Universität Bielefeld/IMW

Working Papers
Institute of Mathematical Economics

Arbeiten aus dem
Institut für Mathematische Wirtschaftsforschung

Nr. 69

Reinhard Selten und Werner Güth

Game Theoretical Analysis of Wage Bargaining in a Simple Business Cycle Model

September 1978

H. G. Bergenthal

Institut für Mathematische Wirtschaftsforschung
an der
Universität Bielefeld
Adresse/Address:
Universitätsstraße
4800 Bielefeld 1
Bundesrepublik Deutschland
Federal Republic of Germany

Game Theoretical Analysis of Wage Bargaining in a Simple Business Cycle Model

by

Reinhard Selten und Werner Güth

It is the purpose of this paper to investigate the problem of wage bargaining within the context of a simple business cycle model. Wage bargaining is assumed to be centralized. There is one union and one employers' association who periodically bargain on wages.[1] It is assumed that both bargainers are rational decision makers with complete knowledge of the economic structure.

The result of the bargaining does not only determine the functional distribution but also the level of national income. The bargainers must consider this effect. Moreover, they must also take into account that the result of present period wage bargaining influences future incomes and bargaining positions. The bargainers are players in a dynamic game.

Unlike earlier studies of wage bargaining with game theoretical methods [Stahl, 1972; Krelle, 1975; Rice,1976; Clemhout-Leitman-Wan, 1975; Shubik, 1952] this paper focusses on the macro-economic and dynamic aspects of the problem [Güth, 1978]. The analysis is performed for a very simple linear difference equation model of second degree. We intentionally selected an easily manageable type of business cycle model in order to be able to show the structure of the problem and its solution in a relatively simple way.

---

[1] There are some countries, e.g. Sweden where wage bargaining is highly centralized.

It is assumed that the players cannot commit themselves for more than one period. Nevertheless, they do pursue long run goals. The union wants to maximize the discounted stream of real wages and the employers' association the discounted stream of real profits. Our game theoretical solution concept combines the local application of Nash's two person bargaining theory with the idea of subgame consistency [Nash, 1950; Selten, 1977]. A global application of Nash's two person bargaining theory would be adequate for a situation where an infinite sequence of wages for all future periods were fixed once and for all by one binding contract. Since the players cannot commit themselves for more than one period, it is necessary to consider a sequence of interdependent separate bargaining problems, one for each period rather than one global problem. In this sense Nash's theory is applied locally and not globally.

In each period the players must expect that bargaining in future periods will follow the prescriptions of the rational solution even if a deviation from rationality should occur now. This is meant by the considerations of subgame perfectness mentioned above.

It is shown that under certain assumptions on the parameters of the model the game theoretic solution exists and is uniquely determined. It has the form of a linear wage bargaining function which connects wages of period t to the national income of period $t-1$ and $t-2$. A somewhat unexpected result is obtained with respect to the influence of a discount factor q which expresses the time preferences of both the union and the employers' association. As q approaches one, the bargaining behaviour prescribed by the solution becomes very similar to the myopic behaviour which results for values of q near zero.

1. The model

Apart from the fact that the functional distribution of income is explicitly considered, the model has essentially the same structure as well-known simple business cycle models of the multiplier-accelerator type developed by Samuelson [1939] and Hicks [1949]. All variables are in real terms. The subscript t is used in order to indicate the period. The basic variables are as follows:

Y_t : national income
C_t : consumption
I_t : investment
L_t : wages
P_t : profits

The economic relationships depend on whether agreement is reached in wage bargaining or not. Therefore, we introduce a dummy variable δ_t with

$$\text{(1)} \qquad \delta_t = \begin{cases} 1 & \text{if agreement is reached in period } t \\ 0 & \text{in case of conflict in period } t \end{cases}$$

The economic relationships are as follows:

$$\text{(2)} \qquad Y_t = C_t + I_t$$

$$\text{(3)} \qquad Y_t = P_t + L_t$$

$$\text{(4)} \qquad C_t = c_1Y_t + c_2L_t$$

$$\text{(5)} \qquad I_t = A + a(Y_{t-1}-Y_{t-2}) + \delta_tD$$

$$\text{(6)} \qquad L_t = vY_t \quad \text{for } \delta_t = 0$$

Here A, a, c_1, c_2, D,v are positive constants with $c_1+c_2<1$. Additional restrictions on these parameters will be introduced later. $c_2>0$ means that the consumption rate is greater for wages than for profits, a familiar assumption in economic theory [Kaldor, 1960].

