

Essays on Limited Liability in Economics

Dissertation zur Erlangung des Doktorgrades
der Wirtschaftswissenschaften (Dr. rer. pol.)
an der Fakultät für Wirtschaftswissenschaften
der Universität Bielefeld

vorgelegt von
M.Sc. Frederik Diermann

Februar 2016

Gutachter

Erster Gutachter: Prof. Dr. Frank Riedel
Zweiter Gutachter: J.-Prof. Dr. Jan-Henrik Steg

Adresse

Universität Bielefeld
Fakultät für Wirtschaftswissenschaften
Universitätsstraße 25
33615 Bielefeld Germany

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Part I.

General Introduction

Limited Liability is a fundamental principle of modern society. Its origins go back as far as to Ancient Greece where Solon, around 600 BC, abolished debt bondage in Athens.¹ Solon's reforms came at a time where many small farmers were threatened by debt bondage due to excessive debt accumulation over the past years. Though abolished in Athens, debt bondage persisted in many regions in the world and modern versions of it still do.² ³ Nowadays, sophisticated societies often provide personal bankruptcy laws to offer some form of debt relief in case of excessive debt accumulation. During a period of time the debtor gives up most of his property and income in order to try best to repay pending debt. If certain requirements are achieved, often the final goal for the debtor is a complete relief of the remaining debt after the proceedings.

But not only on the individual level limited liability is a fundamental principle in society. The introduction of limited liability into the corporate sector has a long history. A milestone in this matter was the Limited Liability Act of 1855 which followed the Joint Stock Companies Act 1844 and introduced limited liability for companies of more than 25 members in the United Kingdom. Regulations for limited liability were extended in the Companies Act 1862 and the most recent version of 2006 where a key renewal was the abolishment of unlimited liability for directors of English companies. The European Union has set a general framework 1989 in their 'Twelfth Council Company Law Directive 89/667/EEC of 21 December 1989 on single-member private limited-liability companies'.

¹Croix, G. E. M. D. S., Croix, G. D. S. (1989)

²Kara, S. (2009). Sex trafficking: Inside the business of modern slavery. Columbia University Press.

³According to the Anti-Slavery Society: 'Pawnage or pawn slavery is a form of servitude akin to bonded labor under which the debtor provides another human being as security or collateral for the debt. Until the debt (including interest on it) is paid off, the creditor has the use of the labor of the pawn.'

In economic theory, the merits and weaknesses of limited liability in corporation law have been under severe discussion since the introduction of limited liability in the nineteenth century and still persist as one of the most debated topics. Limited liability guarantees investors not to lose more than they invest. There are mainly two different kinds of capital that is raised in order to run an enterprise, equity and debt. The main difference from a liability perspective is that in case of a bankruptcy equity is lost first. The investors are consequently the first to suffer from bad enterprise performance. In order to limit their loss, limited liability sets a bound to the loss and shifts part of the risk to creditors. The questions of why it is useful to bound the risk for equity investors and why one should shift risk from one party to another arise. If there was a reason for differentiating between equity and debt investment in order to have debt investment to be less risky as it is lost last, why should debt investment not be protected even more? There are distinct opinions why it is useful to shift risk to creditors. While Posner (1976) argues that creditors are less risk averse than stockholders as well as better informed and therefore better targets for risk taking, Easterbrook and Fischel (1985) set against that creditors accept lower rates of return on investment as well as the higher variance of shareholders' returns.

There are two major reasonings why limited liability is advantageous to unlimited liability. The first is given by Manne (1967). Manne argues that if investors were unlimitedly liable for each investment they undertake, investors would concentrate on few or even a single investment with a large exposure rather than multiple small investments. The reasoning is quite simple. If an investor was unlimitedly liable, he could lose all his wealth if only a single of all the projects he invested into fails. Therefore risk clustering is enhanced by unlimited liability rather than diversification. Besides the itself already positive effect of diversification, holders of diversified portfolios are more likely to realize any projects with positive net present value as risks can be hedged in the overall portfolio. This is a welfare enhancing behavior and desired from a social point of view. Halpern (1980) gives a second important reason for the preservation of limited liability. Limited liability is a necessary tool for the proper functioning of security markets. If there was no limited liability, the value of an asset would significantly depend on the wealth of the holder of the share. This would make the evaluation and thus the trading of shares much more complicated

and handicap efficient risk allocation on financial markets. This distinct valuation of asset values according to equity levels is an important feature of Part II. of this thesis and has important consequences for risk clustering.

While limited liability is sometimes criticized as a benefit the state gives to investors, indeed many of the large, successful companies are public held limitedly liable corporations rather than private, owner run companies. There are several advantages for public held corporations. It is easier to separate the demands of capital, management knowledge and labor. Workers who lack capital and enterprise knowledge, managers without capital, and investors with lack of special production or investment skills can join in order to run a project. While the advantage of resource allocation comes at certain costs such as monitoring of the manager by the investor and an agency loss, the survival of large public corporations shows an outbalancing effect of the advantages. Limited liability further reduces some of these costs as average monitoring costs gets lowered. It might require less intense monitoring since investors are protected by limited liability. Monitoring, therefore, is only necessary up to a certain point where maximal liability is reached. Especially under the above mentioned more diversified portfolios of investors under limited liability, the reduction of necessary monitoring might be substantial. Easterbrook et al.(1985) mention that not only managers, but also other shareholders would be needed to be monitored under unlimited liability as their private wealth level directly influences the enterprise under unlimited liability. This is an important reduction of monitoring costs via limited liability. Further they mention the advantages of cheaper company takeover possibilities on more efficient markets. The efficiency of the market itself yields a better information distribution due to more accurate prices (Lorie and Hamilton (1973)). Therefore, the costs of searching for appropriate investment opportunities is decreased.

On a more general view one of the most important functions of the financial market is the one of risk allocation. As discussed, limited liability is a very important tool for the financial market to be efficient and, hence, be efficient for risk allocation. Limited liability itself, in a broad sense, is a reallocation of risk as it shifts risk from equity investors to debt investors. From a welfare perspective and due to risk

aversion it is often desired that risk is spread between multiple parties within society. Limited liability serves exactly as a risk distribution tool away from maximal risk bearing by the equity investor. Therefore limited liability can be seen as a welfare enhancing risk sharing mechanism.

Having understood that limited liability is a type of redistribution of risk, one could ask the question whether limited liability is needed at all as it could arise endogenously on a financial market via the negotiation of insurance contracts. For instance a debt insurance bought by the company and offered by the creditor might be close to mirroring limited liability. Third party offers of insurances could even improve the risk redistribution. Limited liability simplifies this procedure at a first step without any transaction or bargaining costs.

In between private bankruptcy regulations and corporate limited liability laws, the lower bound on private welfare losses enhances entrepreneurship and the startup of new companies. The often risk averse market participants' concave utility gets (locally) convexified by the lower guaranteed bound of utility. Therefore, the socially desired effect of business formations gets enhanced which is seen as a driving force of the evolution of technology and economic growth.

Besides the individual and the corporate level the third level where limited liability has to be taken into account is the level of countries. While there is a severe lack of regulation for sovereign default the list of sovereign debt defaults or debt restructuring is remarkable. As sovereigns' balance sheets generally contain much larger volumes than company balance sheets, consequences of sovereign defaults might be immense. Economically, nearly every part of the society is involved as creditors, often private investors and companies, have a haircut on, or completely lose, principal and interest. Further, a former bankruptcy decreases the access to liquidity and makes future credits dearer for the government. Due to the impact of the bankruptcy of a government, a sovereign default is often accompanied by further crises such as a banking crisis or an economic crisis.

Aside from regulatory limited liability, not only sovereigns but, especially in nowadays current financial asset management, also private individuals are able to accu-

mulate debt volumes that they, de facto, might never be able to even up.

In this thesis I investigate three different economic scenarios where limited liability plays an impactful role.

In part II., I investigate an investment problem where investors incorporate in their decision making that they are limitedly liable. While in the literature ambiguity aversion is an established explanation for limited market participation, I show that ambiguity might also have positive effects on market participation when limited liability is anticipated. A key result is that, when incorporating limited liability, more ambiguity in expected return can lead to less market participation and risk clustering whereas more ambiguity in volatility might lead to more market participation and less risk clustering in the same model. An important part of the mechanics in this model is that the evaluation of assets under limited liability heavily depends on the liability level of an agent. The idea goes back to Merton (1974) when valuating debt. The loss in the case of default is represented by a put option with strike price equal to the maximal liability. Investors that invest in a risky project might now account for an additional free put option in case of their own default when evaluating the investment. This is a problematic market characteristic as it leads to overrating of, especially, risky assets by low equity market participants. The problem of betting for resurrection is closely related, where financial institutes and funds increase their risky position when they are close to bankruptcy. Ambiguity adds an interesting insight on the distribution of risk and market participation when interacting with limited liability.

Part III. discusses the impact of government-run bailout policies for financial institutes. The presence of a possible bailout takes liability away from financial institutes and creates a moral hazard problem with respect to investment decisions and incentive setting for managers. While induced excessive risk taking might harm overall welfare, coordination improvements due to increased counterparty trust induced by possible bailouts in case of default can outbalance the negative effects. The possibilities of risk monitoring play an important role with respect to welfare when communicating certain bailout expectations. The amount of creditors is crucial as well as the already expected level of bailout probability when a government thinks

about increasing their bailout efforts.

Part IV. investigates the problem of a fund manager managing a portfolio on order of an investor. The investor incentivizes the fund manager via benchmarks and bonus payments based on portfolio success. The only observable variable for the investor is the portfolio performance. The principal agent problem for delegated portfolio management is special because the agent has two choice variables that the principal cannot observe, the choice of effort and the choice of portfolio structure. The agent can exert more or less effort in order to acquire information that is important for the investment decision and therefore have access to better investment opportunities or lower the variance of a portfolio. After doing so, the agent structures the portfolio in order to maximize his utility. Since increasing the bonus payment probability can often be achieved by both, exerting more effort but also by taking more risk, the agent might shirk and simply choose a riskier portfolio. Therefore, while bonus contracts are useful to incentivize an appropriate effort by the manager, they might also result in excessive risk taking in order to reach benchmarks, while the manager is not liable for the taken risks in case of default. In order to limit risk taking, besides the bonus and the benchmark, an appropriate risk measure has to be chosen and an according risk level has to be satisfied. I show that in a model where the market allows the manager to replicate measurable payoff structures the size of the bonus is not the key cause for more risk taking. The setting of the benchmark rather decides the probability of a portfolio default, whereas an excessive bonus would not change the portfolio structure compared to an optimal bonus setting.

The following three parts II-IV are written in a self-contained manner and can be read independently.

Part II.

**On Different Effects of Ambiguity
towards Market Participation under
Limited Liability**

1. Introduction

Limited participation is a well observed phenomenon in stock markets. While the classic portfolio theory expects a much higher share of the population investing in risky assets, empirical studies show that the majority of households do not hold any stock. Mankiw and Zeldes (1991) report that the holding of stock is increasing in the household's liquid assets, but even for households with liquid assets of \$100,000 less than half of them own stock. Campbell (2006) reports similar findings.

Explanations for this have been tried to be found in many directions. Participation costs, as in Paiello (2007) or entry costs as in Allen and Gale (1994) or Yaron and Zhang (2000), have not been found to be fully explaining this phenomenon, as well as risk aversion, heterogeneous beliefs or minimum investment requirements.

Considering risk aversion as a possible explanation, it is a natural question to ask whether ambiguity aversion might explain the observed behavior. In economic theory, risk and (Knightian) uncertainty are two distinct driving features of agents' and firms' observed behavior. The difference of risk and ambiguity can be described as follows. When an investor faces an asset with multiple possible payoffs and given probabilities for these different outcomes, one generally refers to a risky asset. In contrast, when facing uncertainty, the investor is not even sure about the underlying probability distribution. Practically, it is not always clear whether a situation is risky or ambiguous and it has to be seen more as a fluent passage from the one case to the other. While for a (fair) coin toss there is nearly no doubt that the probabilities are known, the probability for a country going bankrupt or an atomic reactor to explode might be highly in question. In between, problems like car insurance provide a large amount of data and give a very good idea about the underlying probabilities, but still leave some ambiguity.

Knight (1921) first theoretically distinguished uncertainty and risk in the behavior of decision making. While he originally differentiated between uncertainty that is measurable and uncertainty that is not measurable (in terms of probability), Ellsberg (1961), in his famous urn experiment, shows that decision makers violate the inde-

pendence axiom of classical expected utility theory by Savage (1954). Gilboa and Schmeidler (1989) give axiomatic requirements (including an axiom that captures uncertainty aversion) for preferences that induce a utility representation known as the maxmin expected utility. This representation features a set of probability measures, rather than a single one, for the decision maker and is a widely used approach to introduce ambiguity into economic models. Since then, several representations of preferences have been found that explain how a decision maker behaves when ambiguity is part of a decision problem.

I will give a short overview over the most important papers that take a step back from subjective expected utility and go towards ambiguity. In the stream of literature with incomplete preferences, Bewley (1986) gives a representation that indicates that agents prefer an alternative x to y only if the expected utility is better under all probability measures of a set of priors. Having a model with endowments as a status quo, agents will only change their status quo if they will be better off under all priors. Several famous representations can be found when assuming that preferences are complete. Gilboa and Schmeidler (1989) give a representation called Maxmin Expected Utility (MEU). They drop the classical independence axiom and add uncertainty aversion as well as constant independence. Their representation shows that agents evaluate alternatives as expected utility under several probability measures from a set of priors but only consider the worst case. The Choquet Expected Utility, Schmeidler (1989), which uses capacities rather than probabilities, is closely related as long as the capacity is convex. Maccheroni, Marinacci, Rustichini (2006) introduce Variational Preferences, a representation where, again, alternatives are evaluated under multiple priors, but each evaluation incorporates a cost function depending on the prior. In this sense, unlikely priors might be devalued compared to more likely ones. Overall, again, the worst case is considered by the agents. This specific representation breaks down to the MEU case in case of a cost function which is constantly zero. Another famous cost function, the relative entropy, yields the Multiplier Preference model of Hansen and Sargent (2001). The relative entropy is a measure for the distance of probability measures and punishes unlikely scenarios for being far away compared to a 'most likely' benchmark scenario. Finally, the relative Gini concentration as a distance of probability measures as cost function,

breaks down the Variational Preferences to Mean-Variance Preferences. Klibanoff, Marinacci, Mukerji (2005) introduce Smooth Ambiguity Preferences. They can be seen as a second order belief representation with a probability distribution over the probability measures in the set of priors.

The first explanation that finds ambiguity aversion as a cause for nonparticipation in financial markets is given by Dow and Werlang (1992). Dow and Werlang show that, under ambiguity aversion, evaluating a long position and a short position of a risky asset is done under different probability measures. Therefore, they find a range of prices where an investor is neither interested in holding positive nor negative positions of risky assets. Cao, Wang and Zhang (2005) show that with heterogeneous agents, differing in their perceived ambiguity about the mean return, a higher ambiguity dispersion leads to a smaller fraction of agents participating in the financial market. Easley and O'Hara (2009) consider professional traders that know the exact prior and private investors that perceive ambiguity in the form of a set of priors. They show that high perceived ambiguity about the mean return might make the private investors not participate in the market, while they do not find any influence of ambiguity with respect to volatility.

The incorporation of limited liability into portfolio theory has a long history. Merton (1974) already treats the equity of a company as a call option on its assets when pricing debt. More recently Gollier (1997) investigates optimal portfolio choices with utility functions that are bounded from below. In this spirit Ross (2004) generally investigates whether convex transformations 'convexify' concave utility functions; i.e. make a decision maker more risk loving. Wilson (2010) points out problems the 'free' put option creates when running bailout or 'bad bank' programs like TARP and PPIP. The free put option accounts for limited liability in case of default. The evaluation of the price of risky, or even more, highly toxic assets, is different for agents with different liability. Institutions close to bankruptcy might evaluate these assets at a higher price than institutions with sufficient equity. This naturally leads to risk clustering at institutions that are already struggling. Governmental programs in order to buy toxic assets at market price to relax banks' balance sheets or even subsidized programs buying these assets at prices that are above the market price

might at first address banks that are well equipped in terms of equity. In this way subsidies directly go to the profits of high equity banks and leave 'stressed' banks unaffected. Only excessive high subsidies and thus, excessive overpricing will address stressed banks. Very closely related to this behavior is the well known phenomenon of 'betting for resurrection'. Troubled institutes increase the amount of risky positions when they are close to bankruptcy, because this maximizes the probability of getting back into a profitable state while there is no difference of going bankrupt or 'even more bankrupt'.

In my model, I will show that ambiguity in expected return and in the variance of the risky asset has different effects in terms of market participation when agents differ in their equity and explicitly take into consideration that they are limitedly liable. For ambiguity in the mean payoff I find similar results as Cao, Wang and Zhang and Easley and O'Hara. The more ambiguity is in the market, it might happen that some agents do not hold any risky assets anymore. In contrast to their papers, I find an effect of volatility uncertainty that is opposing the effect of return uncertainty. The explanation behind this is as follows: A limitedly liable agent holds a free put option in his portfolio that accounts for the possible default case when limited liability comes into play. This option differs in value depending on the equity of the agent, hence, the risky asset might have different values for different agents. The value of this put option heavily depends on the volatility of the risky asset. Increasing ambiguity in the volatility decreases the value of the put option and makes the evaluation of the risky asset in between the different agents more even. Therefore, more agents might be participating in the market. The focus of my analysis is the case where there is one bank with high equity and one bank with low equity in order to point out the most striking results representing a market with 'healthy' and 'stressed' banks.

The paper is structured as follows. In Chapter 2, I give the setup for the model and derive the optimal behavior for the agents. Chapter 3 takes a look at the equilibria of this model and the impact of ambiguity. In Chapter 4, I discuss the model with respect to risk punishment, heterogeneous perception of ambiguity and governmental policies. Chapter 5 concludes.

2. The Model

2.1. Modeling Ambiguity

Ambiguity in this model follows the Maxmin Expected Utility representation of Gilboa and Schmeidler. The utility representation for a random variable X can be written in the following way:

$$\min_{Q \in \mathcal{P}} \int Eu(X)dQ$$

The decision maker evaluates the expected utility under all probability measures given by a set \mathcal{P} and then takes the worst case into account. While this behavior has a nice interpretation in terms of a decision maker identifying all possible scenarios with the set \mathcal{P} and then being very pessimistic about the outcome, this is not necessarily the case. The axioms of Gilboa and Schmeidler only provide the existence of such a set and the observed behavior of agents from the outside looks as if they behave accordingly pessimistic. Whether agents only consider priors in \mathcal{P} cannot clearly be stated as well as the set \mathcal{P} being the remainder after removing all scenarios that can be excluded. Some more insight in the underlying behavior and the interpretation of a set \mathcal{P} and the consideration of the worst case might be found in Gajdos, Hayashi, Tallon, and Vergnaud (2008).

2.2. General Setup

I consider a two stage model with $t \in \{0, 1\}$ and two risk neutral agents $i = 1, 2$. The agents can choose to buy a desired amount θ_i of risky assets at price P with exogenous supply \bar{x} in $t = 0$. In $t = 1$ the return of the risky asset X is realized. The distribution of the return is ambiguous and characterized in the following way. Let

$$(\omega_1, \omega_2, \omega_3) \in \Omega = \{h, l\} \times \mathbb{R} \times \mathbb{R}$$

and let P_1 be given by

$$P_1(w_1 = h) = P_1(w_1 = l) = \frac{1}{2}.$$

Further, I define

$$\mathcal{P}_1 = \{P_1\}$$

$$\mathcal{P}_2 = \{\lambda \in M_1(\mathbb{R}) | \lambda((-\infty, \underline{\mu})) = 0 \wedge \lambda((\bar{\mu}, \infty)) = 0\}$$

$$\mathcal{P}_3 = \{\lambda \in M_1(\mathbb{R}) | \lambda((-\infty, \underline{\sigma})) = 0 \wedge \lambda((\bar{\sigma}, \infty)) = 0\}^4$$

and

$$\mathcal{P} = \{Q = P_1 \otimes P_2 \otimes P_3 | P_2 \in \mathcal{P}_2 \wedge P_3 \in \mathcal{P}_3\}.$$

The perceived payoff of the risky asset is then given by

$$X(\omega = (h, \mu, \sigma)) = \mu + \sigma$$

and

$$X(\omega = (l, \mu, \sigma)) = \mu - \sigma.$$

I assume that the distributions are independent and thus, ambiguity in return and volatility do not depend on each other as well as on the state of the economy given by $\Omega_1 = \{h, l\}$. This assumption of separability into two distinct dimensions of ambiguity enables us to get specific results for each type and point out the difference between these two dimensions. I assume

$$\underline{\sigma} > \underline{\mu} > 0.$$

This will later guarantee a positive expected return and a negative payoff in the case of the low state of the economy. The agents are certain about the probabilities whether the economy is in a high or low state. The payoff in the particular cases, however, is subject to uncertainty as the payoff distributions in the two states cannot be completely specified and only be reduced to the cases contained in \mathcal{P}_2 and \mathcal{P}_3 .

