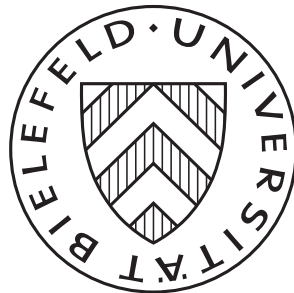


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Immigration, occupational choice and public employment

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Abstract

This paper investigates the theoretical effects of immigration in an occupational choice model with three sectors: a low-skilled, a high-skilled and a public sector. The originality of our approach is to consider (i) intersectoral mobility of labor and (ii) public employment. We highlight the fact that including a public sector is crucial, since omitting it implies that low-skilled immigration unambiguously reduces wages and welfare of *all* workers. However, when public employment is considered, we demonstrate that immigration increases wages in the high-skilled and the public sectors, provided that the immigrant workforce is not too large and the access to public jobs is not too easy. The average wage of natives may also increase accordingly. Moreover, immigration may improve workers' welfare in each sector. Finally, the mechanism underlying these results does not require complementarity between natives and immigrants.

Keywords: Immigration, occupational choice model, public employment

JEL Classification: J24, J61, J45, H44

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1 Introduction

Immigration creates fierce public debate in advanced countries and is a recurrent issue in political campaigns. It is also one of the most controversial topics in economics. Immigration affects natives' wages, but also impacts their occupational choices. For example, natives may adapt their educational decisions or relocate their activity to sectors being less exposed to competition from immigrants. The public sector offers typically this kind of protection since immigrants qualify less easily for public job requirements. At the same time, immigrant workers contribute to tax revenues that are used to finance public expenditures. Accordingly, immigration can generate an additional demand for public goods and thus an additional demand for civil servants.

Empirically, it is important to account for natives' responses to immigration when estimating the wage impact of immigration (Peri, 2011). However, to the best of our knowledge, there has been no research on the impact of immigration on natives' economic outcomes when they can adapt their occupational choices and have access to public jobs. The focus of the present paper is precisely to theoretically explore this aspect of immigration.

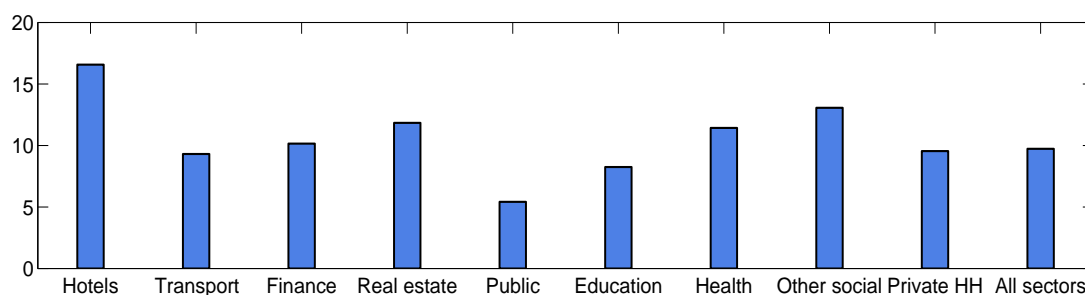
Our main finding is that low-skilled immigration unambiguously reduces wages and welfare of all workers in the absence of a public sector, while it can have positive effects in the presence of public employment, when immigrants do not induce a too large relocation of natives.¹ Before providing more details about our model and its results, let us discuss the evidence on how natives can respond to immigration. In this context we also highlight the empirical importance of public employment and the fiscal effects of immigration.

Natives adapt to immigration in different manners: by moving out to other locations (Borjas et al., 1996; Card and DiNardo, 2000), by switching tasks within the same industry (Ottaviano et al., 2013) or by relocating to different occupations (Ortega and Verdugo, 2011). Recent empirical studies show that immigration affects - in various ways - the educational decisions of natives and their skill composition in general. For instance, immigration is found to raise the labor supply of high-skilled native women (Cortès and Tessada, 2011) and the probability that natives complete high

¹Note that we follow the literature in focusing on the effects on low-skilled immigration because the inflow of immigrants to developed countries has been much larger among low-skilled workers (Gonzalez and Ortega, 2011; Ottaviano and Peri, 2012) though high-skilled immigrants have in recent years become increasingly important (Docquier and Rapoport, 2012). Moreover, high-skilled immigration is often perceived as beneficial for the destination country (see e.g., Storesletten, 2000) and is thus less controversial. Nevertheless, we discuss the effects of high-skilled immigration in Section 6.

school (Hunt, 2012).² By adapting their educational choices, natives can end up in occupations where they face less competition with immigrants. Several studies confirm that immigrants and natives tend to work in different occupations. For example, natives are more likely to work in communication-intensive jobs (Peri and Sparber, 2009; Schoellman, 2010). Figure 1 supports these findings illustrating that immigrants in OECD countries are not evenly distributed across sectors.

Figure 1: Percentage of foreign-born employment by sector (total OECD)



Source: OECD (2008). Figure 1 shows the share of foreign-born employment in the following sectors: *Hotels and restaurants (=Hotels)*; *Transport, storage and communications (=Transport)*; *Financial intermediation (=Finance)*; *Real estate, renting and business activities (=Real Estate)*; *Public administration and defence; compulsory social security (=Public)*; *Education (=Education)*; *Health and social work (=Health)*; *Other community, social and personal service activities (=Other social)*; *Private households with employed persons (=Private HH)*; *All sectors (=All sectors)*. The figure excludes employment in *Extra-territorial organizations and bodies*, where foreign-born workers represent more than 50% of employment.

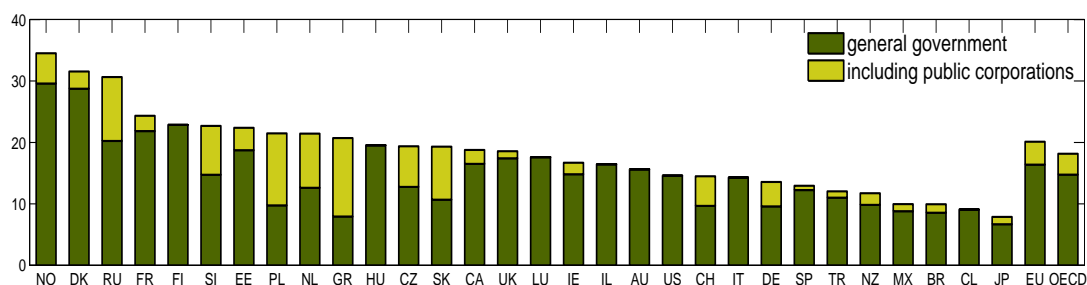
Another observation emerging from Figure 1 is the relative small share of foreign-born workers in sectors which have a majority of public employees such as the public administration and the education sector. Foreign-born workers represent 10% of employment on average in OECD countries but only 5% of employment in the public sector. This stylized fact is also verified for individual country data and different datasets (see Figures 7 and 8 in Appendix A.1 for further evidence). Foreign-born workers are even underrepresented in free movement areas such as the EU (OECD, 2010, p.172-4). The reason is that often only natives can fulfill specific public job requirements, as e.g. citizenship.³ These specific requirements may also have practical reasons. For

²Betts and Fairlie (2003) find evidence that immigration raises local parents' propensity to send their children to private schools at the secondary level of education. Moreover, even the brain drain literature claims that migrants affect human capital accumulation, by raising the incentives of the people remaining in the country of *origin* to invest in education (Docquier and Rapoport, 2012).

³There are many reasons for the underrepresentation of immigrants in the public sector: e.g. various access restrictions (citizenship, knowledge of local languages), specific requirements for certain jobs (degrees in country-specific fields like administrative law), the public sector attaching a higher value

example, George Orwell mentioned that the British Empire was employing locals as civil servants during its colonial period in Burma, because, among other reasons, natives “have a better idea of the workings of their fellow countrymen’s minds, and this helps them to settle legal disputes more easily” (Blair, 1929). In many countries, the State is an important employer in terms of the total workforce employed. Figure 2 shows that employment in the public sector represents a non-negligible share of total employment in advanced countries: on average, 15% in OECD countries in 2008, and even 18% when including public corporations (see Figure 2, last column). The share of public employees can be even larger because it includes civil servants working in the education, health and social work sectors.

Figure 2: Employment in the public sector as a percentage of the labor force (2008)



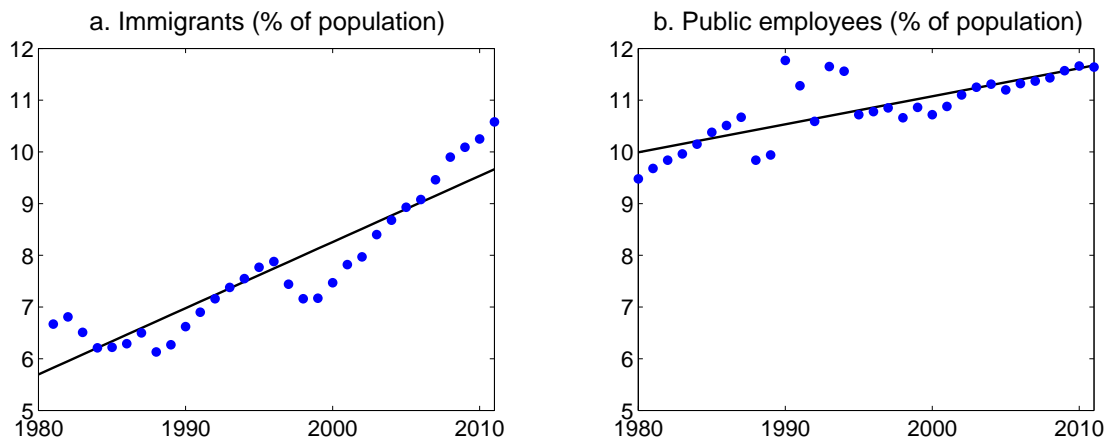
Source: OECD (2011, Table 21.2). Figures for Norway, Denmark, Russian Federation, France, Finland, Slovenia, Estonia, Poland, Netherlands, Greece, Hungary, Czech Republic, Slovak Republic, Canada, United Kingdom, Luxembourg, Ireland, Israel, Australia, United States, Switzerland, Italy, Germany, Spain, Turkey, New Zealand, Mexico, Brazil, Chile, Japan, European Union and OECD average.