We imagine that conflict leads to a strike of limited length which reduces autonomeous investment from A+D to A. Moreover, we assume that in case of conflict wages will be a fixed proportion of income. The problem of choosing optimal strike strategies is excluded by this simplifying assumption. The threat of conflict is modelled as a fixed threat.

Note that for $\delta_t = 1$ nothing is implied for the value of L_t by (1) to (6). The agreement wage level is a result of bargaining which is determined by strategic considerations.

In period t the union wants to maximize the discounted sum of real wages

$$(7) \qquad \Phi_t = \sum_{\tau=t}^{\infty} q^{\tau-t} L_\tau$$

and the employer's association the discounted sum of real profits

$$(8) \qquad \Psi_t = \sum_{\tau=t}^{\infty} q^{\tau-t} P_\tau$$

For the sake of simplicity we assume that the discount factor q with $0<q<1$ is the same for both players.

With the help of the notation

$$(9) \qquad T = \frac{c_2}{1-c_1}$$

$$(10) \qquad R = \frac{a}{1-c_1}$$

$$(11) \qquad S = \frac{A}{1-c_1}$$

$$(12) \qquad U = \frac{D}{1-c_1}$$

the following equation is obtained from (2), (4) and (5):

$$(13) \qquad Y_t = TL_t + R(Y_{t-1}-Y_{1-2}) + S + \delta_t U$$

It is convenient to consider a separate notation for levels of income, wages, and profits in case of conflict. For this purpose we use the symbols $\bar{Y}_t$, $\bar{L}_t$, and $\bar{P}_t$, respectively.

Obviously, we have:

$$(14) \quad \bar{Y}_t = \frac{1}{1-Tv} [R(Y_{t-1}-Y_{t-2})+S]$$

$$(15) \quad \bar{L}_t = \frac{v}{1-Tv} [R(Y_{t-1}-Y_{t-2})+S]$$

$$(16) \quad \bar{P}_t = \frac{1-v}{1-Tv} [R(Y_{t-1}-Y_{t-2})+S]$$

It can be seen easily that the bargainers are always able to find agreements on L_t with $L_t > \bar{L}_t$ and $P_t > \bar{P}_t$. In order to see this it is sufficient to consider the case $L_t = vY_t$.

The model, as it stands, does not present the possibility of negative income levels or even negative wages. Obviously, one wants to exclude such economically nonsensical situations. For the moment we shall neglect this problem which will be discussed again, after we shall have introduced the additional restrictions on the parameters.

2. Game structure

The model is looked upon as a dynamic game beginning in period 0 with specified initial conditions Y_{-1} and Y_{-2}. Both bargainers have complete knowledge of the rules of the game. They know the structural relations and the goal functions (7) and (8). At the beginning of the period t both of them are fully informed about the previous history of the play. On the basis of this knowledge they bargain on L_t.

The game is played over infinitely many periods t = 0,1,2,... A past history of the game up to period t-1 may be characterized by a sequence

$$(17) \quad H_{t-1} = (B_0, B_1, \ldots, B_{t-1})$$

where B_τ, the result of bargaining for period τ, is L_τ if agreement at L_τ is reached in period τ or the symbol $\emptyset$ in case of conflict in period τ. Clearly, the development of Y_τ, L_τ, and P_τ for periods $\tau = 0,\ldots,t-1$ can be computed on the basis of this information.

The game may be characterized as a dynamic game with short run cooperation. As usual in cooperative games individual behaviour is left unspecified and only collective behaviour is formally represented. In our case all possible forms of collective behaviour can be described by wage bargaining functions which are defined as follows: A wage bargaining function is a function f which assigns a bargaining result

(18) $B_t = f(H_{t-1})$

to every possible past history H_{t-1} of the game; here B_t is either a non-negative wage level L_t with

(19) $\bar{L}_t < L_t < Y_t - \bar{P}_t$

or the conflict symbol $\emptyset$. Condition (19) excludes agreements which are not above the conflict levels $\bar{L}_t$ and $\bar{P}_t$ for at least one of both players. Agreements which violate (19) are assumed to be forbidden by the rules of the game regardless whether they are profitable in terms of the goal functions (7) and (8) or not. We may imagine that such agreements are rejected by the members of the organization whose conflict level fails to be attained.