⁴ $M_1(\mathbb{R})$ denotes the set of all probability measures on \mathbb{R} and I assume $0 < \underline{\mu} < \bar{\mu}$ as well as $0 < \underline{\sigma} < \bar{\sigma}$.

The variables μ and σ serve as proxies for return and volatility, respectively. The agents have endowment in terms of illiquid equity which equivalently will be the maximal amount an agent is liable with. I denote the equity of the agents by $L_1 > 0$ and $L_2 > 0$, respectively. Further, there is an infinite supply of liquidity at a constant interest rate r in order to finance the investment. For simplicity I assume $r = 0$. The infinite supply of liquidity is a common feature of many financial models and an important property for markets to be efficient. The constant interest rate is often a simplification that comes along with the assumption of infinite supply, which is highly in question. In addition, the agents face a cost function $c(\theta_i)$ depending on the amount of risky assets in their portfolio. This cost function might capture the fact that liquidity, especially as demands become extremely large, is not available at constant costs. Moreover, possible punishments for risk taking might be included in this function. For the further analysis I continue with the simple case of $c(\theta_i) = \gamma\theta_i^2 P^2$. This case resembles increasing unit punishment for increasing required liquidity. I will later show that qualitatively similar results can be obtained in case of a risk management that punishes risk taking according to the entropic risk measure. In order to focus on the most important results I only allow nonnegative amounts of risky assets $\theta_1, \theta_2 \geq 0$. The agents aim at maximizing their $t = 1$ portfolio value.

2.3. Optimal Demand

The period 1 portfolio value for agent i is given by

$$\max(0, \theta_i(x - P) + L_i - \gamma\theta_i^2 P^2).$$

The objective for agent i is thus

$$\max_{\theta_i \geq 0} \min_{Q \in \mathcal{P}} E(\max(0, \theta_i(x - P) + L_i - \gamma\theta_i^2 P^2)).$$

The optimization is not completely straightforward as the boundedness from below by 0 leaves this problem nonconcave. The portfolio defaults if

$$x < \frac{\gamma\theta_i^2 P^2 - L_i}{\theta_i} + P.$$

2.3.1. Identifying the Worst Case

In classic portfolio theory agents prefer high returns and try to avoid volatility. A crucial assumption that is the driving factor in most cases for preferring low volatility is risk aversion. While risk aversion is modeled via a concave utility function, incorporating limited liability does not leave agents with concave objectives any longer. Ross (2004) discusses whether convex incentive schemes convexify concave utility functionals. This is a closely related problem as call-type incentive schemes as $f(x) = \max(-L_i, x)$ are strongly related to our portfolio problem. Gollier (1997) analyzes the impact of imposing a lower bound on the utility function in order to incorporate limited liability into portfolio choice. A striking result is that, even in the case of risk aversion, high volatility might be preferred.

Coming back to the agents problem, we can easily identify the case $P_2(\mu = \underline{\mu}) = 1$ as the worst case for every possible portfolio choice of the agent. As we only allowed long positions, a low μ yields the lowest expected returns in every case. The worst case with respect to volatility can be identified as $P_3(\sigma = \underline{\sigma}) = 1$. At first the agents are risk neutral putting them at a volatility neutral attitude in the case of full liability. If agents are not able to fully bear possible losses, increased volatility⁵ increases expected returns as only the positive state $\omega_1 = h$ is affected from their point of view. Hence, $P_3(\sigma = \underline{\sigma}) = 1$ can be identified as the worst case, at least in a weak sense. The decision makers in this model, through the distortion of limited liability, can thus be viewed as weakly volatility loving.

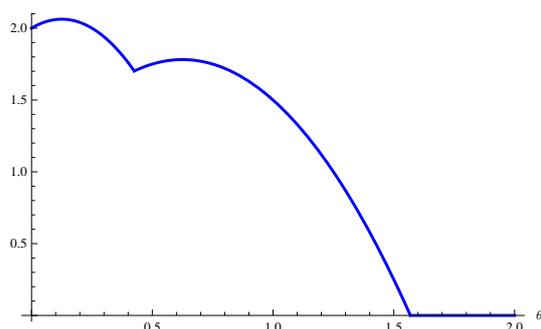
⁵in the sense of a mean preserving spread

2.3.2. Worst Case Optimization

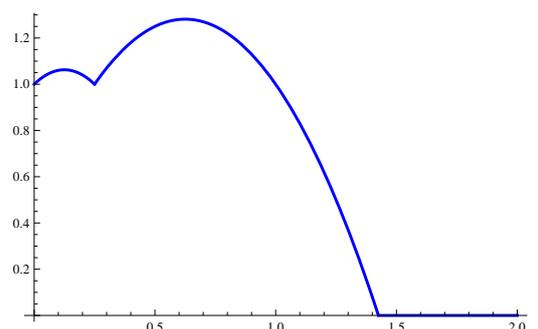
Having identified the probability measures that yield the pair $(\underline{\mu}, \underline{\sigma})$ as the worst case, we can rewrite the objective as:

$$\max_{\theta_i \geq 0} \max(\theta_i(\underline{\mu} - P) - c(\theta_i P) + L_i, 1/2(\theta_i(\underline{\mu} + \underline{\sigma} - P) - c(\theta_i P) + L_i), 0)$$

The expectation is still bounded from below by 0. Moreover, in this decomposition we can see how the limited liability takes effect on the optimal behavior. Either the agent is able to bear all losses and he is in the first case of the maximum function, or the agent is only interested in the high state as he is not able to bear the losses in the low state given his choice θ_i and the second argument of the maximum function is dominating. This decomposition further shows that the objective might have two local maxima.



A typical expected utility curve depending on the amount of risky asset for high equity; the left optimum dominates



The same situation with decreased equity; an agent would demand a higher amount of risky asset

In the graphic a typical curve for the expected utility depending on the amount of risky asset is shown. Starting at 0 risky assets every additional unit yields its expected return at the cost of the price and the risk/liquidity punishment. After costs get too high, utility is decreasing. When the amount of risky assets becomes sufficiently large, limited liability is a driving factor. Any additional unit now only affects the terminal portfolio value in the high state. This might yield an increasing utility, again, until the costs dominate. I will denote the right local maximum as the limitedly liable optimum (lim) and the left optimum as the fully liable optimum

(full). The height of the maxima heavily depend on volatility and equity. It might very well be that the lim optimum is below the corresponding value of the fully liable parabola. The maximizers might even be negative, which is a priori excluded. In order to choose optimally, the agents have to compare whether they are better off with a safe choice or with the risk of running default compared with the third alternative of holding a risky position of zero units. I collect the decision rules in the function ϕ , which is defined in the following theorem. As a result we can state:

Theorem 2.1. *The optimal demand is given by*

$$\theta_i^*(L_i, P, \underline{\mu}, \underline{\sigma}) = \begin{cases} \frac{\underline{\mu} - P}{2\gamma P^2} & \text{if } \underline{\mu} > P \text{ and } L_i > \phi_1(P) \\ 0 & \text{if } \underline{\mu} + \sigma \leq P \text{ or } (L_i > \phi_2(P) \text{ and } \underline{\mu} \leq P \leq \underline{\mu} + \sigma) \\ \frac{\underline{\mu} + \underline{\sigma} - P}{2\gamma P^2} & \text{else} \end{cases}$$

where

$$\phi_1(P) = \frac{(\underline{\mu} + \underline{\sigma} - P)^2 - 2(\underline{\mu} - P)^2}{4\gamma P^2}$$

and

$$\phi_2(P) = \frac{(\underline{\mu} + \underline{\sigma} - P)^2}{4\gamma P^2}.$$

Defining $\phi(P) :=$

$$\begin{cases} \phi_1(P) & \text{if } \underline{\mu} > P \\ \phi_2(P) & \text{if } \underline{\mu} \leq P \end{cases}$$

the optimal demand can be rewritten as

$$\theta_i^*(P) = \begin{cases} \max\left(0, \frac{\underline{\mu} - P}{2\gamma P^2}\right) & \text{if } \phi(P) - L_i < 0 \\ \max\left(0, \frac{\underline{\mu} + \underline{\sigma} - P}{2\gamma P^2}\right) & \text{if } \phi(P) - L_i \geq 0 \end{cases}$$

Proof. Maximizing the two quadratic functions separately yields

$$\theta_{i_full}^* = \frac{\underline{\mu} - P}{2\gamma P^2}$$

for the fully liable case and

$$\theta_{lim}^* = \frac{\underline{\mu} + \underline{\sigma} - P}{2\gamma P^2}$$

for the limitedly liable case. The first result is that for $P \geq \underline{\mu} + \underline{\sigma}$ the optimal demand is 0. For $P \leq \underline{\mu}$ the fully liable optimum is always better than demanding 0 units of the risky asset. Plugging in the two maximizers and comparing the maxima yields that the fully liable choice yields higher expected returns if and only if

$$L_i \geq \frac{(\underline{\mu} + \underline{\sigma} - P)^2}{4\gamma P^2} - \frac{(\underline{\mu} - P)^2}{2\gamma P^2}.$$

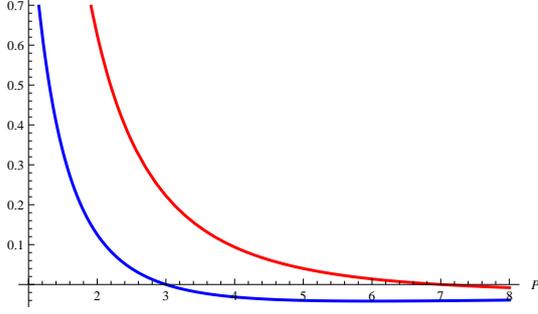
Hence, in this case the limitedly liable optimum is the global optimum if $L_i \leq \phi_1(P)$. The last case is the case of $\underline{\mu} \leq P \leq \underline{\mu} + \underline{\sigma}$. In this case at most the limitedly liable optimum might be the global one. The necessary condition to yield higher expected returns than holding 0 risky assets is then given by $L_i < \phi_2(P)$ \square

An overview about the optimal choices can be found in the following table:

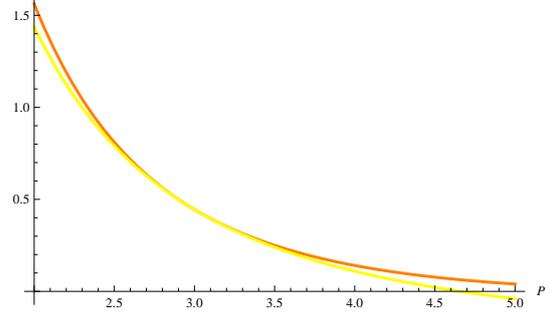
	$P \leq \underline{\mu}$	$\underline{\mu} \leq P \leq \underline{\mu} + \underline{\sigma}$	$\underline{\mu} + \underline{\sigma} \leq P$
$L_i < \phi_1(P)$	<i>lim</i>	<i>lim</i>	0
$\phi_1(P) < L_i < \phi_2(P)$	<i>full</i>	<i>lim</i>	0
$\phi_2(P) < L_i$	<i>full</i>	0	0

The last column is 0 as the price is higher than an asset's expected marginal return, even in the case of limited liability. For low prices, we find that the limitedly liable optimum yields higher returns if the equity is sufficiently low. The medium column has a similar interpretation. At most the limitedly liable optimum can yield a better payoff than nonparticipation since $P \geq \underline{\mu}$. This is only the case if equity is low enough.

2 THE MODEL

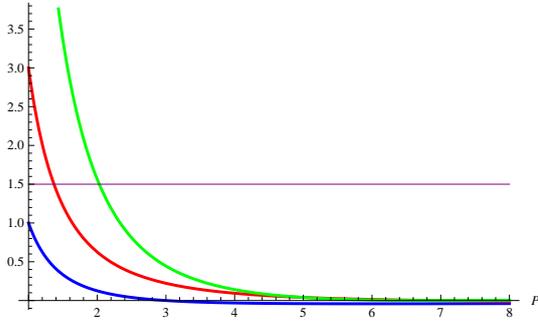


The two local optimizers $\theta_{i_{full}}^*$ (blue) and $\theta_{i_{lim}}^*$ (red) depending on the price

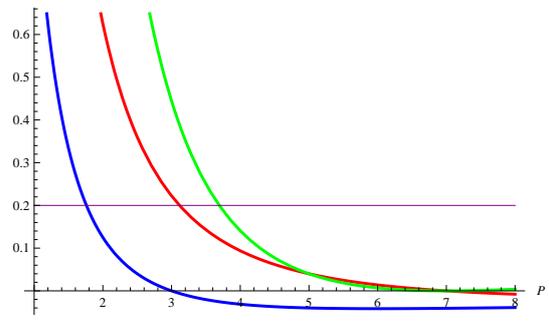


The two sufficient conditions for optimality $\phi_1(P)$ (yellow) and $\phi_2(P)$ (orange)

The left picture above shows $\theta_{i_{full}}^*$ (blue) and $\theta_{i_{lim}}^*$ (red) depending on the price. The right picture shows $\phi_1(P)$ (yellow) and $\phi_2(P)$ (orange). The curve ϕ_2 always lies above ϕ_1 and they have a common tangential point at $P = \underline{\mu}$. The overall relevant decision rule ϕ is given by ϕ_1 until the tangential point and by ϕ_2 afterwards. If equity is below ϕ , the 'lim' case is the relevant optimal amount, else the 'full' case is the optimal amount. In the following picture the optimal demands depending on the price are plotted together with the conditions $\phi(P)$ (green) that compare which optimizer yields higher expected return (ϕ_1) and whether the 'lim' optimizer yields higher expected return than holding 0 risky assets (ϕ_2):



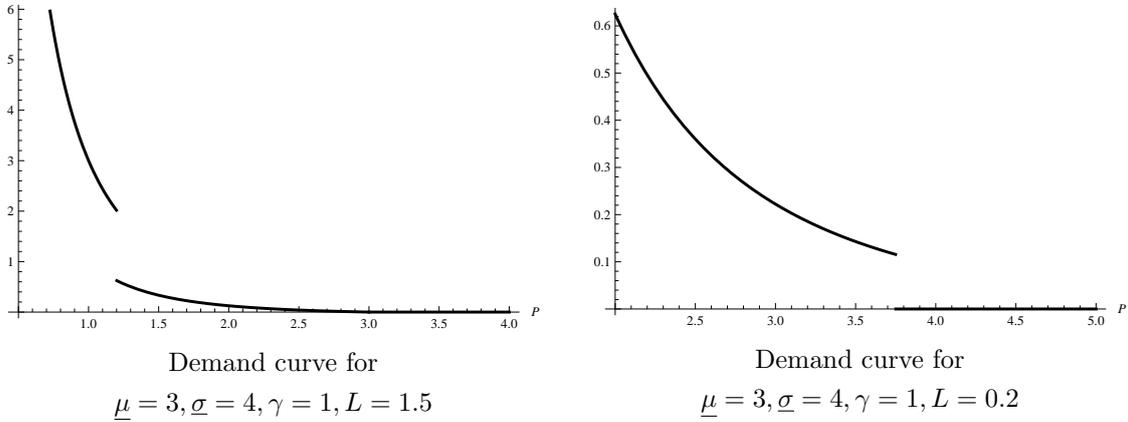
$\theta_{full}^*, \theta_{lim}^*$ with ϕ for
 $\underline{\mu} = 3, \underline{\sigma} = 4, \gamma = 1, L = 1.5$



$\theta_{full}^*, \theta_{lim}^*$ with ϕ for
 $\underline{\mu} = 3, \underline{\sigma} = 4, \gamma = 1, L = 0.2$

In both pictures above, the red curve is the optimal demand until ϕ (green) and equity (purple) cross. Afterwards the fully liable optimizer (blue) gives the optimal demand. Overall demand is bounded by 0. These conditions yield a discontinuous demand curve. The according demand curves for the risky asset depending on the

price P are given in the following graphics:



We see that the demand is decreasing but also discontinuous with respect to the price. This is due to the risk/liquidity punishment and the optimizer jumping from the 'lim' case to the 'full' case when increasing the price. Under certain market conditions, especially if equity is low, the optimal demand might directly jump to zero from the 'lim' case when prices increase. If the equity is low, it crosses ϕ at a very high price. If this price is above $\underline{\mu}$, the demand directly goes to zero. If the equity is high, the crossing will occur at a price where the 'full' case still yields a strictly positive optimizer. The aggregate demand with two agents might, thus, even have two jumps. For an exogenous supply of risky assets it is not always possible to clear the market. In the equilibrium analysis I will further investigate the impact of equity and ambiguity on market clearing.

3. Equilibrium Analysis

Many different equilibrium outcomes might arise with respect to which agent participates and whether they demand an amount where they can fully cover losses or not, depending on the parameters of the model. While the most striking results are fairly general even in this setup, I will, nonetheless, start with a full characterization of all possible equilibrium outcomes in order to describe the model completely.

3.1. A full Characterization

I define an equilibrium as a pair of optimal demand θ^* and a price P^* such that the market clearing condition

$$\theta_1^*(L_1, P^*) + \theta_2^*(L_2, P^*) = \bar{x}$$

is satisfied. Several equilibria might occur in this setup. A single agent might clear the market with an amount that puts him at a possible loss exposure he cannot fully cover. I will denote this equilibrium with the subscript l . For market clearing via an agent who can fully cover possible losses I use the subscript f . For the cases where both agents demand positive amounts, I use the subscripts ll , ff and lf to distinguish whether these agents demand volumes such that both cannot fully cover losses, both are able to fully cover losses or the combined case where only one of the agents can bear all losses. Having the market clearing condition, one can calculate the equilibrium prices for the cases. The equilibrium prices for the respective cases are:

$$\begin{aligned} \bullet P_l &= -\frac{1}{4\gamma\bar{x}} + \sqrt{\frac{1}{16\gamma^2\bar{x}^2} + \frac{\mu+\sigma}{2\gamma\bar{x}}} \\ \bullet P_f &= -\frac{1}{4\gamma\bar{x}} + \sqrt{\frac{1}{16\gamma^2\bar{x}^2} + \frac{\mu}{2\gamma\bar{x}}} \\ \bullet P_{fl} &= -\frac{1}{2\gamma\bar{x}} + \sqrt{\frac{1}{4\gamma^2\bar{x}^2} + \frac{\mu+\sigma/2}{\gamma\bar{x}}} \\ \bullet P_{ff} &= -\frac{1}{2\gamma\bar{x}} + \sqrt{\frac{1}{4\gamma^2\bar{x}^2} + \frac{\mu}{\gamma\bar{x}}} \\ \bullet P_{ll} &= -\frac{1}{2\gamma\bar{x}} + \sqrt{\frac{1}{4\gamma^2\bar{x}^2} + \frac{\mu+\sigma}{\gamma\bar{x}}} \end{aligned}$$

I denote the respective equilibria with E and the according subscript. In order to identify which equilibrium will occur, I start with the following result:

Proposition 3.1. *E_f is impossible.*

Proof. As we can see in the table for a participant to demand a positive amount of risky asset that leaves him fully liable we must have $P \leq \underline{\mu}$. Also in the table we

can see that for this price any participant would demand a strictly positive amount of the risky asset. Hence, E_f is not possible. \square

We proceed with the following result.

Proposition 3.2. *A necessary condition for E_l is $\underline{\mu}^2 \leq \frac{\sigma}{2\gamma\bar{x}}$ and a necessary condition for E_{fl} is $\underline{\mu}^2 \geq \frac{\sigma}{2\gamma\bar{x}}$.*

Proof. In E_l it is necessary to have $P_l \geq \underline{\mu}$. If this was not the case, the second agent would demand a strictly positive amount. In a similar fashion in E_{fl} we must have $P_{fl} \leq \underline{\mu}$. These requirements yield the above conditions and show that with respect to the parameters these two types of equilibria are mutually exclusive.⁶

\square

In the following I will give a full characterization of the equilibria that arise depending on the given parameters. Afterwards, I will discuss the results with a focus on the case where one agent has high equity and the other agent has low equity. This represents the case where a fraction of banks in the market is stressed while others fulfill high equity requirements. Without loss of generality I will assume $L_1 \leq L_2$ for the rest of the paper.

