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A CONTRIBUTION TO THE MACRO THEORY OF COMPARATIVE ECONOMIC SYSTEMS*

by

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Abstract. Within the framework of the general equilibrium model of the firm developed by the author, which features (i) the neoclassical market mechanism for allocation of non–human resources and a cooperative game for allocation of human resources, (ii) a separation of ownership (stockholding) and control (management) of the firm in the capitalistic economy, and (iii) endogenous formation of firms, the following theorem is established: If the socialistic economy can be decomposed into several subeconomies, each satisfying increasing returns with respect to the firm size, then for each equilibrium of the capitalistic economy there exists an equilibrium of the socialistic economy such that the former is Pareto superior to the latter.

1. INTRODUCTION

Parallel to the analytical deepening of the neoclassical paradigm, specialists in the theory of the firm have accumulated a wealth of thoughts and observations on the firm activities during the past half century. Three of the strands in the theory of the firm will be briefly recalled. The first strand, which is the most basic, addresses the raison d'être of the firm: If the information on the labor market is complete, there is no need for a firm to be organized. A resource–holder as producer can find the right type and amount of labor in the market, and hire laborers on a free–lance basis given the prevailing wage rate. In reality, however, the information on the labor market is incomplete, so a producer cannot identify the right labor and wage rate. Instead of relying on the labor market, therefore, a producer and laborers form an organization called a firm, and decide on labor allocation and wage distribution within the organization. In short, a firm is formed as a human–resource allocation mechanism, which serves as an alternative to the neoclassical labor market. This is the basic idea of the seminal work of Coase (1937). Subsequent works illuminated specific features of the firm as a resource allocation mechanism. Alchian and Demsetz (1972) presented the view that production activities are considered coalitional activities, where various human– or non–human–resource holders constitute a coalition. Arrow (1974) presented merits of the firm as an information processor.

The second strand is based on the empirical observation of the present–day capitalistic economy. By carefully studying the stockholding structures of the top U.S. corporations, Berle and Means (1932) argued that these corporations on the one hand had been able to grow by collecting their capital from innumerable small investors, and that these small capitalists on the other hand had been attracted to invest in growing corporations for high rates of return. The majority of the small stockholders (owners of the firm) no longer actively participate in the management, and it is the laborers who manage the firm they work for. Here the term "laborer" should be interpreted broadly; it simply means "human–resource holder", so it includes specifically "manager" as well as "manual laborer". In short, there is a separation of ownership (stockholding) and control (management) of the firm. Laborers hire capital in the present–day capitalistic economy, whereas capitalists hire labor in the neoclassical paradigm.

The third strand analyzes the behaviour of the firm in a certain socialistic economy. Since the stock market does not exist in this economy, the neoclassical hypothesis of profit–maximization does not make sense. Ward (1958) and Domar (1966) postulated the per–capita value–added maximization hypothesis, and studied its implications regarding the output–supply curve. See Vanek (1970) for further work.

In both the second and the third strands, it has been argued that a firm is managed by the laborers who work for it. A new behavioral principle can now be introduced: Each laborer, given his own incentive, coordinates his strategy–choice with the other laborers in a firm, because by doing so he and his colleagues can better serve their diverse incentives; that is, the laborers play a cooperative game. This cooperative game could be one explicit interpretation of the Coasian human–resource allocation mechanism that replaces the neoclassical labor market (the first strand).

Some time ago, the present author constructed a general economic equilibrium model with production, proposed a new equilibrium concept, and established its existence theorem (Ichiishi, 1982, 1985). Among the theoretical characteristics of the model are: First, the model embodies both the neoclassical market mechanism for allocation of non–human resources and a cooperative game played by the laborers for allocation of human resources, and the equilibrium concept is a hybrid of the competitive equilibrium (a version of the typical non–cooperative solution concept) and the core (the typical descriptive cooperative solution concept). Indeed, the equilibrium existence theorem was established by applying the result of Ichiishi (1981) which synthesizes Nash's equilibrium existence theorem (Nash, 1950) and Scarf's theorem for nonemptiness of the core (Scarf, 1967). Second, a firm is managed by the laborers who work for it. By specifying whether or not firms have access to the capital market, the model is defined as capitalistic or socialistic. Third, the formation of firms is endogenously determined in equilibrium.