Obviously, the concept of a wage bargaining function is similar to that of a strategy in a non-cooperative dynamic game. A wage bargaining function may be described as a complete plan of collective behaviour for all possible situations which may arise in the course of the game.

Consider a past history H_{t-1}. The part of the game which still has to be played in periods t,t+1,..., is called the subgame after H_{t-1}.

It is clear that the strategically relevant features at period t do not depend on the whole previous history H_{t-1}, but only on the income levels Y_{t-2} and Y_{t-1}, immediately preceding the beginning of the subgame. Without going into the details of formal definitions it can be said that two subgames which may begin at different periods are homomorphic if they agree with respect to these preceding income levels.

3. Assumptions on the parameters

We shall restrict our attention to systems of parameters and initial conditions which guarantee that regardless of the specific wage bargaining function employed the income levels Y_t always remain above a positive lower level g and below an upper limit G. Moreover, we want to secure that the game-theoretical solution, to be introduced later, always exists. It will be shown that the following set of conditions is sufficient for these purposes:

$$c_1+c_2v+2a<1 \tag{20}$$

$$c_1+\frac{4}{3}c_2\leq 1 \tag{21}$$

$$D < \frac{(1-c_1-c_2)(1-c_1-c_2v-2a)}{a(1-c_1-c_2v)}A \tag{22}$$

$$g \leq Y_{-2} \leq G \tag{23}$$

$$g \leq Y_{-1} \leq G \tag{24}$$

where

$$g = \frac{S}{1-Tv} - \frac{R}{1-Tv-2R}\cdot\frac{U}{1-T} \tag{25}$$

$$G = \frac{S}{1-Tv} + \left(1+\frac{R}{1-Tv-2R}\right)\frac{U}{1-T} \tag{26}$$

(20), (21) and (22) can be rewritten as follows:

$$1-Tv-2R > 0 \tag{27}$$

$$T \leq \frac{3}{4} \tag{28}$$

$$U < \frac{1-T}{1-Tv}(1-Tv-2R)\frac{S}{R} \tag{29}$$

Condition (21) requires that the rate of saving $1-c_1-c_2$ out of wages is at least one quarter of the rate of saving $1-c_1$ out of profits. This condition will be used only in connection with the existence of the game-theoretical solution.

Condition (22) requires that the reduction D of autonomeous investment is sufficiently small compared to total autonomeous investment. Since we have $c_1+c_2<1$ und $0<v<1$, it follows by (20) that the coefficient of A in (22) is positive. It can be seen immediately with the help of (25) and (29) that g is positive. For this purpose we have made assumption (22). The role of condition (20) will become clear by the following proposition:

<u>Proposition 1</u>: Let f be any wage bargaining function and let Y_{-2} and Y_{-1} be initial conditions satisfying (23) and (24), respectively. Let $Y_o, Y_1, \ldots$ be the sequence of income levels generated by the model together with f. Under condition (20) together with the general assumptions on the parameters made in section 1 the inequality

$$(30) \qquad g \leq Y_t \leq G$$

is satisfied for $t = 0,1,2,\ldots$

<u>Proof</u>: For $t = -1,-2$ inequality (30) is satisfied by assumption. We want to show that (30) holds for period t if it holds for all earlier periods. Y_t cannot be lower than the conflict income $\bar{Y}_t$. It follows by (30) that we must have

$$(31) \qquad Y_t \geq \frac{1}{1-Tv}(S-R(G-g))$$

(25) and (26) yield

$$(32) \qquad G-g = \frac{1-Tv}{1-Tv-2R} \cdot \frac{U}{1-T}$$

It follows by (20) that G-g is positive. With the help of (25) and (32) it can be seen immediately that the right-hand side of (31) is nothing else than g. It remains to be shown that we also have $Y_t \leq G$. Y_t is increased if L_t is increased. On the other hand, P_t cannot be smaller than $\bar{P}_t$. Therefore, an upper bound $\hat{Y}_t$ for Y_t can be obtained from the equation

$$(33) \qquad \hat{Y}_t = T(\hat{Y}_t-\bar{P}_t) + R(Y_{t-1}-Y_{t-2}) + S + U$$

This follows by (13) if we insert $\hat{Y}_t - \bar{P}_t$ for L_t. With the help of (16) we obtain

$$(34) \qquad \hat{Y}_t = T\hat{Y}_t + \frac{1-T}{1-Tv} R(Y_{t-1} - Y_{t-2}) + \frac{1-T}{1-Tv} S + U$$