Theorem 3.1. • *Case 1: Let $\underline{\mu}^2 < \frac{\sigma}{2\gamma\bar{x}}$*

- *If $L_1, L_2 \leq \phi_2(P_u)$ then the unique market equilibrium is of type E_u*
- *If $L_1 \leq \phi_2(P_l) \leq L_2$ then the unique market equilibrium is of type E_l*
- *If $L_1, L_2 \geq \phi_1(P_{ff})$ then the unique market equilibrium is of type E_{ff}*
- *It is $0 < \phi_2(P_u) < \phi_2(P_l) < \phi_1(P_{ff})$*

• *Case 2: Let $\frac{\sigma}{2\gamma\bar{x}} < \underline{\mu}^2 < \frac{\sigma}{\gamma\bar{x}}$*

- *If $L_1, L_2 \leq \phi_2(P_u)$ then the unique market equilibrium is of type E_u*

⁶except for the case $\underline{\mu}^2 = \frac{\sigma}{2\gamma\bar{x}}$

3 EQUILIBRIUM ANALYSIS

- If $L_1 \leq \phi_1(P_{fl}) \leq L_2$ then the unique market equilibrium is of type E_{fl}
- If $L_1, L_2 \geq \phi_1(P_{ff})$ then the unique market equilibrium is of type E_{ff}
- It is $0 < \phi_2(P_u) < \phi_1(P_{fl}) < \phi_1(P_{ff})$

• Case 3: Let $\underline{\mu}^2 > \frac{\sigma}{\gamma\bar{x}}$

- If $L_1, L_2 \leq \phi_1(P_u)$ then the unique market equilibrium is of type E_u
- If $L_1 \leq \phi_1(P_{fl}) \leq L_2$ then the unique market equilibrium is of type E_{fl}
- If $L_1, L_2 \geq \phi_1(P_{ff})$ then the unique market equilibrium is of type E_{ff}
- It is $\phi_1(P_u) < \phi_1(P_{fl}) < \phi_1(P_{ff})$

In addition, it is

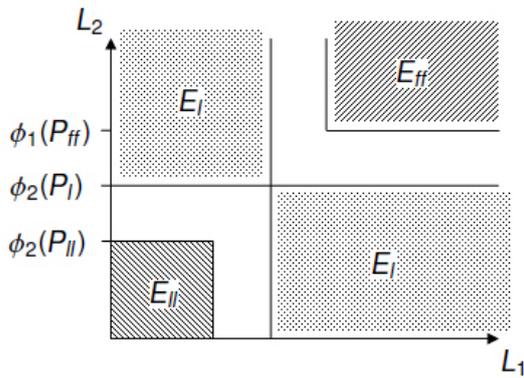
$$\phi_1(P_{fl}) = \phi_2(P_l)$$

for $\underline{\mu}^2 = \frac{\sigma}{2\gamma\bar{x}}$ and

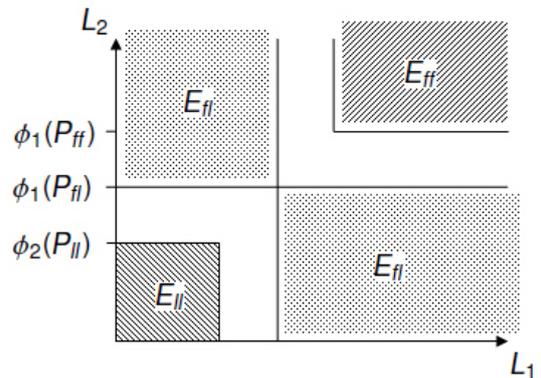
$$\phi_1(P_u) = \phi_2(P_u)$$

for $\underline{\mu}^2 = \frac{\sigma}{\gamma\bar{x}}$.

Proof. The proof can be found in the appendix. □



Types of equilibria depending on the equity of the two agents for $\underline{\mu}^2 \leq \frac{\sigma}{2\gamma\bar{x}}$.



Types of equilibria depending on the equity of the two agents for $\frac{\sigma}{2\gamma\bar{x}} \geq \underline{\mu}^2 \geq \frac{\sigma}{\gamma\bar{x}}$.

Although the full characterization shows that the kind of equilibrium that will arise heavily depends on the parameters of the model, general results can be pointed out. If both agents have fairly high equity at their disposal, the unique equilibrium is of type E_3 . This means that if banks fulfill high equity requirements, in equilibrium they will demand amounts of risky assets so that they are able to bear any possible losses attached to the demanded position. On the other hand, if both banks have equity that is close to 0, the equilibrium outcome is one where both banks cannot fully cover losses in case of a bad state outcome. The most interesting case is the one where one bank has fairly high equity while the other bank's equity is close to zero. In this case it depends on the amount of perceived ambiguity whether both banks will hold risky stock or whether only one bank will hold all available amounts of the risky asset.

Corollary 3.1. *$\exists a, b \in \mathbb{R}$ such that for $L_1 \leq a$ and $L_2 \geq b$ a unique equilibrium exists. The low equity agent participates with certainty, not being able to cover all losses.*

The participation of the high equity agent is subject to ambiguity and is discussed in the comparative statics section.

The results that $\phi_1(P_{fl}) = \phi_2(P_l)$ for $\underline{\mu}^2 = \frac{\sigma}{2\gamma\bar{x}}$ and $\phi_1(P_u) = \phi_2(P_u)$ for $\underline{\mu}^2 = \frac{\sigma}{\gamma\bar{x}}$ show the smoothness of the borders of the equilibrium areas when going from case 1 to 2 and from case 2 to 3. The blank areas show equity combinations for which no equilibrium exists. We can state the following corollary for a policy that demands high equity requirements.

Corollary 3.2. *$\exists a \in \mathbb{R}$ such that for $L_1, L_2 \geq a$ a unique equilibrium exists and the unique equilibrium guarantees that agents can cover losses completely.*

I will conclude the equilibrium analysis with comparative statics to show how ambiguity influences market participation.

3.2. Comparative Statics

When I talk about an increase of ambiguity in expected returns or volatility, I understand a symmetric increase of the sets

$$\mathcal{P}_2 = \{\lambda \in M_1(\mathbb{R}) \mid \lambda((-\infty, \underline{\mu})) = 0 \wedge \lambda((\bar{\mu}, \infty)) = 0\}$$

and

$$\mathcal{P}_3 = \{\lambda \in M_1(\mathbb{R}) \mid \lambda((-\infty, \underline{\sigma})) = 0 \wedge \lambda((\bar{\sigma}, \infty)) = 0\}.$$

More ambiguity in μ means an increase of $\bar{\mu}$ as well as a decrease of $\underline{\mu}$. The same holds for σ when I talk about volatility. Let us start with a situation of a stressed bank and a bank that is well equipped in terms of equity. Let $\underline{\mu}^2 > \frac{\sigma}{2\gamma\bar{x}}$. We then observe an equilibrium of type E_{fl} . Both banks hold positive amounts of risky assets. If there now was a sufficient increase in ambiguity about μ , such that $\underline{\mu}^2 < \frac{\sigma}{2\gamma\bar{x}}$, equilibrium E_l would be observed and only the stressed bank would hold all the amount of risky stock available. We can clearly identify that more ambiguity in the mean return might make agents in the market not participate.⁷ Moreover, this kind of ambiguity might lead to the clustering of risks at already stressed institutions. The result of an increase in ambiguity about returns leading to less participation can also be found in Easley and O'Hara as well as Cao, Wang and Zhang. On the other hand, in this model the opposite happens when ambiguity about the volatility is increased. Assume L_1 is sufficiently low and L_2 is sufficiently high. Again it is easy to see from our equilibrium analysis that, if we are in E_l , a sufficient increase in volatility ambiguity leads to equilibrium E_{fl} . This means that more perceived volatility ambiguity gets back well equipped banks into the market for risky or even toxic assets.

Proposition 3.3. $\exists L_1, L_2$ such that the following holds: Let parameters be given such that the unique equilibrium is of type E_l . A sufficient increase in volatility uncertainty yields a unique equilibrium of type E_{fl} .

⁷It might be necessary to increase the ambiguity in volatility simultaneously to stay within the model setup. Still the result holds that an increase in return uncertainty might make agents, who participated before, not participate.

Proof. Let σ_1 and σ_2 be such that $\underline{\mu}^2 < \frac{\sigma_1}{2\gamma x}$ and $\underline{\mu}^2 > \frac{\sigma_2}{2\gamma x}$. Then $\exists L_{11}, L_{21}$ such that a unique equilibrium of type E_l exists and $\exists L_{12}, L_{22}$ such that the unique equilibrium is of type E_{fl} . Choosing $L_1 = \min(L_{11}, L_{12})$ and $L_2 = \max(L_{21}, L_{22})$ guarantees the result for $\underline{\sigma}$ moving from σ_1 to σ_2 . \square

What has to be pointed out is that the perceived ambiguity is the same for all market participants in this model. While in Easley and O'Hara only one of the two types of agents was affected by a variation of ambiguity and, hence, directly made the evaluations of the risky asset even more distinct, in my model both types of agents are affected in the same way by a change of ambiguity. Only the effect of their distinct levels of equity leads to different evaluations. In the case of volatility uncertainty this, then, leads to the complete opposite reaction; i.e. making evaluations of the risky position more even among the heterogeneous types of agents.

Remark 3.1. *In equilibrium E_{fl} we have the local results*

$$\frac{\delta\theta_2^*}{\delta\underline{\sigma}} < 0,$$

$$\frac{\delta\theta_1^*}{\delta\underline{\sigma}} > 0.$$

This remarks shows that not only a sufficient increase in volatility ambiguity leads to the entrance of the high equity bank into the market, but also that a further increase leads to a distribution of risky assets away from the stressed bank to the high equity bank.

Remark 3.2. *In equilibrium we have the local results:*

$$\frac{\partial P^*}{\partial \underline{\mu}} > 0$$

$$\frac{\partial P^*}{\partial \underline{\sigma}} \geq 0$$

The intuition is that a decrease of the worst case volatility decreases the value of the free put option and therefore the valuation for the low equity agent while

there is no influence on the valuation for the high equity agent. Therefore, in the aggregate, evaluations decrease and, thus, the equilibrium price decreases. For the worst case return it is even more clear. A decrease decreases the valuation of both agents and therefore the price. Thus, for little variations in ambiguity that do not change the type of equilibrium an increase in ambiguity in either dimension has the same effect.

An increase of prices due to an increase in ambiguity is a well discussed phenomenon as the existence of an ambiguity premium in addition to the well known risk premium is assumed to be a driving factor of stock prices (see Ui (2010) or Cao, Wang and Zhang (2005)).

4. Discussion

4.1. Entropic Risk Punishment

The driving feature for the results of chapter 3 are the shape and the behavior of the demand function. Having two local maxima and guaranteeing that the right maximum dominates for low equity and the left maximum dominates for high equity is the key for my results. I will show in the following that with the entropic risk measure as a cost function I can obtain a similar demand function.

4.1.1. The Entropic Risk Measure

The entropic risk measure for a bounded random variable X on a probability space (Ω, F, P) is given by

$$\rho(X) = \frac{1}{\beta} \log(E_P(e^{-\beta X}))$$

with $\beta > 0$. It is a widely discussed risk measure as it satisfied the most reasonable requirements on a risk measure such as monotonicity, cash-invariance and (quasi-)convexity. The relaxation of fulfilling convexity rather than positive homogeneity

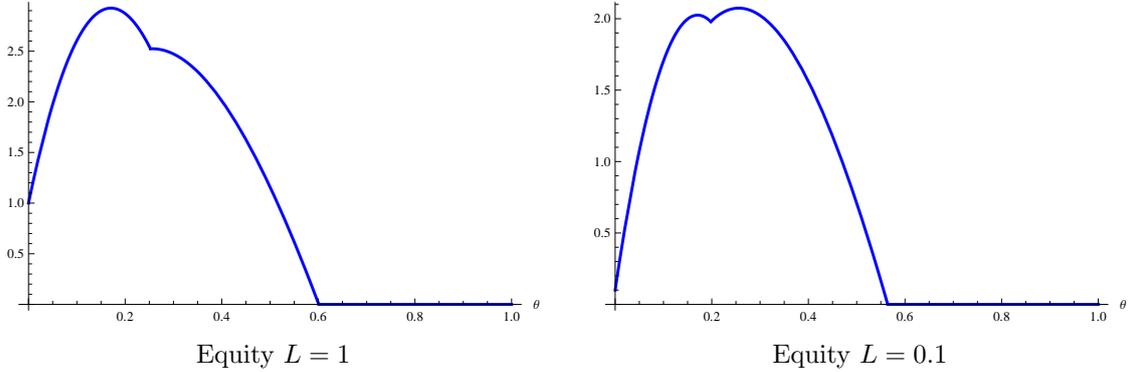
compared to coherent risk measures allows non-trivial solutions in the this model.

4.1.2. Demand Function

Assume now that risk is measured in terms of the entropic risk measure and punished by a cost factor c . The objective for agent i is thus

$$\max_{\theta_i \geq 0} \min_{Q \in \mathcal{P}} E[\max(0, \theta_i(X - P) + L_i - \frac{c}{\beta} \log(E_P(e^{-\beta X})))].$$

The risk punishment is convex and its limit unit punishment for an additional unit of risky asset is given by $-c(\underline{\mu} - \underline{\sigma})$. Hence, for large c this problem is non-trivial and due to the convexity of the entropic risk measure we have a similar behavior as before: The expected return of an additional unit of risky asset net its price has a positive jump when losses cannot fully be covered anymore. The risk punishment is concave and increasing for large demands. This leads to the possible occurrence of two local maxima that depend on the equity of the agent. Below, the objectives are plotted for high and low equity with entropic risk punishment for $\mu = 3; P = 1; \sigma = 10; \beta = 0.2; c = 7$:



The use of risk punishment via other appropriate risk measures in this model is also possible without severely distorting the results. The most important demands on a risk measure to be appropriate for risk controlling with respect to this model is its convexity. If it is strictly convex, the risk measure alone might leave this setup non-trivial. For the more restricted class of coherent risk measures, positive

homogeneity is no drawback to the quality of results as long as other assumptions keep the problem finite.

4.2. Risk Measures as Cost Function

The incorporation of a risk measure as a cost function is without any doubt a feature that is quickly at hand when modeling portfolio decisions. Banks are obliged to fulfill high standards of risk monitoring and the outcome gives feedback on solvency capital or equity requirements. As equity is costly from the perspective of a financial institute, risk directly causes costs. A key feature of the above model is that agents differ in their equity. When evaluating the cost for risk or liquidity, equity is not taken into account. Especially high equity might be a reason for reducing risks or getting better conditions on liquidity demands. The evaluation of the portfolio is, thus, made in a way of narrow framing. Still, the idea of the approach holds true as equity in this model might be identified with 'soft' values like reputation and brand value. These are not to be taken into account for the portfolio problem as these values only exist as long as the portfolio does not default. Moreover, these values are only taken into account by the operator of the business for internal analyses rather than by a regulatory framework. In this sense the use of risk measures as a cost function without the incorporation of the heterogeneity in 'equity' is still meaningful.

4.3. Heterogeneous Beliefs

The formation of the set of priors depends on many factors. As described before, the interpretation of this set is not trivial. Nonetheless, the amount of available information and what an individual can read out of it might influence the set of priors significantly. As not all people have the same access to information or the same ability to draw conclusions out of given information, it is very likely that the set of priors may be different for different individuals. I will briefly discuss how different beliefs might affect the results of this model.

4.3.1. Heterogeneity in Expected Return

In this section I investigate which equilibria will occur if the perceived ambiguity between agents varies. Therefore, I start with dispersion in the mean return keeping the volatility fixed; i.e. let $\underline{\mu}_2 > \underline{\mu}_1$ and $\underline{\sigma}_2 = \underline{\sigma}_1 = \underline{\sigma}$. Agent 2 perceives less ambiguity than agent 1. Keeping the average amount of ambiguity fixed, talking about more ambiguity dispersion means increasing $\underline{\mu}_2$ and decreasing $\underline{\mu}_1$ by the same amount. The following necessary conditions for the possibility of equilibria hold:

Proposition 4.1. *Necessary conditions for the different types of equilibria are given by:*

- E_f : $\underline{\mu}_2 \geq \underline{\mu}_1 + 2\gamma\bar{x}\underline{\mu}_1^2$
- E_l : $\underline{\mu}_2 \geq \underline{\mu}_1 + 2\gamma\bar{x}\underline{\mu}_1^2 - \underline{\sigma}$
- E_{ll} : $\underline{\mu}_2 \leq \underline{\mu}_1 + 2\gamma\bar{x}(\underline{\mu}_1 + \underline{\sigma})^2$
- E_{ff} : $\underline{\mu}_2 \leq \underline{\mu}_1 + 2\gamma\bar{x}\underline{\mu}_1^2$
- E_{fl} : $\underline{\mu}_2 \leq \underline{\mu}_1 + 2\gamma\bar{x}\underline{\mu}_1^2 - \underline{\sigma}$

Corollary 4.1. *High ambiguity dispersion with respect to the mean return is a cause for nonparticipation.*

Proof. The proof of this proposition as well as sufficient conditions are given in the appendix. □

The result has to be read in the following way: The more disperse the perceived ambiguity is, only equilibria with single agents participating are possible to occur. This result shows that, with respect to the mean return, uneven information or knowledge distribution within society might be a factor that causes agents not to hold any risky assets and, thus, cause risk clustering. I will investigate the impacts of volatility uncertainty dispersion in the following chapter.

4.3.2. Heterogeneity in Perceived Volatility

Assume now that $\underline{\mu}_2 = \underline{\mu}_1 = \underline{\mu}$ but the perceived volatility levels may be distinct. Perceived volatility does not influence the equilibrium prices for the types of equilibria where no agent is gambling on going bankrupt. Therefore, with undispersed expected return ambiguity, E_f is not possible for the same reasons as in chapter 3. E_{ff} is possible with the same price as in chapter 3, only the necessary conditions for the minimum amount of required equity have to be fulfilled separately according to the individual perceived volatility level. E_{ll} is possible if equity is low enough for both agents and the price corresponds to the price of chapter 3 with the average worst case ambiguity level.

The most interesting cases, once again, are the equilibria E_l and E_{fl} . In chapter 3 E_l was possible with agent 1 holding all the risky assets as I assumed $L_1 \leq L_2$. Keep the average amount of ambiguity fixed and recall that necessary conditions were given by the relation of $\underline{\mu}^2$ and $\frac{\sigma}{2\gamma\bar{x}}$. The transition from E_l to E_{fl} was possible for an increase in volatility ambiguity if equity was disperse enough. Denote the average worst case volatility with $\underline{\sigma} = \frac{\sigma_1 + \sigma_2}{2}$.

If $\underline{\mu}^2 > \frac{\sigma}{2\gamma\bar{x}}$, E_{fl} is possible and E_l is not. For the case of no dispersion, sufficient conditions are given in chapter 3. If dispersion arises, the equilibrium price for this type of equilibrium is not affected. Only the necessary and sufficient conditions with respect to equity might change. A change to an equilibrium with a single participant is not possible.

If $\underline{\mu}^2 < \frac{\sigma}{2\gamma\bar{x}}$, E_l is possible and E_{fl} is not. Again, for the case of no dispersion, sufficient conditions can be found in chapter 3 and the low equity agent holds all the risky assets. If dispersion arises, the equilibrium price is affected via the perceived volatility of the agent that holds all the risky assets. The main difference in this case is that both, the high and the low equity agent, might be the single participant in the market, depending on both, the different levels of perceived ambiguity and the different levels of equity. If the low equity agent perceives less ambiguity, he can be the only one to hold all the risky assets with sufficient equity conditions. The reasoning is that due to both, the lower ambiguity as well as the lower equity, the

low equity agent will always evaluate the asset higher than the high equity agent. If the low equity agent perceives more volatility ambiguity, the high equity agent might as well be the single participant. The equilibrium, nonetheless, stays unique. Assume that parameters were that equilibria with both agents were possible. For agent 2 it must then hold that

$$L_2 > \phi_2(\sigma_2, P_l(\sigma_1))$$

if agent 1 holds all the risky assets as well as

$$L_2 < \phi_2(\sigma_2, P_l(\sigma_2))$$

if agent 2 holds all the risky assets. Since ϕ_2 is decreasing in P these conditions cannot hold at the same time and the equilibrium must be unique. Still, the high equity agent might be the one holding all the risky assets in the case of $\underline{\sigma}_1 < \underline{\sigma}_2$ if $\underline{\sigma}_2$ is sufficiently large. Therefore, more dispersion might change the market participant in this case. A numerical example is given in the following: Let $x = 1; \gamma = 1; \underline{\mu} = 1; L_1 = 1; L_2 = 4$. Let the dispersion be given by $\underline{\sigma}_1 = 6 - \delta$ and $\underline{\sigma}_2 = 6 + \delta$. One can show that for $\delta = 0.5$ agent 1 will hold all the risky assets, while for $\delta = 4$ agent 2 will hold all the risky assets. Hence, with this new phenomenon of a shift of the market participant due to more ambiguity it is possible to distribute risky assets away from the low equity agent to the high equity agent. This phenomenon only occurs if perceived ambiguity dispersion is high and the high equity agent perceives significantly less ambiguity than the low equity agent.

