Efficient Metric Learning
for the Analysis of Motion Data

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Abstract
We investigate metric learning in the context of dynamic time warping (DTW), the by far most popular dissimilarity measure used for the comparison and analysis of motion capture data. While metric learning enables a problem-adapted representation of data, the majority of methods has been proposed for vectorial data only. In this contribution, we extend the popular principle offered by the large margin nearest neighbours learner (LMNN) to DTW by treating the resulting component-wise dissimilarity values as features. We demonstrate, that this principle greatly enhances the classification accuracy in several benchmarks. Further, we show that recent auxiliary concepts such as metric regularisation can be transferred from the vectorial case to component-wise DTW in a similar way. We illustrate, that metric regularisation constitutes a crucial prerequisite for the interpretation of the resulting relevance profiles.

1 Introduction
Motion capture (Mocap) systems rely on a variety of different principles such as magnetic or mechanical sensors, optical markers, poseable mannequins, dedicated technology for hand or facial expression tracking, and low-cost marker-less technology [2, 3]. Powerful analysis software enables the reconstruction of the underlying skeleton when dealing with human motion [4–6]. These developments cause an increasingly important role of Mocap in diverse areas such as entertainment, sports, or medical applications. When Mocap information is used in complex systems such as virtual trainers or interactive Mocap databases for medical analysis, intelligent data analysis and machine learning technology become necessary. Proposed methods range from independent component analysis (ICA) up to deep learning [7, 8].

Within such systems, distance-based methods are often used for the initial analysis or motion retrieval [9–11]. Due to its capability of adjusting to different durations, dynamic time warping (DTW) constitutes the by far most popular dissimilarity measure in this context [12–14].
Like any other dissimilarity measure or metric, the results of DTW crucially rely on the choice of its intrinsic parameters; Crucial metric parameters for DTW and similar measures such as alignment are the parameters which determine the way in which two single sequence entries are compared. This is often referred to as the scoring matrix provided a discrete alphabet is chosen, such as DNA or protein sequences. When it comes to sequence alignment in bioinformatics, lots of effort has been made for its correct choice based on biological insight [15]. Provided such insight is not always available, so-called inverse alignment can help to infer metric parameters from given, optimum alignments [16] [17]. In general, no such information is present, rather only weak learning signals such as motion labelling or grouping are available. In such cases, the adaptation of metric parameters can be solved by metric learning within a machine learning framework.

Metric learning constitutes a matured field of research for the standard vectorial setting (data represented by feature vectors), see e.g. [18–21]. In these approaches, usually some type of quadratic form is inferred from the given auxiliary information. This vectorial metric adaptation does not only provide an increased model accuracy, but it also greatly facilitates model interpretability, and it can lead to additional functionalities such as a direct data visualisation [22–24]. Recently, some researches have focussed on the validity of the interpretation of metric parameters as relevance weights, and it has been shown that there exist problems in particular for high-dimensional or highly correlated data [25,26]. It is possible to avoid these problems by an efficient form of metric regularisation as detailed in the approaches [25,26].

These developments mainly focus on the context of vectorial data, therefore they are not applicable to distance-based measures, i.e. in the case of DTW. Recently, a few approaches have been proposed which address metric parameter learning for complex non-vectorial data, in particular sequences and sequence alignment [18,27,28]. While these approaches lead to an increased model accuracy and interpretability, they have the drawback that their training complexity is very costly: typically, these techniques adapt metric parameters within sequence alignment, such that pairwise distances of all data samples have to be recomputed after every metric adaptation step.

In this contribution, we take a different point of view for the sake of a greatly reduced computational load: we rely on a representation of sequential data in terms of pairwise dissimilarity vectors as provided by component-wise DTW. This is similar to the popular treatment of dissimilarity data as ‘features’, which is detailed in the monograph [29]. This mathematical formulation enables us to transfer the powerful large margin nearest neighbours (LMNN) metric learner [21] to a metric adaptation for DTW, which is able to adjust the relevance of single joints and their correlations in the Mocap data according to a given specific classification task. Further, we demonstrate that a recent metric regularisation framework can be transferred to Mocap DTW results based on the same principle, this way guaranteeing a valid interpretation of the resulting relevance profiles. We demonstrate the efficiency and effectiveness of the proposed methodology for different benchmark datasets.

