi-definite and hence

\[
R_{kk} < 0, \quad R_{ee} < 0, \quad \text{det}(\text{hess}(R)) = R_{kk}R_{ee} - R_{ke}^2 > 0. \tag{10}
\]

Obviously, from (9) we get the partial derivatives of factor demand w.r.t. its prices which are given by

\[
\begin{align*}
\frac{\partial K}{\partial p_k} &= \frac{R_{ee}}{\text{det}(\text{hess}(R))}, & \frac{\partial K}{\partial \tau_e} &= -\frac{R_{ke}}{\text{det}(\text{hess}(R))}, \\
\frac{\partial E}{\partial p_k} &= -\frac{R_{ke}}{\text{det}(\text{hess}(R))}, & \frac{\partial E}{\partial \tau_e} &= \frac{R_{kk}}{\text{det}(\text{hess}(R))}.
\end{align*} \tag{11}
\]

Often, however, we may be interested in the dependence of factor demands on tax rates rather than on factor prices. In particular, this may be the case if \( \rho \) is not constantly given but depends on local tax rates. By using

\[
dp_k = \left(1 + \frac{\partial \rho}{\partial \tau_k}\right) d\tau_k + \frac{\partial \rho}{\partial \tau_e} d\tau_e
\]

we can evaluate the total effects of tax rates on demand by solving (9) for the desired derivatives:

\[
\begin{align*}
\frac{dK}{d\tau_k} &= \frac{R_{ee}}{\text{det}(\text{hess}(R))} \left(1 + \frac{\partial \rho}{\partial \tau_k}\right), \tag{13} \\
\frac{dK}{d\tau_e} &= -\frac{R_{ke}}{\text{det}(\text{hess}(R))} \left(1 - \frac{R_{ee}}{R_{ke}} \frac{\partial \rho}{\partial \tau_e}\right), \tag{14} \\
\frac{dE}{d\tau_k} &= -\frac{R_{ke}}{\text{det}(\text{hess}(R))} \left(1 + \frac{\partial \rho}{\partial \tau_k}\right), \tag{15} \\
\frac{dE}{d\tau_e} &= \frac{R_{kk}}{\text{det}(\text{hess}(R))} \left(1 - \frac{R_{ke}}{R_{kk}} \frac{\partial \rho}{\partial \tau_e}\right).
\end{align*} \tag{16}
\]

Note that (11) and (12) already represent the total effects of variations of the tax rates, as given in (13) to (16), if we deal with a small region so that the nation-wide net return to capital does not depend on local policy tools. In this case we have \( dK/d\tau_k \big|_{\tau_e=\text{const.}} = K_{p_k} \) and \( dE/d\tau_k \big|_{\tau_e=\text{const.}} = E_{p_k} \).

To find out the signs of these (total) derivatives we state the following lemma

**Lemma 2.1**

\(^{11}\)Alternatively, in the case where \( \rho \) is constantly given we can write the factor demands as functions of the tax rates, \( K(\tau_k, \tau_e, \cdot) \) and \( E(\tau_k, \tau_e, \cdot) \), so that the partial derivatives w.r.t. \( \tau_e \) exist also yielding (11) and (12).
• $R_{ke}$ is unambiguously non-positive,

• $\frac{\partial \rho}{\partial \tau_k} \leq 0$, and $\frac{\partial \rho}{\partial \tau_e} \geq 0$.

**Proof:** $R_{ke} \leq 0$ follows from $F_{ke} \leq 0$ and (4). To see this, write $R_{ke}$ explicitly as

$$R_{ke} = (2p' + Qp'')F_eF_k + (p + Qp')F_{ke},$$

which is clearly non-positive. To show the second part of our claim, we have to consider the market clearing condition of capital. If the region is not too small, its policy has some impact on the equilibrium net return rate of capital. Let $\bar{K}$ denote the fixed aggregate supply of capital. Market clearing on the national capital market,

$$\sum_{j=1}^{n} K_j^i (\rho(\cdot) + \tau_k^i, \tau_e^i, P_k^i) = \bar{K},$$

implicitly defines the equilibrium net rate of capital, as a function of all local tax rates and supply levels of public goods, $\rho((\tau_k^1, \ldots, \tau_k^n), (\tau_e^1, \ldots, \tau_e^n), (P_k^1, \ldots, P_k^n))$.

To see how in equilibrium the nation-wide net rate of capital depends on the local tax rate on capital, differentiate (18), for fixed provision levels of industrial public goods, w.r.t. $\tau_k^i$ yielding

$$-1 \leq \frac{\partial \rho}{\partial \tau_k^i} = -\frac{K_j^i}{\sum_i K_j^i} \leq 0.$$  

(19)

Because we know from (11) and from the first part of Lemma 2.1 that the partial derivative $K_{\tau_k}$ is non-negative, the derivative of $\rho$ w.r.t. $\tau_k^i$ is also non-negative. To see this, differentiate (18) w.r.t. $\tau_e^i$

$$\frac{\partial \rho}{\partial \tau_e^i} = \frac{K_j^i}{\sum_i K_j^i} \geq 0.$$  

(20)

This completes our proof.

Using Lemma 2.1 and evaluating the (total) derivatives of the factor demands w.r.t. $\tau_k$ we get

$$\frac{dK}{d\tau_k} \leq 0, \quad \text{and} \quad \frac{dE}{d\tau_k} \geq 0.$$  

(21)

A rise of the tax rate on capital unambiguously induces a substitution of capital by environmental inputs. However, the reverse need not be true for an increase of
the tax rate on environmental inputs, since an increase of the emission tax rate also affects the price of capital. Consequently, we cannot determine the signs of the (total) derivatives of factor demand w.r.t. the emission tax. All we can do is to give conditions for which a rise of \( \tau_e \) implies substitution of emissions by capital. But we cannot rule out, in general, that capital is substituted by environmental inputs or that the emission tax depresses both factor demands. In detail we have

\[
\frac{dK}{d\tau_e} > 0 \quad \Leftrightarrow \quad \det(\text{hess}(R)) > R_{ee} \frac{1}{\sum K_{p_k}},
\]

(22)

\[
\frac{dE}{d\tau_e} \quad < 0 \quad \Leftrightarrow \quad \det(\text{hess}(R)) > \frac{R_{ke}^2}{R_{kk} \sum K_{p_k}}.
\]

(23)

Note that \( \det(\text{hess}(R)) > R_{ee}/\sum K_{p_k} \) is already sufficient for \( \det(\text{hess}(R)) > R_{ke}^2/(R_{kk} \sum K_{p_k}) \). Thus, three demand regimes may occur:

1. \( \det(\text{hess}(R)) > R_{ee}/\sum K_{p_k} \Leftrightarrow dK/d\tau_e > 0 \land dE/d\tau_e < 0 \).

2. \( R_{ke}^2/(R_{kk} \sum K_{p_k}) < \det(\text{hess}(R)) < R_{ee}/\sum K_{p_k} \)

\( \Leftrightarrow dK/d\tau_e < 0 \land dE/d\tau_e < 0 \).

3. \( \det(\text{hess}(R)) < R_{ke}^2/(R_{kk} \sum K_{p_k}) \Leftrightarrow dK/d\tau_e < 0 \land dE/d\tau_e > 0 \).

