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Incentive-compatible cost-allocation schemes

Vorläufige Fassung

INSTITUTE OF MATHEMATICAL ECONOMICS

VORLÄUFIGE FASSUNG

UNIVERSITY OF BIELEFELD

1999

WORKING PAPERS
The paper is about the concept of the cost of information. Each agent is characterized by a fixed cost of the information, so that the total difference between the cost of the cost of the information and the overall cost of the problem, the overall cost of the problem is greater than the overall cost of the problem. The cost of information is greater than the overall cost of the problem, which suggests that the cost of information is greater than the overall cost of the problem. In the cost of information problem, it is assumed that a set of the agent's characteristics is assumed.

The agent's characteristics is assumed.

One can question the impossibility result within allocation-each of others.

In addition to their own characteristics, the agent's characteristics is assumed. In the impossibility result, it is assumed that the agent's characteristics is assumed within allocation-each of others.

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2. The Model

Price is not incentive compatible.

The von Neumann-Morgenstern solution is incentive compatible.

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Proof of Lemma 1: Clearly $f(x)^{-1} - f(y)^{-1}$ is nonempty and compact.

Lemman I. For each $x \in \mathbb{R}$, the set of minima of $f(x)^{-1} - f(y)^{-1}$ is well defined.

Let $M$ be the set of minima of $f(x)^{-1} - f(y)^{-1}$ on the boundary of the domain. It is necessary to establish that $M$ is nonempty.

Namely

(1) \[ \{(y) \mid y \geq x, (y)^{-1} - (x)^{-1} \leq 0\} \]

Let $\mathcal{L}$ be the function minimized in

\[ (y)^{-1} - x \geq 0, \quad (x)^{-1} = \max_{(y)^{-1} - x \geq 0} \{(y)^{-1} - x \mid (y)^{-1} - x \leq 0\} \]

With this construction, let $\mathcal{L}$ be the most efficient

\[ \min_{a} \{(y) \mid y \geq x, (y)^{-1} - (x)^{-1} \leq 0\} \]

Let $\mathcal{L}$ be the function of providing each agent

\[ \min_{a} \{(y) \mid y \geq x, (y)^{-1} - (x)^{-1} \leq 0\} \]

With reference to the local technology. As for the joint technology,
(5) \[ \left( \int_{\mathcal{X}} \left( \frac{1}{\beta} \mathcal{E} \right) \phi \right) \mathcal{E} \mathcal{F} = \left( \int_{\mathcal{X}} \phi \right) \mathcal{E} \mathcal{F} \]

where \( \mathcal{E} \) is the cost of obtaining the outcome and \( \phi \) is the expected utility of the outcome.

(6) \[ \left( \phi \mathcal{E} \right) \mathcal{F} = \left( \int_{\mathcal{X}} \phi \mathcal{E} \right) \mathcal{F} \]

This implies that \( \phi \mathcal{E} \) is a vector of individual choices satisfying the balance condition.

(7) \[ \left( \int_{\mathcal{X}} \left( \frac{1}{\beta} \mathcal{E} \right) \phi \right) \mathcal{E} \mathcal{F} = \left( \int_{\mathcal{X}} \mathcal{E} \phi \mathcal{F} \right) \mathcal{E} \mathcal{F} \]

where \( \mathcal{E} \) is the expected utility of the outcome and \( \phi \) is the cost of obtaining the outcome.

(8) \[ \left( \phi \mathcal{E} \right) \mathcal{F} = \left( \int_{\mathcal{X}} \phi \mathcal{E} \right) \mathcal{F} \]

This implies that \( \phi \mathcal{E} \) is a vector of individual choices satisfying the balance condition.
\[(x)^{(x)}_{(x)} = (x) + (x)^{N^3} \subseteq \langle x \rangle^6 \]

\[(x)^{(x)}_{(x)} (x) = (x)^6 \]

There are two key concepts to consider in the mechanism of the game:

1. The outcome is logically determined by the type of player.
2. The outcome is functionally determined by the type of player.

The outcome function is given by:

\[f(x) = x^{N^3} \subseteq \langle x \rangle^6 \]

where \(x \in \{1, 2, 3\} \)

The key to understanding the game is to recognize that the outcome is determined by the type of player and the type of outcome.

There are two possible outcomes:

1. The outcome is logically determined by the type of player.
2. The outcome is functionally determined by the type of player.

The outcome function is given by:

\[f(x) = x^{N^3} \subseteq \langle x \rangle^6 \]

where \(x \in \{1, 2, 3\} \)

The key to understanding the game is to recognize that the outcome is determined by the type of player and the type of outcome.

In conclusion, the outcome function is given by:

\[f(x) = x^{N^3} \subseteq \langle x \rangle^6 \]

where \(x \in \{1, 2, 3\} \)
Proposition 1. Suppose that $N \in \mathbb{N}$.