4.4. Policy Implications

The general consensus of many theoretical findings is that a reduction of ambiguity leads to better performances of the market. Easley and O'Hara point out that nonparticipation caused by ambiguity imposes costs on the economy via the effect on the equity premium. More general, all nonparticipation results necessarily lead to a clustering of risks at the remaining market participants. This, especially in times

with stressed markets and many troubled institutes, is not desired from a welfare perspective. Governmental interventions during the last financial crisis showed that, especially in times where several banks are low on equity, one of the main focuses is freeing the balance sheets of low equity banks from risky or toxic assets. While many models already pointed out the benefits of reducing mean uncertainty in order to achieve a better risk distribution among institutes, this cannot be transferred to reducing volatility uncertainty. Reducing ambiguity can generally be achieved via higher reporting standards. Higher standards lead to better information and reduce the set of priors, especially when this set of priors arises from robust statistics. But pure information on its own is not the the only major influence on perceived sets of priors. Especially for naive investors a better knowledge of how to use information in order to make the right investment decision is a substantial part for transferring the increased information to a reduction of the set of priors. Hence, educating investors on financial markets is a key feature to reduce the set of priors. From our findings, we can see that increasing the information about expected future returns should be the focus of reporting standards. Supplying more information and, thus, reducing ambiguity in this dimension leads to a better distribution of risk within the society. On the other hand, we have seen that volatility uncertainty in markets might be desirable from a risk distribution perspective, especially when there are banks with low equity. The liability point of view is new to nonparticipation models and shows that ambiguity in markets might have a positive effect. Hence, when there are many troubled banks, reducing uncertainty in this dimension might not be the target of governmental policy. Especially when running programs as TARP or PPIP, sufficient volatility uncertainty might be crucial for these programs to achieve the desired effect. Higher uncertainty in volatility leads to less overpricing of toxic assets by stressed banks, which makes it easier for subsidized governmental or private partnership asset buying programs to relief toxic assets from stressed banks' balance sheets. But not only effectiveness might be achieved as higher efficiency makes these programs much cheaper through the reduced overpricing. When running these programs, governments clearly define (toxic) assets that are eligible for subsidized buying. In this sense one can identify for which assets the above mentioned reporting standard policies are useful.

Having seen that more dispersion of perceived ambiguity between the agents might also be a cause for nonparticipation, governmental policies should aim at an even information distribution as well as at an even knowledge standard with respect to financial markets, in order to establish a similar perceived ambiguity between all possible market participants. This is a key feature to make more agents participate and points at the importance of an even information distribution and its use, which is in any case desirable, rather than the sheer amount of available information. Regarding volatility uncertainty it is not as important to achieve a uniform information distribution but to provide sufficient information to high equity institutes.

5. Conclusion

Most models that combine ambiguity and nonparticipation find that more ambiguity leads to less participation. In this model I have shown that ambiguity in mean return and volatility can have different effects on market participation. While more ambiguity in the mean return might lead to less participation, which is a result of many former models, more ambiguity about the volatility might lead to more agents participating in the market, which is a new result. Moreover, more volatility uncertainty leads to a better distribution of risk between the agents. The most important driving feature is the incorporation of limited liability. Heterogeneous agents, differing in their equity, evaluate risky assets differently. More volatility uncertainty makes these different evaluations more even. Therefore, overpricing via the free put option gets less and more agents might participate in the market. The jumps in the optimal demand depending on the price might cause a nonexistence of equilibrium as the aggregate demand is discontinuous. More volatility uncertainty reduces the range where there is no equilibrium. While uncertainty is often considered to be harmful to markets and welfare, I have shown that some ambiguity in markets might have a positive effect and therefore, ambiguity might even be desirable. For recent governmental interventions ambiguity facilitates the effectiveness and might even make required governmental subsidies much cheaper. As not all agents necessarily perceive the same amount of ambiguity, I investigate that a more diverse perception

of ambiguity might also be a cause for nonparticipation. This is in line with the findings of Cao, Wang and Zhang and recommends an even information distribution among society.

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Part III.

Bonuses, Bailouts and Creditor Coordination

1. Introduction

The discourse about the appropriate size and structure of manager bonuses is one of the most discussed topics in the delegated portfolio choice literature. While in many cases it is without any doubt useful to incentivize a manager to exert more effort in order to achieve a better performance, many observers called the continued bonus payout in times where a lot of banks were struggling into question. This criticism got enhanced when troubled institutes had to be saved by governments and became the subject of a bailout policy. In the view of many the government subsidies were directly redistributed to the managers bonus payments. But not only the managers were seen as the profiteers of governmental intervention as the banks themselves benefited by several taken measures.

The delegated portfolio choice problem is a special kind in the field of principal-agent problems. While in the most simple principal-agent setting an agent has private information about his effort choice and the principal can base his payment only on an observed outcome, in the delegated portfolio choice problem there is a second dimension of asymmetric information. The agent privately chooses his effort first as well as the structure of the portfolio afterwards while the principal is still only able to observe the outcome of the portfolio. Although a high incentive in most cases might induce the agent to exert more effort which is desired by the principal, a too big sized bonus might be harmful to the portfolio structure choice.

Given that most working contracts contain a payment structure that is bounded from below and moreover the simple fact that every agent is effectively limitedly liable, it is obvious that not only a mean preserving spread, but, from a classic portfolio perspective even worse, a mean decreasing and variance increasing transformation of the portfolio might be desired by the agent and induced by high bonus payments.

In current literature the incorporation of these two private choices of the agent is often approached in the following way. The effort choice is a choice in two dimensions, risk and return, and perfectly separable.

Knowing about the two dimensions of asymmetric information in this internal agency problem between the principal and the agent, it is worthwhile noticing that, as many banks raise money from creditors, there also exists an external agency problem as creditors might observe and anticipate the payment contracts offered by the principal. If creditors observe an incentive scheme that induces a higher default probability, they will charge a higher interest rate on their loans in order to compensate the increased default probability.

Anticipating a bailout strategy by the government, the creditors might demand a lower interest rate on their loans supporting a default increasing incentive setting. While the negative effects of a bailout strategy due to moral hazard are quick off the mark, there are well known reasons for doing so. The first reason is the one of a domino effect. Financial institutes are highly interconnected to not only other financial institutes but also nearly every company in the economy. A bankruptcy of an institute, especially if it is a large one, might be followed up by several more bankruptcies of connected companies. The second reason can broadly be defined as confidence. While there are several works on how important confidence in the financial system is in order for it to work optimally and sustain crises, a special characteristic of the lack of confidence can be seen in the problem of coordination failure between creditors. Coordination failure might cause premature foreclosure of credits or denial of refinancing. Often creditors have invested in a project that is on the verge of failing and a necessary condition for the project success is that sufficient credits are prolonged. For the creditors this decision is not obvious. If enough creditors stay in the project, it is best to stay because it is likely to get back the full loan and interest. If many creditors leave the project, it is better to leave early with a reduced return because the project is likely to fail eventually.

I present a simple model that incorporates the internal principal agent problem, the external problem with creditors, a bailout strategy by the government and the problem of creditor coordination. My findings show first that bonus contracts arise endogenously in the principal agent framework. There is an important difference whether a bank faces a classical effort problem or a risk shifting problem. While both problems lead to a bonus contract, the size of the bonus may be either too high or too

low from a social perspective, depending on the type of the problem. A high bailout probability of the bank will push the bonus size and the managerial effort further away from the desired state. The reasoning behind this is that a bailout strategy primarily influences the creditors in case of default. The creditors therefore might charge lower returns on their loans. While in both problems a shift of probability mass away from the default state becomes less attractive the two problems differ in their incentive strategy. In the classical case higher incentives shift probability away from the default state, while in the risk shifting problem incentives increase this probability. Given this, a cap on manager bonuses, as it is a recent topic in theoretical analysis and political discussion, may thus only be appropriate if it is used in the right context or if there are additional measures that can enforce such a context. The question of financing a bailout strategy comes at hand. While for modeling purposes it is often assumed that a bailout is financed externally or by an indirect tax, one might at least identify the benefit recipients of such a policy in order to address a possible taxation strategy. However, findings show that a bailout strategy might be overall harmful to the net value of the whole problem and there are no profiteers. The multiplicity of creditors plays an important role in delegated portfolio problems if there is a coordination problem. I show that multiplicity of creditors might push the size of the bonus in the opposite direction as a bailout strategy for both types of effort problems. Further, I show that for low bailout probabilities, there is no effect of changing the bailout probability if there is a single creditor, while there is a coordination effect in the multiple creditor case. For high bailout probabilities the effects coincide independent of the number of creditors.

There is vast literature in the field of delegated portfolio management. Holmstrom and Milgrom (1987) analyze the optimal structure of manager compensation. They find that a linear contract might be the optimal contract. However, many findings show that linear contracts are not optimal since an underinvestment in effort can be observed in many models. A famous example is the work of Admati and Pfleiderer (1997) who show that an agent cannot be incentivized to the benefit of the principal. While Starks (1987) argues that symmetric contracts are desirable, it is often difficult to implement appropriate punishment for bad outcomes into working contracts. Most payment contracts are bounded from below and two types of contracts are

predominant. The simple bonus contract that pays a bonus if a benchmark is reached and the call type compensation contract that pays a fixed amount and a share of the profits if profits exceed a certain benchmark. Even if the principal claims big losses from the agent according to the compensation contract, current volumes and risks of portfolios can quickly lead to a loss the agent is not able to bear. Ross (2004) and Gollier (1997) analyze the distortion of risk taking by a convex compensation scheme and find that there are multiple effects. Ross shows that a convex compensation scheme not necessarily increases the risk taking as scale effects might also effect the expected utility, while Gollier shows that mean variance inefficient portfolios in the classical sense might be selected. John and John (1993) show that a compensation scheme might be able to reestablish the first best level of risk taking.

Empirically, there are different results on risk taking behavior. While Houston and James (1995) find no excessive risk taking incentives in the financial sector compared to other sectors, Cheng, Hong and Scheinkman (2010) report a significant connection. Besley and Ghatak (2013) analyze an optimal bonus contract and optimal taxation if the manager is able to influence the risk and the variance of a portfolio. The importance of creditors is barely analyzed in the literature of delegated portfolio choice. Recently, Hakenes and Schnabel (2014) incorporate creditors into the agency problem and analyze the effect of bailouts. The main feature is that creditors are able to observe the bonus contract and anticipate it via the demand for the interest rate on their loans, hence, becoming an active part of the agency problem. They find that the possibilities of influencing return and risk lead to different results in the agency problem.

Creditors often face the problem of coordination when engaged in an investment. Coordination and the selection of equilibria is an important field of game theory and has a close link to the value of debt and default. While in a coordination game there are classically multiple equilibria, the selection of the right one is a big field of research. Harsanyi and Selten (1988) introduce risk dominance in order to justify the selection of a certain equilibrium. When it is observed that in the same problem different equilibria are chosen with a certain frequency, a stream of literature models this with so called 'sunspots'. The assumption is that an extrinsic, non fun-

damental variable influences expectations and thus, the selection of an equilibrium. Cass and Shell (1983) introduce sunspots into economic theory and Diamond and Dybvig (1983) use this approach in order to justify equilibrium selection in a coordination problem in financial markets. A more endogenous approach of multiple possible equilibrium selection in coordination games is opened up by the field of global games. Using higher order beliefs and incomplete information the selection of a single equilibrium based on the outcome of an endogenous random variable can be justified. The seminal work is by Carlsson and van Damme (1993) and a general approach can be found in Morris and Shin (2001). Applications to financial markets are widespread. Goldstein and Pauzner (2005) model the equilibrium selection in the case of a bank run. The advantage of the global game approach is that the driving random variable for the equilibrium selection is a feasible one and thus equilibrium selection probabilities can be specified. Morris and Shin (2004) employ a global games framework on the problem of creditor coordination in order to price debt.

The paper is structured as follows. In section 2, I describe the driving forces of the model. I give the setup and timing of the problem between principal(bank), agent(manager) and creditors. A closer look has to be taken at the difference of the cases of a single creditor and multiple creditors. Chapters 3 and 4 specify the managerial effort and discuss the classical effort problem and the risk shifting problem, respectively. The main interest of my analysis are effects on bonus sizes, expected utility, default probabilities and welfare. In section 5, I discuss the closely related paper of Hakenes and Schnabel, the appearance of both types of effort problems and alternative effort modeling. Section 6 concludes.

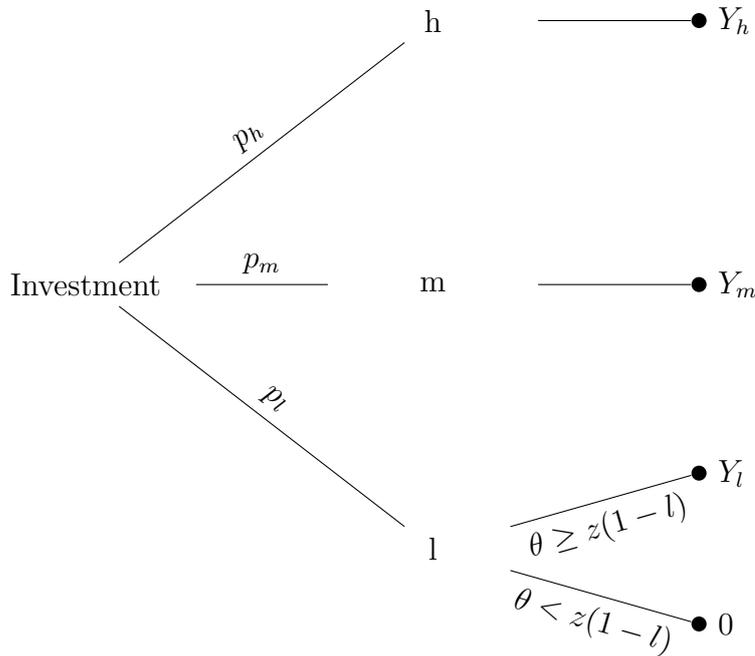
2. The Model

2.1. The Model Setup

Consider a three stage model with $t = 0, 1, 2$. A bank is endowed with equity k and has to collect a loan L from creditors in $t = 0$ in order to run a desired risky investment that yields some payoff in $t = 2$. The interest rate for this loan is contractible and specified later. In $t = 1$ the creditors have the possibility to review their investment given the state of both, the project and the economy. The loan can either be provided until $t = 2$ or the creditors have the possibility to withdraw a collateral K instead.⁸ The investment can only be operated by the manager and the success of the project is subject to the effort the manager exerts. I assume that both the bank and the manager are limitedly liable; i.e. the bank is only liable with respect to its equity while the manager cannot be punished for losses by employing a negative wage. All decision makers in this model are assumed to be risk neutral. The states of the world in $t = 1$ are given by $\Omega = \Omega_1 \times \Omega_2$ where $\Omega_1 = \{h, m, l\}$ and $\Omega_2 = \mathbb{R}$. Ω_1 denotes the possible states for the project in $t = 1$ and Ω_2 are the possible states of the overall economy. The states regarding the project can take four values in $t = 1$ according to the states h, m, l : $Y_h, Y_m, (Y_l, 0)$ with $Y_h > Y_m > Y_l > 0$ while the state of the economy in $t = 1$ is described by θ with continuous distribution function F and support on \mathbb{R} . I assume that project performance and overall economic performance can be separated perfectly and, hence, the state of the project is independent from the state of the economy. In the high state the project will be successful and yield Y_h in $t = 2$. In the medium state the project will be successful and yield Y_m in $t = 2$. For the low state it is not certain whether the project will yield Y_l or fail and yield 0. In l the success of the project depends on two things; the state of the overall economy $\theta \in \mathbb{R}$ and whether the creditors are willing to stay in the project. If the state of the economy is below 0, the project will fail for sure, while if the state of the economy is higher than $z > 0$, the investment is a sure success. For values in $(0, z)$ the project success depends on

⁸I generally refer to K as the value of the collateral in $t = 2$

the provision of liquidity by the creditors. Let R_1 denote the contractible interest rate for the loan given the state is high or medium and R_2 the exogenously given interest rate in case of a success in the low state. The payoff for the creditors is the following: In the states h and m they receive R_1 while in state l they receive R_2 if the project is successful and they stay, 0 if they stay and the project fails.⁹ If they quit, they receive K in any case. The outside option for creditors is given by the market interest rate $r > 0$.



The decision rule in the low state is explained in chapter 2.3.

⁹Keeping R_2 exogenous simplifies the problem and prevents excessive interest rate agreements. Further R_2 is assumed to be sufficiently small in order to guarantee a positive R_1

2.2. The Timing of the Model

- In $t = 0$ the bank offers a contract to the manager
- The creditors observe the offer and the bank repays an exogenously given amount R_2 if the project is in the low state and successful, creditors demand rate R_1 for the states h and m
- The bank collects money L from creditors and invests
- The manager chooses effort¹⁰
- In $t = 1$ the state of the project and the economy is revealed, the agent receives payment based on the state of the project, the creditors decide whether to foreclose the loan or to stay in the project
- In $t = 2$ project success is revealed and loans plus interest are paid back if possible

2.3. The Coordination Game

In $t = 1$ the creditors have to decide whether to stay in the project or to foreclose the loan and receive some collateral K where I assume that $R_2 > K > 0$. While the project success depends on the one hand on the state θ , it also depends on the amount of liquidity that is provided in $t = 1$. The decision for a creditor in the single creditor case depends only on θ . In the multiple creditor case the project success and thus the creditors's payoffs also depend on the decision of the other creditors. We assume that the amount of required liquidity depends in a negative way on the state of the economy; The better the state of the economy is, the less liquidity is required. In both cases, if θ is below 0 the project will fail with certainty, while if the state of the economy is higher than z the investment is a sure success. In between the amount of provided liquidity is of importance and the project will be successful

¹⁰The outside option for the manager is set to 0, hence, he always chooses to participate in the project

if $z(1 - l) < \theta$ where $1 - l$ denotes the share of loans that are not prolonged and z is a parameter that indicates in how far the project is sensitive to credit prolongation and its volume. In the global games literature this parameter is often interpreted as 'the severity of disruption caused by the inability to coordinate'.

2.3.1. The Single Creditor Case

If the loan is provided by a single creditor the optimal behavior of the creditor is determined as follows. If the signal is greater 0 the creditor will prolong the loan since this guarantees project success and $R_2 > K$. If $\theta \leq 0$ the project will fail with certainty and he chooses to foreclose his loan and take K as $K > 0$. Thus, in the single creditor case the low state outcome is a random variable that takes the values 0 and Y_l with probability $F(0)$ and $1 - F(0)$, respectively.

2.3.2. The Multiple Creditor Setup

In the multiple creditor case the payoff of each creditor depends on θ and the decisions of all players. I assume a continuum of creditors $[0, 1]$ and the project success is given as before. Let $u(x, l, \theta)$ denote the payoff for a creditor depending on his action $x \in \{0, 1\}$, where 1 means that the loan is rolled over, while 0 means that the creditor quits the project. The share of creditors that roll over the loan is given by $l \in [0, 1]$ and the state of the economy is given by θ . The payoff for the creditors is then:

$$\bullet u(1, l, \theta) = \begin{cases} R_2 & \text{if } z(1 - l) \leq \theta \\ 0 & \text{if } z(1 - l) > \theta, \end{cases}$$

$$\bullet u(0, l, \theta) = K$$

Given this payoff structure a creditor wants to stay in the project if the fraction of creditors that stay is high enough and quit if the fraction is too low. This is a well known phenomenon when dealing with multiplicity of creditors. The coordination game has two pure strategy equilibria and the selection of one of these equilibria

has a vast literature. Assume now that the creditors observe a noisy signal of θ ; $y_i = \theta + \varepsilon_i$. The ε_i are i.i.d. and normally distributed with mean 0 and standard deviation σ . This is a coordination game with incomplete information and is part of the literature of global games.

2.3.3. Global Games

The literature of global games was started 1993 by Carlsson and van Damme. They showed that in a 2 player coordination game a small amount of noise in privately observing an economic fundamental leads to a unique strategy selection. Morris and Shin (2001) adopted to this. Similarly to the previously described coordination problem two investors evaluate whether to invest or not invest while receiving a noisy signal with normally distributed noise. The decisive underlying fundamental θ in their case has an improper uniform distribution on the real line. As only conditional distributions are relevant for the analysis of this problem there is no drawback in this assumption (see Hartigan 1983). Specifically, the payoff to both investors is θ if both invest. If an investor does not invest he gets 0 irrespectively of what the other investor does. If an investor invests, but the other does not, his payoff is $\theta - 1$. In this sense, the optimal behavior would be clear if there was no noise and θ was not in $[0, 1]$. If θ was greater 1, investing would be optimal, if θ was smaller 0 not investing would be the choice. In between two pure Nash equilibria exist, but the selection is not clear. Assume now that the standard deviation of the independent noise terms ε_i is σ . When player i observes $x_i = \theta + \varepsilon_i$ the updated belief about the distribution of the other player's observation is a normal distribution with a mean of x and standard deviation $\sqrt{2}\sigma$. Morris and Shin then analyze switching strategies at some threshold k ; i.e. if a player's signal $x_i > k$ he will invest, else he will not. Assume now player 1 observes x_1 and believes that his opponent switches at k . His expectation for θ will be x_1 and the expected probability for player 2 observing a signal less than k is $\Phi\left(\frac{k-x}{\sqrt{2}\sigma}\right)$.¹¹ For the case that $x_1 = k$ he assumes his counterpart to be investing or not investing equally likely. Let us now take a look at the switching strategy at $\frac{1}{2}$ for both players. It is easy to see that this strategy

¹¹ Φ denotes the standard normal distribution function

is an equilibrium. If player 1 receives signal x_1 his expected payoff is

$$x_1 - \Phi\left(\frac{\frac{1}{2} - x_1}{\sqrt{2}\sigma}\right).$$

The structure of this payoff is increasing in x_1 and breaking 0 exactly if $x_1 = \frac{1}{2}$. Therefore, investing is optimal exactly if $x_1 \geq \frac{1}{2}$. In fact Morris and Shin show that switching at $\frac{1}{2}$ is the only strategy surviving iterated deletion of strictly dominated strategies. Assume player 2 switches at k . Define the best response of a switching point for player 1 with $b(k)$ as the unique solution for x given the equation:

$$x - \Phi\left(\frac{k - x}{\sqrt{(2)}\sigma}\right) = 0.$$

$b(k)$ is strictly increasing and takes values between 0 and 1 with $b(0) > 0$ and $b(1) < 1$. Moreover, there is exactly one fixpoint $k = \frac{1}{2}$. Define now the strategy

$$s(x) = \begin{cases} \text{invest} & \text{if } x > b^{n-1}(1) \\ \text{not invest} & \text{if } x < b^{n-1}(0) \end{cases}$$

for n rounds of iterated deletion of dominated strategies. For $n = 1$ and if player 2 never invests it is optimal for player 1 to invest if $x_1 \geq 1$ and, conversely, we get that not invest is optimal if $x_1 < 0$. Proceeding by induction and simultaneously for both players, if s yields the only remaining strategies after n rounds of deletion, we have that player 1 will not invest if his signal was below $b^n(0) = b(b^{n-1}(0))$ given that he knows that player 2 will (in the worst case) not invest if he observes a signal lower than $b^{n-1}(0)$. Hence, with every round of deletion the interval where there is no dominated strategy becomes smaller. With b being strictly increasing and having the only fix point at $\frac{1}{2}$, both values $b^{n-1}(1)$ and $b^{n-1}(0)$ tend to $\frac{1}{2}$ for n becoming large. Therefore, $\frac{1}{2}$ is the uniquely selected switching point.