In the first case, substitution takes place, whereas in the second case, a rise of \( \tau_e \) implies a decrease of both demands, and thereby a reduction of the output level. The third case might seem strange, for emission demand rises and capital demand falls as the emission tax is increased. This results from the fact that a rise of the emission tax increases the net return to capital, \( \rho \), and therefore, ceteris paribus, the capital costs. This may, to some extent, offset the induced substitution of emissions by capital. In extreme cases, the price of capital reaches such high levels that emissions become relatively cheaper, so that the substitution reverses, and the use of environmental inputs is extended rather than reduced.

Although we know that all three demand regimes may occur, \( dE/d\tau_e \) is negative and \( dK/d\tau_e \) is positive, if regions are ‘not too different’. To see this, consider the case of symmetric regions. In this case, we have from (19), (20), (16), and (14)

\[
\frac{\partial \rho}{\partial \tau_k} = -\frac{1}{n}, \quad \frac{\partial \rho}{\partial \tau_e} = -\frac{1}{n} \frac{K_{\tau_e}}{K_{p_k}},
\]

\[
\frac{dE}{d\tau_e} = \frac{1}{R_{ee}} \frac{R_{kk} R_{ee} - \frac{1}{n} R_{ke}^2}{R_{kk} R_{ee} - R_{ke}^2} \leq \frac{1}{R_{ee}} < 0,
\]

(24)

\[
\frac{dK}{d\tau_e} = -\frac{R_{ke}}{R_{kk} R_{ee} - R_{ke}^2} \frac{n-1}{n} > 0.
\]

(25)
In the case of symmetric regions, an increase of the price of environmental inputs unambiguously induces substitution of emissions by capital.

3 Welfare Analysis

In this section we derive the f.o.c.s of a local government which maximizes the utility of its representative resident, taking the behavior of the other local governments as given. The resulting system of f.o.c.s implicitly defines the vector of equilibrium tax rates – the Nash tax rates of interjurisdictional tax competition.

Particularly, we refer to the case of a small region which has no influence on national prices, namely, on the net return to capital, \( \rho \). Additionally, we also consider the short run where capital is assumed to be fix.

3.1 Equilibrium Policy of the Local Government

By means of the previous analysis we are now well prepared to solve the optimization problem of the local regulator. He/she maximizes the utility of the representative resident subject to the private (3) and the public budget constraint (1) w.r.t. its policy tools – the tax rates on capital and emissions, the provision level of the residential and the industrial local public good – taking the behavior of the agents of all other regions as given. Substituting (3) into (2) and differentiating the Lagrangian w.r.t. \( \tau_k \), \( \tau_e \), \( P_i \), and \( P_r \) yields

\[
\frac{\partial L}{\partial \tau_k} = U_x (\beta \Pi_{pk} (1 + \rho_{rk}) + \bar{K} \theta \rho_{rk}) + U_g' E_{pk} (1 + \rho_{rk}) + \lambda [E_{pk} (1 + \rho_{rk}) \tau_e + K_{pk} (1 + \rho_{rk}) \tau_k + K] = 0, \tag{26}
\]

\[
\frac{\partial L}{\partial \tau_e} = U_x (\beta (\Pi_{re} + \Pi_{pk} \rho_{re}) + \bar{K} \theta \rho_{re}) + U_g' (E_{re} + E_{pk} \rho_{re}) + \lambda [\tau_e (E_{re} + E_{pk} \rho_{re}) + \tau_k (K_{re} + K_{pk} \rho_{re}) + E] = 0, \tag{27}
\]

\[
\frac{\partial L}{\partial P_i} = U_x (\beta (\Pi_{P_i} + \Pi_{pk} \rho_{P_i}) + \bar{K} \theta \rho_{P_i}) + U_g' (E_{P_i} + E_{pk} \rho_{P_i}) + \lambda [-1 + \tau_e (E_{P_i} + E_{pk} \rho_{P_i}) + \tau_k (K_{P_i} + K_{pk} \rho_{P_i})] = 0, \tag{28}
\]

\[
\frac{\partial L}{\partial P_r} = -\lambda + U_p = 0, \tag{29}
\]

where \( \lambda \) is the Lagrangian multiplier of the public budget constraint. Note that the terms in square brackets within these equations represent the marginal revenue of public funds stemming from a rise of \( \tau_k \) and \( \tau_e \) and the social marginal cost
of providing an additional unit of the industrial public good, respectively: From firm’s optimization conditions we know by envelope theorem (‘Hotelling’s lemma’) that

\[ \Pi_{p_k} = -K, \]  
\[ \Pi_{\tau_e} = -E, \]  

and additionally that

\[ \Pi_{P_t} = F_{P_t}(p + p'Q), \]  

which can be substituted into (26), (27), and (28). In the forward analysis we will use these resulting conditions to derive the marginal rates of substitution between the private good and the residential public good, on the one hand, and between the private good and the prevailing environmental quality, on the other hand. In addition, for the purpose of comparative statics (and for expository purpose), we also solve the f.o.c.s for the corresponding tax rates.

Solving (26), (27), and (28) for the marginal rates of substitution of the private good for environmental quality and for the public good, respectively, we get

\[ \frac{\partial U}{\partial U} = -\beta \left( E \frac{dp_k}{d\tau_k} - K \frac{dp_k}{d\tau_k} \right) + (\beta K - \theta \dot{K}) \left( -\rho_{\tau_e} \frac{dp_k}{d\tau_k} + \rho_{\tau_e} \frac{dp_k}{d\tau_k} \right) \frac{1}{g'} \]

\[ \frac{\partial U}{\partial X} = \beta \left( \frac{E_{p_k} E_{\tau_e}}{E_{K}} \left[ (1 + \rho_{\tau_e}) \right] + \frac{E_{p_k} E_{\tau_e}}{E_{\tau_e}} \frac{\theta \dot{K}}{\beta K} \right) \frac{1}{1 + (1 + \rho_{\tau_e}) \frac{K_{p_k}}{E_{\tau_e}} \left( (1 + \rho_{\tau_e}) \frac{E_{p_k}}{E_{\tau_e}} \frac{\theta \dot{K}}{\beta K} \right) - \rho_{\tau_e}} \]

\[ = \beta \left( \frac{1}{1 - \frac{E_{p_k}}{E_{\tau_e}} (1 + \rho_{\tau_e}) + \frac{E_{p_k} E_{\tau_e}}{E_{\tau_e} \rho_{\tau_e}} + \frac{\tau_{\tau_e}}{E_{\tau_e}} (1 + \rho_{\tau_e}) [K_{p_k} E_{\tau_e} - E_{p_k} K_{\tau_e}]} \right) \]

Since these expressions depend on, among others, the interregional distribution of capital income and on the influence of the policy instruments on the nationwide net return to capital, they look a bit clumsy. However, this model which is, to the best of our knowledge, the most general approach of interjurisdictional tax competition to date, exhibits some important features which do not appear simultaneously in simpler ones, if ever. To show the connection to related, though less general, models we simplify our approach slightly later on. First, however, we make some remarks concerning the f.o.c.s of our general approach.