Immigration has not only effects on the labor market, but also on public finance. On the one hand, immigrant workers contribute to tax revenues which are used to finance public expenditures. On the other hand, immigrants benefit from public transfers and have a demand for public services provided by civil servants.⁴ Despite finding that immigration reduces the *share* of government spending on public education (using 2000 data for 80 countries), Mavisakalyan (2011) does not exclude that immigration may induce a higher total demand for public services, like education, health and “other publicly provided goods, such as infrastructure or public housing”.⁵ Moreover, many countries created specific public entities (and related jobs) for education acquired in the host country, different preferences for public sector jobs between natives and immigrants.

⁴In a recent study, Dustmann and Frattini (2013) find that immigrants having arrived between 2000 and 2012 to the UK had a positive fiscal impact (in contrast to natives), contributing more to tax and welfare systems than benefiting from them.

⁵Several recent studies point at a negative relationship between immigration (or ethnic diverse so-

Figure 3: Immigrants and public employees in EU-15 countries



The figure presents the trend over time of average foreign population and average number of public employees (public administration, education, health and social work) in EU-15 countries (both series expressed as a percentage of total population). The observations are fitted with a regression line. The data source is the OECD National Accounts.

directly linked to immigration issues such as ministries of immigration (e.g. Canada, France). Alesina et al. (2000) find that public employment is higher in more ethnically diverse US cities, consistent with their earlier finding (in Alesina et al., 1999) that (total) public spending is higher in those cities.⁶ Dustmann and Frattini (2011) document that, between 1994 and 2010, public employment and the working age population increased in the UK (Figure 9 in Appendix confirms these trends for EU-15 countries). Immigrants constituted an essential part of the population growth (and in private employment) but only a minority of the newly created public jobs accrued to them, confirming our previous discussion.⁷ Speciale (2012) shows that, while the immigrant

cities) and public good provision. They find a negative effect of the immigration on specific public goods like education as a share of total public spending (Mavisakalyan, 2011) or on specific public spending in per capita terms (Razin et al., 2002; Speciale, 2012). Some other studies mitigate these findings. Gerdes (2013) finds that the inflow of refugee migrants from 1995-2001 had no effects on per capita public goods consumption in Denmark. Relatedly, Alesina et al. (1999) find that more ethnically diverse jurisdictions in the United States dedicate a lower share of [public] spending on roads, education and health but are at the same time associated with higher public expenditures per capita and deficits/debt per capita. These empirical results, which are not inconsistent with our model, are irrelevant for our purpose. Indeed, our focus is on the impact of immigration on the total provision of public goods rather than on per capita expenditures or on specific public goods as a fraction of total expenditures.

⁶Alesina et al. (2000)'s explanation is that public employment acts as an implicit subsidy to ethnically defined interest groups.

⁷Immigration has been responsible for 65% and 58% of the growth in the working age population

share in population rose (as shown in Figure 3.a), the public education expenditure per student (to GDP per capita) declined in EU-15 countries over the period 1980-2000. Based on the same sample of countries, Figure 3.b indicates that this is not true for the share of public employees in total population, which has remained constant (and did even slightly increase). Thus, public employment has adjusted to changes in population and indirectly to variations in immigration.

In the present paper, we analyze the effects of immigration in a tractable and static occupational model with three sectors: a low-skilled, a high-skilled and a public sector. Natives differ in their natural learning ability which is continuously distributed over a given support and they may work in either of the aforementioned sectors, depending on their educational decisions. The public sector offers medium-skilled jobs to natives only⁸ and provides public goods, from which individuals derive utility. The originality of the model lies in the combination of inter-sectoral mobility and public employment.⁹ Our approach is justified by the evidence mentioned above. Natives can respond to immigration through their educational decisions and public sector employment can affect these decisions. The direct effect of low-skilled immigration is to depress wages in the low-skilled sector. This crowds out the most talented low-skilled natives who decide to take a medium-skilled public job. However, immigration also generates additional tax revenues which finance new public sector jobs providing increased public services. As a consequence, immigration raises both the public labor supply and demand and has ambiguous effects on public wages. We demonstrate that high-skilled and public wages move in the same direction. When immigration pushes up public salaries, there is a shift of high-skilled natives to the public sector. When immigration depresses public salaries, the most talented civil employees decide to join the high-skilled sector. In the first case, high-skilled and public wages increase, while they decrease in the second case.

Our results can be summarized as follows. We demonstrate that omitting a public sector means that immigration unambiguously reduces the wages and welfare of

and in employment, respectively, between 1994 and 2010. Employment has grown by 13% and public employment (comprising public administration, education, health and social work) by 19%, which is more than private employment (12%). Immigration has contributed to 29% and 73% of public and private employment growth, respectively.

⁸We make this simplifying assumption based on the evidence that natives have privileged access to public jobs. Assuming that low-skilled immigrants have the same opportunities as natives to enter public jobs as natives would not alter the main conclusions of our analysis (as discussed in Section 6).

⁹In our paper, inter-sectoral mobility is viewed as a long-term process based on changes in educational decisions. However, in a short-term perspective, e.g. at the quarterly rate, intersectoral mobility costs can be high implying slow intersectoral adjustments (Artuc et al., 2010).

all workers. Indeed, if we only consider inter-sectoral mobility, immigration would raise employment and reduce wages in the low-skilled sector and thereby displace low-skilled native workers to the high-skilled sector. This would reduce high-skilled wages because of diminishing returns. Furthermore, without public sector, the welfare loss caused by the fall in wages would not be compensated by the utility gain resulting from increased public services. In contrast, when public employment is considered, the paper demonstrates that immigration may augment the wages of civil servants and high-skilled workers, though low-skilled wages always decrease. This occurs when recruitment of civil servants is selective and the share of immigrants is low. In this case the shift of native workers to the public sector is moderate. The average native wage may increase accordingly depending on the distribution of the workforce. A calibration of the model with reasonable parameter values leads to a moderate displacement of low-skilled natives and also shows that average wages may increase when the public sector is elitist and the immigrant workforce is small. The model also demonstrates that immigration may be welfare-improving for all workers. The reason is that immigration increases the utility derived from the provision of public services net of taxes. As a consequence, when the access to public sector jobs is restrictive and the size of the immigrant workforce is moderate, civil servants and high-skilled workers necessarily gain from immigration. Because immigration increases the supply of public services, the welfare of low-skilled workers may increase although they incur a wage reduction. Moreover, a calibration of the model shows that *average* native welfare may improve.

Finally, our main results are robust to several alternative assumptions. For instance, whether immigrants have access to jobs in the public sector or whether low-skilled natives and immigrants are imperfect substitutes does not fundamentally change the above conclusions. In particular, our model demonstrates that immigration can be beneficial to natives even when they are perfectly substitutable to low-skilled immigrants. This shows that our result should not be impaired by the current disagreement in the recent empirical literature concerning the complementarity degree between both groups and consequently on the wage effects of low-skilled immigration. Those who find a negative wage impact of immigration argue that low-skilled immigrants and natives are perfect substitutes (e.g. Borjas et al., 2011). Others challenge this view, like Ottaviano and Peri (2012), who point to a small but positive impact on average native wages and find immigrants and natives with high school degree or less to be imperfect substitutes.¹⁰ Assuming complementarity between low-skilled

¹⁰Other studies find a negative impact on natives' wages and perfect substitution between immigrants and natives Borjas (2003), Jaeger (2007) and Bratsberg and Raaum (2012). However, Manacorda

immigrants and low-skilled natives would make more likely the beneficial effects of immigration that are highlighted in our model. Everything else equal, the higher the complementarity between natives and immigrants, the less low-skilled wages will decrease and the less natives will have an incentive to move from low-skilled to public sector jobs.

As mentioned above, the originality of our approach is to consider inter-sectoral mobility of labor and public employment. The standard textbook model, with homogeneous labor and a fixed capital stock, predicts that an immigrant influx lowers the wages of competing workers while raising capital owners' income (Borjas, 1995). Alternative models incorporate extensions or long-term adjustments to the traditional model (such as capital adjustment or product demand effects), which may slightly alter the negative effect on wages (see e.g. Borjas, 2009). However, there are very few models accounting for inter-sectoral mobility or human capital responses to immigration. These studies are even omitted in the surveys of the theoretical literature (see, e.g., Bodvarsson and Van den Berg, 2009). Most likely the first contribution analyzing endogenous human capital adjustments to immigration is that of Chiswick (1989), who finds that an immigration policy that is beneficial to natives is generally accompanied by an increase in human capital investments by natives. Eberhard (2012) calibrates a general equilibrium model with endogenous educational choices (to the US economy). His simulations show that the direct effect of immigration on average wages is negative but is more than compensated by the positive indirect effect on human capital accumulation. These studies, however, do not include a public sector and ignore related fiscal and employment effects. In contrast, Razin et al. (2002) and Dottori et al. (2013) analyze how immigration affects the tax base and human capital formation. Using a political economy model, they focus on different questions than ours and do not consider the role of public employment. To our knowledge, the only related study distinguishing between private and public employment is Pierrard (2008), who however, focuses on the effects of cross-border commuting on unemployment in a search and matching framework without taxes and public goods.¹¹

This paper is organized as follows. Section 2 presents the model. The impact of low-skilled immigration is highlighted in Section 3 and its welfare implications in Section 4. Section 5 provides an illustration of our results by way of a numerical exercise,

et al. (2012) stress that immigrants have little impact on wages of native-born while Dustmann et al. (2013) emphasize the positive wage impact of immigration.

¹¹For theoretical studies on immigration and unemployment, see e.g., Kemnitz (2003) or Müller (2003).

while Section 6 discusses several assumptions of our model and how they affect our main results. Section 7 concludes.