With this approach, Morris and Shin (2001) give more general results for games with finitely and infinitely many players and more general payoff functions. The fashion of the logics behind stays the same. Only the payoffs are adjusted depending

on the amount of players that invest. Players then consider a posterior belief about the amount of other players that observe a signal in order to make them invest. Again, there are unique best replies for high and low observed noisy values of θ and iterated deletion of dominated strategies with higher order beliefs leaves the agents with a single switching threshold. One important difference when going to infinitely many players is that the precision of the signal plays an important role. If precision gets high enough, the existence of a unique equilibrium is guaranteed. Moreover, in the limit of the signal becoming infinitely precise, the failure probability of the investment project can be determined depending on the distribution of the fundamental variable which, for instance, can be used to price debt (see Morris and Shin 2004).

2.3.4. The Multiple Creditor Case

Returning to the credit prolongation problem of section 2.3.2 and taking a look at the limit case as information becomes infinitely precise, I define $\pi(l, \theta) = u(1, l, \theta) - u(0, l, \theta)$

$$\bullet \pi(l, \theta) = \begin{cases} R_2 - K & \text{if } z(1-l) \leq \theta \\ -K & \text{if } z(1-l) > \theta, \end{cases}$$

$$\bullet \int_{l=0}^1 \pi(l, \theta) dl = \begin{cases} -K & \text{if } \theta \leq 0 \\ R_2 - K & \text{if } z \leq \theta \\ \frac{\theta R_2}{z} - K & \text{else .} \end{cases}$$

Theorem 2.1. *The unique switching point is given by $\theta^* = \frac{Kz}{R_2}$.*

- *Proof.* The proof is a special case of Morris and Shin 2001. In order to make their result applicable, I have to ensure that the following assumptions are satisfied.
 - $\pi(l, \theta)$ is non decreasing in l
 - $\pi(l, \theta)$ is non decreasing in θ

- $\exists! \theta^*$ with $\int_{l=0}^1 \pi(l, \theta^*) dl = 0$
- There exist $\underline{\theta} \in \mathbb{R}, \bar{\theta} \in \mathbb{R}$ and $\varepsilon > 0$ such that $\pi(l, \theta) \leq -\varepsilon$ for all $l \in [0, 1]$ and $\theta \leq \underline{\theta}$ and $\pi(l, \theta) > \varepsilon$ for all $l \in [0, 1]$ and $\theta \geq \bar{\theta}$
- $\int_{l=0}^1 g(l) \pi(l, x) dl$ is continuous with respect to x and weakly continuous with respect to a density g
- $\int_{z=-\infty}^{\infty} z g(z) dz$ is well defined.

The first two points are satisfied as R_2 is positive and the third point is also true only for the case $\theta^* = \frac{Kz}{R_2}$. The fourth point is satisfied since $K > 0$ and $R_2 - K > 0$. The fifth point is satisfied as the payoff function has only a single jump. The sixth point is satisfied as we have chosen a normal distribution. We can thus use the results of Morris and Shin 2001. As σ becomes sufficiently small the optimal strategy for the creditor is to roll over the loan exactly if $x_i \geq \frac{Kz}{R_2}$ as the unique switching point is determined via the unique solution of $\int_{l=0}^1 \pi(l, \theta^*) dl = 0$. Morris and Shin 2004 show that in the limit of σ going to 0 the state where the project is on the verge of failing coincides with $\theta^* = \frac{Kz}{R_2}$ and the project will succeed if and only if $\theta > \theta^*$. \square

We see that compared to the single creditor case we have a coordination loss in the lower state. The value of this coordination loss can be specified as $(F(\frac{Kz}{R_2}) - F(0))Y_l$. Government measures to reestablish coordination might thus have a welfare enhancing effect if there is a coordination problem.

3. The Effort Case

Assume now that the manager can shift probabilities in the following way:

$$p_h(e) = p_h^0 + e$$

$$p_m(e) = p_m^0$$

$$p_l(e) = p_l^0 - e$$

with $p_h + p_m + p_l = 1$ and effort e such that all probabilities are within $(0, 1)$. The manager can exert effort to shift probability mass from the low state to the high state at a cost $c(e) = \frac{1}{2}\eta e^2$ with $\eta > 0$ and gets a compensation w_h, w_m or w_l by the principal depending on the outcome of the project. When exerting this kind of effort the probability distribution is shifted to another one that first order stochastically dominates the former one. The cost can be seen as the cost for investigating different investment opportunities with decreasing improvement as more alternatives are analyzed. As we assume principal and agent to be limitedly liable we have that $w_h, w_m, w_l \geq 0$.

The manager wants to maximize his expected utility

$$p_h w_h + p_m w_m + p_l w_l - \frac{1}{2}\eta e^2.$$

3.1. A Single Creditor

I assume that there is a contractible repayment R_1 for the high and medium state with payoffs Y_h and Y_m large enough such that we have an interior solution and R_1 can be refunded, while the repayment R_2 in the low state is exogenously given and repayable in case of Y_l . Due to limited liability, 0 is paid back in the case of default. Keeping R_2 in the low state exogenous simplifies the problem. If the interest rate for the low state was contractible and even paid back in case of a bailout, exorbitant interest rates would distort the problem severely. For the bailout case the government will repay the nominal value L . The common expected bailout probability is denoted by β . The risk neutral creditor participates if the expected return of the project is at least the market return $1 + r$

$$(1 + r)L = (p_h + p_m)R_1 + p_l(1 - F(0))R_2 + p_l F(0) \max(K, \beta L)$$

The maximum arises from the fact that the creditor will leave the project early for some collateral of value K as long as the bailout probability is not high enough. If the critical point $\beta = \frac{K}{L}$ is reached, the default value for the creditor is increasing.

Any bailout strategy below this value has no effect in the single creditor problem. Above this value, a further increase of β will make the creditor demand less R_1 . In general, the kink is no drawback in the optimization problem. The demanded minimal return R_1 can thus be calculated as

$$R_1 = \frac{(1+r)L - p_l(1-F(0))R_2 + p_l F(0) \max(K, \beta L)}{p_h + p_m}.$$

We assume that the creditors participate in the project if the expected return equals the market interest rate. The principal wants to maximize his expected profits

$$\Pi = p_h(Y_h - R_1 - w_h) + p_m(Y_m - R_1 - w_m) + p_l(1 - F(0))(Y_l - R_2 - w_l).$$

Lemma 3.1. $w_l^* = w_m^* = 0$

Proof. Since the payments to the manager have to be nonnegative and the project return might be 0 in the low state, we must have that $w_l^* = 0$. As the probability of the medium state cannot be influenced by the agent it is optimal for the principal to set $w_m^* = 0$. \square

In the following we will denote w_h by w .

The optimal effort decision for the agent given the bonus payment w is then

$$e^* = \frac{w}{\eta}.$$

We can see that higher bonuses induce more effort while a higher cost factor η reduces the effort level. The optimal bonus payment for the principal is

$$w^* = \max \left[0, \frac{1}{2}(Y_h - \eta p_h - (1 - F(0))Y_l - F(0)(\max(K, \beta L))) \right]$$

and the optimal effort is given by

$$e^* = \max \left[0, \frac{1}{2\eta}(Y_h - \eta p_h - (1 - F(0))Y_l - F(0) \max(K, \beta L)) \right].$$

If the high state has a sufficiently high return, the optimal bonus is always positive. On the other hand, if effort is sufficiently costly, the optimal bonus and effort are 0. Later we will see that too small values of η also do not admit interior solutions as they might push the probabilities to the boundary of 0 or 1. We can also already observe that a bailout strategy by the government might lead to a flat compensation scheme of 0.

Proposition 3.1. *In an interior solution of the single creditor effort problem and if $\beta \geq \frac{K}{L}$ an increase in bailout probability*

- *decreases the manager bonus*
- *decreases the effort of the manager*
- *increases the default probability of the bank*
- *decreases the expected utility of the manager.*

If $\beta < \frac{K}{L}$ an increase in bailout probability has locally no effect.

Proof. Taking the derivative of w^* with respect to β at an interior solution yields

$$-\frac{1}{2}F(0)L < 0.$$

Hence the manager bonus is decreasing in expected bailout probability. As a direct consequence the effort is decreasing since $e^* = \frac{w}{\eta}$. Having the default probability $F(0)(p_l^0 - e^*)$, the bank's default probability is increasing in expected bailout probability. The agent's expected utility is given by

$$\left(p_h^0 + \frac{w}{\eta}\right)w - \frac{1}{2}\eta\left(\frac{w}{\eta}\right)^2 = p_h^0w + \frac{w^2}{2\eta}$$

which is increasing in w . Hence, the expected return for the agent is decreasing in increased bailout probability. The case $\beta < \frac{K}{L}$ is trivial since $\max(K, \beta L)$ stays constant. \square

The rationale behind this is that a bailout strategy by the government indirectly

increases the value of the low state. As the creditor might get bailed out in the low state, he demands less interest for his loan and the punishment for the principal for not inducing effort to shift probability away from the low state gets weaker. Therefore, the principal has less reasoning for incentivizing the agent to exert effort. As a consequence, wage and effort decrease.

3.2. Multiple Creditors

In the multiple creditor case creditors face the former mentioned coordination problem. Incorporating an expected bailout probability modifies the coordination problem of section 2.3.2 by

$$\bullet u(1, l, \theta) = \begin{cases} R_2 & \text{if } z(1-l) \leq \theta \\ \beta L & \text{if } z(1-l) > \theta, \end{cases}$$

$$\bullet u(0, l, \theta) = K$$

In case of a default the expected return for the creditors is now βL instead of 0. If $\beta L \geq K$ there is no coordination problem as staying in the project is always the optimal choice for the creditors. If $\beta L < K$ our assumptions of section 2 are again satisfied and we have that

$$\hat{\theta} = \frac{(K - \beta L)z}{R_2 - \beta L}.$$

Thus, define

$$\theta^* = \begin{cases} \frac{(K - \beta L)z}{R_2 - \beta L} & \text{if } \beta L < K \\ 0 & \text{if } \beta L \geq K. \end{cases}$$

Hence, when increasing the bailout probability from 0 to $\frac{K}{L}$ we decrease the coordination loss from $Y_l \left(F \left(\frac{Kz}{R_2} \right) - F(0) \right)$ to 0. The coordination loss depending on β is given by

$$Y_l \left(F \left(\frac{(K - \beta L)z}{R_2 - \beta L} \right) - F(0) \right).$$

The creditors participation constraint is now

$$(1 + r)L = (p_h + p_m)R_1 + p_l(1 - F(\theta^*))R_2 + p_lF(\theta^*) \max(K, \beta L).$$

The demanded minimal return R_1 is then given by

$$R_1 = \frac{(1 + r)L - p_l(1 - F(\theta^*))R_2 + p_lF(\theta^*) \max(K, \beta L)}{p_h + p_m}.$$

The principal wants to maximize expected profits

$$\Pi = p_h(Y_h - R_1 - w) + p_m(Y_m - R_1) + p_l(1 - F(\theta^*))(Y_l - R_2).$$

The optimal bonus payment for the principal is

$$w^* = \max \left[0, \frac{1}{2} (Y_h - \eta p_h^0 - (1 - F(\theta^*))Y_l - F(\theta^*) \max(K, \beta L)) \right]$$

and the optimal effort is given by

$$e^* = \max \left[0, \frac{1}{2\eta} (Y_h - \eta p_h^0 - (1 - F(\theta^*))Y_l - F(\theta^*) \max(K, \beta L)) \right].$$

This time we have two different effects of an increase of a bailout policy. Starting at 0 the value θ^* gets shifted from $\frac{Kz}{R_2}$ to 0 until β reaches the kink value $\frac{K}{L}$. Until reaching this value, the project collapses if θ is below this value and else succeeds. After reaching this point, credits will always be prolonged as the expected bailout value dominates the collateral.

Proposition 3.2. *In an interior solution of the multiple creditor effort problem an increase in bailout probability*

- *decreases the manager bonus*
- *decreases the effort of the manager*
- *decreases the expected utility of the manager.*

Proof. Again, taking the derivative of w^* with respect to β at an interior solution yields

$$-\frac{1}{2}F(\theta^*)L < 0.$$

Hence, the manager bonus is decreasing in expected bailout probability and so is the effort. As seen before the agents expected utility is increasing in w . Thus, the expected return for the agent is decreasing in increased bailout probability. \square

The qualitative effects on effort and bonus payments of the multiple creditor case can also be found in the single creditor case, at least if bailout probabilities are sufficiently high. Only the effect on default probability becomes ambiguous when introducing a coordination problem. We will analyze the effect of multiplicity of creditors in the following.

3.3. The Effect of Multiplicity of Creditors

Having analyzed the optimal manager bonuses as well as the implications of a bailout strategy for both cases we can now take a look at what the impact of multiplicity of creditors is compared to a single creditor. I will use the subscripts MC and SC for the multiple creditor case and the single creditor case, respectively.

Proposition 3.3. *The multiple creditor case wage is at least the single creditor case wage $w_{MC}^* \geq w_{SC}^*$. Especially, if there is no coordination problem (i.e. $\beta L \geq K$) we have $w_{MC}^* = w_{SC}^*$ and if there is a coordination problem we have $w_{MC}^* > w_{SC}^*$.*

Proof. The optimal wages only differ in the term of the distribution function of θ

$$w_{MC}^* = \frac{1}{2} (Y_h - \eta p_h - (1 - F(\theta^*))Y_l - F(\theta^*) \max(K, \beta L))$$

$$w_{SC}^* = \frac{1}{2} (Y_h - \eta p_h - (1 - F(0))Y_l - F(0) \max(K, \beta L))$$

It is easy to show that the first part of the term dominates $\max(K, \beta L)$

$$\begin{aligned} w_{SC}^* - w_{MC}^* &= (1 - F(\theta^*))Y_l - (1 - F(0))Y_l + F(\theta^*) \max(K, \beta L) \\ &\quad - F(0) \max(K, \beta L) \\ &= (F(\theta^*) - F(0))(\max(K, \beta L) - Y_l) \leq 0. \end{aligned}$$

While the former term is at least 0, the latter term is negative. The former term is 0 exactly when there is no coordination problem and positive if there is a coordination problem. \square

Corollary 3.1. *The multiple creditor case effort is at least the single creditor case effort $e_{MC}^* \geq e_{SC}^*$. Especially, if there is no coordination problem (i.e. $\beta L \geq K$) we have $e_{MC}^* = e_{SC}^*$ and if there is a coordination problem we have $e_{MC}^* > e_{SC}^*$.*

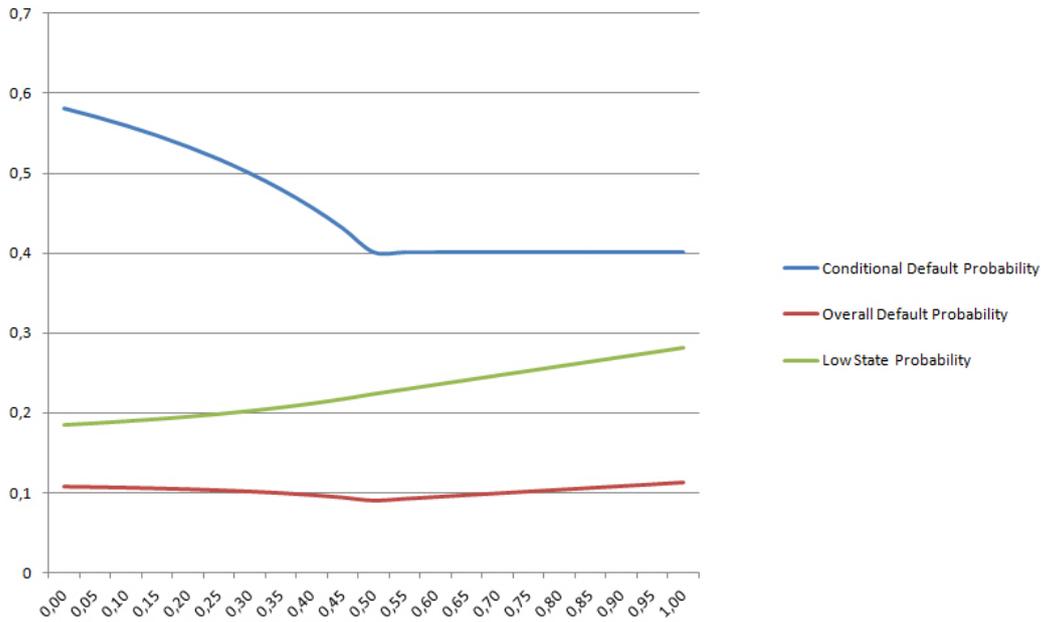
We see that multiplicity of creditors increases the manager bonus if there is an effort problem in the agency setting. The mechanism behind this is that multiplicity of creditors devalues the lower state due to the coordination problem. The difference between the high and the low state increases which makes it more valuable for the principal to invest in incentivizing a probability shift away from the low state to the high state. Multiplicity of creditors has the opposite effect of a bailout policy since the bailout policy aims at enhancing the value of the low state. Nonetheless, we observe a bonus decrease in both settings with respect to an increased bailout probability. The effect of an increased bailout probability on bonuses in the multiple creditor case can even be observed below the point of full coordination where both scenarios are identical. Below the critical value, the coordination effect is the driving force which is only influencing the problem in the multiple creditor case. Above the critical value, there is no coordination effect and we have exactly the same behavior in both cases. An important difference lies in the effect of the bailout strategy with respect to the failure probability of the project. While in the single creditor model bailout interventions have a clear negative effect on the default probability, in the multiple creditor case this effect is ambiguous. For low β the lower wage induces a probability shift from the high state to the low state. On the other hand the conditional probability at the low state is shifted away from default. The effect on

3 THE EFFORT CASE

the failure probability heavily depends on the structure of the distribution function of θ . Any increase in bailout probability above $\frac{K}{L}$ only has the negative effect of an increase in default probability. Whether an increase in bailout probability increases or decreases the default probability up to the critical point depends on the severeness of the coordination problem. Especially if the ratio $\frac{F\left(\frac{Kz}{R_2}\right)}{F(0)}$ gets large, the coordination problem is severe.

Proposition 3.4. $\exists c > 0$: if $F(0) < c$ then the default probability for $\beta = \frac{K}{L}$ is lower than for $\beta = 0$.

Proof. Denoting the bailout case $\beta = \frac{K}{L}$ with BO the default probabilities are given by $(p_l^0 - w_{MC})F\left(\frac{Kz}{R_2}\right)$ and $(p_l^0 - w_{BO})F(0)$. Both default probabilities are positive. Taking the limit in the bailout case of $F(0)$ towards 0 the default probability tends to 0. Hence, a lower default probability for the bailout case can be observed than for the case where $\beta = 0$. \square



The default probability depending on beta

In the graphic we see that the overall default probability is decreasing in expected bailout probability for low expected bailout probabilities since the coordination effect

in the low state is stronger than the probability shifting effect to the low state caused by moral hazard. The kink is the point where $\beta = \frac{K}{L}$ and we have reached full coordination. From that point on, any increase in bailout probability only has the effect of probability shifting to the low state.

3.4. Welfare Analysis and Policy Implications

Having analyzed multiple effects of this simple model we will take a look at policy implications. The main question from a welfare perspective apart from the failure probability is whether the net value of this project increases or decreases.