Since G-g is an upper bound for $Y_{t-1} - Y_{t-2}$ this yields

$$(35) \qquad \hat{Y}_t \leq \frac{1}{1-Tv} (S + R(G-g)) + \frac{U}{1-T}$$

With the help of (26) and (32) it can be seen that the right-hand side of (35) is nothing else than G.

4. Solution Concept

The solution concept which will be applied in this paper will take the form of a wage bargaining function satisfying four intuitively desirable requirements to be explained below.

Subgame consistency: A wage bargaining function f is called subgame consistent if it is invariant with respect to homomorphisms between subgames. This means that essentially the same behaviour is prescribed in homomorphic subgames. For the game at hand subgame consistency has the immediate consequence that the bargaining result B_t depends on Y_{t-1} and Y_{t-2} only. Since we make this requirement we can impose the equation

$$(36) \qquad B_t = f(Y_{t-1}, Y_{t-2})$$

Maximal cooperativeness: A wage bargaining function f is called maximally cooperative if we have $f(H_{t-1}) \neq \emptyset$ everywhere.

By imposing this requirement we neglect the possibility that conflict is sought in order to improve future profits or wages at the expense of present ones. This may not be profitable anyhow, but we do not explore whether this is the case or not. Conflict is looked upon as a threat possibility whose execution is avoided as much as possible.

Linearity: A subgame consistent wage bargaining function f is called linear if the wage level L_t assigned by f is a linear function

(37) $\quad L_t = W_1 Y_{t-1} + W_2 Y_{t-2} + W_3$

on the whole region in the (Y_{t-1}, Y_{t-2})-plane where agreement is reached by f.

A justification for the linearity requirement may be seen in the fact that linearity is the simplest type of functional relationship. It is not unreasonable to suppose that expectations will converge to the most simple wage bargaining function which satisfies the other requirements.

Before we can procede to the fourth and most important requirement, the Nash-bargaining property, we must first introduce some further notations. Let f be a subgame consistent, maximally cooperative wage bargaining function. For fixed t let $\tilde{L}_t, \tilde{L}_{t+1}, \ldots$ and $\tilde{P}_t, \tilde{P}_{t+1}, \ldots$ be the values obtained for $L_{t+\tau}$ and $P_{t+\tau}$, respectively, from the system (2) to (6) with $\delta_\tau = 1$ for $\tau = t, t+1, \ldots$ together with f starting with the initial conditions Y_{t-2}, Y_{t-1}. Define

(38) $\quad \tilde{\Phi}_t = \sum_{\tau=0}^{\infty} q^\tau \tilde{L}_{t+\tau}$

(39) $\quad \tilde{\Psi}_t = \sum_{\tau=0}^{\infty} q^\tau \tilde{P}_{t+\tau}$

The variables $\tilde{\Phi}_t$ and $\tilde{\Psi}_t$ are the utilities to be obtained by the application of f in all future periods t,t+1,...

Under the assumptions of proposition 1 the values of $\tilde{L}_{t+\tau}$ and $\tilde{P}_{t+\tau}$ must be within lower and upper bounds, easily derivable from g and G. Since q is positive and smaller than 1, this has the consequence that the infinite sums on the right-hand side of (38) and (39) exist. Therefore, under the assumptions of proposition 1 $\tilde{\Phi}_t$ and $\tilde{\Psi}_t$ can be written as functions of Y_{t-2} and Y_{t-1}:

$$(40) \qquad \tilde{\Phi}_t = \varphi(Y_{t-1}, Y_{t-2})$$

$$(41) \qquad \tilde{\Psi}_t = \psi(Y_{t-1}, Y_{t-2})$$

It is clear that φ and ψ are completely determined by f together with the system (2) to (6).

For given initial conditions Y_{-2}, Y_{-1} a value of Y_t will be called attainable if it is generated by at least one wage bargaining function.