2 The model

Assume that natives of a given country can choose their occupation. Each option is supposed to correspond to a specific sector. As in Galor and Zeira (1993), there are two technologies producing the same final good. A first sector denoted by h uses only high-skilled personnel, while a second sector denoted by l uses only low-skilled people. In addition, there is a public sector p which is only accessible for natives. As is standard in occupational choice models (Docquier and Rapoport, 2012), we assume that individuals are distributed according to their born ability and that entering an occupation entails a monetary cost, here $\gamma\theta_i$, where γ is the difficulty to learn (ability) and θ_i is the uniform cost to get a job in sector i (with $i = h, l, p$). For simplicity, we posit that the citizens are evenly spread, with density N , along the segment $[0, 1]$ according to their born ability γ to get a job in the different sectors. An individual with poor learning ability (high γ) can always get a job in the low-skilled sector at no cost ($\theta_l = 0$) and earn a wage w_l . However, an individual of type γ , $\gamma \in [0, 1]$ who opts for a public job, earns w_p but incurs a cost of γe depending on her individual ability γ times a uniform cost $e (= \theta_p)$ reflecting the difficulty to access a civil service position (specific training, recruitment exams,...). An individual of type γ , $\gamma \in [0, 1]$ who opts for a skilled job, earns w_h but has to incur a total cost of $c\gamma$ where $c (= \theta_h)$ is a uniform training and education cost. In the sequel, we consider that the education and training requirements to access a job are generally highest in the high-skilled sector.

Assumption 1 $c > e$.

Finally, an individual who chooses a low-skilled job earns w_l without incurring any job access cost. Further, we assume that people consume their wages after having paid a lump sum tax t to finance a public good G . Moreover, individuals derive utility from public spending. We posit quadratic costs in the utility from public goods to ensure tractability, as is common in the literature (see e.g. Fershtman and Nitzan, 1991). The utility function of an individual of type γ , $\gamma \in [0, 1]$ is given by

$$U_i(\gamma) = w_i - \gamma\theta_i - t + g \left(G - \frac{G^2}{2} \right), \quad i = h, l, p, \quad (1)$$

with $\theta_l = 0$, $\theta_p = e$, $\theta_h = c$ and g is a utility weight on public goods/services. For simplicity, we set $g = 1$ as this does not affect our results.

Because we focus on the economic impact of immigration, we assume that total native labor supply does not change and we normalize it to one ($N = 1$). In addition to natives, the labor force is augmented by m low-skilled immigrants, with $m < 1$. Without loss of generality, we assume that there are no high-skilled immigrants and in the following we only shall refer to immigration.¹² Finally, the total labor supply is $S = 1 + m$.

2.1 Occupational choices and sectoral labor supplies

Domestic workers prefer the high-tech sector to a job in the public sector if $U_h > U_p$, which is the case for all individuals of type $\gamma \in (0, \bar{\gamma}]$ with $\bar{\gamma} = \frac{w_h - w_p}{c - e}$. It follows that the supply of high-skilled domestic workers is

$$N_h^s(w_h, w_p) = \frac{w_h - w_p}{c - e}.$$

Since $c > e$, we have $N_h^s > 0$ iff $w_h > w_p$. All the workers for which $\gamma \in (\bar{\gamma}, 1]$ with $\bar{\gamma} = \frac{w_p - w_l}{e}$ prefer the public sector to a job in the low-skilled sector if $U_p > U_l$. The resulting supply of low-skilled workers is

$$N_l^s(w_l, w_p) = \left(1 - \frac{w_p - w_l}{e}\right).$$

Consequently, the supply of civil servants is

$$N_p^s(w_p, w_h, w_l) = \left(\frac{w_p - w_l}{e} - \frac{w_h - w_p}{c - e}\right),$$

provided that the subset $[\bar{\gamma}, \bar{\bar{\gamma}}]$ is non-empty.¹³

Finally, total labor supply in each sector equals $L_h^s = N_h^s$, $L_p^s = N_p^s$ and $L_l^s = N_l^s + m$. The latter implies that low-skilled natives and immigrants are perfect substitutes, as found in Borjas et al. (2011) and in Bratsberg and Raaum (2012).

2.2 Sectoral labor demands and labor market equilibria

Both high-skilled and low-skilled sectors are composed of identical competitive firms producing an homogeneous final good. This good is consumed by the residents and

¹²Our results hold also true if high-skilled immigration increases the labor supply in the high-skilled sector. Moreover, in our framework, an increase in high-skilled immigration leads to analogous effects than a rise in low-skilled immigration (see discussion in Section 6).

¹³This is the case if $\frac{c}{e} > \frac{w_h - w_l}{w_p - w_l} + 1$.

exported to a given world price normalized to one. The production function of the representative firm is $X_h = \alpha L_h - \frac{\alpha}{2} L_h^2$ ($\alpha \geq 1$) in the high-skilled industry and $X_l = L_l - \frac{1}{2} L_l^2$ in the low-skilled sector. In each sector, firms maximize their profit by choosing the appropriate level of labor input. Firms use labor as the sole input characterized by decreasing returns to scale. The latter is a common hypothesis in theoretical models of immigration (Facchini and Willmann, 2005; Marchiori and Schumacher, 2011) and is also supported by empirical evidence (Basu and Fernald, 1997). Moreover, we assume that firms are foreign-owned, which allows to ignore firms' profits (like e.g. Scholten and Thum, 1996) and to focus on how immigration affects the situation of the working population.

The equilibrium wages w_h and w_l are competitively determined by confronting the aggregated labor demand to the domestic labor supply augmented by immigrants. The public good is produced using a linear technology with public labor L_p^s only. For simplicity, the production function of the public good is given by $G = G(L_p) = L_p$.

High-tech sector

The representative firm of the high-skilled sector maximizes its profit by choosing L_h

$$\underset{L_h}{Max} \Pi_h = X_h - w_h L_h.$$

The labor demand for high-skilled workers resulting from the FOC equals $L_h^d = 1 - \frac{w_h}{\alpha}$.¹⁴ The market equilibrium in sector h is obtained by equating the aggregate demand $L_h^d(w_h)$ to the domestic labor supply $N_h^s(w_h)$. The market equilibrium, for given values of w_p and w_l , yields the equilibrium wage rate in the high-skilled sector

$$w_h^* = \alpha \frac{w_p + (c - e)}{\alpha + c - e},$$

where $w_h^* > 0$ since $c > e$.

Low-skilled sector

The representative firm in the low-skilled sector maximizes its profit by choosing L_l

$$\underset{L_l}{Max} \Pi_l = X_l - w_l L_l.$$

The labor demand for low-skilled workers resulting from the FOC equals $L_l^d = 1 - w_l$. The market equilibrium is obtained by equating the labor demand $L_l^d(w_l)$ to the

¹⁴In order to have a positive demand, we impose that $\alpha > w_h$ and thus $L_h^d < 1$.

¹⁵ $L_h^d = N_h^s$ implies $1 - \frac{w_h}{\alpha} = \frac{w_p - w_h}{e - c}$.

domestic labor supply $L_i^s(w_i)$ augmented by an exogenous supply of immigrants m (with $m < 1$). The market equilibrium yields

$$w_i^* = \frac{w_p - me}{1 + e}.$$

We assume that $w_p > me$. It follows that the wage rate w_i^* is positive. Moreover,

$$\begin{aligned} w_h^* - w_p &= \frac{(c - e)(\alpha - w_p)}{\alpha + c - e}, \\ w_p - w_i^* &= \frac{e(m + w_p)}{1 + e}. \end{aligned}$$

Consequently, we have $w_h^* > w_p^* > w_i^*$. This result is consistent with the observation that in many advanced countries the wage dispersion is higher in the private sector and with the fact that the wage-skill profile in the public sector is flatter than in the private sector (evidence is found in Giordano et al. (2011) for Euro Area countries and in ONS (2012) for the United Kingdom).¹⁶

Public sector

Assume that each civil servant provides one unit of public good ($L_p = G$). Then we write the budget constraint of the public sector as

$$w_p G = t(1 + m),$$

where the right hand side represents the taxes levied on the total working population including the natives and the immigrants, and the left hand side represents the wage cost of the public good. It follows that

$$t = \frac{w_p G}{S}. \quad (2)$$

We consider that the jobs in the public sector are occupied by medium-skilled natives. We also suppose that the native population chooses the level of public good provision by direct majority voting. Since the utility functions are single-peaked in G , this is equivalent to the case of a policy maker who maximizes the utility of the median voter¹⁷ characterized by the parameter θ_M :

$$\max_G U_i(G) = w_i - \gamma\theta_M - \frac{Gw_p}{1 + m} + G - \frac{G^2}{2} \quad (3)$$

¹⁶According to these studies, low-qualified workers are paid more in the public sector and high-qualified employees more in the private sector. Similarly, workers at the bottom of the pay scale have a higher salary in the public sector, while workers at the top have a higher wage in the private sector. These studies also indicate that the representative public sector employee earns a higher wage than the representative private sector employee.

¹⁷We assume that only natives have the right to vote. However, we could extend the right to vote to immigrants without changing the demand of public goods. Indeed, in our model, the equilibrium demand of public goods is independent of whether individuals are residents or immigrants.

The first order condition yields the equilibrium demand of public goods $G^* = 1 - \frac{w_p}{S}$ which equals the demand for public labor denoted by L_p^d . Thus we can write

$$L_p^d = 1 - \frac{w_p}{S} \quad (4)$$

and the total tax revenue

$$T = tS = \frac{w_p}{S} \left(1 - \frac{w_p}{S}\right) S = w_p - \frac{w_p^2}{S}. \quad (5)$$

Equating the labor demand to the labor supply in the public sector ($L_p^d = N_p^s$) leads to the following expression

$$1 - \frac{w_p}{S} = \frac{w_p - w_l}{e} - \frac{w_h - w_p}{c - e}.$$

Taking account of the equilibrium wage rates in the high-skilled and low-skilled sectors, we deduce the wage rate w_p^* from the following equation

$$1 - \frac{w_p}{S} = \frac{m + w_p}{e + 1} - \frac{\alpha - w_p}{\alpha + c - e}. \quad (6)$$

3 Immigration, domestic wage and occupational choice

In this section, we analyze how immigration impacts the equilibrium of the host economy with a special focus on how the wage structure is affected.

3.1 General effects of immigration

Before explaining the effects of immigration resulting from our model, we highlight the importance of accounting for *inter-sectoral mobility* and *public employment*. In a simple model with two sectors (low- and high-skilled), but *without inter-sectoral mobility* and *without public employment* (as in Borjas, 1995), immigration increases the labor supply in the low-skilled sector and drives down the corresponding wage rate, because of diminishing returns. The high-skilled sector remains however unaffected. In a two-sector model *without public employment* but *with inter-sectoral mobility*, low-skilled immigration depresses low-skilled wages and crowds out low-skilled natives who decide to join the high-skilled sector. Accordingly, the labor supply of high-skilled workers increases and their wage rate declines (This simple model is sketched in Appendix A.2 and is referred to as “ $G=0$ ” in Table 1).