The present paper continues the analysis of the new model of Ichiishi (1982, 1985). The purpose here is to compare the performance of the capitalistic version with the performance of the socialistic version. In order to formulate this comparative economic systems problem as clearly as possible, the model is greatly simplified here, while retaining all the essential ingredients. It is an improved version of the abridged model of Ichiishi (1981, Appendix 3). Also, in order to see clearly the implications of the nature of the firm as a cooperative resource allocation mechanism, the effects of a change in relative market prices of commodities are abstracted away; in this sense, the model is macroeconomic. The comparative study of two systems is not new (see, e.g., Sertel (1982) for a modern treatment). What is new here is the comparison of two systems in terms of a new descriptive equilibrium concept which is based on cooperative behavior of the agents.

The main result of this paper is that if the socialistic economy can be de composed into several subeconomies, each satisfying increasing returns with respect to the firm size, then for each equilibrium of the capitalistic economy there exists an equilibrium of the socialistic economy such that the former is Pareto superior to the latter. It is precisely in this sense that the capitalistic economy works more efficiently than the socialistic economy. There are other criteria for relative efficiency, but many of them are not applicable to the present economic context. It is not necessarily true, for example, that for each equilibrium of the socialistic economy there exists an equilibrium of the capitalistic economy such that the latter is Pareto superior to the former.

The next section will present the model, in both the capitalistic and the socialistic versions. Section 3 will state the main theorem and show an economically meaningful example which satisfies all the assumptions of the theorem. Proofs will be given in Secton 4. The main theorem will be proved by applying a game–theoretical foundation developed by Ichiishi (1988).

2. MODELS

2.1. Three Roles of Economic Agents. There are n economic agents; denote by N the set of economic agents. A subset S of N, called a coalition, is identified with a potential firm; denote by $\mathcal{N}$ the family of all nonempty coalitions, $2^N \setminus \{\phi\}$. When firm S is actually formed, agent j works for S as a full–time laborer if and only if $j \in S$. Here, the term "laborer" should be interpreted broadly; it simply means "human–resource holder", so it includes not only "manual laborer" but also "highly skilled laborer" such as "engineer" and "manager". A coalition structure is a partition of N; it describes realization and coexistence of firms. Certain coalition structures may be forbidden by law as inadmissible. Let $\mathcal{T}$ be the nonempty family of admissible coalition structures on N. The models here will explain which coalition structure in $\mathcal{T}$ is realized in equilibrium. Each agent j plays three roles in the economy: that of consumer, that of member of a production unit as a laborer, and that of an owner of production units. The economy is called socialistic, if an agent can be an owner only of the firm he works for. The economy is called capitalistic, if each firm is allowed to issue an asset, called a stock, and ownership of the firm is defined as stockholding.

2.2. Capitalistic Economy. The model of the capitalistic economy constructed in this paper is macroeconomic, in the sense that the microeconomic effects of changes in the relative prices of marketed commodities and services are abstracted away from the analysis. To be specific, there is only one marketed "commodity" in the model. Besides the market for this "commodity", there are stock markets. Since there are $2^n - 1$ (= $\# \mathcal{N}$) potential firms, each having the potential to issue its own stock, there are $2^n - 1$ types of stocks. The economy has, therefore $\ell := 1 + (2^n - 1)$ markets, indexed by $h \in \{c\} \cup \mathcal{N}$. Commodity c is the marketed "commodity", and commodity $h \in \mathcal{N}$ is the stock of firm h. The price domain is given by the simplex, $\Delta := \{p \in \mathbf{R}^{\ell}_{+} \mid \Sigma_{h \in \{c\} \cup \mathcal{N}} p_h = 1\}$. There is no market for labor. Indeed, the nature of the firm is that the firm is formed as a cooperative human–resource allocation mechanism, which replaces the neoclassical labor market. The labor allocation is determined not through the market mechanism (a typical example of a non–cooperative game), but through the society–wide cooperative game played by the laborers. Wages are not prices but (a part of the) strategies in the game.

Formally the model under construction is static, but it is best interpreted as a model of temporary equilibrium. There are two periods, today and future. All the endogenous variables are determined today, based upon the agents' subjective expectations about future events. It takes a firm one period to produce outputs (random variables). In order to run the firm, funds are raised first through the stock market, and these funds are used to pay wages and buy marketed inputs today. Negative wages are possible. In the future, the revenue (i.e., the value of the outputs and portfolio) will be distributed to the laborers as future wages and to the owners as dividends.

