Universität Bielefeld/IMW

Working Papers
Institute of Mathematical Economics

Arbeiten aus dem
Institut für Mathematische Wirtschaftsforschung

Nr. 72

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Structural Characteristics of Economic Models: A Study in Complexity

December 1978

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1. Introduction
In the past twenty years we have experienced a tremendous advancement of tools and techniques in analyzing, diagnosing, predicting and controlling economic processes. Sophisticated models have been built that claim to predict future economic trends of micro and macro-economic variables, largely facilitated by the availability of large-scale computational resources. These models also form the basis of policy recommendations such as the Brookings models or similar large-scale econometric models. There have been some major improvements in the design, estimation, statistical structure, testability of economic model-building as have been in decentralized decision-making, distributed computation and hierarchical control. Yet, there is still one major link missing which we consider as one of the most fundamental properties of any model-building in the large that one has not come to grips with the structural constraint of complexity, e.g. the infor-
mation processing limits arising in the control of dynamic systems. Also in a recent survey on large-scale systems (N.R. Sandell et al [1]) it is stated that an adequate measure of complexity is lacking for control systems and that a major task of constructing a unified theory of decentralized control is to include a formal measure of system complexity. We focus on this structural constraint of complexity by designing models that explicitly cope with this issue.

2. Preliminary Considerations

From a methodological view there are two ways to understand and predict the behavior of (discrete-time) dynamic systems. The first is to evaluate changes in the future by past performance, to project realizations in the past into the future, what may be summed up by 'prediction by trends'. Intuitively, this is based on the idea that any system's behavior 'cannot escape the past to shape the future', that any evolving system develops its own memory which conditions it to past events. The more data are compiled from the past the more the memory activates its own self-organized dynamics. This appears to exclude purely random behavior in the past, exhibited by a random sequence, hence the occurrences of events do not seem to be statistically independent. The second is by constructing a model which is considered to be a representative mapping of the system under investigation. A model attempts to capture the basic technical, structural and behavior characteristics of the underlying system and on this ground estimates its potentialities for future development. This is referred to as 'prediction by models'. Now 'prediction
by trends' assumes regularity of the process as if social and economic processes satisfy laws of statistical regularity. It comes very close to conceiving the world of being in a state of 'disorganized complexity' according to W. Weaver [2], meaning that we can view courses of events essentially as 'random sequences' and that the statistical law of large numbers is the basic concept to predict future behavior. Already F.A. Hayek [3] remarked that economic processes cannot be observed in a statistical fashion. The problem with a statistical assessment of complexity, or disorganized complexity, lies in the attempt to average over a large number of random variables, as represented by the mean or expected value, which, indeed, treats all phenomena of the system alike or as uniform whereas in reality there exist different structural relationships between elements that bear different characteristics and that may have a non-negligible impact on the behavior of the entire system. Hence, it is very doubtful that economic systems can be explained on the basis of pure random phenomena. I have shown elsewhere (Gottinger [4]) that this implies that economic systems would perform as if they were infinitely large systems. Instead, I have pointed out that economic systems are much more akin to 'finite complex systems' (FCS), i.e. systems that are much too complex to get explicit solutions for them, as is the case in 'simple' systems, nor is the number of parts large enough or are the parts homogeneous enough to be able to pass to the limit as in infinitely large systems. An FCS has some peculiar properties, not shared by its better worked out counterparts that makes it difficult to analyze, understand and predict its behavior. It appears to be (1) highly sensitive
in responding to changes or disturbances of its environment, (2) strongly interdependent with regard to actions of its components, (3) following a threshold discretionary type of behavior with qualitative jumps, (4) partially or locally controllable inducing global effects. In fact, these properties can be observed in modern economic systems that render them a higher degree of instability as similar behavior patterns are traced in ecosystems (R.M. May [5]). Hence behavior of an FCS is not of a purely random character. The reason for this is that it reveals certain imperfections like lagged responses, maladjustment, non-stochastic dependence between parts, discontinuity that appears not to be compatible with randomness.