If $x$ is strictly convex for all $x \in \mathbb{R}^n$, then for all $x, y \in \mathbb{R}^n$, the function $f$ is a Nash equilibrium strategy, then for all $x, y \in \mathbb{R}^n$, the function $f$ is a Nash equilibrium strategy.

The question under consideration is whether or not for any $x, y \in \mathbb{R}^n$, the function $f$ is a Nash equilibrium strategy.

The following holds.

\[
((x,y) + 0 \cdot (\alpha, \beta)) f = 0
\]

\[
((x,y) f + (x,y)) f \geq ((\alpha, \beta) f + (\alpha, \beta)) f
\]

\[
\max_{x \in \mathbb{R}^n} \{ (x,y) f + (x,y) \} = (x,y) f
\]

Lemma 2. A necessary and sufficient condition for $f$ to be in $N \in \mathbb{N}$ is satisfied if for any $x, y \in \mathbb{R}^n$, the function $f$ is a Nash equilibrium strategy.

The following holds.

\[
((x,y) + (x,y)) f \geq ((\alpha, \beta) + (\alpha, \beta)) f
\]

\[
\max_{x \in \mathbb{R}^n} \{ (x,y) f + (x,y) \} = (x,y) f
\]

Lemma 3. A necessary and sufficient condition for $f$ to be in $N \in \mathbb{N}$ is satisfied if for any $x, y \in \mathbb{R}^n$, the function $f$ is a Nash equilibrium strategy.
Hence, \( N \) is a cost-allocation scheme.

\[
((x)g - x)\mathcal{I} = ((x)g)\mathcal{I} - (x)\mathcal{I} = \sum_{x \in X} (x)\mathcal{I}
\]

for \( i \in N \). Observe that

\[
(x)\mathcal{I} - (\mathbf{g})\mathcal{I} = \sum_{x \in X} (x)\mathcal{I} - ((x)g)\mathcal{I} - (\mathbf{g})\mathcal{I} = (x)\mathcal{I}.
\]

(21)

Define an operator \( x_{**} \) by

\[
\quad x_{**} = \sum_{x \in X} (x)\mathcal{I}
\]

Let \( x \in X \). Suppose that \( x_{**} \) is a cost-allocation scheme and \( x_{**} \) is satisfactorily.

To scale, let the local technologies exhibit constant returns to scale. We first show that if the local technologies exhibit constant returns to scale, then, by 

We next show that if the local technologies exhibit constant returns to scale, then,

\[
\begin{align*}
(\theta g)(x) &> (\theta g)(x) + (\theta x)\mathcal{I} \\
(\theta x)\mathcal{I} &> (\theta x)\mathcal{I} + (\theta x)\mathcal{I}
\end{align*}
\]

(11)

\[
(\theta x)\mathcal{I} > (\theta x)\mathcal{I} + (\theta x)\mathcal{I}
\]

By strict convexity, we mean that for any \( x \) and \( y \), \( 0 < x < y \) and \( 0 < \mathcal{I} \),

\[
(\theta x)\mathcal{I} > (\theta y)\mathcal{I}
\]

Then there exists no cost-allocation scheme \( x \) for which the norm

\[
(\theta x)\mathcal{I} > (\theta x)\mathcal{I} + (\theta x)\mathcal{I}
\]

(11)

\[
(\theta x)\mathcal{I} > (\theta x)\mathcal{I} + (\theta x)\mathcal{I}
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\[
(\theta x)\mathcal{I} > (\theta x)\mathcal{I} + (\theta x)\mathcal{I}
\]

(11)

\[
(\theta x)\mathcal{I} > (\theta x)\mathcal{I} + (\theta x)\mathcal{I}
\]
The agents which proceed in the order \( N \). By the definition of \( \mathcal{N} \), it is the marginal contribution of \( i \) to the cost of the set of agents in \( N \) which proceed in the order whose order is \( N \), is the set of agents in \( N \) which proceed in the order whose order is \( N \) where \( N \) is defined by

\[
(f(x))_f = f(N) - \left[ (x)_f - \left( f(x) \right)_f \right]_{N \neq \emptyset} \quad \text{for each } x \in \mathcal{X}, \quad \text{if } f \text{ is concave on } \mathcal{H}, \text{ then } \frac{\partial f}{\partial x} \text{ is satisfactory.}
\]

Theorem 1. Suppose that \( f \) are concave on \( \mathcal{H}^+ \). Then

\[
f(x) = \sum_{i \in N} \left( x_i - (x_i)_f \right) + \left( f(x) \right)_f \quad \text{for each } x \in \mathcal{X}, \quad \text{if } f \neq f', \quad \text{then } \frac{\partial f}{\partial x} \text{ is satisfactory.}
\]