If we assume *inter-sectoral mobility* and *public employment* (the model is then referred to as “ $G>0$ ” in Table 1), two different effects are induced by immigration.

A first effect is to increase the labor supply in the low-skilled sector and to drive down wages accordingly. This displaces native low-skilled workers to the *public* sector (to the *high-skilled* sector in the “ $G=0$ ” model). The labor supply rises in the public sector consequently. The second effect is to increase the demand of public goods which are financed by additional taxes levied on the new immigrants. This finally induces an additional demand for labor in the public sector. As we shall see, the combination of these two effects on public sector wages is ambiguous. However, the way the public sector wage rate reacts to immigration will help to understand how foreign labor inflow impacts the low- and high-skilled sectors.

In the following, we firstly study the impact of immigration on the public sector wage (summarized in Lemma 1). Then we analyze the effect of immigration on the private economy (summarized in proposition 1).

3.2 Impact of immigration on the public sector

As we highlighted above, immigration increases both the public labor supply and the public labor demand. The effect on labor demand results directly from the fact that $\frac{\partial G^*}{\partial m}$ is positively signed.¹⁸

The wage rate in the public sector w_p^* results from the market clearing condition (6) and satisfies the following expression

$$\left(\frac{1}{e+1} + \frac{1}{\alpha+c-e} + \frac{1}{S} \right) w_p^* = 1 - \frac{m}{e+1} + \frac{\alpha}{\alpha+c-e}. \quad (7)$$

Since immigration increases both the supply and demand for labor in the public sector, the resulting effect on w_p^* is unclear as shown by the following derivative :

$$\frac{\partial w_p^*}{\partial m} = \left(\frac{w_p^*}{S^2} - \frac{1}{e+1} \right) / \left(\frac{1}{e+1} + \frac{1}{\alpha+c-e} + \frac{1}{S} \right). \quad (8)$$

This leads to the following lemma (the proof is given in Appendix A.3):

¹⁸Because we know that $G^* = 1 - \frac{w_p^*}{1+m}$ we deduce that $\frac{\partial G^*}{\partial m} = \frac{w_p^*}{S^2} - \frac{\partial w_p^*}{\partial m} / S$. Using (7) and after rearranging, we obtain

$$\frac{\partial G^*}{\partial m} = \left[1 - \frac{1}{1+S \left(\frac{1}{e+1} + \frac{1}{\alpha+c-e} \right)} \right] \frac{w_p^*}{S^2} + \frac{1}{(e+1)S \left(\frac{1}{e+1} + \frac{1}{\alpha+c-e} + \frac{1}{S} \right)},$$

which is always positive.

Lemma 1 Suppose that $c > e \geq 0$. There exists $\hat{e} > 0$, such that:

(1.) for $0 < e < \min\{c, \hat{e}\}$, we have $\frac{\partial w_p^*}{\partial m} < 0, \forall m \in [0, 1)$;

(2.) for $\hat{e} < e < c$, there exists $\hat{m} > 0$, such that

$$\begin{cases} (a.) \frac{\partial w_p^*}{\partial m} > 0, & \text{if } 0 < m < \min\{\hat{m}, 1\}, \\ (b.) \frac{\partial w_p^*}{\partial m} < 0, & \text{if } \hat{m} < m < 1. \end{cases}$$

The intuition underlying Lemma 1 is straightforward. Immigration pushes down the public sector wage if the resulting increase in the public labor supply is larger than the resulting increase in the public labor demand. This occurs when the immigrant population is large ($m > \hat{m}$) and/or when jobs in the public sector are relatively easy to access ($e < \hat{e}$, case 1). In other words, a large immigrant population and/or *non-selective* recruitment of civil servants facilitate the displacement of native workers to the public sector. However, if the access to public sector jobs is difficult and the share of immigrants is low ($e > \hat{e}$ and $\hat{m} > m$, case 2a), the induced public labor supply will be insufficiently high to meet the increased demand for public labor. As a consequence, the public sector wage rate will increase.

3.3 Impact of immigration on the low- and high-skilled sectors

Table 1 summarizes the effects of immigration in the three sectors of our model. The following expressions highlight the impact of immigration on the equilibrium wage and the equilibrium employment level in the low-skilled sector

$$\frac{\partial w_l^*}{\partial m} = \frac{1}{1+e} \left(\frac{\partial w_p^*}{\partial m} - e \right) < 0, \quad (9)$$

$$\frac{\partial L_l^*}{\partial m} = 1 + \frac{\partial N_l^*}{\partial m} = 1 - \frac{1}{1+e} \left(\frac{\partial w_p^*}{\partial m} + 1 \right) > 0. \quad (10)$$

Equation (9) states that immigration leads to an unambiguous decrease of the equilibrium low-skilled wage (as $\partial w_p^*/\partial m < e$), while equation (10) reveals that immigration has two opposing effects on low-skilled employment (all the proofs are provided in Appendix A.4). The first term of the equation means that immigration leads to a one-to-one increase in the labor supply and the second term indicates that immigration crowds out low-skilled natives ($\partial N_l^*/\partial m < 0$). Finally, immigration always increases employment in the low-skilled sector because the flow of additional immigrants exceeds the number of displaced low-skilled natives. Diminishing marginal utility in

the provision of public goods (equation (3)) limits the positive impact of immigration on public labor demand. Indeed, the possible wage-gain of a low-skilled native who joins the public sector following additional immigration does not exceed, at equilibrium, the cost for accessing to this job ($\frac{\partial w_p^*}{\partial m} < e$).

Table 1: Increase in low-skilled immigration

	Model	Case		w_p	L_p^*	w_l	N_l^*	L_l^*	w_h	L_h^*	U_p	U_l	U_h	
$m \nearrow$	$G = 0$			n.a.	n.a.	-	-	+	-	+	n.a.	-	-	
	$G > 0$	$e < \hat{e}$		-	+	-	-	+	-	+	?	?	?	
		$e > \hat{e}$	$m < \hat{m}$		+	+	-	-	+	+	-	+	?	+
			$m > \hat{m}$		-	+	-	-	+	-	+	?	?	?

m is the share of low-skilled migrants. w_p , w_l and w_h are wages in the public, low- and high-skilled sectors, respectively. L_p^* , L_l^* and L_h^* are the equilibrium employment levels in the public, low- and high-skilled sectors, respectively. N_l^* is the equilibrium level of native labor in the low-skilled sector. U_p , U_l and U_h are the utility levels in the public, low- and high-skilled sectors, respectively. \hat{e} and \hat{m} originate from the solution of the polynomial defined in lemma 1 and represent the threshold entry cost into public jobs and the threshold of immigrant share in the total population, respectively. $G = 0$ stands for the case without public employment (sketched in Appendix A.2). “-”, “+” and “?” indicate respectively a negative, positive and ambiguous reaction of the concerned variable to a positive change in the migrant share. “n.a.” means not applicable.

Differentiating the equilibrium wage rate and the employment level in the high-skilled sector with respect to m , yields

$$\frac{\partial w_h^*}{\partial m} = \frac{\alpha}{\alpha + c - e} \left(\frac{\partial w_p^*}{\partial m} \right), \quad (11)$$

$$\frac{\partial L_h^*}{\partial m} = \frac{\partial N_h^{s*}}{\partial m} = -\frac{1}{\alpha + c - e} \left(\frac{\partial w_p^*}{\partial m} \right). \quad (12)$$

Note that when immigration increases, the equilibrium wage in the high-skilled moves in the same direction as in the public sector, but the impact on the latter is larger. High-skilled employment and public wages move however in opposite directions. Recalling Lemma 1 let us clarify the effect of immigration on w_h^* and L_h^* .

If the immigrant population is large ($m > \hat{m}$) and/or the access to public jobs is easy ($e < \hat{e}$), the crowding-out effect caused by immigration is substantial and public wages decrease. Reduced public sector wages keep high-skilled natives away from

low(er)-paid public sector jobs and encourage the more talented among the medium-skilled natives to leave the public sector in favor of the high-skilled sector. As a consequence, employment in the high-skilled sector increases and high-skilled wages decline (though less than public wages).

If the immigrant population is low ($m < \hat{m}$) and access to public jobs is difficult ($0 < \hat{e} < e$), the shift of low-skilled natives to the public sector is moderate and public sector wages increase. High-skilled natives are attracted by public sector jobs and this exerts upward pressure on high-skilled wages.

We conclude the above analysis by the following proposition.

Proposition 1 *The effects of immigration can be summarized as follows.*

- *Immigration always raises tax revenue. It also increases the demand for public services and for public labor accordingly.*
- *Immigration always decreases the low-skilled wage and displaces native workers from the low-skilled sector to the public sector.*
- *When the sectoral displacement of native workers is substantial, i.e. public sector accessibility is easy and/or the immigrant population is large, then public sector and high-skilled wages decrease and labor increases in the high-skilled sector.*
- *When the sectoral displacement of native workers is moderate, i.e. public sector accessibility is difficult and the size of the immigrant population is moderate, then public and high-skilled wages increase and labor decreases in the high-skilled sector.*

4 Welfare analysis

In this section, we focus on the welfare effects of immigration (as e.g. Schmitt and Soubeyran, 2006), which can be useful for policy purposes.¹⁹ More exactly, we focus on how immigration impacts individual utility levels. Toward this end we use equation (1), rewritten in the following way

$$U_i(\gamma) = [w_i - \theta_i \gamma] + \Omega, \quad (13)$$

¹⁹Schmitt and Soubeyran (2006) study the (welfare) effects of high-skilled migration in a model with workers or entrepreneurs. Some analyses also look at Pareto-improving transfer payments in the presence of immigration (see e.g. Wildasin, 1994).

where $i = h, l, p$ and $\Omega \equiv \left(G - \frac{G^2}{2}\right) - t$ is the benefit from public goods net of taxes. In what follows, we refer to Ω as the *net public benefit* which can be simplified to $\Omega = \frac{G^2}{2}$ given equations (2) and (4).