3. A General Approach to Dynamic Systems and Complexity

The approach starts with the basic identification problem. The technicalities have been treated elsewhere (Gottinger [6], [7]). Given some natural complex system, a black box, where we only observe outputs and inputs over time but we are ignorant about what is happening 'inside', e.g. about local properties or parameters of transition. Is it possible to find an 'artificial' system that simulates the original natural system? Systems we have in mind in this context are those which respond in real-time to their environments just to stay alive (bull fighting a bear). Ecological systems (bird colonies) or biological systems (metabolism of cells) constitute systems striving for survival, also all types of competitive economic systems challenged by adversary environments belong to this category. In general, extreme notions, such as survival or non-survival (death), which are characteristic for pure forms of competitive systems
are involved. Here interest is focused on global properties of
dynamic systems. The design orientation follows from the ident-
tification process, e.g. by taking interconnecting part of the
artificial system 'hooking' them together in order to simulate
the 'black box'. The approach is algebraic, since it starts from
finite-state dynamic systems, e.g. sequential machines, the ge-
neral characteristics of which are described by:

1. A set of inputs, e.g. those changing parameters of the envi-
ronment which will affect the system behavior in a predictable
way.

2. A set of outputs, i.e. those parameters which act upon the
environment leaving observable changes in the relationship
between the system and the environment.

3. A set of states, i.e. those internal parameters which deter-
mine the relationship between inputs and outputs and which
may reveal all necessary information embodied in the past.

4. The state transition function which determines the dynamics
of how the state will change when the system is fed by va-
rious inputs.

5. The output function which determines what output the system
will yield with a given input when in a given state.

There are some obvious advantages, theoretical and practical
ones, to use an algebraic approach. First, algebra is a na-
tural tool because it emphasizes the design of a system as a
collection of objects. Second, algebra is natural for computa-
tional work, and this is an important factor in applications.
Modern computers accept digital instructions and those in turn
require an algebraization of systems. Third, algebraic system
theory emphasizes qualitative aspects of systems to the extent
that we are interested in properties such as survival or break-
down. This is achieved by determining complexity bounds of the
system's design (design complexity). By the fact that algebraic
system theory is related to computational structures we are
in the position to construct a complexity theory for dynamic
systems which is also amenable to applications.
Systems of that sort reveal a natural bound of complexity.
Complexity here has two facets, computational complexity and
structural complexity.
Structural complexity refers to the inherent capability of
parts of the system, of what they are able to perform.
Computational complexity refers to the length of computation
given by the extent of interconnection between parts. The
most important distinction is that between design and con-
trol complexity.
Under design complexity it is understood that complexity (num-
ber) associated to the transformation process in which full
use of the system potential is made. Design complexity can
only be achieved if a system is working without faults, if the
components live up to computability requirements, if the parts
function reliably. Under control complexity we understand that
specific complexity number which keeps the entire system or
at least part of it under complete control. Only if design and
control complexity coincide stable configurations in the state
and output space will result, or the system runs smoothly.
The relation between design and control complexity is an in-
dication for stability or harmony of a dynamic system.

4. Decomposable Systems
We wish to explore the nature of the interdependence in economic
systems, to show how these systems might be decomposed into
their component parts for analytic purposes, and to relate
the results to the choice of models in policy analysis and projection.

By the previous arguments and those provided by the algebraic theory of machines we are advised that there must be general principles of decomposition which arise in the design process of algebraic (computational) systems. From these general principles of decomposition we emerge with a natural theory and measure of complexity which is structural (intrinsically related to the parts, the basic building blocks) and computational, e.g. addressed to the computational links between these parts. We will see how this framework is useful for approaching more specific type questions in economic model building.

The core for the investigation in the present section is provided by the Ando-Fisher theorem [8] on the decomposibility and independence in dynamic economic systems. The Ando-Fisher theorem and related results deal with general ways how to decompose a system into its component subsystems for purposes of analysis and prediction. They distinguish between (completely) decomposable and nearly (completely) decomposable systems. In the latter case, a nearly decomposable system consists of subsystems where each is causally dependent on the rest of the system but the rest of the system is only weakly dependent (or weakly coupled) on (with) each subsystem. In contrast, a decomposable system only allows for within systems dependencies but ignores intersystems dependencies altogether. The Ando-Fisher theorem asserts, for linear dynamic systems, that
(1) provided inter-systems dependencies are sufficiently weak (up to a negligible degree) relative to intra-systems dependencies, then, in the short run, the relative behavior of a nearly decomposable system becomes almost indistinguishable to that of a decomposable system.