This contribution is structured as follows: first, we introduce LMNN and its
transfer to component-wise DTW, dubbed DTW-LMNN. Afterwards, we discuss how the resulting metric can be regularised to avoid its dependency on random effects caused by correlations in the observed data. Finally, we demonstrate the suitability of the proposed methodology in a couple of benchmark Mocap data, where we demonstrate the increased classification accuracy of DTW-LMNN in comparison to alternatives. Finally we demonstrate the increased interpretability and robustness of the results when metric regularisation takes place.

2 Dissimilarity based metric learning

In this section we review the LMNN algorithm and the DTW distance computation. Then we introduce the proposed algorithm to use the DTW metric along with metric learning optimisation, and we discuss how to regularise the resulting dissimilarities to diminish random effects caused by data correlations. Note that DTW is not a metric since the triangle inequality does not hold, rather it is a pairwise symmetric distance function, which can serve as a data dissimilarity measure. Nevertheless, we will carelessly refer to DTW as a metric in some places, although strong metric properties do not apply.

2.1 Large margin nearest neighbours metric learning

LMNN is a metric learning algorithm which learns a quadratic form from given labelled data \((\vec{x}_i, y_i) \in \mathbb{R}^n \times \{1, \ldots, c\}, i = 1, \ldots, m, c = \text{number of classes}\), in order to improve the classification accuracy of the well-known \(k\)-nearest neighbours (KNN) method. As a distance-based approach, the accuracy of KNN fundamentally relies on its underlying distance measure which defines the \(k\) nearest neighbours of a given data point. LMNN tries to robustly adjust this neighbourhood structure by learning a parameterised form

\[
D(\vec{x}_i, \vec{x}_j) = (\vec{x}_i - \vec{x}_j)^\top L^\top L(\vec{x}_i - \vec{x}_j) = (\vec{x}_i - \vec{x}_j)^\top M (\vec{x}_i - \vec{x}_j)
\]

with adjustable linear transformation matrix \(L\) which induces a quadratic form characterised by \(M := L^\top L\). The objective function of LMNN is based on a fixed \(k\)-neighborhood structure characterised by \(\eta_{ij} \in \{0, 1\}\) where \(\eta_{ij} = 1\) iff point \(\vec{x}_j\) is within the \(k\) closest neighbours of \(\vec{x}_i\). Based on the intuition of having compact neighbourhoods, while maximising distances of a data point to its neighbours with different labelling, the costs of LMNN become

\[
\epsilon(L) := \sum_{ij} \eta_{ij} D(\vec{x}_i, \vec{x}_j) + c \sum_{ijl} \eta_{ij} (1 - \delta_{y_l}) \left[ 1 + D(\vec{x}_i, \vec{x}_j) - D(\vec{x}_i, \vec{x}_l) \right]_+
\]

where \([\cdot]_+\) refers to the Hinge loss and \(c > 0\) is an adjustable meta parameter. This objective can be interpreted as the goal to adjust the metric such that all points with different class labels are located outside of the data neighborhood with a margin. It has been shown in [21] that this optimization problem is
equivalent to the following semi-definite optimization:

\[
\begin{align*}
\min & \quad \sum_{ij} D(\vec{x}_i, \vec{x}_j) + c \sum_{ijl} \eta_{ijl}(1 - \delta_{y_l}^{ij}) \xi_{ijl} \\
\text{where } & \quad D(\vec{x}_i, \vec{x}_j) - D(\vec{x}_i', \vec{x}_j') \geq 1 - \xi_{ijl} \\
\xi_{ijl} & \geq 0 \\
M & \geq 0
\end{align*}
\]

which can be optimized efficiently w.r.t. the matrix \(M\). Note that it is possible to choose a low-rank matrix \(M\) which corresponds to a low-dimensional projection \(L\) of the data vectors.