3.4.1. Single Creditor

A social planner who wants to maximize the net value faces the problem of maximizing

$$(p_h^0 + e)Y_h + (p_m^0)Y_m + (p_h^0 - e)(1 - F(0))Y_l - \frac{1}{2}\eta e^2.$$

The optimal effort he would implement is then given by

$$e_{FB}^* = \frac{Y_h - (1 - F(0))Y_l}{\eta}.$$

Comparing this to the second best effort at an interior solution

$$e_{SB}^* = \frac{1}{2\eta}(Y_h - \eta p_h - (1 - F(0))Y_l - F(0) \max(K, \beta L))$$

we can state that the agency problem causes underinvestment of effort. As the objective for the social planner is a quadratic function of effort, even lower effort, as caused by a bailout policy, exacerbates the underinvestment problem. Since governmental costs of the bailout are just a redistribution in this model we observe a worsening of the net value of the project. Having analyzed the net value it is natural to ask whether single parties of this model benefit from a bailout. We have seen in the previous section that the expected utility for the agent is decreasing. As the creditor

in this model is always kept at the break even point for participating, he neither benefits nor is a bailout to his detriment. A bailout policy is not desired by the social planner as it has negative net value effects and worsens the underinvestment problem.

3.4.2. Multiple Creditors

In the multiple creditor case the optimal effort he would implement is then given by

$$e_{FB}^* = \frac{Y_h - (1 - F(\theta^*))Y_l}{\eta}.$$

Comparing this to the second best effort at an interior solution

$$e_{SB}^* = \frac{1}{2\eta}(Y_h - \eta p_h - (1 - F(\theta^*))Y_l - F(\theta^*) \max(K, \beta L)),$$

again, as in the single creditor case, we observe an underinvestment of effort compared to the first best case.

If we compare the difference of the first best effort to the second best effort for the single creditor case and the multiple creditor case, we see that the underinvestment problem is more severe in the multiple creditor case. In this sense, the coordination failure problem not only has the classical direct effect on welfare via the coordination loss, but further an indirect effect that worsens the effort choice problem in this principal agent setting.

4. The Risk Effort Case

In contrast to the preceding chapter I assume that the manager can shift probability mass from the medium state to both, the high and the low state and vice versa. The idea is that a manager usually is not only able to influence the return but also the variance of a portfolio. It is often criticized that managed portfolios do not

outperform benchmarks and managers, instead of acting in the desired way of the principal, even increase risk in order to maximize bonus payments. The controlling of the risk choice is often not possible for the bank or investor. Assume now that the manager can shift probabilities in the following way:

$$p_h(a) = p_h^0 + a$$

$$p_m(a) = p_m^0 - (1 + \kappa)a$$

$$p_l(a) = p_l^0 + \kappa a$$

with $p_h + p_m + p_l = 1$, $\kappa > 0$ and effort a such that all probabilities are within $(0, 1)$. As the aim is to model an increase in the variance, an appropriate choice of κ would create a mean preserving spread of the original distribution.

Proposition 4.1. *If Y_l is 0, $\kappa = \frac{Y_h - Y_m}{Y_m}$ yields a mean preserving spread. For positive Y_l , $\kappa = \frac{Y_h - Y_m}{Y_m - (1 - F(0))Y_l}$ yields a mean preserving spread in the full coordination case and decreases the mean in the other cases.*

As the value in the lower state depends on the coordination of creditors we can not choose a parameter such that we create a mean preserving spread in any case. The choice of $\kappa = \frac{Y_h - Y_m}{Y_m - (1 - F(0))Y_l}$ ensures that we do not have an increase in return and is a safe way to separate our results from the effects of the previous section where the mean was increased by managerial effort. The manager can exert effort to shift probability mass at a cost $c(a) = \frac{1}{2}\alpha a^2$ and as in the previous chapter gets a compensation w_h, w_m or w_l by the principal depending on the outcome of the project. The agent remains limitedly liable such that we have $w_h, w_m, w_l \geq 0$.

The manager wants to maximize his expected utility $E[u(a)] = p_h w_h + p_m w_m + p_l w_l - \frac{1}{2}\alpha a^2$.

4.1. A Single Creditor

The break even condition for the creditors and the demanded return are the same as in the previous section and the principal wants to maximize

$$\Pi = p_h(Y_h - R_1 - w_h) + p_m(Y_m - R_1 - w_m) + p_l(1 - F(0))(Y_l - R_2 - w_l).$$

As in Hakenes and Schnabel we set $w_l^* = w_m^* = 0$. In the following we will denote w_h by w . The optimal effort decision for the agent given the bonus payment w is then

$$a^* = \frac{w}{\alpha}.$$

The optimal bonus payment for the principal is

$$w^* = \max \left[0, \frac{1}{2}(Y_h - \alpha p_h - (1 + \kappa)Y_m + (1 - F(0))\kappa Y_l + \kappa F(0)(\max(K, \beta L)) \right]$$

and the optimal effort is given by

$$a^* = \max \left[0, \frac{1}{2\alpha}(Y_h - \alpha p_h - (1 + \kappa)Y_m + (1 - F(0))\kappa Y_l + \kappa F(0)(\max(K, \beta L)) \right].$$

Again, if effort is sufficiently costly, the optimal bonus and effort are 0. Choosing $\kappa = \frac{Y_h - Y_m}{Y_m - (1 - F(0))Y_L}$ as proposed earlier, this simplifies to

$$w^* = \max \left[0, \frac{1}{2}(F(0)) \left(\frac{Y_h - Y_m}{Y_m - (1 - F(0))Y_L} \max(K, \beta L) \right) - \alpha p_h \right].$$

One can observe that a necessary condition for the wage to be positive is that the collateral or bailout probability is sufficiently high. Obviously, as the incentivizing would be at best mean preserving, there have to be other factors in order to achieve a positivity result for the bonus. The driving force is that risk can be shifted towards the government or the insurance of the collateral in this agency setting. In the proceeding of this chapter we assume that we are at conditions that guarantee a positive bonus. Due to the maximum function in the optimum we again only focus on bailout probabilities that exceed the critical value. Below this value, an increase

of bailout probability has no influence.

Proposition 4.2. *In an interior solution of the single creditor risk effort problem an increase in bailout probability*

- *increases the manager bonus*
- *increases the effort of the manager*
- *increases the default probability of the bank*
- *increases the expected utility of the manager.*

Proof. Taking the derivative of w^* with respect to β at an interior solution yields

$$\frac{\kappa F(0)L}{2} > 0.$$

Hence the manager bonus is increasing in expected bailout probability. As a direct consequence the effort is increasing since $e^* = \frac{w}{\alpha}$. Having the default probability $F(0)(p_l^0 + \kappa a^*)$, the banks default probability is increasing in expected bailout probability. The agent's expected utility is again increasing in w . Hence, the expected return for the agent is increasing in bailout probability. \square

This time, the rationale behind this is that a bailout strategy by the government, as before, indirectly increases the value of the low state. As the creditor might get bailed out in the low state, he demands less interest for his loan. The difference to the previous chapter is that the principal originally incentivized the agent to shift probability mass away from the low state. In the risk shifting problem, the principal wants the manager to shift risk to the low state. If now the punishment for the low state via the return demand of the creditors gets lowered, the principal might finance even higher effort taking by the agent via incentivizing even more. According to that, bonus and effort increase.

4.2. Multiple Creditors

We proceed as before and receive the optimal bonus payment for the principal as

$$w^* = \max \left[0, \frac{1}{2}(Y_h - \alpha p_h - (1 + \kappa)Y_m + (1 - F(\theta^*))\kappa Y_l + \kappa F(\theta^*) \max(K, \beta L)) \right]$$

and the optimal effort is given by

$$a^* = \max \left[0, \frac{1}{2\alpha}(Y_h - \alpha p_h - (1 + \kappa)Y_m + (1 - F(\theta^*))\kappa Y_l + \kappa F(\theta^*) \max(K, \beta L)) \right].$$

Proposition 4.3. *In an interior solution of the single creditor effort problem an increase in bailout probability*

- *increases the manager bonus*
- *increases the effort of the manager*
- *increases the expected utility of the manager.*

Proof. The proof follows the reasoning of the previous chapter. □

4.3. The Effect of Multiplicity of Creditors

Having analyzed the optimal manager bonuses as well as the implications of a bailout strategy for both cases we can now take a look at what the impact of multiplicity of creditors is compared to a single creditor.

Proposition 4.4. *The multiple creditor case wage is at most the single creditor case wage $w_{MC}^* \leq w_{SC}^*$. Especially, if there is no coordination problem (i.e. $\beta L \geq K$) we have $w_{MC}^* = w_{SC}^*$ and if there is a coordination problem we have $w_{MC}^* < w_{SC}^*$.*

Proof. The optimal wages only differ in the term of the distribution function of θ

$$w_{MC}^* = \frac{1}{2}(Y_h - \alpha p_h - (1 + \kappa)Y_m + (1 - F(\theta^*))\kappa Y_l + \kappa F(\theta^*) \max(K, \beta L))$$

$$w_{SC}^* = \frac{1}{2}(Y_h - \alpha p_h - (1 + \kappa)Y_m + (1 - F(0))\kappa Y_l + \kappa F(0) \max(K, \beta L))$$

It is easy to show that the Y_l term dominates $((1 - \beta)K + \beta L)$

$$\begin{aligned} w_{SC}^* - w_{MC}^* &= (1 - F(0))\kappa Y_l - (1 - F(\theta^*))\kappa Y_l \\ &\quad - \kappa F(\theta^*) \max(K, \beta L) + \kappa F(0) \max(K, \beta L) \\ &= (F(\theta^*) - F(0))(Y_l - \max(K, L)) \geq 0. \end{aligned}$$

While the former term is at least 0 the latter term is positive. The former term is positive exactly if there is a coordination problem. \square

We see that multiplicity of creditors decreases the manager bonus if there is a risk shifting problem in the agency setting. The mechanism behind this is that multiplicity of creditors devalues the lower state due to the coordination problem. This time, however, the principal wants to incentivize the low state. As the low state now, via the higher demanded repayment for creditors, becomes less valuable, incentivizing the same return becomes more costly. Therefore, incentives set are lower in the multiple creditor case. Nonetheless, an increase in bailout probability increases the manager bonus in both setting. Again, above the critical value the effects coincide. Here, the enhanced value of the low state due to the bailout policy is the driving force. Below the critical value, there is no effect in the single creditor case, but a coordination effect in the multiple creditor case. The coordination effect acts in the same direction as the effect above the critical value.

4.4. Welfare Analysis and Policy Implications

Having analyzed multiple effects of this simple model we will take a look at policy implications. The main question from a welfare perspective apart from the failure probability is whether the net value of this project increases or decreases.

4.4.1. Single Creditor

A social planner who wants to maximize the net value faces the problem of maximizing

$$(p_h^0 + a)Y_h + (p_m^0 - (1 + \kappa)a)Y_m + (p_h^0 + \kappa a)(1 - F(0))Y_l - \frac{1}{2}\alpha a^2.$$

The optimizing effort he would implement is then given by

$$\frac{Y_h - (1 + \kappa)Y_m + \kappa(1 - F(0))Y_l}{\alpha}.$$

By construction of κ the project net value is not increasing in costly effort and it is easily computed that any effort is not desirable in this context. As negative effort is excluded it is

$$a_{FB}^* = 0.$$

Comparing this to the second best effort at an interior solution

$$a_{SB}^* = \max \left[0, \frac{1}{2\alpha} (Y_h - \alpha p_h - (1 + \kappa)Y_m + (1 - F(0))\kappa Y_l + \kappa F(0)(\max(K, \beta L)) \right]$$

we can state that the agency problem causes overinvestment of effort. As the objective for the social planner is a quadratic function of effort, even higher effort, as caused by a bailout policy, exacerbates the overinvestment problem. Since governmental costs of the bailout are just a redistribution in this model we observe a worsening of the net value of the project. Having analyzed the net value, again, it is natural to ask whether single parties of this model benefit from a bailout. We have seen in the previous section that the expected utility for the agent is increasing. The creditor in this model, as previously stated, is always kept at the break even point for participating, he neither benefits nor is a bailout to his detriment. A bailout policy is not desired by the social planner as it has only negative net value effects and worsens the overinvestment problem.

4.4.2. Multiple Creditors

In the multiple creditor case the optimal effort he would implement is again given by

$$a_{FB}^* = 0.$$

Comparing this to the second best effort at an interior solution

$$a_{SB}^* = \max \left[0, \frac{1}{2\alpha} (Y_h - \alpha p_h - (1 + \kappa) Y_m + (1 - F(\theta^*)) \kappa Y_l + \kappa F(\theta^*) \max(K, \beta L)) \right],$$

again, as in the single creditor case, we observe an overinvestment of effort compared to the first best case.

If we compare the difference of the first best effort to the second best effort for the single creditor case and the multiple creditor case, this time we see that the overinvestment problem is more severe in the single creditor case. This is due to the fact that the first best effort is at its boundary value 0 and the second best effort is higher in the single creditor case. The coordination problem weakens the overinvestment induced by the agency problem.

5. Discussion

5.1. Comparison with Hakenes and Schnabel

Hakenes and Schnabel 2014 consider the internal and external agency problem in the trinomial setup. They utilize a two period model in order to show the most important effects of the moral hazard problem and the effects an increased expectation of bailout probability has on wages and managerial effort. The most important differences to their paper are the introduction of collateral, credit prolongation and the case of multiple creditors. As soon as we reach the critical value for the bailout probability $\beta = KL$ we have full coordination and a decision of the creditors to prolong the loan. Thus, for high values of β that exceed this critical value, our re-

sults are consistent with Hakenes and Schnabel. For lower values we have different effects depending on whether we have a coordination problem or a single creditor. In the single creditor case the option of falling back on insured collateral leads to no effect of increased bailout probabilities if these are low enough. Creditors will still leave early in $t = 1$ and therefore still demand the same return for the repayment R_1 . This keeps the behavior of the other participants in our model constant. In the multiple creditor case, there is a coordination effect for low values of β . This coordination effect can be observed exactly until the critical state $\beta = \frac{K}{L}$. The coordination effect shifts R_1 in the same direction as the bailout effect in Hakenes and Schnabel but for completely different reasons. In Hakenes and Schnabel the increased value in case of a default is what shifts R_1 and eventually leads to an increase of default probability. In my model an increase of default value leads to better coordination and therefore shifts probability away from the default state as a first reaction. As a result, the lower states overall gets more value but not necessarily the case of default. Only the feedback of the principal agent problem via the shifted demanded repayment R_1 might lead to increased default probabilities if β is low. While in Hakenes and Schnabel the participants only “gamble” on a default at the cost of the government (which is equal to the value of the default state), in my model and if β is low they take the increased risk of reaching the low state due to better coordination probabilities and even for cases of less expected value of the default state. For higher β the reasonings for the participants’ behavior fall back to the case of Hakenes and Schnabel. Depositors in Hakenes and Schnabel fulfill a similar role as the government as long as the risk premium for the deposit insurance is fixed. Therefore, I do not focus on depositors.

5.2. The Combined Case

As different effects can be observed in the effort and the risk shifting case, the general agency problem might very likely include both issues. Calculations show that one driving factor are the cost parameters for the types of effort. If risk shifting becomes expensive, the effort problem dominates and vice versa. While the wage and effort conclusions become ambiguous in the combined case, the impact of an increased

bailout probability on the default probability for high bailout probabilities persists in a negative manner. Only for low bailout probabilities the default probability might decrease. Thus the combined model offers no new insight except for having the effects of both single models at once. Hakenes and Schnabel argue that in the combined case the parameter space of the cost parameters for which risk shifting dominates and bonus caps are useful is increasing in β . This is also true in my model for large increases as for large β collateral and coordination effects vanish. Still, if β is sufficiently low, we cannot conclude that the parameter space is increasing.

5.3. Alternative Managerial Effort and Costs

The quadratic cost function for exerting effort guarantees an easy way to ensure a finite solution due to its convex structure when combined with a linear influence of effort on the probabilities. In our model, as in the model of Hakenes and Schnabel and the model of Besley and Ghatak, the parameter space for the effort therefore has to be restricted in order that the probabilities do not exceed the boundaries 0 and 1. We will briefly give an example in the effort case of an alternative way of modeling the influence of effort, keeping the parameter space less restricted and showing that a convex cost structure is not needed. I model the managers influence on the probabilities in the following way:

$$p_h(e) = p_h^0 + \frac{e}{1+e}p$$

$$p_m(e) = p_m^0$$

$$p_l(e) = p_l^0 - \frac{e}{1+e}p$$

with $p = \min(p_l^0, 1 - p_h^0)$ and a cost function

$$c(e) = -\eta p^2 \frac{1+2e}{2(e+1)^2}.$$

In this fashion we can allow the choice variable e to take values in the interval $[0, \infty)$. The optimization problem of the agent is then the maximization of

$$\left(p_h^0 + \frac{e}{1+e}p\right)w + \eta p^2 \frac{1+2e}{2(e+1)^2}.$$

The optimality condition for the agent is

$$\frac{e}{1+e}p = \frac{w}{\eta}.$$

We see that an offered wage greater or equal to the cost parameter $\frac{\eta}{p}$ pushes effort to infinity. This is in a way consistent with the previous modeling as in the original model a too high wage pushes probability out of the admissible parameter space. Having had

$$p_h(e) = p_h^0 + \frac{w}{\eta}$$

at the optimal effort in the original model we can observe the same result (Assuming that $p_h(e)$ is binding instead of $p_l(e)$). Our alternative approach is constructed such that the objective for the principal, after substituting the optimal behavior of the agent and the participation constraint of the creditors, is the same as in the original model and thus yields the same results.

6. Conclusion

If there are sufficient tools for risk monitoring and controlling for the government the endogenously generated bonus contracts in a principal agent framework sets a too low incentive for optimal manager behavior. An increase in bailout probability by the government has either no effect or worsens the underinvestment of effort. Bonus caps are thus inefficient or even harmful from a welfare perspective. Multiplicity of creditors induces a coordination problem between the creditors having the opposite effect on the bonus as a bailout policy. Therefore, the effect of a bailout strategy is even bigger in the multiple creditor case since the original effect of the bailout is accompanied by the reestablishment of coordination. Overall, there is a destruction

of net value when there is a bailout policy.

In case of a lack of risk controlling possibilities and the manager can exert effort in order to increase risk, things change drastically. The endogenously generated bonus becomes too big in terms of welfare. An increase in bailout probability again worsens the problem. This time however, as bonuses are too high, bonus caps might be useful to accompany the bailout policy. Multiplicity of creditors, again, is a counterdirected force to the bailout and thus strengthens the effects of an increase in bailout probability.

As long as governments are not able to ensure a good risk management to control the risk of financial institutions, bonus caps are a recommended tool especially if the government is open to bailing out troubled banks.

In either setup, bailouts are ineffective or even harmful with respect to the bank's default probability in the single creditor case. In the multiple creditor case, up to a critical value of bailout probability, the coordination effect might dominate the negative effect of the agency problem. For high bailout probabilities only the destructive effect remains. Thus, ignoring other effects like domino effects of large institutes, a government should never indicate too high bailout expectations. Keeping the bailout expectations low, a bailout only has an effect if there is a coordination problem. Setting the expected bailout probability just as high as to induce full coordination might be the best from a social planner's perspective.

In this sense, an alternative to a bailout policy might be any policy that helps to establish confidence in the financial market in order to reestablish coordination between creditors. Better signals and information structures due to higher risk reporting standards might help to coordinate to the socially better equilibrium without the negative side effects a bailout strategy brings in via the agency problem.

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Part IV.

Bonus and Benchmark Setting for Limitedly Liable Portfolio Managers with Portfolio Insurance

1. Introduction

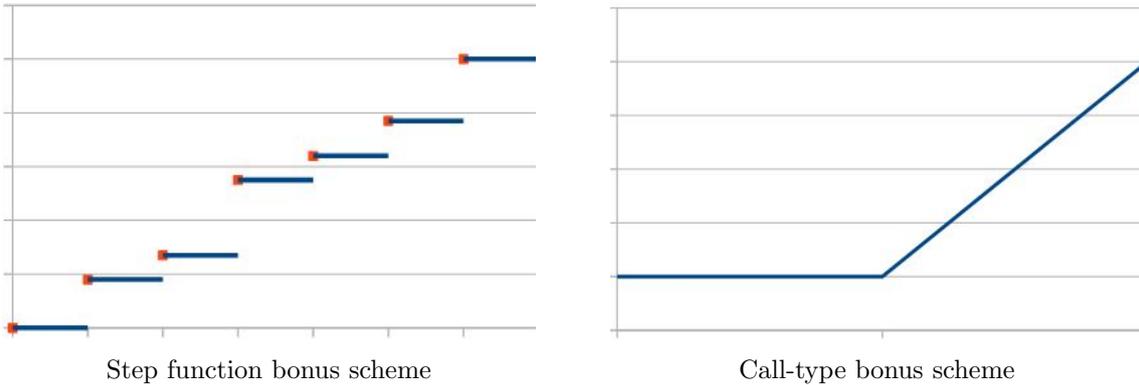
Incentivizing a manager with bonus payments is a wide-spread concept to transfer a company owner's objectives to the execution of the manager. Especially in financial markets, the discourse about the appropriate size and structure of manager bonuses is steadily persisting in economic theory as well as on the society level. During the last years, excessive bonuses and bonus payout in times where financial institutes were experiencing losses or even were at the verge of going bankrupt, raised the question whether the current handling of bonus payments is appropriate for the financial institutes themselves and, moreover, for all connected enterprises and the overall society. In particular when the population had to cover banks' losses or bear risk via government subsidies, the general question of who was benefitting by the taken measures arose and whether the architecture of the current financial system and their bonus payouts incentivized excessive risk taking. Specifically on markets with complex financial products, where derivatives allow for nearly arbitrary portfolio payoff structures, the current bonus policy was criticized. The advantages of bonus payments are multilayered. Bonus contracts might make an agent exert effort at a better performance level in order to achieve a bonus payment. Furthermore, a structured payment contract might separate high skilled from low skilled agents when allocating human capital to appropriate management positions. More general, an agent's utility function might be transferred via the bonus contract to resemble the principal's utility. Often times, the agent is more risk averse than the principal. This can be counteracted by a convex bonus scheme. An obvious disadvantage though is the problem of moral hazard induced by bonus schemes with lower bounds due to the limited liability aspect of the manager.