The marginal rates of substitution depend on the curvature of the Laffer-curves, \( dP_r/d\tau_k \) and \( dP_r/d\tau_e \), on the elasticities of local factor demand, on the sensitivities of the net rate of capital, \( \rho_{\tau_k} \) and \( \rho_{\tau_e} \), and on the (perceived) local
excess demand of capital, $\beta K - \theta K$. The term $\rho_{tr}(\beta K - \theta K)/(\beta K)$ represents the cost change of the (effective) net capital import per unit resulting from a rise of $\tau_r$. Correspondingly, the ratio of initial capital endowment to effective capital demand $(\theta K)/(\beta K)$ reflects the relative interregional distributive characteristics. Note that, as long as $\beta < 1$, local consumers do not bear the full costs of industrial capital. Thus, the capital costs perceived by the residents are lower than its social costs. However, they receive the full profits from their initial capital endowment, $\theta K$.

Consider the marginal rate of substitution of the private for the public good, given in (35). The second term of the numerator as well as of the denominator reflects the fact that raising an additional unit of public funds by increasing the tax rate on capital has an indirect effect on emissions, because it alters the industrial input ratios due to factor substitution. As we already know, $1 + \rho_{tr}$ is positive, i.e., rising the tax rate on capital increases its consumer price and therefore induces factor substitution away from capital towards emissions which in turn worsens the environmental quality. Thus, rising $\tau_r$ has indirectly negative welfare effects through lowering the environmental quality. This raises the (social) price of providing local public goods through capital taxation. However, this effect is also present in the denominator. Hence, if no other effects would occur, the marginal rate of substitution between the private and the public good would be equal to $\beta$, the perceived cost of providing an additional unit of the (residential) local public good.\textsuperscript{12}

Even if we consider the special case of $\beta = 1$, there are two effects causing the marginal rate of substitution between the private and the public good to differ from marginal rate of transformation, which is equal to unity. The third and the fourth term of the numerator as well as of the denominator of (35) are responsible for an inefficient provision level of the local public good. First, if we inspect the third parts, we see that the numerator falls more than the denominator if $\theta K/\beta K > 1$, i.e., the provision of the public good becomes less expensive. If we deal with a net-capital-'exporting' region, the rise of the net return to capital resulting from an increase of $\tau_r$ is partially borne by non-residents. Or in other words, residents gain relatively much from an increase of $\rho$, because the net return has to be paid by non-residents to some extent. This means that non-residents

\textsuperscript{12}Only if $\beta$ is equal to unity, the (locally) perceived marginal cost of public goods are identical to its social (i.e., national) marginal cost, because local residents bear the full provision costs. If, however, we allow for pollutant transmission resulting from local industrial emissions, this is no longer true due to real externalities.
have to pay a positive portion of the provision cost of local public goods acting as a subsidy for the residents of net-capital-‘exporting’ regions. (The opposite is true for net-capital-‘importing’ regions.)

The interpretation of the forth part of the numerator is analogous to the previous analysis of the third part. The only difference is that we are concerned with an increase of the capital tax rather than of the emission tax. Since \( \rho \) depends negatively on \( \tau_k \), capital owners have to pay a part of the provision cost of public goods. Hence, its provision becomes less expensive for net-capital-‘importing’ regions, whereas the opposite is true for net-capital-‘exporting’ regions.

The second effect responsible for the inefficient provision of local public goods is due to distortional capital taxation represented by the forth term of the denominator, which does not (directly) depend on national distributional characteristics. In order to work out the following proposition, we need to determine the sign of that last expression within the denominator. Obviously, while \( E_{\tau_k} \) is negative by (12) and we have \( 0 \leq 1 + \rho_{\tau_k} \leq 1 \) by (19), the sign of the forth term crucially depends on the bracket term. However, this term, \( K_{p_k} E_{\tau_k} - E_{p_k} K_{\tau_k} \), is equal to the inverse of the determinant of the Hessian of the revenue, \( 1/(R_{kk} R_{ee} - R_{ke}^2) \), which is positive since \( R \) is concave.\(^{13}\) Hence, the denominator of (35) is smaller than \( 1 - \frac{E_{p_k} E_{\tau_k}}{E_{\tau_k} K} (1 + \rho_{\tau_k}) \) if \( \tau_k \) is positive.\(^{14}\) The numerator, however, may become greater than this expression by the last two terms even for positive tax rates. This enables us to derive a necessary and sufficient condition for under- and over-provision of public goods, respectively. Let \( \varepsilon_{p_k}^k := (\rho + \tau_k) K_{p_k}/K \) the (own-price) elasticity of capital demand.

**Proposition 3.1** If local residents hold all stocks of the local firm, \( \beta = 1 \), over-(under)provision of local public goods occurs if and only if initial capital endowment satisfies

\[
\frac{\theta K^0}{K} \left( > \right) \frac{1 - \varepsilon_{p_k}^k}{\varepsilon_{p_k}^k} \frac{\tau_k}{\rho + \tau_k} \frac{1 + \rho_{\tau_k}}{\rho_{\tau_k}}.
\]

\(^{13}\)The positivity of this term within the brackets means that the cross-price effects of the factor demands do not outweigh its own-price effects. Or, roughly speaking, that each price change induces the strongest demand effect on its own item.

\(^{14}\)More precisely, the denominator of (35) is smaller than \( 1 - \frac{E_{p_k} E_{\tau_k}}{E_{\tau_k} K} (1 + \rho_{\tau_k}) \) if and only if

\[
\tau_k > -\frac{E_{p_k} \rho_{\tau_k} K}{1 + \rho_{\tau_k}} \frac{\det(Hess(K))}{1 + \rho_{\tau_k}} < 0.
\]
Clearly if the right hand side of (36) is negative, overprovision of public goods does not emerge, regardless of the initial capital endowment. Provided that the tax rate on capital is positive, overprovision of local public goods is only possible for capital-‘importing’ regions. Note that the right hand side may not only be positive, but even be greater than one if \( r_k \) is negative. This implies that overprovision of public goods may also occur in capital-‘exporting’ regions. On the other hand, for capital-‘importing’ regions underprovision can only emerge if \( r_k \) is positive. If the elasticity of capital demand is equal to zero, local public goods are over- (under)provided if and only if \( \theta \bar{K} / K < 1 (> 1) \). Each net-capital-‘importing’ (‘exporting’) jurisdiction overprovides (underprovides) local public services.

Roughly speaking, Proposition 3.1 says that overprovision is the more likely the lower the (relative) initial capital endowment. The intuition behind this is as follows. Capital-well-endowed regions suffer extra costs from capital taxation by inducing a decrease of the net return to capital. The contrary is true for capital-poor-endowed regions. Because a relatively small portion of the national capital stock is held by its residents, this decline of the net return to capital resulting from an increase of the local capital tax rate does not bother them much. Moreover, the decline of the interest on capital acts as a subsidy for capital-poor-endowed regions. If this effect is sufficiently strong, the local government tends to raise the tax rate on capital so far that tax revenues may ‘skyrocket’ beyond the point of efficient provision of local public goods and thus overprovision occurs.

This result shows that we cannot rule out overprovision of local public goods, in general. Especially, if the regional distribution of capital is sufficient unequal, overprovision may emerge in the capital-worst-endowed region. However, for the special case where in each region the excess demand of capital equals to zero, \( \theta \bar{K} = K \), we have the following corollary of Proposition 3.1.