Consequently, immigration m impacts the utility of an individual of type i ($= h, l, p$) through his/her wage and through the net public benefit:

$$\frac{\partial U_i}{\partial m} = \frac{\partial w_i}{\partial m} + \frac{\partial \Omega}{\partial m}. \quad (14)$$

Let us first investigate how immigration modifies the net public benefit. For that purpose we write

$$\frac{\partial \Omega}{\partial m} = G \frac{\partial G}{\partial m} = -G \frac{\partial (w_p^*/S)}{\partial m} = -\frac{G}{S} \left(\frac{\partial w_p^*}{\partial m} - \frac{w_p^*}{S} \right). \quad (15)$$

It is easy to verify that $\frac{\partial w_p^*}{\partial m} < \frac{w_p^*}{S}$, using equation (8), and hence the effect of low skilled immigration on the net public benefit Ω is always positive: $\frac{\partial \Omega}{\partial m} > 0$.

The following proposition can thus be stated.

Proposition 2 *Immigration increases the net benefit of public goods (Ω).*

We now analyze the welfare implications of immigration for each sector separately. More precisely, we consider how workers' individual utility is affected by immigration in each sector. Welfare in the public sector is impacted by immigration in the following way

$$\frac{\partial U_p}{\partial m} = \frac{\partial w_p^*}{\partial m} + \frac{\partial \Omega}{\partial m}. \quad (16)$$

Given Proposition 2, it appears from equation (16) that immigration increases a public employee's welfare if $\frac{\partial w_p^*}{\partial m} > 0$. This occurs when the access to public jobs is restrictive and the immigrant population not too large (see Lemma 1). In this case, the crowding-out effect caused by immigration is moderate. However, if the crowding-out effect is substantial (implying $\frac{\partial w_p^*}{\partial m} < 0$), immigration does not necessarily deteriorate a public employee's welfare. This only occurs when the increased net public benefit overcompensates for the loss of utility from reduced wages or when the crowding-out effect caused by immigration is high enough.²⁰

The impact of immigration on the welfare of a high-skilled worker can be seen from

$$\frac{\partial U_h}{\partial m} = \frac{\alpha}{\alpha + c - e} \frac{\partial w_p^*}{\partial m} + \frac{\partial \Omega}{\partial m} = \frac{1}{\alpha + c - e} \left[\alpha \frac{\partial U_p}{\partial m} + (c - e) \frac{\partial \Omega}{\partial m} \right].$$

²⁰The determination of the parameter configuration for which this result occurs is cumbersome and does not offer more insights than the qualitative explanation we provide.

Given Proposition 2, it results that immigration makes a high-skilled worker better off when a public servant is made better off. It also appears that a high-skilled worker could benefit from immigration even when the welfare of a public employee deteriorates.

The impact of immigration on the welfare of a low-skilled worker is given by

$$\frac{\partial U_l}{\partial m} = \frac{1}{1+e} \left(\frac{\partial w_p^*}{\partial m} - e \right) + \frac{\partial \Omega}{\partial m} = \frac{\partial U_p}{\partial m} - \frac{e}{1+e} \left(\frac{\partial w_p^*}{\partial m} + 1 \right),$$

where $\frac{\partial w_p^*}{\partial m} + 1 > 0$ (see Appendix A.4). When immigration decreases the welfare of a public employee (a necessary condition is $\frac{\partial w_p^*}{\partial m} < 0$), the welfare of a low-skilled worker decreases too. Moreover, immigration can reduce a low-skilled worker's welfare even if public employees are made better off. This occurs when the impact of immigration on a public employee's welfare is moderate.

The welfare effects of immigration can be summarized in the following proposition.

Proposition 3 *Increased immigration affects the welfare of different types of natives in the following way:*

- *The welfare of a public employee increases ($\partial U_p / \partial m > 0$) if the access to public sector jobs is selective and the size of the immigrant population is moderate, i.e. $\hat{e} < e$ and $\hat{m} < m < 1$. However, if the crowding-out effect caused by immigration is high enough, the welfare of a public employee decreases ($\partial U_p / \partial m < 0$).*
- *The welfare of high-skilled workers increases when public employees' welfare increase or decrease moderately.*
- *The welfare of low-skilled workers decreases when public employees' welfare decrease or increase moderately.*

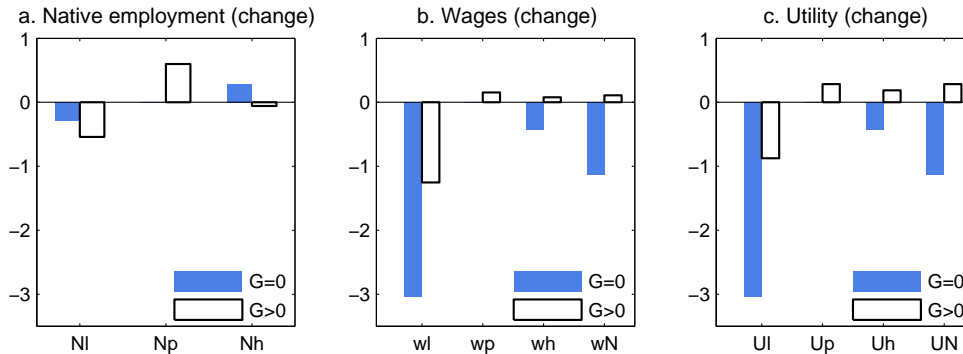
5 Numerical analysis

For illustrative purposes, we calibrate our model for a representative advanced country. We set the skill premium to 2.5, the share of public sector to total native population to 20%, the low-to-high-skilled native population ratio to 2 and the share of low-skilled immigrants to total native population to 10%. These values are comprised in the data range for advanced countries.²¹ They are consistent with a public job accessibility parameter (e) of 1.1, which represents 13% of the education cost (c). The

²¹The sources - for the range of values in OECD countries we refer to - are: Zhu (2005) for the skill premium in the manufacturing sector, the OECD (2011) for the share of public employment and

same parameter values are used to calibrate the model without public sector ($G=0$). We suppose that the immigration inflow raises the share of foreign to native population from 10% to 11%.

Figure 4: Impact of immigration without ($G=0$) and with public sector ($G>0$)



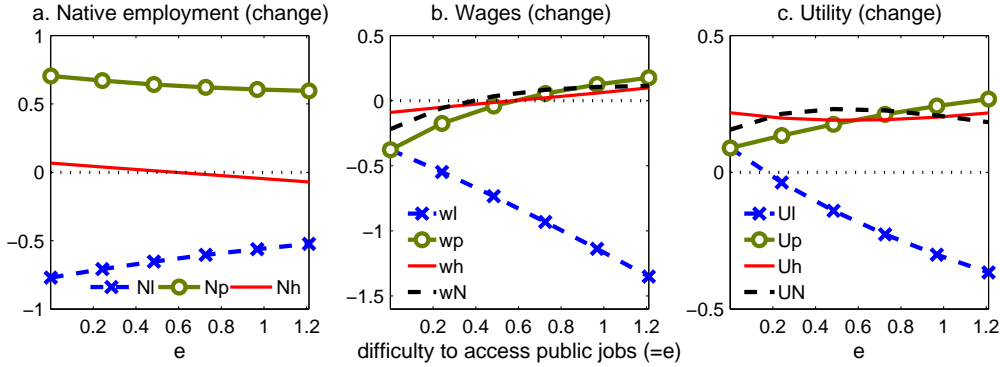
Panel *a*: changes in the percentage points. Panels *b* and *c*: relative changes (%). N_p , N_l and N_h are the equilibrium levels of native labor in the public, low- and high-skilled sectors, respectively. w_p , w_l and w_h are wages in the public, low- and high-skilled sectors, respectively. U_p , U_l and U_h are the utility levels in the public, low- and high-skilled sectors, respectively. w_N and U_N stand for average native wage and utility, respectively.

Figure 4 illustrates the impact of additional immigration without public sector ($G=0$) and with public sector ($G>0$). In the absence of public sector, immigration displaces low-skilled natives to the high-skilled sector (panel *a*) and reduces wages and utility (panels *b* and *c*), which are identical in this case.²² The picture radically changes if we account for a public sector. In this case there is a shift of low-skilled workers to the public sector and the wages and welfare of public employees increase accordingly. Public jobs attract high-skilled workers and wages in the high-skilled sector rise. Overall, immigration is beneficial for the average native worker who experiences an increase in his/her wage and in welfare. Clearly, we are in case 2.a of Lemma 1, where the size of the immigrant population is not too large and access to public sector jobs is relatively difficult. While the model without public sector ($G=0$) is compatible with studies highlighting the negative wage impact of immigration (Borjas, 2003; Borjas et al., 2011), the model which accounts for a public sector ($G>0$) is consistent Docquier et al. (2009) for the share of low-skilled immigrants and for the share of low-to-high-skilled natives.

²²Numbers in figure 4 should be read in the following way. An increase of one percentage point in the share of foreign-born from 10% to 11% decreases the share of low-skilled natives by about 0.25 percentage points and depresses their wages and welfare by 3% (blue bars for “NI”, “wl” and “Ul” in $G=0$ model).

with the findings that point to a small but positive impact on average native wages (Ottaviano and Peri, 2012; Dustmann et al., 2013).

Figure 5: Impact of immigration for different values of e



Panel *a*: changes in the percentage points. Panels *b* and *c*: relative changes (%). N_p , N_l and N_h are the equilibrium levels of native labor in the public, low- and high-skilled sectors, respectively. w_p , w_l and w_h are wages in the public, low- and high-skilled sectors, respectively. U_p , U_l and U_h are the utility levels in the public, low- and high-skilled sectors, respectively. w_N and U_N stand for average native wage and utility, respectively.