As mentioned in part III., the delegated portfolio choice problem is special in the field of principal-agent problems. In the portfolio choice problem there are two dimensions of asymmetric information. The agent privately chooses his effort first as well as the structure of the portfolio afterwards, while the principal is only able to observe the outcome of the portfolio. Although a high incentive in most cases might induce the agent to exert more effort which is desired by the principal, a too

big sized bonus might be harmful to the portfolio structure choice.

Empirically, most working contracts contain a payment structure that is bounded from below. De facto, every agent is effectively limitedly liable, since he cannot cover losses that might be arbitrary high. This leads to the possibility that, in contrast to classic portfolio theory, an agent might not only desire volatility, but, depending on the available financial products, prefer a portfolio with, both, low return and high variance to a portfolio with higher return and lower variance.

The most common compensation contracts either consist of discrete benchmarks and pay out increasing bonus amounts for each benchmark reached, so called step function contracts, or they pay out a certain fraction of every additional profit above a certain benchmark, so called call-type bonus contracts. Upper caps on both contract types might prevent excessive risk taking but are also reasons for suboptimal portfolio structures.



Not only the structure of the payment contract but also the type of possible investment is crucial to the analysis of optimal bonus setting and of major importance to identify where excessive risk taking might have its origin. Is the investment choice a simple option between investing a share into a risky asset or project and the remaining capital into a bond? Is further risk taking by exceeding the liquidity of the initial capital possible? For fund managers who have access to sufficiently complex financial products or the access to advanced trading strategies arbitrary payoff structures depending on fundamental variables can be replicated. In a more

restricted market, the manager might only be able to influence return and volatility by exerting effort as in Hakenes and Schnabel (2014).

Other important driving features are the type of the benchmark and the time of evaluation. Benchmarks might be static to achieve given goals or they might be dynamic in order to make success comparable to average market success to determine the managers performance. Static benchmarks might function as an insurance since they are less likely to pay bonuses when overall market performance is low. In reverse, they do not account for underperforming, market-driven success. Dynamic benchmarks, like the comparison with indices, better identify the performance of a manager but might also pay out bonuses when losses are realized and, thus, burden an already stressed balance sheet.

The focus of the paper is a setup of an investor handing out money to an agent to invest. The agent has access to sufficiently complex financial products meaning that he can buy or replicate arbitrary measurable payoff structures.. The investor buys a portfolio insurance to cover possible losses and incentivizes the manager in the most simple form of a one step bonus contract. The insurance demands the portfolio to not exceed a given risk level. In this way the problem can be divided into two parts; optimal risk taking and optimal payoff structuring for the portfolio. Through the risk taking in terms of possible losses, which are covered by the insurance, the agent can generate higher payoffs in the case of success. Due to the flat bonus scheme, the agent has no incentives to exceed the benchmark. Therefore, the investor has to carefully choose the benchmark. As the structuring of the portfolio in order to achieve higher benchmarks is more costly, the agent has to generate more liquidity from risk taking. This additional generating of payoff in the success case comes at the cost of a decreased success probability. Moreover, as the agent's bonus is only payed out in case of success, a higher bonus has to accompany a higher benchmark setting in order to keep the agent incentivized. I show that in this setting excessive risk taking is not driven by the managers bonus but the bonus is a necessary tool for the investor to keep the agent incentivized. Apart from this, it is even of interest of the investor to keep the bonus low. The key feature for lowering the success probability is rather given by the benchmark setting of the investor than the bonus

setting.

The literature in the field of delegated portfolio management is vast. Holmstrom and Milgrom (1987) analyze the optimal structure of manager compensation. They find that a linear contract might be the optimal contract. However, many findings show that linear contracts are not optimal since an underinvestment in effort can be observed in many models. A famous example is the work of Admati and Pfleiderer (1997) who show that an agent cannot be incentivized to the benefit of the principal. While Starks (1987) argues that symmetric contracts are desirable, it is often difficult to implement appropriate punishment for bad outcomes into working contracts. Most payment contracts are bounded from below and two types of contracts are predominant. The simple bonus contract that pays a bonus if a benchmark is reached and the call type compensation contract that pays a fixed amount and a share of the profits if profits exceed a certain benchmark. Even if the principal claims big losses from the agent according to the compensation contract, current volumes and risks of portfolios can quickly lead to a loss the agent is not able to bear. Ross (2004) and Gollier (1997) analyze the distortion of risk taking by a convex compensation scheme and find that there are multiple effects. Ross (2004) shows that a convex compensation scheme not necessarily increases the risk taking as scale effects might also affect the expected utility, while Gollier shows that mean variance inefficient portfolios in the classical sense might be selected. John and John (1993) show that a compensation scheme might be able to reestablish the first best level of risk taking. Empirically, there are different results on risk taking behavior. While Houston and James (1995) find no excessive risk taking incentives in the financial sector compared to other sectors, Cheng, Hong and Scheinkman (2010) report a significant connection.

If financial market completeness allows for replication strategies, arbitrary portfolio payoffs might be generated. Therefore, portfolio insurance and risk constraints for the manager are a necessary tool in order to prevent excessive risk taking. Two of the most common portfolio insurance approaches are the Option Based Portfolio Insurance (OBPI) and the Constant Proportion Portfolio Insurance (CPPI). The OBPI was introduced by Leland and Rubinstein (1976). Its idea is to cover a

portfolio invested in a risky asset by a put written on it. This guarantees that the value of the portfolio at some terminal date will be greater than the strike price of the put. Perold (1986) introduced the CPPI that requires a trading strategy to hold the portfolio above a certain level at any time. As these strategies might not be applicable if markets allow jump processes, more modern approaches try to keep portfolio values above a benchmark with a certain confidence level or impose a risk measure constraint to a portfolio. In particular, regulatory frameworks focus on quantiles and risk measures in order to regulate financial markets.

Portfolio optimization under a risk measure constraint is studied in Emmer et al. (2001) for a capital-at-risk constraint. Boyle and Tian (2007) consider the closely related portfolio choice problem of outperforming a benchmark with a certain confidence level. Föllmer and Leukert (1999) investigate the optimal hedging of a portfolio up to a certain confidence level. A new approach is followed by De Carmine and Tankov via separating the problems of positive portfolio payoff structuring and optimal risk taking. As a portfolio insurance only covers losses, the risk constraint is only applied to the nonpositive part of the random variable that represents the terminal portfolio value. The evaluation of utility of the portfolio manager is restricted to the positive part as the insurance covers arbitrary losses. In this fashion, the problem can be split into two convex problems and explicit solutions can be obtained.

The paper is structured as follows. In section 2, I describe the setup of the model. I introduce the relation between principal, manager and the insurance and their objectives. Chapter 3 describes the optimal behavior of the manager for given incentive schemes. The agent has to choose an optimal scheme for gains and for losses. In section 4 the optimal bonus scheme is determined. I conclude with my results in chapter 5.

2. The Model

I consider the problem of an investor who wants to incentivize a fund manager in order to optimally analyze a market and structure a portfolio. The common issue with this kind of investment problem is that the investor has incomplete or no information about the effort choice of the fund manager and whether or not the manager structures the portfolio in his sense. If markets allow for complex financial investment opportunities, possible losses might be substantial. In order to avoid substantial capital losses that might affect the operating business in other areas, a portfolio insurance has to be effected. The insurance takes a premium as well as giving a specification for a risk level that must not be exceeded by the portfolio manager. The investor offers a bonus payment if a desired benchmark is achieved. The fund manager anticipates bonus, benchmark and risk constraint in order to choose the structure of the portfolio to maximize his utility. Therefore, the bonus and the benchmark have to be chosen carefully by the investor in order to incentivize an appropriate portfolio structure.

2.1. Setup

Let $B \geq 0$ denote the bonus payment when the managed portfolio achieves benchmark $b > 0$. The manager/agent is equipped with an increasing and strictly concave utility function u and reservation utility/outside option u_0 .¹² Without loss of generality we set the fixed part of the salary to zero.¹³ The terminal portfolio value is denoted by X_T . Therefore, the payment of the agent z is

$$z(X_T) = \begin{cases} B & \text{if } X_T \geq b \\ 0 & \text{else.} \end{cases}$$

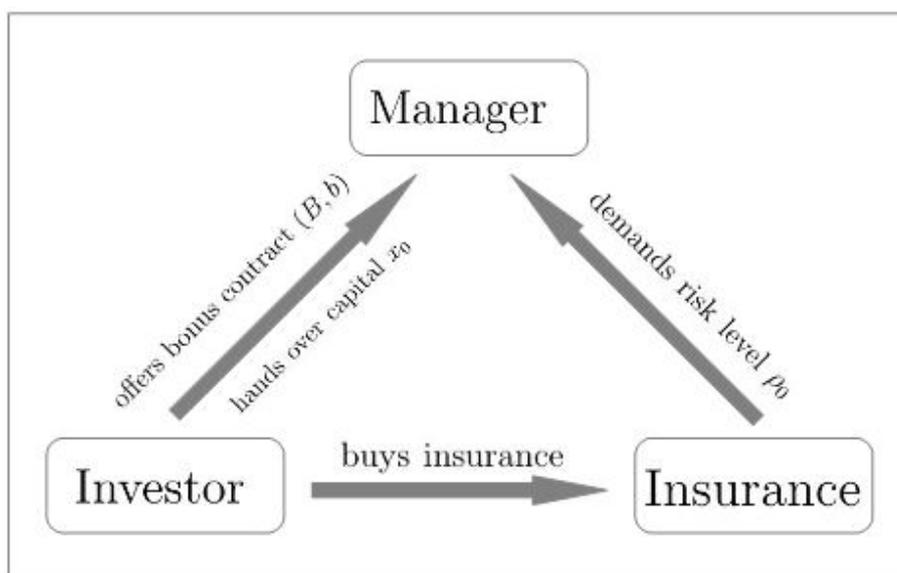
¹²the outside option might represent the general value of other working opportunities and total rejection of the offer. It might also be seen as the value of taking the offer and shirking. Since optimal effort choice is not the main focus of this paper, I assume that overall market conditions are such that the investor is always interested in preventing shirking.

¹³else the utility function is simply transformed

I assume that the insurance covers any terminal losses incurred by the portfolio and imposes a risk constraint ρ_0 specified by the entropic risk measure for a portfolio choice X :

$$\rho(X) = \frac{1}{\beta} \log(E_P(e^{-\beta X}))$$

for some $\beta > 0$. The investor/principal hands over x_0 to the agent and offers a contract (B, b) .¹⁴



The investment occurs on an arbitrage free, complete financial market. Let (Ω, F, F_t, P) denote the filtered probability space. For simplicity, let F_0 be trivial. The adapted process of the risky asset is given by $(S_t)_{0 \leq t \leq T}$. For simplicity the interest rate is set to 0 and $S_t^0 = 1$ denotes a bond. An admissible, self-financing strategy is denoted by initial capital $x_0 \geq 0$ and a predictable process ζ such that

$$X_t = x_0 + \int_0^t \zeta_s dS_s.$$

¹⁴I assume that the investor covers the insurance and x_0 is the net investment

For the risk taking problem there is no special requirement for the admissability of the trading strategy, while for the investment part we will temporarily demand

$$X_t = x_0 + \int_0^t \zeta_s dS_s \geq 0 \forall t \in [0, T] \text{ } P - a.s.$$

when talking about quantile hedging. The unique martingale measure on the complete market is denoted by P^* and the according density for the change of measure is denoted by $\xi = \frac{dP^*}{dP}$. Market completeness enables the agent to replicate any measurable (with respect to P) random variable and therefore turns the problem of finding an optimal trading strategy into a maximization problem on the space of integrable random variables. Therefore, in the following, I will simply use X instead of X_T .

3. The Agent's Problem

The agent tries to maximize his expected utility with respect to his compensation.

$$E[u(z(X_T))].$$

Since the payment to the agent is given by $z = \begin{cases} B & \text{if } X_T \geq b \\ 0 & \text{else} \end{cases}$ and the market is complete, this problem can be rewritten as

$$\max_{X \in L^1(P)} P(X \geq b)u(B)$$

such that

$$\rho(\min(X, 0)) \leq \rho_0$$

and

$$E(\xi X) \leq x_0.$$

For the risk constraint only the negative part of the random variable is applied as the insurance is only interested in possible losses. This is a key feature for the

separation of the portfolio choice problem into the gains and losses part. Therefore, the agent's problem is reduced to the problem of maximizing the probability that the portfolio achieves the benchmark while keeping the risk below the given boundary. Following the idea of De Franco and Tankov the portfolio problem can be divided into two parts. First, I investigate optimal investment structures for investments that yield a nonnegative payoff if the payoff is restricted to a subset of the state space. This is shown by utilizing a result of Föllmer and Leukert in the context of quantile hedging. Second, the optimal risk taking can be determined by a result of De Franco and Tankov.

3.1. Quantile Hedging

Föllmer and Leukert (1999) investigate the optimal, up to a certain probability incomplete, hedging of a portfolio. The basic idea is that a portfolio manager is looking for the cheapest hedge while ignoring a quantile of the underlying probability space. This problem might, for instance, be motivated by aiming to fulfill a value at risk constraint that is imposed on the portfolio. The huge difference of quantile hedging, compared to an optimal portfolio choice problem, is that the hedge should almost surely have a positive payoff. In this way, the problem is mathematically well defined and does not permit arbitrary speculations on the financial market. Although the problem of quantile hedging finds its major motivation in incomplete markets where complete hedges are rather expensive and quantile hedging might be a more executable alternative, the method works the same in complete markets. In the following I will assume that the market is complete.

As before, (Ω, F, P) be a probability space with filtration (F_t) and $S = (S_t)_{t \in [0, T]}$ ¹⁵ a discounted price process on this space. An admissible, self-financing strategy is given by initial capital $V_0 \geq 0$ and a predictable process ζ that fulfill

$$V_t = V_0 + \int_0^t \zeta_s dS_s \geq 0 \quad \forall t \in [0, T] \quad P - a.s.$$

¹⁵In this part I will put a focus on the Black and Scholes model, Föllmer and Leukert give results for general semi-martingales

3 THE AGENT'S PROBLEM

Let P^* denote the unique equivalent martingale measure on the complete market and $H \in L^1(P)$ the perfect hedge for portfolio that is to be hedged. Föllmer and Leukert show that the problem of finding the optimal trading strategy for hedging the portfolio up to a certain probability is equivalent to the problem of finding a subset of the state space where hedging is the cheapest. The transformed problem can be stated in the following way:

$$\min_{A \in \mathcal{F}_T} E^*[H1_A]$$

such that

$$P[A] \geq 1 - \varepsilon$$

for some $\varepsilon \in (0, 1)$.

A is the part of the state space where the portfolio is perfectly hedged. Now A should be chosen such that the incomplete hedge $H1_A$ is as cheap as possible while keeping the failure probability of the overall hedge at most at ε . Let Q^* be given by

$$\frac{dQ^*}{dP^*} = \frac{H}{E^*[H]}$$

and

$$\tilde{b} = \inf \left\{ b : P \left[\frac{dQ^*}{dP} > b \right] \leq \varepsilon \right\}.$$

Moreover, let the success ratio be defined as

$$\phi = 1_{H_T} + \frac{V_T}{H} 1_{V_T < H},$$

and let

$$\tilde{\phi} = 1_{\frac{dQ^*}{dP} < \tilde{b}} + \gamma 1_{\frac{dQ^*}{dP} = \tilde{b}}$$

as well as

$$\gamma = \frac{(1 - \varepsilon) - P\left[\frac{dQ^*}{dP} < \tilde{b}\right]}{P\left[\frac{dQ^*}{dP} = \tilde{b}\right]}.$$

Föllmer and Leukert show that

Theorem 3.1 (Föllmer and Leukert). *Let $\tilde{\zeta}$ be the perfect hedge for $\tilde{H} = H\tilde{\phi}$ and define $\tilde{V}_0 = E^*[\tilde{H}]$. Then $(\tilde{V}_0, \tilde{\zeta})$ has minimal cost under all admissible strategies (V_0, ζ) with $E[\phi] \geq 1 - \varepsilon$ and we have $E[\tilde{\phi}] = 1 - \varepsilon$.*

The problem of finding the cheapest hedge for a given failure probability is closely related to the problem of finding the hedge that minimizes the failure probability while keeping the costs of the hedge below a given threshold. If a perfect hedge cannot be afforded due to liquidity constraints or is economically not desired¹⁶, a portfolio manager might search for the part of the state space where hedging is the cheapest. Föllmer and Leukert show that the problem of finding the optimal trading strategy is, again, closely related to finding the optimal part of the state space where the hedge should be effective. Let \tilde{V}_0 denote the available liquidity. The transformed problem can then be stated as follows:

$$\max_{A \in \mathcal{F}_T} P(A)$$

such that

$$E[\xi H 1_A] \leq \tilde{V}_0.$$

Theorem 3.2 (Föllmer and Leukert). *Let*

$$\tilde{a} = \inf \left\{ a : Q^* \left[\frac{dP}{dP^*} > aH \right] \leq \frac{V_0}{H_0} \right\}$$

and

$$\gamma = \frac{\tilde{V}_0/H_0 - Q^*[\frac{dP}{dP^*} > \tilde{a}H]}{Q^*[\frac{dP}{dP^*} = \tilde{a}H]}.$$

The optimal trading strategy for the hedging problem is given by $(\tilde{V}_0, \tilde{\zeta})$ where $\tilde{\zeta}$ replicates the option $\tilde{H} = H\tilde{\phi}$ with $\tilde{\phi} = 1_{\frac{dP}{dP^} > \tilde{a}H} + \gamma 1_{\frac{dP}{dP^*} = \tilde{a}H}$.*

From Föllmer and Leukert we know that the problem of finding an optimal trading strategy is equivalent to the problem of finding the best part of the state space where

¹⁶On incomplete markets hedging might be overly expensive

the perfect hedge should be effective. The part of the state space where hedging is the cheapest is characterized by the density $\frac{dP^*}{dP}$ and the boundary \tilde{b} . The correction term γ adjusts for the (probabilistic unlikely) case that $P[\frac{dQ^*}{dP} < \tilde{b}] < 1 - \varepsilon$. The idea of finding the best hedging region is very similar to finding the region that yields the best test in statistics when testing for a hypothesis. In fact, the proof of Föllmer and Leukert is very similar and utilizes the Neymann-Pearson lemma. We will stick to optimizing over subsets of the state space later, when we will analyze optimal risk taking. Before, we will take a closer look at two special applications of these results.

3.1.1. Quantile Hedging in the Black-Scholes Model

In the Black-Scholes model the price process is given by

$$dS_t = S_t(\sigma dW_t + m dt), S_0 = s_0$$

where W is a standard Wiener process and m and $\sigma > 0$ are constant. With the unique equivalent martingale measure given by

$$\frac{dP^*}{dP} = e^{-\frac{m}{\sigma}W_T - \frac{1}{2}(\frac{m}{\sigma})^2T}$$

and the equation for the price process given by

$$S_T = s_0 e^{\sigma W_T + (m - \frac{1}{2}\sigma^2)T}$$

the density can be rewritten depending only on a constant c and the stock price:

$$\frac{dP^*}{dP} = c S_T^{-\frac{m}{\sigma^2}}.$$

From Föllmer and Leukert we know that the optimal set where the portfolio is partially hedged on is given by

$$A = \left\{ \frac{dP}{dP^*} > dH \right\}.$$

Therefore, the optimal hedge can be calculated explicitly depending on the state price density and the structure of the claim. For constant claims the the optimal hedging region can be determined in an easy way.