### 2.2 Dynamic time warping

Rather than vectors, we deal with sequential data \(X^i = (\vec{x}^i(1) \ldots \vec{x}^i(T)) \in (\mathbb{R}^n)^*\) where \(T\) denotes the length of the time series. DTW aligns two time series of possibly different lengths according to warping paths such that the aligned points match as much as possible, respecting the temporal ordering of the sequence entries. Dynamic programming enables an efficient computation of an optimum match in quadratic time with respect to sequence lengths. For the exact formulas as well as ways to speed up the computation, we refer to [30]. The following facts are of interest for us:

**I** Given two time series of possibly different length \(X^i\) and \(X^j\), DTW provides a dissimilarity \(D_{\text{DTW}}(X^i, X^j)\).

**II** There exist two different ways to treat the vectorial nature of the sequence entries:

**II.1** We can directly compute DTW on vectorial sequences. Then the outcome of DTW is determined by choosing the parameters of the metric which is used to compare vectorial sequence entries along the warping path: that means, crucial metric parameters are those involved in computing \(D(\vec{x}^i(t_1), \vec{x}^j(t_2))\) where the time points \((t_1, t_2)\) are decided by the warping path, and \(D : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}\) is a vectorial metric used to compare the vectorial sequences.

**II.2** We can compute DTW separately for every dimension of a given time series \(X^i_k = (x^i_k(1) \ldots x^i_k(T)) \in \mathbb{R}^n\), where \(k \in \{1, \ldots, n\}\) refers to the component \(k\) of the vectorial sequence entries. For two time series, we thus get a vector of distances \(\vec{D}^{ij} := (D_{\text{DTW}}(X^i_1, X^j_1), \ldots, D_{\text{DTW}}(X^i_n, X^j_n)) \in \mathbb{R}^n\) of dimensionality \(n\). A real-valued dissimilarity can be computed thereof by a standard quadratic form:

\[
D_{\text{LMNN-DTW}}(X^i, X^j) := (L \cdot \vec{D}^{ij})^2 = \left(L : (D_{\text{DTW}}(X^i_1, X^j_1), \ldots, D_{\text{DTW}}(X^i_n, X^j_n))\right)^2
\]

which is parameterised by a linear mapping \(L : \mathbb{R}^n \rightarrow \mathbb{R}^n\) (or a low-dimensional counterpart \(L : \mathbb{R}^n \rightarrow \mathbb{R}^{n'}\) where \(n' < n\)). In both cases, metric parameters are in form of a linear transformation \(L\) or corresponding quadratic matrix \(M = L^\top L\), which have to be adapted according to the given problem for an optimal result. Recently, a few approaches have been proposed which deal with
the question how to learn an optimum transformation \(L\) provided DTW is used for vectors (i.e. approach (II.1)), see e.g. [18,27,28]. These techniques, however, face the problem that metric adaptation can change the form of an optimum warping path, i.e. a computationally costly recalculation of the warping path is necessary to obtain stable results.

In the following, we will therefore propose an approach which is based on the strategy (II.2) to deal with vectorial data: we will adapt the metric according to component-wise DTW vectors (5). This formulation has the benefit, that not only LMNN can efficiently be transferred to a novel dissimilarity-based setting, but also recent concepts for metric regularisation are applicable to such problems, as we will see in the following.

### 2.3 DTW-LMNN

For a sequence metric such as \(D_{\text{LMNN-DTW}}\), the LMNN costs (2) become:

\[
\epsilon(L) = \sum_{ij} \eta_{ij} D_{\text{LMNN-DTW}}(X^i, X^j) + c \sum_{ijkl} \eta_{ij} (1 - \delta_{y_iy_j}) \cdot \left[ 1 + D_{\text{LMNN-DTW}}(X^i, X^j) - D_{\text{LMNN-DTW}}(X^i, X^l) \right].
\]

Using the distance computation (Eq. 5), we obtain an optimisation problem which is similar to (Eq. 3):

\[
\begin{align*}
\min \quad & \sum_{ij} (\tilde{D}^{ij})^\top M \tilde{D}^{ij} + c \sum_{ijl} \eta_{ij} (1 - \delta_{y_iy_j}) \xi_{ijl} \\
\text{where} \quad & (\tilde{D}^{ij})^\top M \tilde{D}^{ij} - (\tilde{D}^{ij})^\top M \tilde{D}^{ij} \geq 1 - \xi_{ijl} \\
& \xi_{ijl} \geq 0 \\
& M \geq 0
\end{align*}
\]

This can be solved by means of semi-definite programming. Again, a restriction to low-rank matrices \(M\) and \(L\) is possible, provided the relevant information is located in a low-rank subspace of the full data space only. Note that the computational complexity of this method is the same as for vectorial LMNN, further, the convexity of the problem is preserved.