(2) provided inter-systems dependencies are sufficiently weak relative to intra-system dependencies, then also in the long run relative behavior of the nearly decomposable system and the decomposable system is approximately the same even though their behavior in terms of absolute levels and rates of change may be very different. In other words, if we give a nearly decomposable system enough running time, and that even when influences having been neglected have had time to make themselves fully felt, the relative behavior of the variables within any subsystem will be approximately the same as would have been the case had those influences never existed.

Now there are obviously two critical factors in the validity of the last result:

(a) the degree of approximation depends on the number of computational cycles (e.g. running time) and is certainly a result in the limit,

(b) the degree of approximation depends on a prepostulated, sufficiently weak linkage among the subsystems, below a certain threshold value where the system moves continuously in an orderly manner. This however presupposes a very large number of subsystems, acting uniformly on an equal power base, as in the classical model of economic
equilibrium.

Both assumptions (a) and (b) are hard to defend in the context of analyzing real-life social and economic systems. For these systems, as real-time systems, we simply haven't got enough running time to force this approximation upon the system's behavior. Time is a scarce and valuable resource.

Second, there are only a few, relatively large subsystems, the relative behavior of which have a great potential impact on the entire system, and where the intra-system behavior is not limited to the subsystem itself.

In fact, we would very much argue in the opposite direction. The characteristics of large-scale complex systems are such that they are very sensitive to discretionary behavior of their subsystems and that actions, outcomes and consequences of these subsystems very often induce snowball-effects that pervade other subsystems and enforce actions, perturbations, maladjustments, all sorts of reactions that deeply affect smoothness, regularity, stability, controllability which are highly unpredictable because of the nonlinearities involved.

5. Systems, Modelling and Complexity

Let us start by describing a dynamic system very much like an automaton. The ingredients are given by

an internal state vector \( z_t = [z_1, t, z_2, t, \ldots, z_n, t] \)

an external state vector \( x_t = [x_1, t, \ldots, x_n, t] \)

representing exogeneous factors, driving the system from 'outside', and not incorporated in the decomposition.
An output function $\delta$ that maps strings of inputs, say $(a_1, \ldots, a_n)$ into single outputs $b_1, b_2, \ldots, b_n$ which enter as inputs into other parts (components) of the system, the state transition function $\lambda$ with

$$z_t = \lambda(z_t, z_{t-1}, \ldots, x_t),$$

given in difference equation form. The function $\lambda$, without any specification yet, represents the hypothesis about the process, inferred from the observations of real world systems.

The external state vector $x_t$ can be conceived as a primary input factor (stimulus) which sets the system into motion, but which itself may be suitably partitioned as to which component is primarily affecting the process.

The behavior of the system is generated by the set of time series of the $z_{it}$ which are produced as the model generates successive state descriptions plus the external states given exogeneously.

Now in the chartist approach to studying the future of systems it is assumed that the connections between different components either do not exist or are sufficiently weak that they can be safely ignored. The value of any component $z_{it}$ depends only on its previous values and possibly random disturbances $u$, thus predicting a future course of actions or events means extrapolating past performance.

In a comprehensive modelling approach, as suggested here, at least some of the possible interdependencies among components exist in general. The behavior of each component may depend on its past behavior as well as on other variables in the system, the corresponding exogeneous factors $x_{it}$.
\[ i = 1, \ldots, n, \text{ respectively, and possibly the random disturbances } u, \text{ i.e.} \]

\[ z_{it} = \lambda(z_{i,t-1}, z_{i,t-2}, \ldots, z_{k,t}, z_{k,t-1}, \ldots, x_{i,t}, u) \quad k \neq i. \]

Three immediate problems in the model procedure arise from the Decomposition Scheme exhibited in Fig. 1:

1. As the number of components and sufficiently strong connections among components increase the behavior of the system becomes increasingly obscure by complex interactions which resemble very much non-linearities in total system's behavior in correspondence with size.