3.1.2. Quantile Hedging a Constant Claim

For the proceedings of this paper the quantile hedge of a constant claim is of special interest. The agent in the principal-agent problem is incentivized by a simple bonus contract. As seen before, he is interested in maximizing the probability of at least hitting the benchmark. As we will see later, it is a direct consequence that the agent tries not to outperform this benchmark as this does not increase his expected bonus payments while it would be costly. Therefore, flat payoff derivatives, as digital options, are of special interest. Now assume that the hedge that is to be replicated is of constant payoff $H = b$ for $b \in \mathbb{R}$. The optimal quantile hedge for any $V_0 < E[\xi b] = b$ is then characterized by an acceptance region $A = \{\frac{dP}{dP^*} > cb\}$ for some constant c . As b is also constant, we will replicate the digital option on the part of the state space where state prices are low. The optimal quantile hedge is given by a binary/digital option $b1_{\{\frac{dP}{dP^*} > cb\}}$ with c such that $V_0 = E[\xi b1_{\{\frac{dP}{dP^*} > cb\}}]$. The most important implication of this section is that long positions in a portfolio are preferredly taken in the part of the state space where state prices are low as investing in this area is "cheap per probability". For given capital, ignoring the part of the state space where risk is incurred, it is best from the agents perspective to generate a flat payoff scheme for the portfolio that generates payoff on the part of the state space where state prices are low. The aim for the next part is to show that losses are optimally incurred on the part of the state space where state prices are high. This will finally lead to the optimal structure of the portfolio.

3.2. Optimal Risk Taking

As in the previous part we approach the optimization of the risk structure by an optimization over subsets of the probability space. In order to identify the optimal

structure for negative payoffs I follow the approach of De Franco and Tankov. The problem is defined in the following way:

$$\min_{Y \in J(A)} E[\xi Y]$$

where

$$J(A) = \{Y \in L^1(\xi P) | \rho(Y) \leq \rho_0, Y = 0 \text{ on } A, Y \leq 0 \text{ on } A^c\}.$$

The problem describes that for a given part of the state space the manager is interested in the structure of the portfolio and the part of the state space that yields the most liquidity while keeping the risk below the given level. Define

$$\underline{\xi} = \text{essinf } \xi$$

and

$$\bar{\xi} = \text{esssup } \xi.$$

Further, let

$$v(A) = \inf_{Y \in J(A)} E[\xi Y].$$

Theorem 3.3. *Suppose that the law of ξ has no atom and let $A \in F$. Let $c \in [\underline{\xi}, \bar{\xi}]$ such that $P(\xi \leq c) = P(A)$ and let the risk measure be law invariant. Then*

$$v(A) \geq v(\{\xi \leq c\}).$$

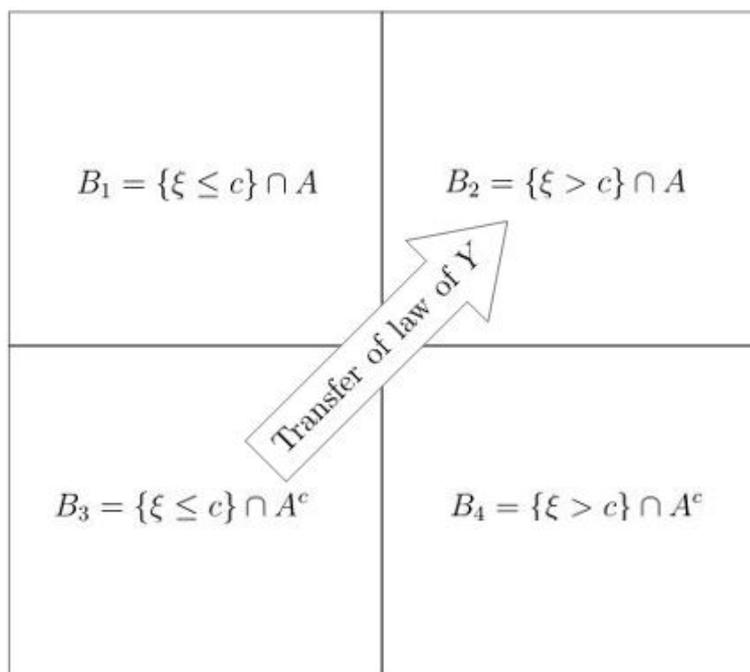
The theorem states that if losses are realized on a part of the state space that does not exceed a given probability boundary, there is no part that yields more liquidity than the part that represents the highest state spaces. In the Black and Scholes model the state price density has no atom. Further the entropic risk measure is law invariant. The idea of the proof is the following: We partition the state space into four parts via

$$B_1 = \{\xi \leq c\} \cap A$$

$$B_2 = \{\xi > c\} \cap A$$

$$B_3 = \{\xi \leq c\} \cap A^c$$

$$B_4 = \{\xi > c\} \cap A^c.$$



The law of Y is kept on B_4 and transferred from B_3 to B_2 . By construction we have

$$P(B_2) = P(B_3).$$

For any given A^c with $0 < P(A^c) < 1$ we keep the part of a variable Y on the state space where $\{\xi > c\}$, i.e. B_4 . The remaining structure gets shifted from B_3 to B_2 in order to generate a new random variable \hat{Y} that incurs losses only on $\{\xi > c\}$, generates the same liquidity and has at most the risk of Y according to the risk measure. The construction is given by the following:

- $f(t) = P(Y \leq t|B_3)$
- $g(t) = P(\xi \leq t|B_2)$
- $Z = g(\xi)$
- $W = f^{-1}(Z)$

In the beginning, let f describe the original law for losses on B_3 . Points 2 and 3 generate a variable Z on B_2 that has a uniform distribution on the unit interval. Finally, the fourth point applies the original law of Y on B_3 to B_2 . In order to ensure that the same amount of liquidity is generated by the new variable we define:

$$k = \begin{cases} 1 & \text{if } W = 0 \text{ on } B_2 \\ \frac{E[\xi Y 1_{B_3}]}{E[\xi W 1_{B_2}]} & \text{else.} \end{cases}$$

Eventually, we can define

$$\hat{Y} = Y 1_{B_4} + kW 1_{B_2}.$$

Let \hat{F}^{-1} and F^{-1} denote the generalized inverses of the distribution functions of \hat{Y} and Y , respectively. Since

- $E[\xi \hat{Y}] = E[\xi Y 1_{B_4}] + E[\xi kW 1_{B_2}] = E[\xi Y]$ by construction,
- $P(A) = P(\xi \leq c)$ by construction,
- $\rho(\hat{Y}) = \rho(-\hat{F}^{-1}(U)) \leq \rho(-F^{-1}(U)) = \rho(Y) \leq \rho_0$ by law invariance of ρ with a uniformly distributed U on the unit interval and since $\hat{F}^{-1}(u) \leq F^{-1}(u)$ for all $u \in [0, 1]$,

we have shown that losses are optimally generated on high state prices as they yield the same amount of liquidity that they would yield on other parts of the state space with the same probability mass, while incurring at most the risk according to the risk measure.

The question of finiteness and whether the problem is well defined comes at hand. It is quite obvious that for risk measures like the value at risk this type of problem is not well defined. With the value at risk the agent can generate arbitrary liquidity on an interval that has probability mass according to the required confidence level. Therefore, large risk clusters are enhanced by this type of risk controlling in the agency problem. But even for law invariant risk measures it is not certain that a risk measure admits a finite generating of liquidity. However, De Franco and Tankov

show that a sufficient condition for

$$\inf_{A \in \mathcal{F}} v(A) > -\infty$$

is given by

$$\gamma_{min}(\xi P) < \infty,$$

where

$$\gamma_{min}(Q) = \sup_{X \in \mathcal{A}_\rho} E^Q[-X]$$

with acceptance set \mathcal{A} of ρ denotes the minimal penalty function in the Fenchel-Legendre representation of convex risk measures. For the entropic risk measure

$$\rho(X) = \frac{1}{\beta} \log(E(e^{-\beta X}))$$

with $\beta > 0$ we have that the entropic risk measure can be represented by

$$\rho(X) = \sup_{Q \ll P, \log\left(\frac{dQ}{dP}\right) \in L^1(Q)} \left(E \left[-\frac{dQ}{dP} X \right] - \beta E \left[\frac{dQ}{dP} \log \left(\frac{dQ}{dP} \right) \right] \right)$$

with

$$\gamma_{min}(\xi P) = \beta E[\xi \log(\xi)].^{17}$$

Therefore, the entropic risk measure guarantees to leave the problem well defined in the Black and Scholes setup. We will proceed with combining the results for positive and negative payoffs and deduce the optimal payoff structure in the following section.

3.3. Optimal Portfolio Choice

It is easy to see that for the agent the optimal gains structure is a flat payoff scheme when state prices are low and the optimal risk taking is realized at high state prices. The following theorem describes the optimal portfolio choice. It is of importance for the following theorem to notice that increasing the probability of reaching the

¹⁷see Föllmer and Schied 2004

benchmark is demanding more liquidity while, on the other hand, increasing the probability mass where losses are generated, while keeping the risk at the boundary level ρ_0 , generates more liquidity.

Theorem 3.4. *Let us start with the trivial case. If $x_0 \geq E(\xi b) = b$ an optimal portfolio choice is given by $X = b$. The more interesting and by the principal enforced case occurs when $x_0 < E(\xi b)$. The optimal portfolio choice is then given by*

$$X^* = b1_{\xi \leq c^*} - \beta \left[\log \left(\frac{\beta}{\eta(c^*)} \xi \right) \right]^+ 1_{\xi > c^*}$$

where

$$\alpha(c) = P(\xi > c)$$

$$v(c) = -\beta E \left[\left(\xi \log \left(\frac{\beta \xi}{\eta(c)} \right) \vee 1 \right) \right]$$

$$\eta(c) \text{ is the unique solution of } E \left[\left(\frac{\beta \xi}{\eta(c)} \vee 1 \right) 1_{\xi > c} \right] = e^{\frac{\rho_0}{\beta}} + \alpha(c) - 1$$

c^* is the unique maximizer of $u(B)(1 - \alpha(c))$ such that $E[\xi b] = x_0 - v(c)$.

Proof. The derivation of the optimal risk structure $-\beta E[(\xi \log(\frac{\beta \xi}{\eta(c)}) \vee 1)]$ is a simple lagrange optimization after restricting the problem to the space where losses are incurred, since the constraint is convex due to the convexity of the risk measure and can be found in De Carmine and Tankov. The function η corresponds to the lagrange parameter of the constraint. It is obvious that $\alpha(c)$ is decreasing in c . For the dependence of v on c we observe the following: From

$$E \left[\left(\frac{\beta \xi}{\eta(c)} \vee 1 \right) 1_{\xi > c} \right] = e^{\frac{\rho_0}{\beta}} + \alpha(c) - 1$$

we can deduce that $\eta(c)$ is decreasing in c . Therefore, $v(c)$ is increasing in c which yields the existence of a unique solution of maximizing $u(B)(1 - \alpha(c))$ such that $E[\xi b] \leq x_0 - v(c)$. Combined with the results of the previous two sections this yields the optimal portfolio choice. \square

I have established the optimal portfolio choice of the agent for any given pair

(B, b) . The bonus payment has no influence on the structure of the portfolio in this decision problem. The investor is interested in the pair that maximizes his expected payoff. His optimal choice is part of the following chapter.

4. Optimal Contract

The investor wants to anticipate the behavior of the manager in order to maximize the net payoff of the portfolio. His objective is given by

$$\max_{(B,b)} E[(\max(X_T, 0) - B)1_{X_T \geq b} + \max(X_T, 0)1_{X_T < b}]$$

such that

$$u(B)P(X_T \geq b) \geq u_0.$$

Incorporating the optimal behavior of the agent and the simple observation that the constraint will be binding in the optimum, the objective can be rewritten as

$$\max_b (b - B^*(b))P(\xi \leq c^*(b)).$$

$B^*(b)$ denotes the necessary bonus if the success probability $1 - \alpha$ is determined in the portfolio choice problem via $c^*(b)$ in order to ensure $u(B)P(X_T \geq b) = u_0$. We note that $B^*(b)$ is an increasing function that is even strictly convex when viewed as a function of $1 - \alpha(c^*(b))$, where we recall that $1 - \alpha(c^*(b))$ is the probability mass where positive payoffs are realized when the agent chooses a portfolio for a given benchmark.

Theorem 4.1. *Let b^* denote the unique solution to the problem*

$$\max_b (b - B^*(b))P(\xi \leq c^*(b)).$$

The optimal contract is given by the pair $(B^(b^*), b^*)$.*

Proof. For an optimal b^* , $B^*(b^*)$ denotes the bonus payment that keeps the partici-

pation constraint binding. Any lower bonus would not make the agent participate, while every higher bonus would not change the portfolio structure but decrease the investor's expected utility. What remains to be shown is that the overall problem

$$\max_b (b - B^*(b))P(\xi \leq c^*(b))$$

is finite. Tankov shows that the portfolio choice problem for a strictly risk averse agent who retains all portfolio payoffs as a compensation is finite. The overall portfolio problem from the investor's point of view has, due to the (with respect to the success probability strictly convex) increased bonus demands induced by the increased benchmark setting, strictly decreasing marginal utility for investment on each marginal state. Therefore, the agency problem also has a finite solution. \square

The risk neutral investor tends to invest only into low state prices and generate extremely high payoffs on the lowest state prices. Via the agency problem the portfolio choice is distorted since the agent tries to reach the benchmark on a maximal subset of the state space. Setting the benchmark higher enforces a portfolio transformation closer to the desired investment structure of the principal. The strictly concave utility of the agent limits this benchmark setting since incentivizing the agent becomes more and more costly.

5. Conclusion

I have analyzed the optimal portfolio choice of an agent who is incentivized by a flat bonus payment and restricted by a risk level measured via the entropic risk measure. A flat bonus scheme leaves the size of the bonus without effect on the portfolio structure, since on a complete market the structure of the portfolio can be chosen such that the benchmark is exactly reached. In other models, even flat bonus schemes might lead to increased risk taking, because payoff structures cannot be replicated exactly and simple scaling of portfolios might increase the probability of achieving the benchmark. In this sense, the often times criticized combination of

complex financial products and high bonuses does not lead to higher risk taking. On the contrary, we have seen that flat bonuses have no influence on the portfolio when financial markets allow for complex products. The simple reason for a high bonus size in this model is to keep the agent incentivized. The driving factor for risk taking is the benchmark that is set by the principal. As a risk neutral principal in general is only interested in the, from his perspective, best state prices, i.e. the low state prices, he tries to set high benchmarks. Increasing the benchmark becomes increasingly expensive as the decreased success probability of the portfolio has to be equalized by increased bonus payments to the agent in order to keep him incentivized. This also becomes increasingly expensive due to the risk aversion of the manager. From the principal's perspective bonuses should be as low as possible as long as the agent accepts the contract.

Therefore, we cannot conclude that high bonuses alone are the reason for excessive risk taking. When discussions about bonus caps arise, one should always keep in mind that benchmarks, as well as appropriate risk controlling are substantial to this kind of principal agent problems. Especially the wrong choice of a risk measure might lead to portfolio choices that induce arbitrary risks on high state prices in order to generate maximal gains on low state prices.

While it is obvious that a call-option type bonus structure would yield different results in terms of the influence of bonus setting on risk taking, portfolio choice models as discussed in part III. reveal that too high bonus setting might induce higher risk taking even for flat bonus schemes. When the agent is able to (costly) control the mean and variance of a portfolio, high bonuses might induce risk effort that is not desired by the investor. As we have seen, this is not the case on a complete financial market.

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Part V.

Appendix

Appendix

A. Proof of Part II. Theorem 3.1

Let $\underline{\mu}^2 \leq \frac{\sigma}{2\gamma\bar{x}}$. From the propositions 3.1 and 3.2 we know that only E_u, E_l and E_{ff} are possible. In order to have E_u it must be that the expected return is higher than L_i . This is exactly given for $L_i \leq \phi_2(P_u)$. Since both agents must satisfy this condition and since $P_u < \underline{\mu} + \underline{\sigma}$, it is necessary and sufficient that $L_1, L_2 \leq \phi_2(P_u)$. For E_{ff} we get a necessary and sufficient condition $L_1, L_2 \geq \phi_1(P_{ff})$ as both agents must participate. This guarantees that the fully liable optimum dominates the limitedly liable one. Since by construction $P_{ff} < \underline{\mu}$, the result follows. Finally for E_l it must be that for one agent the limited optimum dominates and for the other agent the 'full' optimum dominates. As by construction the price yields both optima to dominate nonparticipation, the necessary and sufficient condition is again given by comparing ϕ with L_i . Hence, $L_i \leq \phi_2(P_l) \leq L_j, i, j \in \{1, 2\}, i \neq j$. The logic for the cases $\underline{\mu}^2 \geq \frac{\sigma}{2\gamma\bar{x}}$ and $\underline{\mu}^2 < \frac{\sigma}{\gamma\bar{x}}$ and $\underline{\mu}^2 \geq \frac{\sigma}{\gamma\bar{x}}$ is the same.

Recall that

$$P_u = -\frac{1}{2\gamma\bar{x}} + \sqrt{\frac{1}{4\gamma^2\bar{x}^2} + \frac{\underline{\mu} + \underline{\sigma}}{\gamma\bar{x}}}.$$

Plugging in that $\gamma\bar{x} = \frac{\sigma}{\underline{\mu}^2}$, it follows that

$$P_u = -\frac{\underline{\mu}^2}{2\sigma} + \sqrt{\frac{\underline{\mu}^4}{4\sigma^2} + \frac{\underline{\mu}^3 + \underline{\mu}^2\sigma}{\sigma}} = -\frac{\underline{\mu}^2}{2\sigma} + \sqrt{\left(\frac{\underline{\mu}^2 + 2\mu\sigma}{2\sigma}\right)^2} = \underline{\mu}.$$

Hence, from $P_u = \underline{\mu}$, it is $\phi_1(P_u) = \phi_2(P_u)$, thus, $\phi_1(P_u) = \phi_2(P_u)$ whenever

$$\gamma\bar{x} = \frac{\sigma}{\underline{\mu}^2}.$$

In a similar fashion we have that for $\underline{\mu}^2 = \frac{\sigma}{2\gamma\bar{x}}$:

$$P_l = P_{fl} = \underline{\mu}.$$

Given this, it follows that $\phi_1(P_{fl}) = \phi_2(P_l)$ for $\underline{\mu}^2 = \frac{\sigma}{2\gamma\bar{x}}$.

For the behavior of the boundaries recall that $\phi_1(P) \leq \phi_2(P)$ and $\phi_2(P) \geq 0$. Moreover, $\phi_2(P)$ is decreasing up to $\mu + \sigma$.

For case 1, this immediately yields $0 < \phi_2(P_u)$. With $\mu + \sigma \geq P_u > P_l$ it follows that $\phi_2(P_u) < \phi_2(P_l)$. Finally, $\phi_2(P_l) < \phi_2(\mu) = \phi_1(\mu) < \phi_1(P_{ff})$, as $P_{ff} < \mu < P_l$ and $\phi_1(P)$ is decreasing until μ .

For case 2, $0 < \phi_2(P_u)$ holds for the same reasons. $\phi_2(P_u) < \phi_1(P_{fl})$ since $\phi_2(P_u) < \phi_2(\mu) = \phi_1(\mu) < \phi_1(P_{fl})$. From the monotonicity of $\phi_1(P)$ until μ and $P_{ff} < P_{fl} < \mu$ it is $\phi_1(P_{fl}) < \phi_1(P_{ff})$.

For case 3, $\phi_1(P_{fl}) < \phi_1(P_{ff})$ for the same reasons as in case 2. $\phi_1(P_u) < \phi_1(P_{fl})$ follows from $\phi_1(P_u) < \phi_2(P_u) < \phi_2(\mu) < \phi_1(P_{fl})$.

B. Proof of Part II. Theorem 4.1

A necessary condition for E_f is that the price generated by the agent with the higher worst case expectation, i.e. agent 2, makes agent 1 not participate. For this it must be that $P_f(\underline{\mu}_2) \geq \mu_1$. This condition yields $\underline{\mu}_2 \geq \underline{\mu}_1 + 2\gamma\bar{x}\underline{\mu}_1^2$. One can see that for $\underline{\mu}_2 = \underline{\mu}_1$ this is not possible and therefore this type of equilibrium can only occur with heterogeneous ambiguity. Similarly, for E_l we get $\underline{\mu}_2 \geq \underline{\mu}_1 + 2\gamma\bar{x}\underline{\mu}_1^2 - \sigma$. For E_u it must be ensured that even for the agent who expects less return the price admits a positive optimizer. Therefore, from $\mu_1 \geq P_u$ it follows that $\underline{\mu}_2 \leq \underline{\mu}_1 + 2\gamma\bar{x}(\underline{\mu}_1 + \sigma)^2$. The cases E_{ff} and E_{fl} follow from the same method. Necessary and sufficient additions are the requirements on the equity in the same manner as in chapter 3.

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Short Curriculum Vitae

Frederik Diermann

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|-----------|---|
| 2010-2016 | Member of the Bielefeld Graduate School in Economics and Management (BiGSEM) at Bielefeld University, Germany |
| 2008-2010 | Master of Science in Mathematical Economics at Bielefeld University, Germany Member of the master program Quantitative Economic Models (QEM) at Bielefeld University and Université Paris I Panthéon Sorbonne, France |
| 2005-2008 | Bachelor of Science in Mathematical Economics at Bielefeld University, Germany |
| 2004-2005 | Alternative Civilian Service at the St. Elisabeth Hospital Gütersloh, Germany |
| 2004 | Abitur at the Evangelisch Stiftisches Gymnasium Gütersloh, Germany |