Nash-bargaining property: A subgame consistent, maximally cooperative wage bargaining function f has the Nash-bargaining property if the following conditions are satisfied: The functions φ and ψ exist in the sense that the infinite sums in (38) and (39) are finite for arbitrary attainable values of Y_{t-2} and Y_{t-1}. The wage level $L_t = f(Y_{t-1}, Y_{t-2})$ maximizes the product

$$(42) \qquad \Pi_t = F_t G_t$$

with

$$(43) \qquad F_t = L_t - \bar{L}_t + q(\varphi(Y_t, Y_{t-1}) - \varphi(\bar{Y}_t, Y_{t-1}))$$

and

$$(44) \qquad G_t = P_t - \bar{P}_t + q(\psi(Y_t, Y_{t-1}) - \psi(\bar{Y}_t, Y_{t-1}))$$

Interpretation: Both F_t and G_t, the so-called Nash-dividends, are differences between agreement and conflict payoffs. The agreement payoff has two components, namely the immediate payoff L_t and P_t, respectively, and the future payoff $q\varphi(Y_t, Y_{t-1})$ and $q\psi(Y_t, Y_{t-1})$, respectively. The conflict payoffs are also combined from immediate and future payoffs.

Note that the application of the wage bargaining function f is taken as given with respect to the maximization of the Nash-product Π_t. Only L_t, but not $L_{t+1}, L_{t+2}, \ldots$, is considered as a variable to be chosen optimally. This corresponds to the idea that cooperation is short run and does not extend over more than one period. Nevertheless, the expected results of future cooperation have to be considered but the way in which those are derived is not changed by a deviation in the present period.

Equilibrium wage bargaining function: A wage bargaining function is called an equilibrium wage bargaining function if it is subgame consistent, maximally cooperative, and linear and satisfies the Nash-bargaining property. The equilibrium wage bargaining function is the solution concept used in this paper. Conditions for existence and uniqueness will be explored in the next section.

5. Derivation of the equilibrium wage bargaining function

In the following we shall always assume that the parameters of the system have the properties mentioned in section 1 and satisfy conditions (20), (21) and (22). Moreover, it will be assumed that Y_{-1} and Y_{-2} satisfy (23) and (24), respectively. As we shall see, under these conditions a uniquely determined equilibrium wage bargaining function exists.

As has been shown at the end of section 1, it is always possible to reach an agreement with $L_t > \bar{L}_t$ and $P_t > \bar{P}_t$. Therefore, subgame consistency, maximal cooperativeness, and linearity imply that agreement is reached everywhere and the wage level assigned by the equilibrium wage bargaining function is always given by a linear function of the form (37).

Let f be an equilibrium wage bargaining function. The dynamic system arising from f can be described by (13) with $\delta_t := 1$ together with a wage equation of the form (37):

$$(45) \qquad Y_t = T(W_1 Y_{t-1} + W_2 Y_{t-2} + W_3) + R(Y_{t-1} - Y_{t-2}) + S + U$$

If the characteristic roots x_1 and x_2 of (45) do not coincide the solution of (45) has the form

(46) $$Y_{t+\tau} = \lambda_1 x_1^\tau + \lambda_2 x_2^\tau + Y,$$

where λ_1 and λ_2 are linear functions of the initial conditions Y_{t-1} and Y_{t-2} and Y is the stationary solution of (45). For the double root-case $x_1 = x_2$ a similar statement holds true with τx_1^τ substituted for x_2^τ in (46). With the help of (37) the variables $L_{t+\tau}$ and $P_{t+\tau}$ can be expressed in the same way. If the resulting expressions are inserted for $\tilde{L}_{t+\tau}$ and $\tilde{P}_{t+\tau}$ on the right hand side of (38) and (39), respectively, we obtain infinite series which can be summed up if the absolute value of the characteristic roots x_1 and x_2 is not greater than one. In view of proposition 1 it is clear that this must be the case. It follows that φ and ψ are linear functions:

(47) $$\tilde{\Phi}_t = r_1 Y_{t-1} + r_2 Y_{t-2} + r_3$$

(48) $$\tilde{\Psi}_t = s_1 Y_{t-1} + s_2 Y_{t-2} + s_3$$

The coefficients r_i and s_i in (47) and (48) will be determined together with the parameters W_i: The Nash-dividents F_t and G_t can now be written as follows:

(49) $$F_t = L_t - \bar{L}_t + q r_1 (Y_t - \bar{Y}_t)$$

(50) $$G_t = P_t - \bar{P}_t + q s_1 (Y_t - \bar{Y}_t)$$

Consider the necessary condition for the maximization of the Nash-product Π_t:

(51) $$\frac{d\Pi_t}{dL_t} = \frac{dF_t}{dL_t} G_t + F_t \cdot \frac{dG_t}{dL_t} = 0$$

According to (3) and (13) we have

(52) $$\frac{dY_t}{dL_t} = T$$

(53) $$\frac{dP_t}{dL_t} = T-1$$

Therefore (51) is equivalent to

(54) $$(1+qr_1T)G_t + (T-1+qs_1T)F_t = 0$$

With the help of (3) we obtain

(55) $$L_t-\bar{L}_t = W(Y_t-\bar{Y}_t)$$

with

(56) $$W = \frac{2Tqr_1(1+qs_1)-q(r_1-s_1)+1}{Tq(r_1-s_1)+2-T}$$

Since agreement is always possible and always reached by an equilibrium wage bargaining function, W must be positive if it is connected to an equilibrium wage bargaining function. W can be interpreted as labor's share in the surplus above the conflict income.

A linear relation between $L_t-\bar{L}_t$ and $Y_t-\bar{Y}_t$ can also be obtained from (13) by making use of the fact that (13) also holds with $\bar{Y}_t$ and $\bar{L}_t$ and $\delta_t = 0$:

(57) $$Y_t-\bar{Y}_t = T(L_t-\bar{L}_t) + U$$

Together with (40) this yields

(58) $$L_t-\bar{L}_t = \frac{WU}{1-WT}$$

In view of (15) we have

(59) $$L_t = \frac{Rv}{1-Tv}(Y_{t-1}-Y_{t-2}) + \frac{Sv}{1-Tv} + \frac{WU}{1-WT}$$

This can be inserted in (13) with $\delta_t = 1$. We obtain

(60) $$Y_t = \frac{R}{1-Tv}(Y_{t-1}-Y_{t-2}) + \frac{S}{1-Tv} + \frac{U}{1-TW}$$

Equation (59) shows that we must have

(61) $$W_1 = \frac{Rv}{1-Tv}$$

(62) $$W_2 = -\frac{Rv}{1-Tv}$$

(63) $$W_3 = \frac{Sv}{1-Tv} + \frac{WU}{1-TW}$$

In order to determine W and the coefficients r_i and s_i we make use of the following functional relationship which must hold in equilibrium:

(64) $$\tilde{\phi}_t = L_t + q\tilde{\phi}_{t+1}$$

(65) $$\tilde{\Psi}_t = P_t + q\tilde{\Psi}_{t-1}$$

With the help of (47) and (48) we obtain

(66) $$r_1Y_{t-1} + r_2Y_{t-2} + r_3 = L_t + qr_1Y_t + qr_2Y_{t-1} + qr_3$$

(67) $$s_1Y_{t-1} + s_2Y_{t-2} + s_3 = Y_t - L_t + qs_1Y_t + qs_2Y_{t-1} + qs_3$$

In these equations L_t and Y_t can be substituted by the right-hand side of (59) and (60), respectively. In this way, we obtain conditions on the parameters r_i, s_i and W which must hold regardless of the values of Y_{t-1} and Y_{t-2}. The co-efficients of Y_{t-1} and Y_{t-2} and the absolute terms must be zero. This yields

(68) $$r_1 = \frac{Rv}{1-Tv} + qr_1\frac{R}{1-Tv} + qr_2$$

(69) $$s_1 = \frac{R(1-v)}{1-Tv} + qs_1\frac{R}{1-Tv} + qs_2$$

(70) $$r_2 = -\frac{R(v+qr_1)}{1-Tv}$$

(71) $$s_2 = -\frac{R(1-v+qs_1)}{1-Tv}$$

(72) $$r_3 = \frac{S}{(1-q)(1-Tv)}(v+qr_1) + \frac{U}{(1-q)(1-TW)}(W+qr_1)$$

(73) $$s_3 = \frac{S}{(1-q)(1-Tv)}(1-v+qs_1) + \frac{U}{(1-q)(1-TW)}(1-W+qs_1)$$

Now we combine (68) with (70) and (69) with (71) and obtain

(74) $$r_1 = \frac{R(1-q)v}{1-Tv-q(1-q)R}$$

(75) $$s_1 = \frac{R(1-q)(1-v)}{1-Tv-q(1-q)R}$$

With the help of these formulas it is now possible to compute W and W_3. The coefficients W_1, W_2 and W_3 are determined by (56), (61), (62), (63), (74) and (75).