It is interesting to see how the picture changes when the government becomes more or less selective in the recruitment of civil servants. Figure 5 depicts the impact of immigration for different values of e (from 0 to 2.5). We see that the calibrated value of e (equal to 1.1) is higher than the threshold value \hat{e} (around 0.7), above which immigration has a positive impact on public sector wages. Moreover, when e varies from 0 to 2.5, the percentage decrease of low-skilled wages induced by immigration varies from 0.5% to more than 2% (panel *b*). It also appears that the percentage shift of low-skilled natives to the public sector resulting from immigration varies from 0.45 to 0.75 percentage points (panel *a*). In panel *c* of the same figure, we see that average native welfare rises when it is relatively easy to access public jobs ($e < \hat{e}$) in which case public wages and high-skilled wages decrease. In other words, utility of public service provision compensates for the wage drop. When access to public jobs is restrictive ($e > \hat{e}$), the calibration indicates that immigration generally improves the welfare of the average native worker except for very high values of e (panel *c*). The reason of this welfare reduction is related to decreasing average wages (panel *b*). Indeed, when the access to public sector jobs is very restrictive, immigration decreases low-skilled wages significantly, which pulls down the average wage of the domestic economy.²³

²³It can be noticed that the threshold level of immigration \hat{m} is largely higher than 1 in the example

6 Further Discussion

In this section, we discuss several of our assumptions and how they affect our main results.

6.1 No intersectoral mobility

In section 3, we compared our model ($G > 0$) with a no-public-sector version ($G = 0$), while retaining inter-sectoral mobility in both cases. Consider an equilibrium situation without inter-sectoral mobility. In this case, immigration increases employment at a one-to-one scale in the low-skilled sector. There is no crowding out of natives and low-skilled wages decrease more than with inter-sectoral mobility. In other words, $\partial N_l^*/\partial m = 0$ in equation (10) and thus $\partial L_l^*/\partial m = 1$. Immigration always increases wages in the public sector, because immigrants raise demand for public jobs (through taxes) without increasing public labor supply. Immigration has no impact on wages and employment of high-skilled workers. Welfare of civil servants rises and the one of high skilled workers as well (as they benefit from an increased supply of public goods), while the impact on low-skilled workers' welfare is ambiguous.

6.2 Public wages

In the short run there may exist rigidities that impede wages to adjust instantaneously to foreign worker inflows. In the present paper we only focus on long-run effects of immigration and no adjustment problems are thus addressed. However, downward wage rigidities may persist in the public sector, especially because of institutional arrangements. Accounting for such a complication would not change our results when immigration drives public wages up (see case 2a of Lemma 1). However, in cases 1 and 2b of Lemma 1, public labor demand rises less than supply. It follows that rigid public wages create a persistent excess supply of low-skilled natives who want to leave the low-skilled sector to obtain a public job. As a consequence, low-skilled wages fall more than in the flexible wages case. However, public and high-skilled wages remain unaffected by downward wage rigidities in the public sector.

(for any value of e), so that case 1 of Lemma 1 is unlikely to arise here. We observe also that immigration causes a slight decrease in native average wage before e reaches \hat{e} and when e is very high (close to 2), because of composition effects.

6.3 High-skilled migrant workers

Our analysis has so far ignored high-skilled immigrants. Introducing high-skilled migrants does however not change the results of Section 3, i.e. the way low-skilled immigration affects the economy. Moreover, high-skilled immigration has similar economic effects than low-skilled immigrants. To show these two points, we only provide intuitions (precise calculations are available on request). Suppose the high-skilled labor supply consists of natives and high-skilled immigrants: $L_h^s = N_h^s + M$. Total population equals $S = 1 + m + M$. The wage rate in the high-skilled sector now becomes

$$w_h^* = \alpha \frac{w_p + (c - e)(1 - M)}{\alpha + c - e}.$$

Equations in the low-skilled sector and in the public sector remain unchanged (except that the latter sector benefits from taxes paid by high-skilled migrants). The effects of low-skilled immigration are thus qualitatively the same as in Section 3.

High-skilled immigration will now decrease wages in the high-skilled sector and crowd natives out of this sector. High-skilled immigration also raises tax revenues. Consequently, both labor supply and demand increase in the public sector. We have again two possible outcomes:

- If the high-skilled immigrant population is large (M is above a specific threshold) and/or the access to public jobs is easy (e is below a specific threshold), we observe a substantial crowding-out effect caused by immigration. As a consequence, public wages decrease and the least talented among the medium-skilled natives leave the public sector in favor of the low-skilled sector inducing a decline in low-skilled wages.
- If the high-skilled immigrant population is low (M below a threshold) and access to public jobs is difficult (e above a threshold), the shift of high-skilled natives to the public sector is moderate and public sector wages increase. The most talented low-skilled natives wish to obtain a public job, which pushes up low-skilled wages.

6.4 Foreign-born workers in the public sector

Our analysis considers that low-skilled immigrants do not have the opportunity to enter the public sector. Assume now that immigrants - as natives - are evenly spread, with density m , along the segment $[0, 1]$ according to their born ability γ^m . Moreover, they can occupy either low-skilled or public jobs. We thus have $m = m_l + m_p$. It is

costless to occupy low-skilled jobs but immigrants have to incur a cost $\gamma^m e^m$ (where e^m is a uniform cost specific to immigrants) if they wish to obtain a public job. We assume that foreign-born workers may face a higher uniform cost to occupy public jobs than natives, all else equal. This reflects, for instance, the possible effort associated to native language acquisition. Thus, we write $e^m = e/\epsilon$ with $\epsilon \in [0, 1]$. When $\epsilon = 0$, we are back to the benchmark case where immigrants are excluded from public jobs.

Comparing U_p^m and U_l^m , the migrants' labor supply for low-skilled jobs equals $m_l^s = m - \frac{w_p - w_l}{e^m}$. Equating labor supply and demand in the low-skilled sector $L_l^d = N_l^s + m_l^s$ yields the following low-skilled equilibrium wage

$$w_l^* = \frac{(1 + \epsilon) w_p - m e}{1 + e + \epsilon}.$$

Equating labor supply and demand in the public sector $L_p^d = N_p^s + m_p^s$, yields the following equation which replaces (7)

$$\left(\frac{1 + \epsilon}{e + 1 + \epsilon} + \frac{1}{\alpha + c - e} + \frac{1}{S} \right) w_p^* = 1 - \frac{(1 + \epsilon) m}{e + 1 + \epsilon} + \frac{\alpha}{\alpha + c - e}.$$

Our main conclusions drawn in Section 3 remain qualitatively the same. Immigration still leads to an increase in public wage provided the share of immigrant population is not too large and the access to public jobs is not too easy (proofs provided in Appendix A.5). The only difference is that the thresholds \hat{e} and \hat{m} will change. More precisely, now we have : $\hat{e}(\epsilon) > \hat{e}$ and $\hat{m}(\epsilon) < \hat{m}$, where $\hat{e}(\epsilon)$ and $\hat{m}(\epsilon)$ are the new thresholds when immigrants are eligible for a public job and \hat{m} and \hat{e} when they are not. Allowing immigrants to obtain a public job increases the threshold entry cost to a public job and decreases the threshold share of immigrant population. The underlying intuition is straightforward. More people can respond to immigration and obtain a public job - everything else constant - and thus, increased public labor demand can only dominate public labor supply under stricter conditions.

6.5 Complementarity between low-skilled immigrant and native workers

So far we considered that natives and immigrants are perfect substitutes in the low-skilled sector. While some authors claim that this is a reasonable assumption (e.g., Borjas et al., 2011) others challenge this view, like Ottaviano and Peri (e.g., 2012), who find immigrants and natives with high school degree or less to be imperfect substitutes. Suppose now that the low-skilled labor demand is defined as follows

$$\tilde{L}_l = [a N_l^{\frac{\sigma-1}{\sigma}} + (1 - a) m^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}},$$

where σ is the elasticity of substitution between low-skilled natives and immigrants and ranges from 0 (perfect complements) to $+\infty$ (perfect substitutes, our benchmark). The corresponding production function is $X_l = \tilde{L}_l - \frac{1}{2}\tilde{L}_l^2$. Assuming that the firms maximize their profit, we obtain the following result from the FOCs

$$w_l = a(1 - \tilde{L}_l)^{\frac{\sigma}{\sigma-1}} \left(\frac{\tilde{L}_l}{N_l} \right)^{\frac{1}{\sigma}}.$$

The impact of immigration on the low-skilled wages is

$$\frac{\partial w_l}{\partial m} \begin{cases} > 0, & \text{if } \sigma < \bar{\sigma} \\ = 0, & \text{if } \sigma = \bar{\sigma} \\ < 0, & \text{if } \sigma > \bar{\sigma}, \end{cases}$$

where $\bar{\sigma} \equiv \frac{\tilde{L}_l}{1-\tilde{L}_l}$. Two forces are at play. On the one hand, immigration decreases the marginal productivity of low-skilled labor, but on the other hand, it increases the productivity of the low-skilled natives provided that they are complementary with immigrants (σ is not infinite). Thus, if complementarity is high enough, i.e. σ low enough, it can dominate the decreasing return effect and wages increase. Otherwise they decrease. A high enough increase in low-skilled wages can reverse the crowding-out effect of natives and lead to an inflow of the least talented natives leaving the public sector. Wages in the three sectors can thus increase.

Figure 6 illustrates the effects of immigration assuming four different degrees of complementarity (parameter σ) between natives and immigrants ranging from *no complementarity* (or *perfect substitution*, our benchmark case) to *weak*, *intermediate* and *strong complementarity*. The parameters are calibrated to match the same targets as in Section 5 in each of the four cases we consider. We take specific values for σ (∞ , 6, 2 and 0.2) to illustrate the different possible cases that can arise in our model. Let us notice that the calibration exercise yields qualitatively similar immigration impacts for values of σ comprised between ∞ and 6. Moreover, the elasticities found in the literature are comprised in that range. Borjas et al. (2011) find that the elasticity of substitution is insignificantly different from infinity while Ottaviano and Peri (2012) report an elasticity of substitution of 12.5.