### 2.4 Metric regularisation

The adaptation of a quadratic form as present in LMNN does not only enhance the classification accuracy, but it can also give rise to an increased interpretability of the results. A quadratic form corresponds to the linear data transformation \(\vec{x}^i \mapsto L \vec{x}^i\). Hence the diagonal terms of the matrix \(M\)

\[
M_{kk} = \sum_i L_{ik}^2
\]

summarise the influence of feature \(k\) on the mapping. Due to this observation, metric learners are often accompanied by the relevance profile which is provided by the diagonal entries of \(M\); this gives insight into relevant features for the given task, such as potential biomarkers for medical diagnostics [31]. For DTW-LMNN, this interpretation directly transfers to a relevance profile for
the sequential data related to each feature component, such as single joints in
the case of Mocap data: for the metric (Eq. 5), the entry $M_{kk}$ summarises the
influence of distances which are based on the feature sensor $k$.

It has recently been pointed out that this interpretation has problems pro-
vided high dimensional or highly correlated data are analysed: in such cases,
the relevance profile and the underlying linear transformation $L$ are not unique,
rather data correlations can give rise to random, spurious relevance peaks. We
expect this effect for Mocap data due to a high correlation of neighbouring joints.
For vectorial data, this effect is caused by the following observation, as pointed
out in [26]: assume $X = [\vec{x}_1, \ldots, \vec{x}_m]$ refers to the data matrix. Then two linear
transformations $L_1$ and $L_2$ are equivalent with respect to $X$ iff $L_1X = L_2X$.
This is equivalent to the fact that the difference $(L_1 - L_2)X$ vanishes. Hence,
by considering the squared form

$$(L_1 - L_2)XX^\top(L_1 - L_2)^\top = 0 \quad (9)$$

we can relate this property to the fact that the differences of the rows are given
by vectors which lie in the null space of the data correlation matrix $C := XX^\top$.
This gives us a unique characterisation of the equivalence class of matrix $L$ with
respect to the data transformations for $X$: equivalent matrices, i.e. matrices
which map data $X$ in the same way as matrix $L$, differ from $L$ by multiples of
eigenvectors related to 0 eigenvalues of $C$. Provided the metric learning method
does not take this fact into account, its outcome matrix will be random as
concerns contributions of this null space.

For LMNN, costs are invariant with respect to null space contributions, i.e.
the matrix $L$ is random in this respect. Albeit this property has no effect on the
training data $X$, it influences the result in two aspects: for test data, the null
space is usually different, i.e. the generalisation ability of the model is affected
by random effects of the training data correlation and the initialisation point $L$
for the optimization problem. Second, more severely, random contributions of
the null space of $C$ change the relevance profile $M_{kk}$ and can give rise to spurious
effects such as high values which are not supported by any ‘real’ relevance of
the feature $k$.

Therefore, it is advisable to regularise the matrix $L$ by relying on the rep-
 resentative of the equivalence class of $L$ with smallest Frobenius norm. This
is equivalent to a projection of $L$ to the space of eigenvectors of $C$ with non-
v anishing eigenvalues, or more precisely, the unique transformation

$$\tilde{L} := L\Phi$$

$$\text{where } \Phi := \sum_{j=1}^{J} \vec{u}_j (\vec{u}_j^\top) \text{ with the eigenvectors }$$
$$\vec{u}_1, \ldots, \vec{u}_{J} \text{ of } C \text{ with nonvanishing eigenvalues} \quad (10)$$

For vectorial data, the same effect can be obtained by deleting the null space
from the data vectors in the first place by means of principal component analysis
(PCA), as a very popular preprocessing approach. However, the reformulation
as matrix regularisation has the benefit that this principle can directly be trans-
ferred to more general data such as the alignment vectors $\vec{D}^i$, as we will see in
the following.
For alignment vectors (Eq. 4) and the distance (Eq. 5), we find

\[ L_1 \vec{D}_{ij} = L_2 \vec{D}_{ij} \text{ for all } i, j \]
\[ \iff (L_1 - L_2) \vec{D}_{ij} = 0 \text{ for all } i, j \]  