2. The structure and size of the components themselves present a potential source of complexity depending on whether and to which extent the component system is sensitive to disturbances, errors, threshold phenomena, etc.

3. As the number of components and interdependencies in the system enhances, increasingly longer sequences of calculations are required to deduce the behavior of the system which results in computational complexity.

The solutions of these three problems would enable us to determine the complexity of the system on-line, as it is running from some initial time to some target time in the future. But knowing the complexity would permit us to design control strategies which are effective in guiding the system toward relative stability or harmony.
Fig. 1 Cascade Form of Decomposition
Therefore to understand the complexity of such systems we should be able to understand the strongly connected, coupled nature of its subsystems. For this purpose we need a measure of complexity that reflects the structural performance of each of the connected subsystems in terms of state space configurations plus the number of computational links that are established among the various subsystems and that reflect the richness of state representations in the global trajectory space of the entire system.

**Illustration**

Take the population subsystem \([\text{POP}]\) consisting of initial eight states, e.g. \(z = (z_1, z_2, \ldots, z_8)\), then after inputs are fed into the \([\text{POP}]\) system a new state configuration obtains.

\[
\text{Fig. 2} \quad [\text{POP}] - \text{Subsystem}
\]

A simple measure of structural complexity, in accordance to Gottinger [6], [7], could be given by the number

\[
\#_{s}(z) = \sum_{i=1}^{8} \chi_{s}(z_i) = \sum_{i=1}^{8} \text{ (length of time needed to reach a satisfactory state) } \times \text{ (number of feasible states to be attainable).}
\]
On the other hand, the computational complexity indicates the number of links between various subsystems times the number of interactions that ensue until the computational cycle (in real-time) is completed, as roughly indicated only for one specific subsystem, say [POP], (for all other subsystems likewise) in the following illustration.

In the satellite system, connecting all neighboring subsystems, the computational complexity related to the [POP] subsystem amounts to

\[ \#C([POP]) = (\# \text{links to [EN]} + \# \text{links to [ECO]} + \ldots + \# \text{links to [P]}) \]

\[ = \sum_{i=1}^{n} \# \text{(links to i)}. \]

Then the total computational complexity comprising all links among all subsystems within the satellite system is given by
Fig. 3  Satellite System
\[ \#_{c}(\{\text{POP}, \{\text{EN}, \{\text{ECO}, \{\text{EC}, \{G\}, \{F\}\}\}\}\}\) = \#_{c}(Z_{F}, F) = \sum_{k=1}^{6} \#_{k}(Z_{Fk}, F_{k}) \]

Using the notation, let \( Z_{F} \) be the state space over which the entire finite state system is running, let \( F \) be all feasible semigroups of transformations acting on this space, then in accordance with decomposition results of algebraic finite state systems we establish a comprehensive complexity measure comprising of structural and computational complexity, e.g.

\[ C(Z_{F}, F) = \prod_{k=1}^{m} \#_{s}(Z) \#_{c}(Z_{Fk}, F) \]

and this is the measure we use in analyzing the structure of a particular economic model.

6. Structure of the Model

We wish to explore the nature of interdependencies in societal systems to show how systems might be decomposed into their component parts for analytical purposes and to relate the results to the choice of models in policy analysis and projection. According to what has been illustrated in Fig. 1 we choose a kind of partition of the overall system into parts that comprise the main activities of complex societal systems – this is very much in spirit of earlier attempts at modelling complex political systems for purposes of simulating their behavior, see R.D. Brunner and C.D. Brewer [9]. The components of this model consist of the set of variables and parameters listed under the detailed description later.

\{\text{POP}\} – the population subsystem is relatively specialized to the growth and distribution (density) of the population \( N_{t} \), being composed of the urban and rural population.
[EN] - the energy subsystem is singled out as the major resource sector which could be of mixed private-public activities, and where the energy basket consists of the output $E_t$ of primary and secondary energy resources generated.

[ECO] - the environmental system puts environmental restrictions (water and air pollution) on growth of production in urban and industrial areas.