Immigration leads to a decrease in wages of low-skilled natives when natives and immigrants are weak complements ($\sigma = 6$). This is equivalent to the benchmark case ($\sigma = \infty$). For medium complementarity ($\sigma = 2$), immigration increases low-skilled wages, but not enough to avoid a crowding-out of natives. Again similar results are obtained as in the benchmark case, except that wages and welfare of low-skilled workers do not decrease with immigration. Finally, when low-skilled natives

Figure 6: Impact of immigration under different complementarity degrees between low-skilled natives and immigrants



Panels *a* and *d*: changes in percentage points. Panels *b*, *c*, *e* and *f*: relative changes (%). Natives and immigrants are perfect substitutes when $\sigma = \infty$ (our benchmark case), weak complements when $\sigma = 6$, intermediate complements when $\sigma = 2$ and strong complements when $\sigma = 0.2$. N_p , N_l and N_h are the equilibrium levels of native labor in the public, low- and high-skilled sectors, respectively. w_p , w_l and w_h are wages in the public, low- and high-skilled sectors, respectively. U_p , U_l and U_h are the utility levels in the public, low- and high-skilled sectors, respectively. w_N and U_N stand for average native wage and utility, respectively.

and immigrants are strong complements ($\sigma = 0.2$), immigration raises low-skilled wages. Consequently, public workers are attracted by the low-skilled sector and quit their job (N_l increases). Public wages increase as public labor supply decreases. This attracts the least talented natives of the high-skilled sector where wages will increase as well. The strength of these effects will depend on the elasticity of substitution. The more low-skilled natives are complementary to immigrants, the stronger low-skilled wages will rise and the more natives will be displaced from public jobs. Panels $d - f$ illustrate how important the public sector is when studying the effects of (skilled) immigration. Omitting the role of the public sector, would imply that immigration decreases wages and welfare, except in the case of strong complementarity. In short, assuming imperfect substitution does not invalidate the result that immigration can raise wages and welfare when the immigrant workforce is not too large and the access to public jobs selective. It rather adds an additional welfare-improving channel.

7 Conclusion

The aim of this paper is to provide new insights into the theoretical effects of immigration. The originality of our approach is to consider inter-sectoral mobility and public employment in an occupational choice model with three sectors: a low-skilled, a high-skilled and a public sector. In particular, we show that accounting for a public sector is crucial, because omitting it implies that immigration unambiguously reduces all the workers' wages and welfare. In contrast, when public employment is considered, immigration may raise wages in the high-skilled and the public sectors, provided that the immigrant workforce is not too large and the access to public jobs selective. On average, native wages may also increase. Finally, we find that immigration may improve workers' welfare in each sector.

It is worth noting that our results are based on a tractable model that ignores many economic and non-economic effects of immigration. For instance, our purpose was not to focus on social transfers. Consequently, we did not address the debated issue of whether immigrants constitute net contributors or a burden to the social protection systems of the receiving countries. Moreover, our analysis only considers a homogeneous public good ignoring the effect of immigration on the composition of public expenditures. However, several studies have found that higher ethnic diversity may decrease utility from public goods because individuals of different ethnicities have different preferences concerning public goods and therefore disagree on public policy choices (see e.g., Alesina and Ferrara, 2000). At the same time, greater diversity

may have beneficial effects through higher productivity (Ottaviano and Peri, 2006) or increased knowledge exchanges (Lazear, 2000). Nevertheless, the negative (and positive) effects of diversity may vanish over time as immigrants assimilate to the native population (Chiswick, 1978). Accounting for these highlighted issues would not change our results relative to the impact of immigration on the wage and employment structure of the receiving economy, as long as immigration induces a demand for additional public jobs. Therefore, omitting these features allows us to understand the relevance of natives' occupational choices and public jobs in analyzing the economic impact of immigration. However, focusing on the role of the composition of public spending (social transfers included) would improve the social welfare aspect of the model. Future work should account for this extension.

The paper demonstrates that conditions of access to public-sector employment and the share of immigrants in the resident workforce can be relevant aspects in gauging the effects of immigration on overall wages and welfare. This insight can contribute to explaining the mixed empirical results found in the literature and may be helpful in designing immigration policies.

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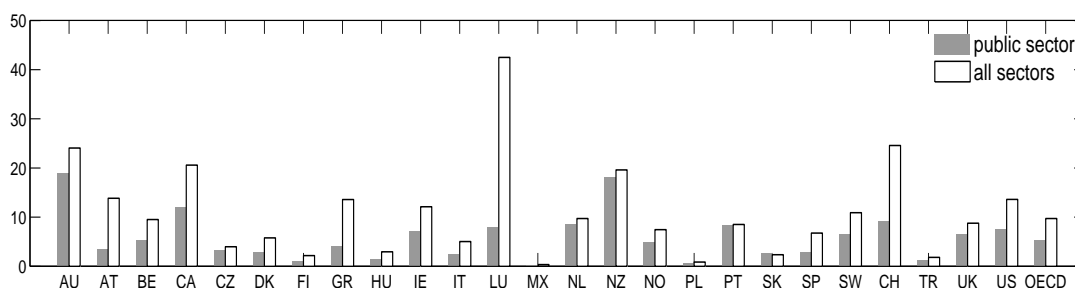
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A Appendix

A.1 Additional figures on foreign-born share in employment

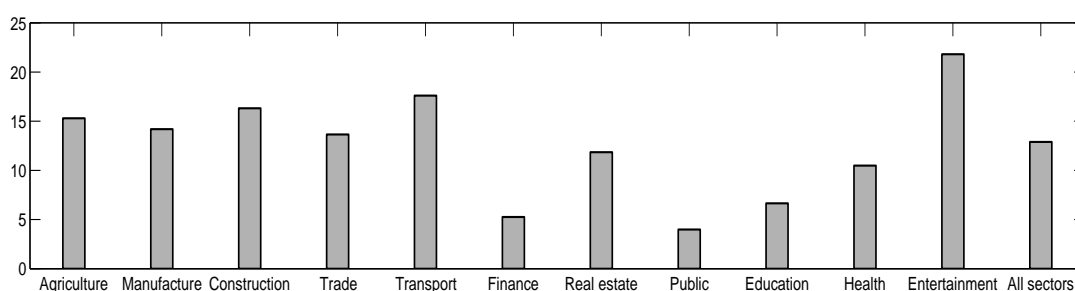
Figure 7 indicates that in each OECD country foreign-born workers are underrepresented in the public sector. Moreover, in all these countries, the public sector is the

Figure 7: Percentage of foreign-born in the public sector and in all sectors, by country



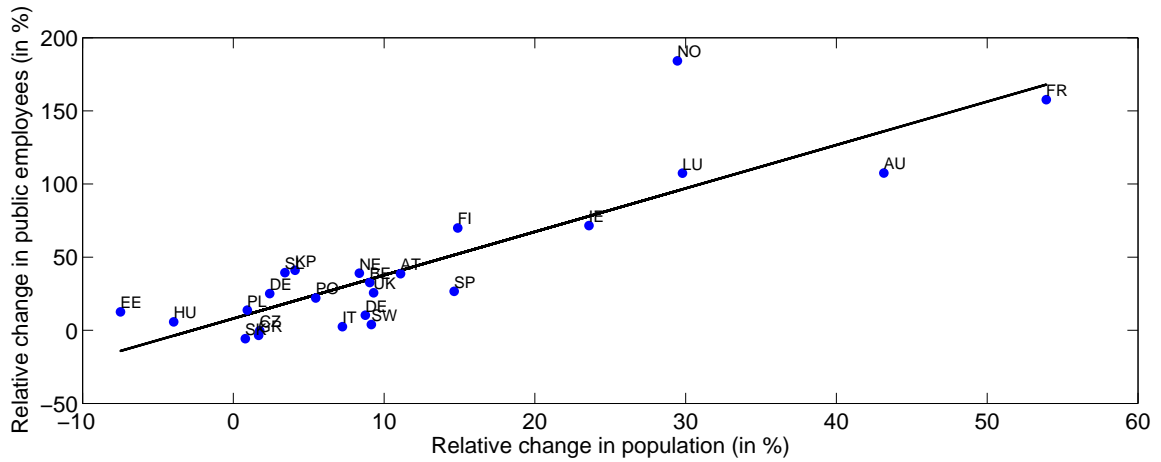
Source: OECD (2008). Figures for Australia, Austria, Belgium, Czech Republic, Denmark, Finland, Greece, Hungary, Ireland, Italy, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States, OECD-total.

Figure 8: Percentage of foreign-born in employment, by sector (country average)



Source: Household Finance and Consumption Network (2013). Weighted average of the following countries: Austria, Belgium, Cyprus, Germany, Finland, Greece, Italy, Luxembourg, Malta, Portugal, Slovenia and Slovak republic (data not provided for other countries in the survey).

Figure 9: Public employment and Population



The figure presents relative changes in the number of public employees (public administration, education, health and social work) in OECD countries compared to relative changes in population. The observations are fitted with a regression line. The data source is the OECD National Accounts. Changes are computed for the following countries as differences between 2012 (except 2011 for AU) and the earliest available year indicated in parentheses: AU(1985), AT(1988), BE(1995), CZ(1993), DE(1990), EE(1995), FI(1975), FR(1950), DE(1991), GR(2005), HU(1995), IE(1998), IT(1992), KP(2004), LU(1995), NE(1995), NO(1970), PL(2004), PO(1995), SK(1995), SL(1995), SP(2000), SW(1993), UK(1994).

sector with one of the lowest (if not the lowest) share of foreign-born. In Italy, Denmark, Sweden, the Slovak and Czech republics, the financial sector has an even lower an even lower share of foreign-born than the public sector. Figure 8 shows that survey data confirm that the public sector is the sector where foreign-born workers least represented. Data originate from the Household Finance and Consumption Network (HFCN).