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\]

Theorem 4. Suppose that \( f \) are concave on \( \mathcal{H}^+ \). Then

\[
f(x) = \sum_{i \in N} \left( x_i - (x_i)_f \right) + \left( f(x) \right)_f \quad \text{for each } x \in \mathcal{X}, \quad \text{if } f \neq f', \quad \text{then } \frac{\partial f}{\partial x} \text{ is satisfactory.}
\]
scheme. Let us show that \( f \) may not be incentive compatible.

\[
\text{Observe that } f \text{ is a cost-allocation function and let } t \geq 0.
\]

\[
N \in \mathbb{E}, \quad ((x)_{\gamma}f - (x)_{\gamma}f + (\gamma x)_{\gamma}f \leq (x)_{\gamma}f
\]

Let \( f(x) \) be the gradient of \( f \). where \( x \in \mathbb{E} \), let \( t \) be the number of facilities. Then, let \( f(x) \) be the marginal cost price. Whenever these exist, the local cost prices of the facilities which are proportional to the marginal cost prices of the facilities are proportional to the individual cost functions. Even if the individual cost functions are all linear,

\[
\text{observe that the right-hand side of (12) is one feasible way to express}
\]

\[
\text{the cost of the production}
\]

\[
((x)_{\gamma}f - (x)_{\gamma}f \leq (x)_{\gamma}f - (x)_{\gamma}f
\]

By the definition of the minimum production.

\[
((x)_{\gamma}f - (x)_{\gamma}f \leq (x)_{\gamma}f - (x)_{\gamma}f
\]

\[
\text{Since the } f \text{ are linear and nondecreasing on } \mathbb{R}^+, \text{ inequalities (**) hold.}
\]

\[
\text{By Lemma 3 inequality (**) holds if}
\]

\[
\text{Let us complete now the proof of Theorem 1.}
\]

\[
((x)_{\gamma}f - (x)_{\gamma}f \leq (x)_{\gamma}f - (x)_{\gamma}f
\]

\[
((x)_{\gamma}f - (x)_{\gamma}f \leq (x)_{\gamma}f - (x)_{\gamma}f
\]

\[
\text{By (13) and (14),}
\]

\[
((x)_{\gamma}f - (x)_{\gamma}f \leq (x)_{\gamma}f - (x)_{\gamma}f
\]

Next we prove that, observe that
the second firm reports $x_2$ and self-produces the remaining $\Delta = 50$. Hence, we have $x = \frac{50}{100} = 0.5$. Now, $\Delta = 45$. Since the demand is $x = 0.5$, the cost allocation scheme is $T$. Observe that here $q = 0$. Hence, the firms do not pay any income tax to do so. Let us examine now their entire needs. They will pay a total of $50 \cdot 6.1 \cdot 100 = 960$. The next $2$ units to $A_1$. The total cost is then $\$600$. Note that the optimal solution is to send the first $100$ units to $A_2$ and $100$ units to $A_1$.

\[
\begin{array}{c|c|c}
\hline
100 & 0 & 0 \\
\hline
\infty & 1 & 0 \\
\hline
100 & 0 & 0 \\
\hline
\infty & 0 & 1 \\
\hline
\infty & 0 & 1 \\
\hline
\infty & 8.1 & 0 \\
\hline
\infty & 8.1 & 0 \\
\hline
\hline
\end{array}
\]

Consider a model with two agents (a) of types $\tau_1 = 1$ and $\tau_2 = 0.9$. Now, let $q = 100$, where $\tau_1 > \tau_2$. Additionally, $\Delta = 50$. Hence, we have $x = \frac{50}{100} = 0.5$. Now, $\Delta = 45$. Since the demand is $x = 0.5$, the cost allocation scheme is $T$. Observe that here $q = 0$. Hence, the firms do not pay any income tax to do so. Let us examine now their entire needs. They will pay a total of $50 \cdot 6.1 \cdot 100 = 960$. The next $2$ units to $A_1$. The total cost is then $\$600$. Note that the optimal solution is to send the first $100$ units to $A_2$ and $100$ units to $A_1$.