[EC] - the economic subsystem is relatively specialized to the production and distribution of economic goods and services within the private sector.

[G] - the government subsystem comprises all activities of the public sector which are directed at providing goods and services demanded by the market place or generated by political considerations.

[P] - the political subsystem is specialized to the production of changes in the size and distribution of mass support for the government $M V_t$ to permit majority rule and conservation of power, furthermore, determines the consumptive and distributive characteristics of government expenditures $G_t$. The relationships in the model are grouped according to subsystems, these relationships are hypotheses about the way in which each variable changes as a function of the others, with the magnitudes of the changes being determined in part by the parameters.

The degree of connectedness or interdependence implied in the relationships can be explored by noting the presence or absence of causal links among the variables. Call this the structural connectedness of the model. In fact, there are direct causal and indirect causal connections (links) between the output and input variables, depending on whether they occur within the components blocks or are cross-connected among different blocks. The static description of the structural connectedness underestimates the degree of connectedness among variables as the model operates through time. For example, since government revenue $R_t$ is merely a function of gross output $Y_{t-1}$, variation in $G_t$ within one real-time cycle is limited to the variation in this variable. However, if the model is operated through time, the number of variables causally connected to $R_t$ increases. Thus while $R_t$ is a direct
function of $Y_{t-1}$ (or some aggregates thereof), it is also
an indirect function through this variable of $C_{t-1}$, $I_{t-1}$,
$G_{t-1}$, and through these variables $R_t$ is an indirect function
of several other variables, etc. Obviously, the power of
the connectedness matrix increases in correspondence with
an increase in the number of real-time cycles of the model.
The analysis of structural connectedness has obvious limi-
tations for the chartist approach to prediction. If the
analysis suggests that even in a system that is loosely
connected in its static description every variable in the
system may ultimately depend upon every other variable as
the system runs on-line through time, then a simple extra-
polation of trends which ignores these dependencies may
indeed be highly misleading. In a situation where the num-
ber of outside factors tend to be increasing to infinity,
we would well expect some statistical regularity in offset-
ting influences among the factors, and, indeed, a chartist
approach may be a good thing to follow by neglecting these
dependencies. This would correspond to view the real si-
tuation as a state of 'disorganized complexity'. However,
in the course of previous arguments, the case of an FCS
would not support this point. Then a serious modelling
and prediction effort would have to take into account in-
direct effects that may have a highly nonlinear character. Of
course, there is the possibility that in a particular system
or in a particular application of the model, certain of these
dependencies may be sufficiently weak that they can be safe-
ly ignored. If this were the case, the chartist's approach
which ignores these dependencies and the comprehensive modelling approach taking them more fully into account may lead approximately to the same conclusions.

7. The Model's basic sets of relationships

(a) \([\text{POP}]\)-Subsystem

Rate of population change

\[ p(\text{rho}) = \alpha_1 (\text{average number of children per family}) + \beta_1 (\text{number of women in child-bearing age}) + \lambda_1 (\text{availability of birth control devices and liberal abortion policies}) + \delta_1 (\text{expected average family income, given some current level of family income}) + \varepsilon_1 (\text{population supporting energy consumption level}) + \eta_1 (\text{population density index level}) + \xi_1 (\text{provision of public goods and services relevant to child rearing activities}) + \theta_1 (\text{level of widespread political satisfaction and trust in political institutions}) \]

(b) \([\text{EN}]\) - Subsystem

Supply of Energy

\[ S(E_L) = \alpha_2 (\text{availability of domestic energy resources}) + \beta_2 (\text{energy provision through imports}) + \lambda_2 (\text{potential of mobilizing new energy sources via technology}) \]
+ $\delta_2$ (price expectations and market prices) \quad \text{[EC]}

+ $\varepsilon_2$ (level of manpower directed toward energy production) \quad \text{[POP]}

+ $n_2$ (upper limits of energy provision by environmental factors, i.e. strip-mining, health and environmental hazards, pollution) \quad \text{[ECO]}

+ $\xi_2$ (degree of government intervention in terms of price or quantity regulation) \quad \text{[G]}

+ $\theta_2$ (political interference in the industrial organization of the energy industry) \quad \text{[P]}

$$D(E_t) = \mu N_t + \lambda Y_t,$$ energy consumption is proportional to total population and to the level of G.N.P.

$$\Delta(D(E_t) - D(E_{t-1})) = \mu(N_t - N_{t-1}),$$ the rate of energy consumption is proportional to the rate of population change.