A.2 The model without public employment

Consider that $L_p = G = 0$ (and thus $t = 0$). Then native the labor supply in the high-skilled and low-skilled sectors are given by $N_h^s = \frac{1}{c} (w_h - w_l)$ and $N_l^s = 1 - \frac{1}{c} (w_h - w_l)$. Matching labor demand with labor supply yields the following equilibrium wage rates

$$w_h^* = \frac{\alpha [(1+c) - m]}{1 + \alpha + c}; \quad w_l^* = \frac{\alpha - (\alpha + c)m}{1 + \alpha + c}$$

The impact of low-skilled immigration is then

$$\begin{aligned} \frac{\partial w_l^*}{\partial m} &= -\frac{\alpha + c}{1 + \alpha + c} (> -1); & \frac{\partial N_l^*}{\partial m} &= -\frac{1}{1 + \alpha + c} (> -1); & \frac{\partial L_l^*}{\partial m} &= \frac{\alpha + c}{1 + \alpha + c} (< 1) \\ \frac{\partial w_h^*}{\partial m} &= -\frac{\alpha}{1 + \alpha + c} (> -1); & \frac{\partial N_h^*}{\partial m} &= \frac{\partial L_h^*}{\partial m} = \frac{1}{1 + \alpha + c} (< 1) \end{aligned}$$

A.3 Proof of Lemma 1

From (7), we get by simple calculations that

$$\frac{\partial w_p^*}{\partial m} = \frac{\alpha + c - e}{D^2} G(m, e),$$

where

$$G(m, e) = -(\alpha + c + 1)m^2 - 2(\alpha + c + 1 + (\alpha + c - e)(e + 1))m + G(0, e),$$

$$D = (\alpha + c + 1)(1 + m) + (e + 1)(\alpha + c - e) \text{ and } G(0, e) = e(1 + e)(\alpha + c - e) + \alpha e(2 + e) - c - 1.$$

Given that D^2 and $\alpha + c - e$ are positive, the sign of $\frac{\partial w_p^*}{\partial m}$ will depend on the sign of $G(m, e)$ in the following way:

$$\begin{cases} \frac{\partial w_p^*}{\partial m} > 0, & \text{if } G(m, e) > 0, \\ \frac{\partial w_p^*}{\partial m} < 0, & \text{if } G(m, e) < 0. \end{cases}$$

For given e , $G(m, e)$ is a second order polynomial in the parameter m which opens downwards. It is easy to check that the maximum value of this polynomial is obtained at $m^* = -1 - \frac{(\alpha+c-e)(e+1)}{\alpha+c+1} < 0$. Thus, for $m > 0$, the polynomial $G(m, e)$ is a strictly decreasing function of m . Therefore, the sign of $G(m, e)$ depends on the sign of $G(0, e)$ in the following way:

- (1) If $G(0, e) < 0$, then for all $m > 0$ and $e \in (0, c)$, we have $G(m, e) < 0$.
- (2) If $G(0, e) > 0$, there exists $\hat{m} > 0$, such that $G(\hat{m}, e) = 0$, since $G(m, e)$ is decreasing in m (if $m > 0$). So, $G(m, e) > 0$ for $0 < m < \hat{m}$ and $G(m, e) < 0$ for $m > \hat{m}$.

The just highlighted cases are now analyzed in detail.

It is easy to see that

$$G(0, e) = -e^3 + (2\alpha + c - 1)e^2 + (3\alpha + c)e - (c + 1),$$

which is a third order polynomial in the parameter e , implying that there are three roots. We only consider the case with three real roots.

To test whether $G(0, e)$ is positive or negative will depend on Descartes' Rule of Signs, which informs on the number of positive and negative real roots of a polynomial.²⁴

It is easy to check that coefficients of polynomial $G(0, e)$ change signs twice and coefficients of $G(0, -e)$ change sign once. Therefore, by Descartes' Rule of Signs, there are two positive roots and one negative root of $G(0, e) = 0$. And we denote the smallest positive root as \hat{e} and the largest as e_1 (that is, $e_1 > \hat{e} > 0$).

Furthermore, it is easy to see that $G(0, 0) = -(c + 1) < 0$. Thus, we have the following conclusion:

$$\begin{cases} G(0, e) < 0, & 0 < e < \hat{e}, \\ G(0, e) > 0, & \hat{e} < e < e_1, \\ G(0, e) < 0, & e > e_1. \end{cases} \quad (17)$$

However, we also notice that $G(0, \alpha) = \alpha^3 + (c + 2)\alpha^2 + c(\alpha - 1) - 1 > (c + 2)\alpha^2 > 0$ due to $\alpha > 1$, which means that $\alpha \in (\hat{e}, e_1)$. Given that $\alpha > w_h > c$, we can conclude

²⁴Descartes' Rule of Signs is a method of determining the maximum number of positive and negative real roots of a polynomial. Let $P(x)$ be a polynomial with real coefficients written in descending order of x . Then it follows that :

- (1) the number of positive roots of $P(x) = 0$ is either equal to the number of variations in sign of $P(x)$, or less than this by an even integer.
- (2) Number of negative roots of $P(x) = 0$ is either equal to the number of variations in sign of $P(-x)$, or less than by an even integer.

that $e_1 > c$, and hence, the last case in (17) can not happen since by definition we have $c > e$.

To finish the proof we need to show that for given $\hat{e} < e < c$, we can have $\hat{m} > 1$ or $\hat{m} < 1$, where \hat{m} is the positive root of $G(\hat{m}, e) = 0$. Toward this aim, let us rewrite $G(m, e)$ as $G(m, e) = -Am^2 - Bm + C$ with $A = \alpha + c + 1 (> 0)$, $B = 2(\alpha + c + 1 + (\alpha + c - e)(e + 1)) (> 0)$ and $C = G(0, e) = e(1 + e)(\alpha + c - e) + \alpha(2 + e)e - c - 1$. Then, \hat{m} is given by

$$\hat{m} = \frac{B - \sqrt{B^2 + 4AC}}{-2A} (> 0).$$

We obtain $\hat{m} > 1$ if and only if $(0 <) B - \sqrt{B^2 + 4AC} < -2A$, which is equivalent to $C > A + B$. That is, $\hat{m} > 1$ if and only if

$$G(0, e) > K \text{ where } K = 3(\alpha + c + 1) + (\alpha + c - e)(e + 1) > 0,$$

Given that $G(0, e) > 0$, it follows that $\hat{m} > 1$ if $G(0, e) > K > 0$ and $\hat{m} < 1$ if $0 < G(0, e) < K$. Thus, we can get $\hat{m} > 1$ or $\hat{m} < 1$ for $\hat{e} < e < c$.

We finish the proof.

A.4 Additional proofs for Section 3

Here we provide the hints to prove that (i) $\frac{\partial w_p^*}{\partial m} > -1$ and (ii) $\frac{\partial w_p^*}{\partial m} < e$. The proof for (i) implies that we always have $\frac{\partial N^*}{\partial m} < 0$, while (ii) guarantees that $\frac{\partial w_p^*}{\partial m} < 0$ and $\frac{\partial L_l^*}{\partial m} > 0$ always hold.

It is easy to show that $\frac{\partial w_p^*}{\partial m} > -1$, since considering equations (8) and (7) yields

$$a^2 b (b + \alpha) > - (2Sb + Sa) (Sb + Sa + ab) - (1 + M) a b^2 - a^2 b [S + \alpha (1 - M) + (c - e)],$$

where $a \equiv 1 + e$ and $b \equiv \alpha + c - e$. Similarly, we can show that $\frac{\partial w_p^*}{\partial m} < e$ using (8) and (7).

$$\begin{aligned} 0 < S^4 (\alpha + c) [b(S^2 - 1) + a] + S^3 a b [\alpha (S - 1) + c + e] \\ + S^3 e b^2 (S - 1) + S^2 b^2 (ea + m) + S^2 b (\alpha M a + e). \end{aligned}$$

A.5 Proof for Section 6.4

The conclusions in Section 6.4 are obtained from the following results, which are derived in the same manner as those in Section 3. The effect of immigration on public

wages is given by

$$\frac{\partial w_p^*}{\partial m} = \frac{\alpha + c - e}{D^2(\epsilon)} G(m, e; \epsilon),$$

where

$$D(\epsilon) = (1 + m)[\alpha + c + e + (1 + \epsilon)(1 - e)] + (\alpha + c - e)(e + 1 + \epsilon)$$

and

$$G(m, e; \epsilon) = -m^2(1 + \epsilon)(\alpha + c + e + (1 + \epsilon)(1 - e)) \\ - 2m(1 + \epsilon)[(\alpha + c + e)(e + 2 + \epsilon) + (1 + \epsilon)(1 - e)] + G(0, e; \epsilon)$$

with

$$G(0, e; \epsilon) = -e^3 + (2\alpha + c - (1 + \epsilon))e^2 + (1 + \epsilon)(3\alpha + c + \epsilon)e + (1 + \epsilon)[(\alpha - 1)(1 + \epsilon) - (\alpha + c)].$$

When $\epsilon = 0$, we have $D(\epsilon = 0) = D$, $G(m, e; \epsilon = 0) = G(m, e)$ and $G(0, e; \epsilon = 0) = G(0, e)$.

The results are analogous to those in Section 3, except that they depend now also on ϵ . We thus focus in the following developments on the impact of ϵ .

First notice that $G(m, e; \epsilon)$ can be rewritten in terms of ϵ in the following way:

$$G(m, e; \epsilon) = [(e + \alpha + 1) - m^2(1 - e) - 2m(\alpha + c + 1)](1 + \epsilon)^2 \\ + [e(3\alpha + c - 1) - e^2 - (\alpha + c) - (m^2 + 2m(e + 1))(c + e + \alpha)](1 + \epsilon) \\ + [(2\alpha + c)e^2 - e^3],$$

which is a second order polynomial of $1 + \epsilon$ or ϵ with a critical point at $\epsilon = 0$ and a critical value at $G(m, e; \epsilon = 0) = G(m, e)$. Therefore, $G(m, e; \epsilon)$ increases or decreases with ϵ depending on the sign of the coefficient of the second order term, that is, $(e + \alpha + 1) - m^2(1 - e) - 2m(\alpha + c + 1)$, which we denote as $P(m, e)$ from now on.

It is easy to see, for $\epsilon > 0$, that $G(m, e; \epsilon)$ is increasing as ϵ increases if and only if $P(m, e) > 0$. Moreover, when m is sufficiently small, for example $m \rightarrow 0$, we always have $P(m, e) > 0$.

More generally,

$$P(m, e) = (e + \alpha + 1) - m^2(1 - e) - 2m(\alpha + c + 1) \\ = e(1 + m^2) + \alpha - 1 - 2m(\alpha + c + 1) \\ \geq 2me + \alpha - 1 - 2m(\alpha + c + 1).$$

Therefore, one sufficient condition for $P(m, e) > 0$ is $2me + \alpha - 1 - 2m(\alpha + c + 1) > 0$, which is equivalent to

$$e > \alpha + c + 1 + \frac{1 - \alpha}{m} \quad \text{and} \quad m < \frac{\alpha - 1}{2(1 + \alpha)},$$

where the second condition guarantees that $e < c$.

These conditions are analogous to those in Lemma 1. Therefore, the claims in Section 6.4 are proved. \diamond