**Corollary 3.1** If capital demand exactly equals local initial capital endowment, over- (under)provision of local public goods occurs if and only if capital is taxed by a negative (positive) rate.

Previous analysis suggests that underprovision is increasing in initial capital endowment; i.e., the marginal rate of substitution between the private and the public good, \( MRS_{xp} \), is a monotone increasing function of \( \theta \bar{K} \). To prove this presumption, differentiate \( MRS_{xp} \) w.r.t. \( \theta \bar{K} \) yielding.

\[
\frac{\partial MRS_{xp}}{\partial \theta \bar{K}} = -\frac{1}{\mathcal{N}} \frac{\rho_r}{\bar{K}} \left( 1 - \frac{\rho_{r_k}}{\rho_{r_k}} \frac{E_{r_k}}{E_{r_\tau}} \right) > 0, \tag{37}
\]
where $N > 0$ denotes the denominator of (34) and (35). This gives us the following proposition, which is closely related to Proposition 3.1.

**Proposition 3.2** The marginal rate of substitution between the private and the public good is monotone increasing in the local initial capital endowment, $\theta \hat{K}$. I.e., underprovision (overprovision) of local public goods is the higher (lower) the higher the capital endowment of the region.

Solving (26) to (29) for $\tau_k$ and $\tau_e$ gives

$$
\tau_k = \frac{(U_{P_P} - \beta U_e)(E E_{p_k} - K E_{\tau_e})}{U_{P_P}(E_{\tau_e} K_{p_k} - E_{p_k} K_{\tau_e})} + \frac{(U_{P_P} K - U_e \theta \hat{K})(E_{\tau_e} \rho_{\tau_k} - E_{p_k} \rho_{\tau_e})}{U_{P_P}(1 + \rho_{\tau_e})(E_{\tau_e} K_{p_k} - E_{p_k} K_{\tau_e})},
$$

$$
\tau_e = -\frac{U_{d} g'}{U_{P_P}} + \frac{(U_{P_P} - \beta U_e)(K K_{\tau_e} - E E_{p_k})}{U_{P_P}(E_{\tau_e} K_{p_k} - E_{p_k} K_{\tau_e})},
$$

(38)

(39)

where we have used the fact that

$$
K_{\tau_e} \rho_{\tau_e} - K_{p_k} \rho_{\tau_k} = K_{\tau_k} \rho_{\tau_k} \left(1 - \frac{K_{p_k} K_{\tau_e}}{K_{\tau_k} K_{p_k}}\right) = 0.
$$

The tax rate on capital is composed of two parts. The first part results from distortionary taxation of industrial inputs, reflecting the extent of inefficient provision of local public goods, $(U_{P_P} - \beta U_e)$. The second part stems from the strategic effect of unequal distribution of capital. This effect, non-internalized external effect of capital taxation, is proportional to the extent of evaluated capital excess demand, $(U_{P_P} K - U_e \theta \hat{K})$. The sign of $\tau_k$ depends on these two first terms within each numerator, because all other terms are positive. Note that the sign of $\tau_k$ plays a crucial role as was already indicated by Proposition 3.1. Moreover, it is easy to show that $\tau_k$ depends negatively on $\theta \hat{K}$,

$$
\frac{\partial \tau_k}{\partial \theta \hat{K}} = -\frac{U_{d} E_{\tau_e} \rho_{\tau_k} \left(1 - \frac{E_{p_k} \rho_{\tau_e}}{E_{\tau_e} \rho_{\tau_k}}\right)}{U_{P_P}(1 + \rho_{\tau_e})(E_{\tau_e} K_{p_k} - E_{p_k} K_{\tau_e})} < 0.
$$

(40)

**Proposition 3.3** The higher the local initial capital endowment the lower, ceteris paribus, the local tax rate on capital.

Capital-well-endowed regions, ceteris paribus, tend to tax capital less heavily than capital-poor-endowed regions.

The emission tax, on the other hand, is made up of the environmental effect and of a corresponding effect of distortionary taxation. The sign of the emission tax rate depends solely on the extent of under- (over-) provision of public goods, $(U_{P_P} - \beta U_e)$, for the marginal social damage of pollutant emission, the first term, is positive. This gives us the following result:
Proposition 3.4 If local residents hold all stocks of the local firm, \( \beta = 1 \), local governments fix the emission tax rate below (above) marginal social damage if and only if local public goods are over- (under) provided.

From (39) we have that \( \tau_e \) is independent of (i.e., not directly dependent on) distributive characteristics. The initial endowment of capital does not (directly) affect the emission tax rate, \( \partial \tau_e / \partial \theta K = 0 \). Consequently, we do not get a similar result for the emission tax as we have in Proposition 3.3 for the tax rate on capital. Rather we get:

Proposition 3.5 The local emission tax rate, ceteris paribus, is independent on the local initial capital endowment.

A higher initial endowment of capital, ceteris paribus, leaves \( \tau_e \) unaffected, but lowers \( \tau_k \). So there is no (direct) substitution of capital taxation by emission taxation as the initial capital endowment increases.

3.2 The Special Case of a Small Region

Thus far we have seen that taxation of productive factors leads, through its influence on the net return to capital, to shifts in income. Hence, fiscal and environmental policy have distributive effects. To remove this topic from our analysis for now, consider the special case of a small region which has no impact on national variables, especially on the nation-wide net return to capital.\(^{15}\) If \( \rho \) is constantly given for this small region, the marginal rates of substitution are no longer (directly) dependent on local excess demand of capital. To see this, imagine a small jurisdiction where the residents hold all stocks of the local firm, i.e., \( \beta = 1 \). In this case, our above stated conditions (33) and (34) reduce to

\[
\frac{\partial U}{\partial U} = \frac{-E}{K} \frac{dP_e}{d\tau_e} + \frac{dP_r}{d\tau_e} \frac{1}{E_{r e} \frac{dP_e}{d\tau_e} - \frac{E_{r e}}{K} \frac{dP_r}{d\tau_e} g'} = \frac{1 - \frac{E}{K} \left( -\frac{d\tau_e}{d\tau_k} \right)}{\frac{E_{r k}}{K} - \frac{E_{r e}}{K} \left( -\frac{d\tau_e}{d\tau_k} \right) g'}
\]

(41)

\[
\frac{\partial U}{\partial P_r} = \frac{1 - \frac{E}{K}}{1 + \tau_k \frac{K_{r k}}{K}} - \frac{\frac{E_{r k}}{E_{r e} \frac{E}{K}}}{1 + \tau_k \frac{K_{r k}}{K}}
\]

(42)

\(^{15}\)Instead of dealing a small region, we may alternatively examine the case of identical regions (including identical initial endowments). In this case, capital demand must meet capital supply in each region. The consequences of dealing with a small region, however, are even stronger than those resulting from competition of identical regions.
where \(-d\tau_k/d\tau_k\) denotes the political marginal rate of substitution between these two fiscal instruments. Now, in both optimality conditions, the local initial capital endowment no longer appears, and distributive effects are erased. The assumption of \(\beta\) being equal to one implies that each region's residents bear the full operating cost of the firm. In this case, the local government has no incentive to tax away the profits of the firm, for there is no capital outflow through distribution of dividends. Moreover, if local residents receive the full profits of the local firm, no welfare effects are induced by taxing profits and redistributing them by lump-sum transfers to residents.