For each agent j, his strategy space X^j is a subset of $\mathbf{R}^{\ell} \times \mathbf{R}^{\ell} \times E^j$, where E^j is a Hausdorff locally convex topological vector space. A generic element x^j of X^j is denoted by $(x^j_I, x^j_{II}, x^j_{III})$, with $x^j_I := (x^j_{Ic}, \{x^j_{Ih}\}_{h \in \mathcal{N}}) \in \mathbf{R}^{\ell}$, $x^j_{II} \in \mathbf{R}^{\ell}$, $x^j_{III} \in E^j$. The "commodity"–stock bundle x^j_I signifies the excess "commodity"–demand and portfolio of j as an individualistic consumer, x^j_{II} signifies the "commodity" input and portfolio of the firm which j works for, and x^j_{III} signifies the amount of labor that j is assigned to supply, the wage he receives today, and his subjective probability measure on his future consumption. No agent holds stocks as an initial endowment, so net demand for (net supply of, resp.) a stock is equal to its demand (supply, resp.). The usual sign convention for demand and supply is adopted; i.e., consumer j's excess demand for (excess supply of, resp.) a commodity is measured by a positive number (negative number, resp.), and firm S's supply of (demand for, resp.) a commodity is measured by a positive number (negative number, resp.).

Define $X^S := \Pi_{j \in S} X^j$ for each $S \in \mathcal{N}$, and set for simplicity $X := X^N$. Suppose $(\bar{x}, \bar{p}) \in X \times \Delta$ is given at the outset. Firm $S \in \mathcal{N}$ is formed, once the members of S agree to cooperate; they agree to choosing the input–portfolio vector $x^{(S)}_{II}$ $(= x^j_{II}$, for all $j \in S)$ for their firm, wage w^j and labor assignment for each member j (the first two components of x^j_{III}), and their future wages and dividend policy (the last two items influence member j's subjective probability measure on his future consumption, viz., the third component of x^j_{III}).

Not all strategies $\{x^S \in X^S \mid x^i_{II} = x^j_{II} \ (=: x^{(S)}_{II})$ for all $i,j \in S\}$ are feasible to firm S. The firm first supplies its stock to receive capital $\bar{p}_S\, x^{(S)}_{IIS}$, then use the capital to purchase input/portfolio and pay wages. Consequently, firm S has the budget constraint,

$$\bar{p}_S\, x^{(S)}_{IIS} \geq \bar{p}_c(-x^{(S)}_{IIc}) + \Sigma_{h \in \mathcal{N} \setminus \{S\}} \bar{p}_h(-x^{(S)}_{IIh}) + \Sigma_{j \in S}\, w^j,$$

or

$$-\bar{p} \cdot x_{II}^{(S)} \leq -\Sigma_{j \in S} w^j,$$

where for two vectors x and y, $x \cdot y$ denotes the inner product $\Sigma_i x_i y_i$.

Once agent j agrees to his wage w^j in his firm S, his income level today is determined. He carries out his net trade x_I^j in the ℓ markets subject to his budget constraint,

$$\bar{p} \cdot x_I^j \leq w^j,$$

by himself (i.e., non–cooperatively) as a consumer. Firm S's future–wage policy and dividend policy (which takes into account its future output), the prevailing stock-holding structure outside firm S ($\{\bar{x}_{Ih}^i, \bar{x}_{IIh}^i\}_{i \in N \setminus S,\, h \in \mathscr{N}}$), and agent j's own portfolio ($\{x_{Ih}^j\}_{h \in \mathscr{N}}$) determine the subjective probability measure on j's future consumption.

The constraints specified in the preceeding two paragraphs are summarized by the <u>feasible strategy set</u> $F^S(\bar{x}, \bar{p})$ $(\subset X^S)$. To re–state some of these constraints, each $x^S \in F^S(\bar{x}, \bar{p})$ has to satisfy

$$x_{Ih}^j \geq 0, \text{ for all } j \in S \text{ and all } h \in \mathscr{N},$$

$$x_{II}^i = x_{II}^j =: x_{II}^{(S)}, \text{ for all } i, j \in S,$$

$$x_{IIh}^{(S)} \begin{cases} \leq 0, \text{ for all } h \in \{c\} \cup \mathscr{N} \setminus \{S\}, \\ \geq 0, \text{ if } h = S, \end{cases}$$

and by summing up all the budget constraints,

$$\bar{p} \cdot (\Sigma_{j \in S}\, x_I^j - x_{II}^{(S)}) \leq 0 \qquad (1)$$

Denote by gr F^S the graph of the correspondence $F^S : X \times \Delta \to X^S$; gr $F^S := \{(\bar{x}, \bar{p}, x^S) \in X \times \Delta \times X^S \mid x^S \in F^S(\bar{x}, \bar{p})\}$. For each member j of S, his <u>utility function</u> $u_S^j : \text{gr } F^S \to \mathbf{R}$ is given.