(c) [ECO] - Subsystem

Abatement Performance (measured in terms of level of exhaust emission)

$$\text{AP} = \alpha_3 \text{ (index of industrial distribution)} \quad \text{[EC]}

+ \beta_3 \text{ (environmental immersion factors)} \quad \text{[EF]}

+ \lambda_3 \text{ (level of environmental technology)} \quad \text{[EF]}

+ \delta_3 \text{ (G.N.P. as related to regional and sectoral industrial activity)} \quad \text{[EC]}

+ \varepsilon_3 \text{ (rate of population change and structural population shifts from rural to urban sectors)} \quad \text{[POP]}

+ \xi_3 \text{ (government regulation and industrial incentive structure in terms of emission limits, waste disposal, land use zoning regulations, noise restrictions)} \quad \text{[G]}.\]
(d) [EC] - Subsystem

Links of G.N.P. aggregates by definition:

\[ Y_t = C_t + I_t + G_t + F_t \] (F_t = foreign trade, given exogeneously).

\[ C_t = \alpha_t (1-\tau) Y_{t-1} \dot{N} \],
\[ I_t = I_{t-1} \dot{N} + r(CN) + f(G_t), \dot{C} = \frac{dC}{dt} \]

where \( f \) represents the extent of government expenditure restricting private investment, to take care of welfare - state limits on private investment activity.

Growth of G.N.P.

\[ \dot{Y} = \alpha_4 \text{ (resource endowment)} \]
\[ + \beta_4 \text{ (R & D level)} \]
\[ + \lambda_4 \text{ (level of technology used)} \] [EF]
\[ + \delta_4 \text{ (minimal level of energy supply needed to support growth)} \] [EN]
\[ + \epsilon_4 \text{ (minimal abatement performance AP = maximal pollution, noise, waste disposal limit)} \] [ECO]
\[ + \eta_4 \text{ (size of government investment activities)} \] [G]
\[ + \xi_4 \text{ (rate of population change related to productive employment)} \] [POP]
\[ + \theta_4 \text{ (government guidance and organizational support)} \] [P]

(e) [G] - Subsystem

Government revenue is proportional to G.N.P. \( Y_t \) via the
proportionality factor $\tau$ (tax rate).

$$R_t = \tau Y_t.$$  

Government expenditure

$$G_t = a_5 \begin{cases} \text{(international structural changes in terms of balance-of-payments surplus/deficit)} & \text{[EF]} \\ + \beta_5 \text{ (rate of population change and age structure of society)} & \text{[POP]} \\ + \lambda_5 \text{ (energy supply } S(E_t) \text{ and R & D expenditures on new energy sources)} & \text{[EN]} \\ + \delta_5 \text{ (abatement performance: safety and environmental regulations)} & \text{[ECO]} \\ + \epsilon_5 \text{ (level of economic and industrial activity)} & \text{[EC]} \\ + \eta_5 \text{ (employment and capacity level)} & \text{[P]} \\ + \xi_5 \text{ (index of inflation)} & \text{[P]} \\ + \theta_5 \text{ (political support by major segments of the population (social groups))} & \text{[P]} \end{cases}$$

(f) [P] - Subsystem

Majority of votes from all or some segments of the population to achieve majority rule in Parliament.

$$MV_t = a_6 \begin{cases} \text{(psychological conditions of trust, honesty and stability in society)} & \text{[EF]} \\ + \beta_6 \text{ (rate of population change)} & \text{[POP]} \\ + \lambda_6 \text{ (change of migration patterns from rural to urban areas)} & \text{[EN]} \\ + \delta_6 \text{ (occupational shifts)} & \text{[POP]} \\ + \epsilon_6 \text{ (energy supply and distribution of energy resources)} & \text{[ECO]} \\ + \eta_6 \text{ (level of pollution affecting major social groups and key industrial areas)} & \text{[ECO]} \end{cases}$$
8. Evaluation of Complexity