If we rearrange terms of (42), we see that the marginal rate of substitution between private consumption and the local residential public good is greater than one if and only if \(\tau_k\) is positive. I.e., an overprovision of local public goods does not occur if capital is taxed by positive amounts. To see this, rewrite (42) as

\[
\frac{\partial U}{\partial P_c} = \frac{1 - \frac{E_{\tau_k}}{K\tau_k \tau_k}}{1 - \frac{E_{\tau_k}}{K\tau_k} + \left[ E_{\tau_k} K_{\tau_k} - E_{\tau_k} K_{\tau_k} \right] \frac{\tau_k}{E_{\tau_k} K}}
\]  

(43)

As we already know from our former analysis, the bracket term of the denominator is positive, which implies for \(\tau_k > 0\) that the denominator is smaller than the numerator, meaning that the marginal rate of substitution is greater than unity. Hence, contrary to our more general model of a variable net rate of capital, we can exclude the possibility of overprovision of local public goods if \(\tau_k > 0\). (The contrary is true for \(\tau_k < 0\).)

**Proposition 3.6** If the jurisdiction under consideration is sufficiently small, so that the nation-wide net return to capital is independent of the level of local tax rates, and residents hold all the stocks of local firm(s), \(\beta = 1\), interjurisdictional tax competition leads to under- (over)provision of local public goods if and only if the tax rate on capital is positive (negative).

Note that Proposition 3.6 gives us the same result as Corollary 3.1 which was derived for the special case where \(\theta K = K\). Hence, there is the same equivalence between the inefficiency of the public sector and capital taxation under the local market clearing condition for capital as under the assumption of a small region.

For a further inspection, solve (41) and (42) which implicitly define the optimal second-best tax rates. (Equivalently evaluate (38) and (39) at \(\rho = \text{const.}\) and \(\beta = 1\).)

\[
\tau_k = \frac{(U_{P_c} - U_{c})(E_{\tau_k} - KE_{\tau_k})}{U_{P_c}(E_{\tau_k} K_{\tau_k} - E_{\tau_k} K_{\tau_k})}
\]

(44)
\[ \tau_e = -\frac{U_u g'}{U_p} + \frac{(U_{P_k} - U_{Q})(KK_{\tau_e} - EK_{\tau_e})}{U_p(E_{\tau_e}K_{\tau_e} - E_{P_k}K_{\tau_e})}. \] (45)

Contrary to our former, more general, model, the signs of \( \tau_k \) and of \( \tau_e \) depend solely on the extent of inefficient provision of public goods, \( U_{P_k} - U_{Q} \). But because (45) is identical to (39) evaluated at \( \beta = 1 \), Proposition 3.4 still holds in the special case of a small region.

Composing Proposition 3.4 and 3.6 we see that only two possible regimes may emerge, if the region is sufficiently small:

**Regime 1:** Under-provision of local public goods \( \Leftrightarrow \tau_k > 0 \Leftrightarrow \tau_e > -\frac{U_u g'}{U_p} \).

**Regime 2:** Over-provision of local public goods \( \Leftrightarrow \tau_k < 0 \Leftrightarrow \tau_e < -\frac{U_u g'}{U_p} \).

Curiously enough, this means that in regime 1 \( \tau_e \) is too high and thus 'over-provision' of environmental quality occurs(), whereas in regime 2 the emission tax is too low implying an 'under-provision' of environmental quality.

Because the local government is by no means able to influence the nationwide net rate of capital, no additional strategic effect arises from taxing capital as against emissions. The only effects which are still present are the distortionary effect of taxing inputs – variables which are under the control of the taxpayer – to provide local public goods and the direct social damage of industrial pollution.\(^{16}\)

In the first-best case, where the local government is entitled to finance any desired level of public goods through head taxation, the optimal head tax is implicitly determined by \( \partial U/\partial P_r = \partial U/\partial X \). The marginal rate of substitution between the (residential) public and the the private good would be equal to the marginal rate of transformation which in turn is equal to unity. In the world of second-best, however, this may emerge only by chance, if the revenues collected from second-best emission tax rate exactly meet the expenditures required for the optimal provision level of local public goods. In this case, no further revenues are needed from capital taxation, and the local government leaves capital untaxed. We can see this from (41) by using the optimal tax rate on capital, \( \tau_k = 0 \), and solving for \( \tau_e \)

\[ \tau_e = -\frac{U_u g'}{U_x} \geq 0. \] (46)

\(^{16}\)We derive a similar result in a model that also exhibits a strategic output effect. Because local residents consume the industrial output of the local firm, positive direct welfare effects accrue from local production. See UPMANN (1995).
In the social optimum, the emission tax rate is equal to the marginal willingness of the residents to pay for a better environment, i.e., the marginal damage. In the special case, where environmental quality is just the negative of aggregated emissions, $g'$ is equal to minus one, we have $\tau_c = U_u/U_e$, which is a widely known familiar result of optimal emission taxation.

3.3 Short Run Analysis

Due to the installation of long-term durable investment goods, industrial capital is fixed in the short run. Tax rates, however, often can be altered on short- or at least on medium-terms. Whenever an unexpected variation of the tax rates emerges, industry is urged to adjust exclusively its emissions (and thereby its output). This gives us reasons to consider a model where industrial capital is fixed and the firm takes, as before, the supply of industrial local public goods as well as tax rates as given. In this case, increasing $\tau_k$ does not alter $K$, and hence $E$ remains unchanged. The only derivative of factor demand which is different from zero is $dE/d\tau_e = 1/R_{ee} < 0$. In general, the capital market is not in equilibrium after the change of the tax rates, and hence $\rho$ is not well-defined by (18), nor are $\partial \rho / \partial \tau_k$ and $\partial \rho / \partial \tau_e$. To avoid this, assume that in long-term rent contracts of capital its net return is fixed for the whole investment period. Then, the net return to capital is independent of local tax rates, $\partial \rho / \partial \tau_k = 0$, $\partial \rho / \partial \tau_e = 0$, and $\partial \rho / \partial P_i = 0$.\textsuperscript{17} Using the resulting f.o.c.s

$$-U_e \beta K + U_P K = 0,$$

$$-U_e \beta E + U_e g' \tau_e + U_P (\tau_e E_e + E) = 0,$$

$$U_e \beta \Pi_P + U_e g' E_P + U_P (-1 + \tau_e E_P) = 0,$$

it becomes clear that $\tau_k$ is adjusted so that the marginal rate of substitution between the private and the (residential) public good, $U_P / U_e$, is equal to its perceived marginal cost, $\beta$. The inelasticity of capital demand means that any distortionary effect of raising public funds is avoided, and taxing capital serves the sole purpose of financing public goods. Or in other words, any desired level of public funds can be raised without affecting production in a distortionary manner. On the other hand, for $\beta = 1$ equation (48) implies, by using (47), that the emission tax rate is set according to its first-best formula, (46). Consequently,\textsuperscript{17} Capital consumers (firms) bear the full cost of increasing the tax rates and thus of extending the supply of local public services. If, on the contrary, we have $\partial \rho / \partial \tau_e = -1$ and $\partial \rho / \partial \tau_k = -1$, capital owners bear the full burden of (additional) taxation.
if capital is demanded inelastically, the local government adjusts the emission tax rate at its first-best, i.e., Pigouvian, level; whereas the capital tax rate acts non-distortionary as a head tax to raise the efficient amount of public funds.