The <u>capitalistic economy</u> is defined as a list of specified data, $\mathscr{E}_c := (\{X^j\}_{j \in N}, \{F^S(\cdot)\}_{S \in \mathscr{N}}, \{u_S^j(\cdot)\}_{j \in S \in \mathscr{N}}, \mathscr{T})$. An <u>equilibrium</u> of the capitalistic economy $\mathscr{E}_c$ is a triple $(x^*, p^*, \mathscr{T}^*)$ of members of X, Δ, $\mathscr{T}$, respecitvely, such that:

i) $x^{*T} \in F^T(x^*, p^*)$ for every $T \in \mathcal{T}^*$;

ii) it is not true that there exist $S \in \mathcal{N}$ and $x^S \in F^S(x^*, p^*)$ such that $u^j_S(x^*, p^*, x^S) > u^j_{T(j)}(x^*, p^*, x^{*T(j)})$ for every $j \in S$, where $T(j)$ is the unique member of $\mathcal{T}^*$ such that $T(j) \ni j$; and

iii) $\Sigma_{j\in N}\, x_I^{*j} - \Sigma_{T\in\mathcal{T}^*}\, x_{II}^{*(T)} \leq 0.$

Given $(\bar{x}, \bar{p})$, the agents play a cooperative game; this changes the strategical part $\bar{x}$ of $(\bar{x}, \bar{p})$ in disequilibrium. At the same time, given $(\bar{x}, \bar{p})$, the market mechanism works in the spot "commodity"/stock markets; this changes the price vector $\bar{p}$ of $(\bar{x}, \bar{p})$ in disequilibrium. An equilibrium (x^*, p^*) is achieved (i.e., the feasibility condition (i)), in which no coalition as a price–taker can bring about by its own effort a higher utility level to each of its members (and specifically no coalition has incentives to change the strategies) (the coalitional stability condition (ii)), and all the "commodity"/stock markets are cleared (the market clearance condition (iii)).

2.3. Socialistic Economy. The socialistic economy is obtained from the capitalistic economy $\mathcal{E}_c$ by allowing no trade of a stock nor its issuance. Funds for operating firm S are collected from its members in the form of negative wages. Given $(\bar{x}, \bar{p})$, the set of all feasible strategies of firm S is then given as:

$$G^S(\bar{x}, \bar{p}) := \left\{ x^S \in F^S(\bar{x}, \bar{p}) \;\middle|\; \begin{array}{l} x^j_{Ih} = x^j_{IIh} = 0, \\ \text{for all } j \in S \text{ and all } h \in \mathcal{N} \end{array} \right\}.$$

Assume $\bar{p}_c > 0$. Since there is no stock market, a wage structure in S completely determines an allocation of the "commodity", $(\{x^j_{Ic}\}_{j\in S}, x^{(S)}_{IIc})$. Indeed, the contraint (1) becomes

$$\Sigma_{j\in S}\, x^j_{Ic} - x^{(S)}_{IIc} \leq 0. \tag{2}$$

Moreover, the input $x^{(S)}_{IIc}$ and a future wage structure alone determine members' probability measures on their future consumption. Thus one may postulate:

ASSUMPTION 1. For each coalition S, the feasible strategy set $G^S(\bar{x}, \bar{p})$ is independent of $(\bar{x}, \bar{p})$. For each member j of S, his utility function $u^j_S(\bar{x}, \bar{p}, x^S)$ is independent of $(\bar{x}, \bar{p})$.