The assumptions on the parameters guarantee that all denominators occurring in (61) to (63) and (68) to (75) are positive. In the case of (74) and (75) this follows by (27) together with the fact that in view of $0<q<1$ we must have

(76) $$q(1-q) \leq \frac{1}{4}$$

It is now clear that an equilibrium wage bargaining function if it exists is uniquely determined by (61) to (63) together with (56) and (70) to (75). Obviously, the way in which we have derived our results guarantees subgame consistency, maximal cooperativeness and linearity. We did not yet show that not only the necessary but also the sufficient condition for the maximization of the Nash-product is satisfied. Moreover, we are not yet sure whether the parameters W_i constitute a wage bargaining function since it still remains to be shown that the agreement yields wages and profits above the conflict levels. It follows by (55) that this is the case if W is a positive number smaller than one.

In order to prove what remains to be seen we need upper bounds for qr_1 and qs_1. It follows by (74) and (75) that r_1 and s_1 are positive. With the help of (27) we obtain

(77) $$1-Tv-q(1-q)R > R(2-q(1-q))$$

This together with (76) yields

(78) $$\frac{q(1-q)R}{1-Tv-q(1-q)R} < \frac{1}{7}$$

Consequently, we have

(79) $$0 < qr_1 < \frac{1}{7}v$$

(80) $$0 < qs_1 < \frac{1}{7}(1-v)$$

In order to examine the sufficient condition for the maximization of the Nash-product we form the second derivative:

(81) $$\frac{d^2\Pi_t}{dL_t^{\,2}} = 2(1+qr_1T)(T-1+qs_1T)$$

Obviously, $1+qr_1T$ is positive. It follows by (28) and (80) that $T-1+qs_1T$ is negative. This shows that the sufficient condition for the maximization of the Nash-product is satisfied.

It remains to be seen that W is positive and smaller than one. It follows by (28), (79) and (80) that the nominater and the denominater on the right-hand side of (56) are positive. Therefore W is positive. The inequality $W<1$ is equivalent to

(82) $$2Tqr_1(1+qs_1)-(1+T)q(r_1-s_1) < 1-T$$

In view of (74) and (75) this can be rewritten as follows:

(83) $$\frac{q(1-q)R}{1-Tv-q(1-q)R}(2Tvqs_1-2v+T+1) < 1-T$$

It can be seen with the help of (78) and (80) that (83) is satisfied if we have:

(84) $$\frac{1}{7}(\frac{2}{7}v(1-v) - T^{-2v} + T + 1) < 1-T$$

This is equivalent to the following condition:

$$(85) \qquad T < \frac{3+v}{4 + \frac{v(1-v)}{7}}$$

(85) can be rewritten as follows:

$$(86) \qquad T < \frac{3}{4} + \frac{v(25+3v)}{4(28+v(1-v))}$$

In view of (28) and $0 < v < 1$ it is clear that this condition is satisfied. Consequently, W is a positive number smaller than one.

The results of this section are summerized by the following proposition:

<u>Proposition 2</u>: Under the conditions (20) to (24) together with the general assumptions on the parameters made in section 1 a uniquely determined equilibrium wage bargaining function f with

$$(87) \qquad f(Y_{t-1}, Y_{t-2}) = W_1 Y_{t-1} + W_2 Y_{t-2} + W_3$$

exists. The parameters W_1, W_2, W_3 can be computed with the help of (56), (61), (62), (63), (74) and (75). The income levels Y_t generated by the model and its equilibrium wage bargaining function satisfy the difference equation (60).

6. Properties of the equilibrium solution

The absolute value x of the characteric roots of (60) is given by

$$(88) \qquad x = \sqrt{\frac{R}{1-Tv}} = \sqrt{\frac{a}{1-c_1-c_2 v}}$$

This shows that only the parameters c_1, c_2, a and v influence the stability of the system. The stability is decreased if any one of these parameters is increased.