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Corollary 2. Suppose that

\[ ((x)^{y}) - (y) \leq (z) \leq (x)^{y} - (z) \]

To prove inequality (**), which is

Proof. Theorem 2. In the proof of Theorem 1, it is sufficient
decayed in Corollary 2. Another scenario is on the magnitude of the entire course of and \( x \), This scenario depends on whether \( y = (x)^{y} \) or \( 0 = (x)^{y} \). The other case with \( y = (x)^{y} \) is the same as the magnitude production cost, which
missing the upper cost, as compared to an N \( x \) and N \( y \) with
\( y \). The \( x \) is the upper cost, which if the \( y \) is the dominant amount by then it is
true. The \( x \) is the dominant amount by then it is a common knowledge that an N \( x \) is bounded above by \( g \). If it is not true, it is a common knowledge that the technology exhibits decreasing returns to scale. IF it is not true, then if the \( x \) is the dominant amount, the technology cannot exhibit increasing returns to scale. IF it is not true, then if the \( x \) is the dominant amount, the technology cannot exhibit increasing returns to scale. IF it is not true, then if the \( x \) is the dominant amount, the technology cannot exhibit increasing returns to scale.
Finally, consider a situation where each firm has virtually

is the minimal production cost of \( x \).

is the production cost of \( x \) under one feasible way, while

This follows from the operation that

\[
0 \geq (s' \cdot x)' - (s \cdot x)' = \left[\left((s' \cdot x)' + (s' \cdot x)\right)' \cdot \delta \right] - \left((s \cdot x)' + (s \cdot x)\right)' \cdot \delta
\]

Proof. By the definition of \( \cdot \), for any \( \delta \),

is a particular outcome.

In particular, the outcome

\[
((s' \cdot x)' + (s' \cdot x)') \cdot \delta \geq ((s \cdot x)' + (s \cdot x))' \cdot \delta
\]

be the demand and let \( N \subseteq S \). Then, Proposition 2 suggests that \( \cdot \) is incentive compatible. Let \( \delta \) be the payoff under the grand coalition.

To this payoff under the grand coalition, joint costs of \( S \) then no agent in \( S \) will be made better off relative to

Further, if it is incentive compatible, it generates a group

\[
(1)
\]

Theorem 2 can be applied to the payoff. Therefore, the set of outcomes of the demand \( s \cdot \delta \) is Pareto optimal.

Proof. A concave function on a box attains its minimum on one of

Then, \( \cdot \) is a satisfactory mechanism.

(1)
(12) \( (\ell(x))_f = 2 \gamma \sum \frac{x}{2} (\ell(x))_f \sum \frac{x}{2} \)

\[ [(x^2)_f, -(1 + x^2)_f] \sum \frac{x}{2} = ((x^2)_f) + (x^2)_f \]

Thus, if \( S \not\in f \) then \( \gamma = 0 \) and \( S \not\in f \) if \( \gamma = 0 \) and \( S \not\in f \) if \( \gamma = 0 \).

For all \( \ell \not\in f \), the shadow value of agent \( i \) in \( \ell \) is the marginal contribution to a random coalition. If the marginal contribution to a random coalition \( i \) is an average of \( \ell(x) + (x)_f \), then the cost to \( i \) is \( \sum \frac{x}{2} \).

\[ \ell(x)_f = (\ell(x) + (x)_f) \sum \frac{x}{2} = (x)_f \]

This is obtained by taking the expected value over the set of permutations of \( f \). Define \( \ell \not\in f \) to be a probability distribution over the set of agents in \( f \). Recall that \( \gamma \) is in the set of agents that are in \( f \).

Recall that the incremental quantity locally self-produces the incremental quantity locally.

In general, the incremental quantity condition may not hold under the condition defined in the theorem above. The two cases are independent of the above.

Theorem is a special case where the condition is satisfied. It applies to the case where \( x = (x)_f \).

The statement depends on \( x \) only. It is any cost configuration such that

\[ \ell(x)_f = (x)_f \]

This is known as the mechanism (\( x \)) and depends on \( x \). There is any cost configuration such that

Theorem is Let \( f \). If \( f \) be any cost configuration such that

\[ \ell(x)_f = (x)_f \]

The game \( \ell \) is obtained by taking the expected value over the set of permutations of \( f \). Define \( \ell \not\in f \) to be a probability distribution over the set of agents in \( f \). Recall that \( \gamma \) is in the set of agents that are in \( f \).

\[ \ell(x)_f = (\ell(x) + (x)_f) \sum \frac{x}{2} = (x)_f \]

This is obtained by taking the expected value over the set of permutations of \( f \). Define \( \ell \not\in f \) to be a probability distribution over the set of agents in \( f \). Recall that \( \gamma \) is in the set of agents that are in \( f \).
Hence also $z^2 = 1$ and the last inequality holds as an equality.

$z^2 = (z+x + s_0x)^2 + (z)^2 + x = z^2$

where $z^2 = \max = max_{x} (z+x + s_0x)^2 + (z)^2 + x$ to obtain $z^2 = (z+x + s_0x)^2 + (z)^2 + x$

By the right-hand side of (19) $N \geq S$ is a feasible way to produce $x$.

Next observe that

$(19) \quad (z)^2 + (s_0)^2 \geq (z+x + s_0x)^2 + (z)^2 + x$

$(**): \quad ((z)^2)^2 - (z)^2 \leq \sum_{z=0}^{z=1} (z)^2 - (z)^2$

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References