This assumption justifies the notation, $G^S := G^S(\bar{x}, \bar{p})$, $u^j_S(x^S) := u^j_S(\bar{x}, \bar{p}, x^S)$,

The socialistic economy is defined as a list of specified data, $\mathscr{E}_s := (\{X^j\}_{j \in N}, \{G^S\}_{S \in \mathscr{N}}, \{u^j_S(\cdot)\}_{j \in S \in \mathscr{N}}, \mathscr{T})$. An equilibrium of the socialistic economy $\mathscr{E}_s$ is a pair $(x^\dagger, \mathscr{T}^\dagger)$ of members of X, $\mathscr{T}$, respectively, such that:

(i) $x^{\dagger T} \in G^T$ for every $T \in \mathscr{T}^\dagger$; and,

(ii) it is not true that there exist $S \in \mathscr{N}$ and $x^S \in G^S$ such that $u^j_S(x^S) > u^j_{U(j)}(x^{\dagger U(j)})$ for every $j \in S$, where $j \in U(j) \in \mathscr{T}^\dagger$.

Notice that in this macro model, the "commodity" market clearance condition is satisfied by condition (2).

2.4. Example. The simplest example of a socialistic economy $\mathscr{E}_s$ might be the case in which (i) utility of each agent j is determined only by his strategy x^j, (ii) labor supply does not enter the utility function (so that each laborer always supplies his maximal labor time to his firm), (iii) there is no uncertainty in the future (so that the future gross consumption allocation $\{x^j_{IIIc}\}_{j \in S}$ is completely determined as today's strategy, and (iv) the technology of S is desribed by a production function, $g^S : \mathbf{R}_+ \to \mathbf{R}_+$. One may suppress the labor–component from E^j in view of (ii). One may also suppress the wage component from E^j in view of the identity, $x^j_{Ic} = w^j$ (here, the marketed "commodity" is the numeraire). The space E^j is, therefore, identified with the one–dimensional future–consumption space. The feasible strategy set is given by

$$G^S = \left\{ x^S \in X^S \;\middle|\; \begin{array}{l} \Sigma_{j \in S}\, x^j_{Ic} - x^{(S)}_{IIc} \le 0 \\ \Sigma_{j \in S}\, x^j_{IIIc} \le g^S(-x^{(S)}_{IIc}) \end{array} \right\},$$

and there exists a function $u^j : \mathbf{R} \times \mathbf{R}_+ \to \mathbf{R}$ such that

$$u^j_S(x^S) = u^j(x^j_{Ic}, x^j_{IIIc}).$$

3. WELFARE COMPARISON OF THE TWO ECONOMIC SYSTEMS

3.1. Main Result. The crucial difference between the two economies $\mathscr{E}_c$ and $\mathscr{E}_s$ is that in the former the capital markets serve as a channel for re–distribution of the initial resources. This fact is summarized by the condition,

$$G^S \subset F^S(\bar{x}, \bar{p}), \text{ for every } S \in \mathscr{N} \text{ and every } (\bar{x}, \bar{p}) \in X \times \Delta \tag{3}$$

The purpose of this section is to establish that under certain economically meaningful conditions, condition (3) results in a more efficient allocation of resources in $\mathscr{E}_c$ than in $\mathscr{E}_s$. The precise content of the phrase, "more efficient" will be specified as the assertion of the Theorem below.

In the socialistic economy $\mathscr{E}_s$, the set of attainable utility allocations of each coalition S is given by :

$$W(S) := \left\{ u \in R^n \;\middle|\; \begin{array}{l} \text{There exists } x^S \in G^S \text{ such that} \\ \text{for each } j \in S,\ u_j \le u_S^j\,(x^S) \end{array} \right\}$$

(Inclusion of the coordinates that correspond to N\S is made simply for notational convenience.) Set $W(\phi) := \phi$. The following Assumption 2 characterizes the increasing returns with respect to the coalition size.

ASSUMPTION 2. For any S, T $\in \mathscr{N}$, $W(S) \cap W(T) \subset W(S \cap T) \cup W(S \cup T)$.

This is a strengthened form of the super–additivity assumpiton. Its precise meaning would be readily understood if utilities are transferable, i.e., if there exists a function $w : \mathscr{N} \to R$ such that

$$W(S) = \{u \in R^n \mid \Sigma_{j \in S}\, u_j \le w(S)\}.$$

In this transferable utility case, Assumption 2 is equivalent to:

$$w(S \cup \{j\}) - w(S) \leq w(T \cup \{j\}) - w(T),$$

for all $j \in N$ and all $S \subset T \subset N \setminus \{j\}$, where $w(\phi) := 0$. The last inequality says that the larger the coalition that j joins is, the higher the marginal worth of j is.

For each coalition S, define

$$\mathbf{W}(S) := \bigcup_{\mathscr{P}} \bigcap_{P \in \mathscr{P}} W(P),$$

where the union in the right–hand side is taken with respect to all the partitions $\mathscr{P}$ of S. The following Assumption 2′ weakens Assumption 2.