The stationary solution of (60) is given by

$$(89) \qquad Y = \frac{S}{1-Tv} + \frac{U}{1-TW}$$

This is equivalent to

$$(90) \qquad Y = \frac{A}{1-c_1-c_2v} + \frac{D}{1-c_1-c_2W}$$

Obviously, Y is increased by an increase of A or D. The influence of the parameters c_1,c_2,a and v on Y is difficult to discuss since W depends on these parameters in a complicated way. We shall not explore this question in detail but it can be said that the sign of the partial derivative of Y with respect to each one of the parameters c_1,c_2 and v will be determined by the first term of the right-hand side of (90) if D is sufficiently small compared to A. If this is the case Y will be increased by an increase of c_1,c_2 or v.

The first term on the right-hand side of (90) can be interpreted as that part of the stationary income which would be reached even in case of permanent conflict. This can be seen by inserting vY_t for L_t into (13) with $\delta_t = 0$. The resulting difference equation yields the first term of the right-hand side of (90) as the stationary solution. We shall call the first term on the right-hand side of (90) the "conflict income".

The second term on the right-hand side of (90) may be called the "agreement increment" since it is due to the avoidance of conflict.

It follows by (61), (62) and (63) that the stationary wage L is equal to W_3:

$$(91) \qquad L = \frac{vA}{1-c_1-c_2v} + \frac{WD}{1-c_1-c_2W}$$

Equation (91) has an obvious interpretation. Labor receives the share v of the conflict income and the share W of the agreement increment.

It is interesting to look at the influence of q. This parameter does not influence the stability of the system but it does influence W and thereby Y and L. In the form (56) for W the coefficients r_1 and s_1 always occur combined with q. W can be thought of as function of T, qr_1 and qs_1. It follows by (74) and (75) that both qr_1 and qs_1 approach zero if either q or 1-q goes to zero. Therefore in view of (56) we have

$$(92) \qquad \lim_{q \to 0} W = \lim_{q \to 1} W = \frac{1}{2-T}$$

Since W_1 and W_2 do not depend on q and $L = W_3$ is influenced by q via W only, (92) has the consequence that the equilibrium wage bargaining function is almost the same for very myopic and very far-sighted goal functions. The greatest deviations of W from 1/(2-T) can be expected in the middle range of q. The surprising similarity between extremely myopic and extremely far-sighted behavior can be understood more easily if one looks at the part of the goal functions which contains future wages or profits, namely $q\varphi(Y_{t-1},Y_t)$ and $q\psi(Y_{t-1},Y_t)$ respectively. These terms may be called the "future terms". Obviously, as q becomes small the influence of the future terms becomes insignificant. On the other hand, as q approaches one the influence of the future terms becomes insignificant, too, since r_1 and s_1 approach zero. This seems to be due to the fact that a change of present income has little influence on the sum of future incomes over a sufficiently long horizon since a deviation in one direction in the immediate future is counterbalanced by opposite influences on later periods. In the middle ranges of q the counterbalancing effect is weak since the immediate future has much more weight than the more remote future. Therefore substantial deviations from myopic behavior may occur for such q.

References:

[1] S.Clemhout-G.Leitman-H.Y.Wan, Bargaining under Strike: A Differential Game View, Journal of Economic Theory 11, 55-67 (1975)

[2] W. Güth, Zur Theorie kollektiver Lohnverhandlungen, Baden-Baden 1978

[3] J.R. Hicks, A Contribution to the Theory of the Trade Cycle, Oxford 1949

[4] N. Kaldor, Alternative Theories of Distribution, Essays on Value and Distribution (1960), 209-236

[5] W. Krelle, A New Theory of Bargaining - Applied to the Problem of Wage Determination and Strikes -, Bonn 1975

[6] J. Nash, The Bargaining Problem, Econometrica 18 , 155-162 (1950)

[7] P. Rice, A Note on the Hicks Theory of Strike Bargaining, Bielefeld 1976

[8] P. Samuelson, Interactions between the Multiplier Analysis and the Principle of Acceleration, Review of Economic Statistics 21, (1939), 75-78

[9] R. Selten, Coalition Probabilities in a Non-Cooperative Model of Three-Person Quota Game Bargaining, November 1977, Bielefeld

[10] M. Shubik, A Business Cycle Model with Organized Labor Considered, Econometrica 20, 284-294 (1952)

[11] J. Ståhl, Bargaining Theory, Stockholm 1972