**Proposition 3.7** In the special case where $\beta$ is equal to one and capital demand as well as its net return are fixed, the local government establishes the first-best solution, although it has no access to lump sum transfers.

Note that although the capital tax rate is not equal to zero, public services are efficiently provided.

If, however, emissions are determined by long-term technological investments but capital goods can be adjusted immediately, a variation of the tax rates induces the firm to vary its capital rather than its emission demand. In this case, $\tau_k$ and $\tau_e$ are adjusted so that

$$\frac{\partial U/\partial P_r}{\partial U/\partial X} = \frac{\beta - \rho \tau_k \frac{1}{K} (\theta K - \beta K)}{1 + (1 + \rho \tau_k) \frac{\tau_k \frac{e^K}{K}}{\rho_k + \tau_k} \frac{e^K}{\rho_k}} = \frac{\beta - \rho \tau_k \frac{1}{K} (\theta K - \beta K)}{1 + \tau_k \frac{e^K}{K} + \tau_k \rho \tau_k \frac{e^K}{\rho_k}}. \quad (50)$$

Both tax rates are adjusted in that way that not ‘too much’ capital is driven out of the region. Contrary to the case of fixed capital, the tax rate of the inelastically demanded factor, here $\tau_e$ does not act, as one might expect, as a lump sum transfer. The reason is that the emission tax has some effect on the net return to capital and thereby on capital demand. This means that both tax rates are distortionary, and as it is obvious from (50), the marginal rate of substitution is, in general, different from unity.

It is worth noting that, by including environmental aspects and correspondingly emission taxation, the special case of our model considered in this subsection still is a generalization of the pure capital taxation models of interjurisdictional tax competition.¹⁸ Or to put it the other way round, if we assume environmental inputs to be fixed, $\rho$ as constantly given, $\beta$ being equal to one, and we do not allow for emission taxation ($\tau_e \equiv 0$), our model reduces to the commonly known model of interjurisdictional tax competition on mobile capital. Referring to equation (42), it represents, as a consequence of including emissions, a generalization of the simple commonly known (implicit) tax formula

$$\frac{\partial U/\partial P_r}{\partial U/\partial X} = \frac{1}{1 + \tau_k \frac{K_{rh}}{K}}. \quad (51)$$

---

Obviously, the marginal rate of substitution differs from unity as long as $\tau_k$ is unequal to zero, i.e., as long as the local government has no access to head taxes. Especially, because local government has no access to other sources for public funds than capital taxation, $\tau_k$ is clearly positive, implying that overprovision of local public goods never occurs. I.e., up to now the economic literature of interjurisdictional tax competition only considers what we call the first regime, whereas the second is ignored.

If we compare (51) and (42), we see that in the numerator as well as in the denominator of (42) additional terms are subtracted which are both unambiguously negative if $\tau_k$ is positive, so that the numerator as well as the denominator is increased. Thus, we are not able to decide whether the marginal rate of substitution between the private and the public good, compared to the simpler model without emissions, rises or falls, provided that capital is taxed by positive amounts. In other words, if the local government decides to assess emissions by laying a proportional emission tax, it is not quite clear whether the situation of inefficient provision of public goods is improved or deteriorated.

However, our analysis is not restricted to the case of a positive tax rate on capital; because welfare improvements can be achieved by local governments, if industrial capital is subsidized by other sources of public funds. Recognizing that the second regime may also emerge, resulting in an overprovision of local public goods, the assertion, as stated by traditional theory of tax competition, of underprovision of local public goods need not to be true.

4 Interference of the Federal Government

We have seen that interjurisdictional tax competition leads either to under- or to overprovision of local public goods, due to the fiscal externality of driving out the tax base (capital) by rising tax rates (distortionary taxation). The constancy of the national capital stock, however, implies that capital flight from one region corresponds to an equal increase of the capital stock of the other regions. On national (or federal) grounds, welfare losses in one region and gains in the others have to be weighed against each other. Therefore, the higher (i.e., federal) government seeks to make the local governments to consider the externalities induced by their policies. A common device is either to set fiscal incentives – subsidies or penalties (taxes) – for the local governments to internalize these effects or to
try to persuade local government to revise their policies in order to come to a cooperative solution.\textsuperscript{19}

### 4.1 Correcting Interjurisdictional Competition

We show that as long as the federal ministry of finance is not forced to balance revenues and expenditures, efficiency can be re-established by subsidizing (i.e., encouraging) capital taxation and taxing (i.e., discouraging) emission taxation if \( \tau_k > 0 \) (regime 1) and vice versa if \( \tau_k < 0 \) (regime 2). To see this, consider for the purpose of tractability, our model of a small jurisdiction as established in section 3.2. Suppose that federal government commits itself to pay a subsidy of \( S_k(\tau_k) \) and \( S_e(\tau_e) \) on capital and emission taxation, respectively. The (local) public budget constraint (1) changes to

\[
P_k + P_e = \tau_k K + \tau_e E + S_k(\tau_k) + S_e(\tau_e).
\]

Because policy variables of higher governments are determined beforehand, the local government treats, in determining its policy, the subsidy schedules \( S_k(\tau_k) \) and \( S_e(\tau_e) \) as given. Consequently, the f.o.c.s of the local government must be modified in so far that within the third bracket term of (26) and (27) we have to add \( S_k'(\tau_k) \) and \( S_e'(\tau_e) \), respectively. By setting these subsidy terms appropriately the federal government can ensure national efficiency.

**Proposition 4.1** Let \( \beta = 1 \) and \( \rho \) constant given. If the federal government pays subsidies to local governments for taxing capital and emissions, where marginal payments satisfy \( S_k'(\tau_k) = -\tau_k K_{p_k} \) and \( S_e'(\tau_e) = -\tau_k K_{p_e} \), first-best is re-established.

**Proof:** Using \( S_k'(\tau_k) = -\tau_k K_{p_k} \) and \( S_e'(\tau_e) = -\tau_k K_{p_e} \) for \( \beta = 1 \) and \( \rho = \text{const.} \) the f.o.c.s for the local government reduce to

\[
-U_s K + U_a g' E_{p_k} + U_{p_k} [E_{p_k} \tau_c + K] = 0,
\]

\[
-U_s E + U_a g' E_{\tau_c} + U_{p_e} [E_{\tau_c} \tau_e + E] = 0.
\]

\textsuperscript{19}Clearly, if the federal government is entitled by law to determine the local tax rates directly, neither a no subsidy scheme nor a persuasion is required. In federal states, however, for a couple of tax rates – especially for capital and emission tax rates – the legislature is given to subordinate jurisdictions.
Instead of (41) and (42) we get

\[ \frac{u_t}{u_x} = -\tau_e \frac{1}{g'} \]  

(55)

\[ \frac{u_F}{u_x} = 1. \]  

(56)

(55) gives the first-best emission tax rate given by (46) and thus the social optimal emission level. On the other hand (56) ensures the optimal provision of local public goods.