ASSUMPTION 2′. For any $S, T \in \mathscr{N}$, $\mathbf{W}(S) \cap \mathbf{W}(T) \subset \mathbf{W}(S \cap T) \cup \mathbf{W}(S \cup T)$.

The next assumption postulates the efficiency of the admissible coalition structures.

ASSUMPTION 3. $\mathbf{W}(N) = \bigcup_{\mathcal{T} \in \mathscr{T}} \bigcap_{T \in \mathcal{T}} W(T)$.

The main result of this paper is the following Theorem; its proof will be given in Section 4.

THEOREM. Let $\mathscr{E}_c$ and $\mathscr{E}_s$ be the capitalistic economy and the socialistic economy. Suppose that $\mathscr{E}_s$ satisfies Assumptions 1, 2′ and 3. Then, for each equilibrium $(x^*, p^*, \mathcal{T}^*)$ of $\mathscr{E}_c$ for which $p_c^* > 0$, there exists an equilibrium $(x^\dagger, \mathcal{T}^\dagger)$ of $\mathscr{E}_s$ such that the former is Pareto superior to the latter, i.e.,

$$u^j_{T(j)}(x^*, p^*, x^{*T(j)}) \geq u^j_{U(j)}(x^{\dagger U(j)}) \text{ for all } j \in N,$$

where $\quad j \in T(j) \in \mathcal{T}^*$ and $j \in U(j) \in \mathcal{T}^\dagger$.

3.2. Remarks. (i) Discussions in Ichiishi (1988, Section 2) show why the assertion of the above Theorem is the only result one can hope for on welfare comparison of the two systems, given condition (3).

(ii) A subfamily $\mathscr{B}$ of $\mathscr{N}$ is called balanced, if there exist nonnegative real numbers $\{\lambda_S\}_{S\in\mathscr{B}}$ such that $\Sigma_{S\in\mathscr{B}:S\ni j}\ \lambda_S = 1$ for every $j \in N$. Define $X^j_{-II} := \{(x^j_I, x^j_{III}) \in R^\ell \times E^j \mid x^j \in X^j\}$, and $\tilde{F}^S_{-II}(\bar{x}, \bar{p}) := \{x_{-II} \in \Pi_{j\in N} X^j_{-II} \mid x^S \in F^S(\bar{x}, \bar{p}),\ x^{N\setminus S} = 0\}$. By applying the social coalitional equilibrium existence theorem of Ichiishi (1981), one can easily prove that there exists an equilibrium of $\mathscr{E}_c$, if X^j is nonempty, compact and convex, F^S is both upper semicontinuous and lower semicontinuous in $X \times \Delta$, each $F^S(\bar{x}, \bar{p})$ is nonempty and closed, $\Sigma_{S\in\mathscr{B}}\ \lambda_S\ \tilde{F}^S_{-II}(\bar{x}, \bar{p}) \subset \cup_{\mathscr{T}\in\mathscr{J}} \Sigma_{T\in\mathscr{T}}\ \tilde{F}^S_{-II}(\bar{x}, \bar{p})$ for every balanced family $\mathscr{B}$ with the associated coefficients $\{\lambda_S\}_{S\in\mathscr{B}}$, and there exists a continuous function $u^j : X \times \Delta \times X^j_{-II} \to \mathbf{R}$ such that $u^j_S(\bar{x}, \bar{p}, x^S) = u^j(\bar{x}, \bar{p}, x^j_{-II})$ and such that each $u^j(\bar{x}, \bar{p}, \cdot)$ is quasi-concave in X^j_{-II}.

3.3. Example. Consider the example of Section 2.4. Define

$$g^\phi(a) := \begin{cases} 0 & \text{if } a = 0, \\ \infty & \text{if } a > 0. \end{cases}$$

The following Claim is an extension of a result of Moulin (1987, Appendix B); Moulin treated the case in which $g^S = g^T$ for all $S, T \in \mathscr{N}$.