Now the static description of the structural connectedness underestimates the degree of interaction among subsystems as the interaction operates through time, this is true since the various feedback- and feedforward mechanisms reinforce the interaction in a nonlinear way. We consider the connected system as a real-time type on-line dynamic system, see Martin [10], that starts operating at a specific initial state for which sufficient information is available, and which operates on an equal length time interval of one year. The selection of one year (in a real-time computational cycle) is arbitrary and used only for illustration purposes although parameter changes in this model appear to have only longer term impacts. Then the number of links to be estimated (pertaining to computational complexity) amounts to the following set-up. The entire system consists of altogether six 'modules', [POP], [EN], [ECO], [EC], [G], [P], plus the external environment, exogeneous factors [EF]. Each module consists of at least five linking factors to other modules, neglecting the exogeneous factors [EF] which are considered to be predetermined, the information of which is only directly accessible by the module concerned. Each of these five linking factors or parameters depend upon at most eight factors
in some other module, therefore indirectly involving [EF].

In the first case we refer to **direct links**, in the second to **indirect links** of each of the modules involved.

For illustration look at the determination of \( G_L \) in Module [G]:

![Diagram of Module [G]](image)

*Fig. 4* Determination of \( G_L \) in Module [G] by direct links to other modules.
With six active decision-making modules operating the total number of structural parameters to be computed amounts to at least 240 structural parameters, e.g. five direct links given by the parameters of each module time eight indirect links for each such parameter in some other module times six, the number of interactive modules.

Now granting that at least 240 structural parameters be estimated, and suppose a response cycle of yearly data for a period of a ten year planning model is used, then in view of the non-linearities of the system interactions as the system operates through time and extends the length of indirect links, we can show, by complete enumeration, that we arrive at a total number of 240 \(^{10}\) structural links which is a dramatic increase. In other words, the same number of structural parameters would have to be estimated in order to reach a control complexity that fits the design complexity of the entire system - a formidable task. (Still, it would be even more dramatic if we relate the response cycle to quarterly or even monthly data.) Now this holds for a problem-solving mechanism acting as a brute-force search, as built into large-scale computer programs, in which

(i) the modules do not provide any structural complexity, i.e. no intrinsic problem-solving power as given by a special heuristic,

(ii) there is complete ignorance on the controller's side concerning the parameter variations in the dynamic process.

In other words, this corresponds to a centralized controller who perceives the modules simply as 'black boxes' with no self-steering capability and who at each real-time response cycle is ignorant about (or does not learn about) parameter variations that ensue.
In the alternative case the controller may activate the problem-solving power, heuristic search capabilities of the modules where each module, as a decentralized unit, by itself intrinsically computes all indirect links, thereby decreasing the computational burden of the controller. Hence the computational process reduces to direct links only, at least five for each module, so that for each response cycle $5 \times 6 = 30$ structural parameters have to be estimated as compared to 240. This amounts to $30^{10}$ structural links in a ten years period. In fact, the reduction of the number $240^{10}$ to $30^{10}$ could be assigned to the 'smartness' or 'structural complexity' of the modules, precisely the structural complexity amounts to $240^{10} - 30^{10}$, the residual in the reduction of computational complexity. Therefore, we see how a tradeoff between structural complexity and computational complexity evolves in such an interactive model.

![Graph showing the tradeoff between structural complexity and computational complexity.](image)
9. Conclusions

The most interesting aspect of this analysis for problem-solving methodologies pertains to the distinction between design and control complexity. If the design complexity of the system amounts to $240^{10}$ structural links, the only way to cope successfully with the control of such systems is to use a decomposition that makes most out of the intrinsic computational capabilities of the modules, their smartness or sophistication, much in the same way as chess playing programs become smarter to match master chess players by building into the programs some sophisticated heuristics. This sheds new light on the question of centralization vs decentralization of decision-making in organizations, in particular in view of these results one could hardly hope to achieve rational centralized economic planning against market-type decentralized mechanisms (see P.A. Hayek [11]). At least the need for decentralization is now widely acknowledged in Soviet work on program-planning. As N.N. Moiseev and A.G. Schmidt [12] put it: 'it is not difficult to see that no matter how advanced the level of the techniques for data processing and data transmission may be, a certain level of decentralization in management will always be necessary.'