At a first glance, it seems to be curious that in the first regime emission taxation is discouraged by imposing a negative marginal subsidy on the emission tax rate by the federal government, and vice versa in the second regime. But if we keep in mind Propositions 3.6 and 3.4 where we found that in the first (second) regime revenues are too low (high) but environmental quality is too high (low) from efficiency viewpoint this result is quite natural.

Note that an increase of \( \tau_e \) induces \( E \) to fall, and thus \( F_k \) to raise, which in turn encourages an ‘import’ of capital as long as the marginal product of capital decreases with the use of environmental inputs, \( \frac{dF_{ke}}{dE} \leq 0 \). But this means that increase of the emission tax induces capital flight from other regions into the ‘domestic’ region. The federal government seeks to offset this effect by imposing a penalty on emission taxation which is at the margin equal to the induced additional tax revenues going back to an influx of capital.

By introducing the subsidy scheme on capital taxation, the local government is compensated for its capital outflow as a consequence of raising its tax rates. Thus, the required tax revenues are increased by an appropriate adjustment of the tax rate on capital which is implicitly given by (53) and (54). Under the subsidy/penalty scheme, local government acts as if there is no capital flight and the local regulator no longer hesitates to tax capital more heavily in order to raise public funds. This enables the local government to provide local public services optimally, although it has no access to head taxation.

If, as it is the case under the second regime, social marginal environmental damage is so high that revenues from first-best emission taxation exceed required first-best revenues considerably, the argumentation goes the other way round. Without federal subsidies, the local government lowers the emission tax below its first-best level and subsidizes, with the aid of the remaining excess funds, capital influx as well as an increase of its provision level of public goods. Again, this effect is offset by the federal government through discouraging capital taxation and subsidizing emission taxation at the margin.
4.2 Intervention of the Federal Government in the Case of a Symmetric Equilibrium

Another possibility for the federal government to overcome the inefficiency problem is to initiate a cooperative solution of the local governments. The higher government has to convince the subordinate, but w.r.t. this policy tool autonomous, governments to vary their tax rates simultaneously by a mutual agreement. But the transmission from Nash equilibrium to a nation-wide Pareto optimum does not necessarily imply a Pareto improvement, i.e., a welfare improvement for every region. If, for example, a raise of all capital tax rates means that there is at least one region which suffers a welfare loss, we have to expect that this (or these) local government(s) offer political resistance against this proposal of the central government. Therefore, it is needed to inspect which welfare effects are induced if in Nash equilibrium all local governments raise (or lower) their tax rates.

Sometimes, regions within one nation are roughly identical w.r.t. most of their characteristics. In these cases, it seems to be quite reasonable to assume that the preferences of the inhabitants do not differ significantly across the regions (or states) and that the technological standard is almost the same everywhere. This justifies our simplifying assumption of identical jurisdictions. We claim that the utility functions of the representative consumers and the local production functions are the same in all regions. Correspondingly, the local demand functions are identical and the effects of local tax rates on the national net return to capital are symmetric. But, as it often occurs, even within one nation, there are regional distributive differences. These distributive characteristics are represented within our model by different regional portions of the national capital stock, $\hat{K}$. Hence, we assume that all regions are identical, except that we allow for $\theta^i \neq \theta^j \forall i \neq j$.

Moreover, not to mix up several distributive effects, we claim that the portions of the residents of the local firms are assumed to be identical as well. To exclude the possibility of providing public goods at the expense of foreigners, we not only have to assume that $\beta^i = \beta^j \forall i, j$ but also that $\beta^i = 1 \forall j$. Within this simplified framework we discuss the issue of re-establishing efficiency by means of an equal and nation-wide variation of local tax rates.\(^{20}\)

Assume that each local government chooses its policy tools under the pre-

\(^{20}\)Clearly, there may be other policy measures which dominate a symmetric variation of all capital and/or emission tax rates. But, symmetric solutions seem to be very popular and often, especially in political bargaining processes, the only enforceable ones. This makes us to restrict our attention to symmetric solutions here.
umption that it cannot affect national prices, namely, the net return to capital. Taking this behavior of each government (see sec. 3.2) for granted, we show that, even if all regions are identical except for their shares of the national capital stock, slight differences of the initial capital endowments may be sufficient to imply opposite welfare effects of an equal increase of all tax rates in different jurisdictions starting from a symmetric Nash equilibrium.\(^{21}\) For each region, however, the welfare effect induced by an equal increase of all tax rates is contrary to that resulting from an equal increase of all emission tax rates. Clearly, as we may expect from the former analysis, the signs of the induced welfare effects depend on the local excess demand of capital and on the question whether we observe over- or underprovision of public goods within the considered region. If, in addition, all regions are identically endowed with capital (in the following 'perfect identical regions') the welfare effects of an equal increase of all tax rates are unique, i.e., identical for all regions.

In the case of different initial capital endowments, there are, as a consequence of a symmetric increase of all local tax rates, loosing and winning regions. To see this, we have to differentiate the utility function of the representative resident w.r.t. the vector of the emission tax rates, \(\tau_e\), and the capital tax rates, \(\tau_k\), respectively, evaluated at the symmetric Nash equilibrium.

First of all remark that we get from the capital market clearing condition

\[
\frac{\partial \rho}{\partial \tau_e} = -\frac{K_{\tau_e}}{K_{\tau_k}} \geq 0, \quad (57)
\]

\[
\frac{\partial \rho}{\partial \tau_k} = -1. \quad (58)
\]

Because we know that \(K_{\tau_e}\) and \(K_{\tau_k}\) have opposite signs, the net rate of capital does unambiguously not fall as the emission tax raises in all regions. On the other hand, if the capital tax rate increases nation-wide by equal amounts, the net rate of capital falls by an equal amount, implying that capital owners bear the full cost of capital taxation and no tax burden can be shifted towards anybody else.

Secondly, we need to know how the factor demands react to symmetric tax rate increase. Therefore we have to differentiate totally the f.o.c.s of the local firm, \(R_k p = \rho + \tau_k\), and \(R_e p = \tau_e\), w.r.t. \(\tau_k\) and \(\tau_e\), respectively, and to evaluate the result at \(dP_i = 0\). Clearly, by symmetry we have that no variation of the tax

\(^{21}\)Note that initial capital endowments do not have any effect on optimal tax rates; given by (44) and (45). Because factor demand functions are the same in all jurisdictions, each local governments imposes the same tax rates and thus provides the same quantity of public services.
rates can affect equilibrium capital demand, i.e. $dK/d\tau_k = 0$ and $dK/d\tau_e = 0$. For emission demand we get

$$\frac{dE}{d\tau_k} = \frac{1}{R_{ke}} \left( 1 + \frac{\partial \rho}{\partial \tau_k} \right) = 0,$$

$$\frac{dE}{d\tau_e} = \frac{1}{R_{ee}} < 0. \quad (59)$$

These results enable us to evaluate the welfare effects of an centrally proposed raise of the emission taxes of all regions,

$$\frac{dU}{d\tau_e} = -U_e E \left( 1 - \frac{U_{Pe}}{U_e} \right) - U_e \frac{\partial \rho}{\partial \tau_e} (K - \theta K) + U_{Pe} \left( \frac{g'U_u}{U_{Pe}} + \tau_e \right) \frac{dE}{d\tau_e}. \quad (61)$$