CLAIM. Suppose the production functions $g^S : \mathbf{R}_+ \to \mathbf{R}_+$, $S \in \mathscr{N}$, satisfy that

$$\left.\begin{aligned} &g^S(0) = 0, \\ &g^S(a) + g^T(b) \le g^{S\cap T}(c) + g^{S\cup T}(a + b - c), \end{aligned}\right\} \tag{4}$$

<u>for all</u> S, T $\in \mathcal{N}$ <u>and any</u> $c \leq \min\{a, b\}$. <u>Suppose also that each utility function</u> $u^j : \mathbf{R} \times \mathbf{R}_+ \to \mathbf{R}$ <u>is non-decreasing in</u> $\mathbf{R} \times \mathbf{R}_+$, <u>and that</u> $\{N\} \in \mathcal{T}$. <u>Then both Assumptions 2 and 3 are satisfied.</u>

This claim will be proved in Section 4. Notice that condition (4) implies <u>both</u> increasing returns to scale for each firm S,

$$g^S(a + h) - g^S(a) \leq g^S(b + h) - g^S(b) \text{ for all } a < a + h \leq b < b + h,$$

<u>and</u> increasing returns with respect to the coalition size (in terms of the production functions),

$$g^S(a) \leq g^T(a) \text{ for all a and all } S \subset T.$$

<u>3.4. Example.</u> A non–trivial coalition structure may be realized in equilibrium of $\mathcal{E}_c$ or $\mathcal{E}_s$. Suppose $\mathcal{T}(\in \mathcal{T})$ has the properties: For each $T \in \mathcal{T}$,

$$W(S) \cap W(S') \subset W(S \cap S') \cup W(S \cup S') \text{ for all } S, S' \subset T,$$

and for any two distinct T, $T' \in \mathcal{T}$,

$$W(S \cup S') \subset W(S) \cap W(S') \text{ for all } S \subset T \text{ and all } S' \subset T'.$$

Then Assumptions 2′ and 3 are satisfied, and $\mathcal{T}$ is realized in equilibrium of $\mathcal{E}_s$.

4. PROOFS

Proof of the Theorem of Section 3.1. Let $(x^*, p^*, \mathcal{T}^*)$ be an equilibrium of $\mathcal{E}_c$ for which $p_c^* > 0$. Define non–side–payment games $V, \bar{V} : \mathcal{N} \to \mathbf{R}^n$ by:

$$V(S) := \left\{ u \in \mathbf{R}^n \,\middle|\, \begin{array}{l} \exists x^S \in F^S(x^*, p^*) : \forall j \in S: \\ u_j \leq u_S^j(x^*, p^*, x^S) \end{array} \right\}$$

$$\bar{V}(S) := \bigcup_{\mathcal{P}} \bigcap_{P \in \mathcal{P}} V(P),$$

where the union in the right–hand side is taken with respect to all the partitions $\mathcal{P}$ of S. Define also

$$H := \bigcup_{\mathcal{T} \in \mathcal{T}} \bigcap_{T \in \mathcal{T}} V(T).$$

Let u^* be the equilibrium utility allocation : $u_j^* := u_{T(j)}^j(x^*, p^*, x^{*S})$, for every $j \in N$. The allocation u^* is in the core of game (V, H), $H \setminus \bigcup_S \overset{\circ}{V}(S)$. To see that it is also in the core of the game $(\bar{V}, H)$, suppose there exists $S \in \mathcal{N}$ and $u \in \bar{V}(S)$ such that $u_j^* < u_j$ for all $j \in S$. There exists a partition $\mathcal{P}$ of S such that $u \in V(P)$ for all $P \in \mathcal{P}$. Each P improves upon u^*, which contradicts the fact that u^* is in the core of game (V, H). By (3), $W(S) \subset \bar{V}(S)$ for every S. Assumptions 2′ and 3 say that by setting $K := \bigcup_{\mathcal{T} \in \mathcal{T}} \bigcap_{T \in \mathcal{T}} W(T)$, (W, K) is an ordinal convex game. Theorem 3.1 of Ichiishi (1988) is now applicable, and there exists a core utility allocation $u^\dagger$ of game (W, K) such that $u^* \geq u^\dagger$. Q.E.D.