Furthermore, in general mixed-type economies, one can conclude that government regulation will easily find its limit of workability because of the computational burden that ensues. Another point worth of consideration is that the analysis of structural connectedness had obvious implications for the chartist approach to projection. Since even in a system that is loosely connected in its static description, every parameter in
the system may ultimately depend upon every other parameter as the system operates through time. A simple extrapolation of trends which ignores these dependencies may indeed be highly misleading. If in a particular system it turns out that most of these dependencies (links) are sufficiently weak so that they can be safely ignored (according to the Ando-Fisher Theorem), the projections of the chartist who neglects these dependencies and the fundamentalist who takes them fully into account may be approximately the same. But even 'simple' systems which are only loosely connected in their static description are highly interconnected in their dynamic behavior.

Let's close with a methodological note.

Throughout this paper we attempted to answer the question how complexity appearing as a structural constraint on large-scale dynamic economic systems can be successfully handled from a controller's point of view. For answering this question it appears necessary to know what units of decomposition are most appropriate for designing and understanding complex systems. The sheer amount of computation, reflected by a measure of computational complexity, prohibits the decision-making or information-processing power of any human or artificial controller, yet to be designed. This indicates that by proper decomposition we should strive for mobilizing the problem-solving potentiality of systems components, that helps to reduce the computational burden of the controller. These systems components are supposed to activate useful heuristics at a local or decentralized level that involves sophistication, creativity, non-routine
problem-solving vs standard operating programs under repetitive situations as required in purely arithmetical tasks. It is reasonable to assume that market-type processes, e.g. market clearing functions, taking place in well-defined sub-units, provide such a useful heuristic: they substitute structural for computational complexity. This is an issue that drives at the core of discussion of human problem-solving and the principles of bounded rationality as presented by March and Simon [13], ch. 7.
ADDENDUM

The results of this paper seem to contradict claims such as that price systems require essentially the same calculations as command systems or that command and price systems are informationally equivalent. According to S.A. Marglin ('Information in price and command systems of planning', in: J. Margolis and H. Guitton, eds., Public Economics, New York: MacMillan 1969, chapter 3, pp. 54-77) the information aspect is not essential for the distinction between price and command systems. Neglecting the incentive aspect, our analysis shows, on the other hand, that except for relatively small systems, in general, decentralized systems are more likely to cope with complex tasks than centralized systems. The reason appears to be that the market is a heuristic device, capable of searching for a 'good solution', whereas the centralized unit has to activate an algorithm that completely relies on an enumeration technique. A more differentiated viewpoint has been taken by H. Oniki ('The Cost of Communication in Economic Organization', Quarterly Journal of Economics 88, Nov. 1974, pp. 529-550) who argues that centralized economic systems may perform better, e.g. with lower cost of communication, if only a low degree of accuracy is required (measured by the 'allocation error' as a departure from the efficient allocation), and if the system is relatively small in the number of interacting units, variables, etc.. However, this applies only to tiny but not real economies so that - for all practicable purposes - command and price mechanisms appear not to be informationally equivalent, agreeing with our lines of reasoning. But we suggest even a stronger statement: not merely 'costs of communication' will be higher in a centralized system but beyond a certain threshold of complexity, centralized systems will not be able to control physically the economy without simultaneously achieving the same performance as a comparable decentralized system (of the same size). K.J. Arrow ('Limited Knowledge and Economic Analysis', American Economic Review 64, March 1974, pp. 1-10) in a presidential address to the American Economic Association reemphasizes the information-economizing effect of decentralized systems. To quote him: 'But what was left obscure is a more definite measure of information and its costs, in terms of which it would be possible to affect superiority of the price system over a centralized alternative'.

This paper is an attempt to provide such a measure, a measure of complexity.
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