The first term reflects the inefficient provision of local residential public goods. Only if public services are provided efficiently, this term drops out. In the ‘regular’ case of underprovision its sign is positive, whereas an overprovision means that its sign is negative. The second term corresponds to whether the region is w.r.t. the nation a net capital ‘exporter’ or ‘importer’. In the first case the sign of the second term is positive and negative in the second one. The third term is determined by the level of the local emission tax. Because $dE/d\tau_e$ is negative, the third part is positive if the emission tax is set below its efficiency level, i.e., the marginal social damage of emissions. However, from (45) we know that the emission tax is set too high if and only if public services are underprovided. Substituting (45) and (60) into (61) yields a more concentrated expression,

$$\frac{dU}{d\tau_e} = \left[ U_e K \left( 1 - \frac{U_{Pe}}{U_e} \right) - U_e \left( K - \theta K \right) \right] \left( -\frac{K\tau_e}{K_{Pe}} \right), \quad (62)$$

which immediately gives us the following proposition.

**Proposition 4.2** If in the Nash equilibrium local public goods are underprovided (overprovided) an equal raise of the emission taxes in all regions implies negative (positive) welfare effects for net-capital-importing (-exporting) regions.

Proposition 4.2 is somehow disheartening because, in general, we cannot hope that regions would consent to an equal decrease (or increase) of overall emission taxes, although national welfare may be improved. Even in the case of identical regions which only differ in their initial capital endowments, such a cooperative solution is not attainable as long as the distribution of capital is sufficient unequal and side-payments are ruled out. If, however, within each region local capital demand meets exactly initial local capital endowment we get a stronger result.
**Corollary 4.1** Let all regions be equally endowed with capital, so that within each region local capital supply meets local demand. Then all regions gain from an symmetrical decrease (increase) of the emission tax rates if and only if local public goods are underprovided (overprovided) in the Nash equilibrium.

Corollary 4.1 in conjunction with equation (45) says that if in Nash equilibrium of perfectly identical regions the provision of local public goods is inefficiently low then the emission tax is set above its efficient level, and the resulting environmental quality is too high (regime 1). The contrary is true for the case of overprovision of public goods (regime 2).

Now consider the welfare effects induced by an equal increase of all local capital tax rates. Similarly, the derivative of the local welfare function w.r.t. $\tilde{\tau}_k$,

$$\frac{dU}{d\tilde{\tau}_k} = -U_x K \left(1 - \frac{U_{px}}{U_x}\right) + U_x (K - \theta K).$$

(63)

exhibits the same terms as in (62) but with opposite signs. Hence, compared to the previous case of an equal raise of the emission tax rates, we get the reversal results of Proposition 4.2 and Corollary 4.1.

**Proposition 4.3** If in the Nash equilibrium local public goods are underprovided (overprovided), an equal raise of the capital taxes in all regions implies positive (negative) welfare effects for net-capital-importing (-exporting) regions.

**Corollary 4.2** Let all regions are equally endowed with capital so that within each region local capital supply meets local demand. Then all regions gain from an symmetrical increase (decrease) of tax capital tax rates if and only if local public goods are underprovided (overprovided) in the Nash equilibrium.

From Propositions 4.2 and 4.3 and by inspection of equation (63) and (62) we can conclude that

**Corollary 4.3** Each region approves (disapproves) an equal rise of all capital tax rates if and only if it disapproves (approves) an equal rise of all emission tax rates.

Clearly, if all regions are perfectly identical and in Nash equilibrium underprovision of public goods occurs (regime 1), a symmetric increase of all capital tax rates is welfare improving everywhere, and vice versa in regime 2.
To summarize, we cannot hope to reach an agreement of (almost) identical jurisdictions to overcome the nation-wide inefficiency as long as we restrict our attention to symmetric variations of tax rates. The reason is that a symmetrical variation of either the emission or the capital tax does not necessarily imply a welfare improvement in each region if regions differ sufficiently w.r.t. their initial capital endowments. Only in the case of perfectly identical regions in equilibrium, under regime 1, emission taxes are set too high and capital taxes are set too low everywhere. The resulting amount of public funds raised is too low, implying an underprovision of local public goods, whereas environmental quality is ‘overprovided’. Thus, an equal decrease of the emission taxes and an equal increase of the capital taxes induces a Pareto improvement on national grounds. (The opposite is true under regime 2.)

5 Concluding Remarks

We have provided a model that integrates, on the one hand, capital taxation as well as the provision of local public goods and, on the other hand, industrial emission regulation within the framework of interjurisdictional competition. The resulting equilibrium tax rates on capital and emissions lead to inefficient provision of local public goods and environmental quality, in general.

It turned out that the provision of local public goods crucially depends, among others, on the local initial capital endowment. Especially, the marginal rate of substitution between the private and the public good is strictly increasing in local capital endowment, whereas the tax rate on capital is strictly decreasing. I.e., overprovision is the more likely the lower the (relative) capital endowment. In the special case where local capital demand exactly meets local capital supply, local public goods are underprovided if and only if the tax rate on capital is positive.

Environmental quality is ‘provided’ at inefficient levels as well, in general, and is closely related to the provision of local public goods. Namely, environmental quality is too high, if and only if local public goods are underprovided. The reason is that the emission tax rate is set above (below) the marginal social damage of pollutants if and only if local public goods are under- (over)provided. Contrary to the capital tax rate, this equivalence neither (directly) depends on local initial capital endowment nor on region’s market power at the national capital market.
For the special case of a small region, we found that only two possible regimes occur. In the first regime, public goods are underprovided, environmental quality is too high, and the tax rate on capital is positive. In the second one, public goods are overprovided, environmental quality is too low, and capital is subsidized. Hence, the assertion of traditional tax competition theory that in equilibrium local public goods are underprovided need not to be true. If we allow for emission taxation, this result rather depends on the curvature of the social damage function. For a sufficiently steep damage function overprovision of public goods is accompanied by an inefficiently high level of environmental quality (and vice versa).

The higher government (federal regulator) can re-establish nation-wide efficiency by applying a subsidy/penalty scheme on local tax rates, if this authority has access to public funds. For the case of a small region, we found that in the first regime the regulator has to subsidize capital taxation and to fine emission taxation. The reason is that local governments are afraid of capital flight, so that they refrain from taxing capital more heavily. On the other side, to collect revenues in order to provide local public goods, although their provision level remains too low, the local government taxes emissions too high. Hence, the central regulator discourages emission taxation by imposing a negative subsidy. (In the second regime the opposite is true.)

If, however, the federal government has less political power and, therefore, cannot enforce policy tools which imply welfare losses in some regions, although national welfare is improved, we can hardly expect that efficiency is re-established. The reason is that, in general, a uniform increase (or decrease) of all tax rates implies welfare effects of different signs for some regions. Even though regions are almost identical – i.e., they only differ w.r.t. their initial capital endowments – local governments prefer different policies. Hence, we get the discouraging result that cooperative symmetric solutions of the inefficiency problem are less likely to be achieved. Only if all regions are perfectly identical, Pareto improvements can be obtained through cooperative symmetric behavior.
References


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