Proof of the Claim of Section 3.3. Step 1. For any a, b, c, d, A, B, C, D $\in \mathbf{R}_+$ and any S, T $\in \mathcal{N}$ such that $A + B \leq g^S(a + b)$ and $C + D \leq g^T(c + d)$, at least one of the following four inequalities holds true: $B \leq g^{S\cap T}(b)$, $C \leq g^{S\cap T}(c)$, $A + B + D \leq g^{S\cup T}(a + b + d)$, and $A + C + D \leq g^{S\cup T}(a + c + d)$. Indeed, suppose $B > g^{S\cap T}(b)$ and $C > g^{S\cap T}(c)$. If $b \leq c$, then $b \leq \min\{a + b, c + d\}$. By (4),

$$\begin{aligned}&A + B + C + D\\ &\le g^S(a + b) + g^T(c + d)\\ &\le g^{S\cap T}(b) + g^{S\cup T}(a + b + c + d - b)\\ &< B + g^{S\cup T}(a + c + d).\end{aligned}$$

Consequently, $A + C + D \le g^{S\cup T}(a + c + d)$. Similarly, if $b \ge c$, then $A + B + D \le g^{S\cup T}(a + b + d)$.

Step 2. One needs to show that the non-side-payment game W is ordinal convex. In view of the monotonicity of the utility functions u^j, it suffices to show that if $x^S \in G^S$ and $y^T \in G^T$, then $x^{S\cap T} \in G^{S\cap T}$, or $y^{S\cap T} \in G^{S\cap T}$, or $(x^S, y^{T\setminus S}) \in G^{S\cup T}$, or $(x^{S\setminus T}, y^T) \in G^{S\cup T}$. Now, $x^S \in G^S(y^T \in G^T$, resp.) means that $\Sigma_{j\in S}\, x^j_{IIIc} \le g^S(\Sigma_{j\in S}(-x^j_{Ic}))$ $(\Sigma_{j\in T}\, y^j_{IIIc} \le g^T(\Sigma_{j\in T}(-y^j_{Ic}))$, resp.). Set $a := \Sigma_{j\in S\setminus T}\,(-x^j_{Ic})$, $b := \Sigma_{j\in S\cap T}\,(-x^j_{Ic})$, $c := \Sigma_{j\in S\cap T}\,(-y^j_{Ic})$, $d := \Sigma_{j\in T\setminus S}\,(-y^j_{Ic})$, $A := \Sigma_{j\in S\setminus T}\, x^j_{IIIc}$, $B := \Sigma_{j\in S\cap T}\, x^j_{IIIc}$, $C := \Sigma_{j\in S\cap T}\, y^j_{IIIc}$, $D := \Sigma_{j\in T\setminus S}\, y^j_{IIIc}$, and apply Step 1. Q.E.D.

REFERENCES

Alchian, A.A., and Demsetz, H., "Production, Information Costs, and Economic Organization." American Economic Review **62**: 777 – 795, 1972.

Arrow, K.J., The Limits of Organization. New York, NY: Norton, 1974.

Berle, Jr., A.A., and Means, G.C., The Modern Corporation and Private Property. New York, NY: Macmillan, 1932.

Coase, R.H., "The Nature of the Firm." Economica **4**: 386 – 405, 1937. Reprinted in K.E. Boulding, and G.J. Stigler, Eds., Readings in Price Theory. Chicago, IL: Irwin, 1952.

Domar, E., "The Soviet Collective Farm." American Economic Review **56**, Part 1: 734 – 757, 1966. Reprinted in J. Vanek, Ed., Self–Management: Economic Liberation of Man. Harmondsworth: Penguin, 1975.

Ichiishi, T., "A Social Coalitional Equilibrium Existence Lemma." Econometrica **49**: 369 – 377, 1981.

Ichiishi, T., "Management Versus Ownership, I." International Economic Review **23**: 323 – 336, 1982.

Ichiishi, T., "Management Versus Ownership, II." European Economic Review **27**: 115 – 138, 1985.

Ichiishi, T., "Comparative Cooperative Game Theory." Working Paper No. 165, Institut für Mathematische Wirtschaftsforschung, Universität Bielefeld, 1988.

Moulin, H., "A Core Selection for Regulating a Single–Output Monopoly." Rand Journal of Economics **18**: 397 – 407, 1987.

Nash, Jr., J.F., "Equilibrium Points in n–Person Game." Proceedings of the National Academy of Sciences of the U.S.A. **36**: 48 – 49, 1950.

Scarf, H., "The Core of an N Person Game." Econometrica **35**: 50 – 69, 1967.

Sertel, M.R., Workers and Incentives. Amsterdam: North–Holland, 1982.

Vanek, J., The General Theory of Labor–Managed Market Economies. Ithaca, NY: Cornell Universityy Press, 1970.

Ward, B., "The Firm in Illyria: Market Syndicalism." American Economic Review **48**: 566 – 589, 1958.