(11)

Hence, similar to (Eq. 9), transformations are equivalent with respect to the given data if their difference lies in the null space of the correlation matrix \( \mathbf{D} \mathbf{D}^\top \) for the distance matrix \( \mathbf{D} := [\vec{D}_{11}, \ldots, \vec{D}_{1m}, \ldots, \vec{D}_{m1}, \ldots, \vec{D}_{mm}] \), consisting of all \( n \)-dimensional vectors of pairwise distances. Note that this observation enables an effective regularisation of the matrix \( \mathbf{L} \) (and \( \mathbf{M} = \mathbf{L}^\top \mathbf{L} \)) in the same way as for the vectorial case, relying on the regularisation (Eq. 10):

\[ \tilde{\mathbf{L}} := \mathbf{L} \Phi \]

where \( \Phi = \sum_{j=1}^{J} \vec{u}_j (\vec{u}_j)^\top \) with the eigenvectors \( \vec{u}_1, \ldots, \vec{u}_J \) of \( \mathbf{D} \mathbf{D}^\top \) with nonvanishing eigenvalues  

(12)

As for the vectorial case, this yields the equivalent matrix \( \tilde{\mathbf{L}} \) of \( \mathbf{L} \) with smallest Frobenius norm, for which an interpretation of the diagonal entries becomes possible. Thereby, this principle is applicable for full-rank matrices as well as low-rank counterparts. We will see in experiments section that matrix regularisation has a strong effect on the variance of the resulting relevance profile. Further, it can also enable a slightly better generalisation ability since it suppresses noise in the given data.

3 Datasets and Experiments

We compare DTW-LMNN in relation to alternatives with a focus on two different aspects: its classification accuracy when used within a KNN method, and its capability to lead to dimensionality reduction by a reduced relevance profile. In this section, we will first specify the different training models which are compared to the method LMNN-DTW. Afterwards, we will explain the used benchmark data. Experiments will be conducted in three steps for these data: first, we compare the classification accuracy of the proposed method. Then we investigate its low-rank counterparts. Finally, we discuss the effect of regularisation for the obtained relevance profiles.

3.1 Methods

We use the following major pipelines for comparison:

- **DTW-LMNN**: We use component-wise DTW (4) together with metric learning by LMNN (6). Based on the found distances (5), a KNN classifier with \( k = 3 \) is evaluated. We will use this technique with a full-rank matrix, as well as a low-rank matrix with rank 3, to investigate LMNN’s ability to infer a low-dimensional representation of the data with a high accuracy.

- **DTW-KNN**: We use the plain DTW distance together with a KNN classifier, without any metric adaptation. Again, \( k = 3 \). This setting
Figure 1: A stick-figure of the human body showing 41 markers (red spheres) placed on different parts of the body in order to capture the motion data. Captured data are related to movements of the different joints during the human’s basic activities. 

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can serve as a baseline to evaluate the improvement obtained by metric learning for sequential data.

- **Euclidean LMNN:** As second comparison, we use LMNN based on the standard Euclidean metric instead of DTW, together with a subsequent KNN classifier with $k = 3$. More precisely, we use the LMNN formulation (7) with the choice
  \[
  \vec{D}_{ij} := (D_{Euc}(\vec{X}_i^1, \vec{X}_j^1), \ldots, D_{Euc}(\vec{X}_i^n, \vec{X}_j^n)) \in \mathbb{R}^n
  \]
  where $D_{Euc}$ refers to the standard Euclidean distance of the vectors $\vec{X}_i^k$.

One problem consists in the fact that the considered time series $X_i$ and $X_j$ have different length, and we have to compute a vectorial representation $\vec{X}_i^k$ and $\vec{X}_j^k$ of equal dimensionality for every component $k$. Here, we simply pick the entries $\vec{X}_i^k := (x_i^k(1), \ldots x_i^k(T))$ for the shorter time series, and we subsample $T$ entries $(x_j^k(t_1), \ldots x_j^k(t_T))$ of $X_j$ at equal time intervals $1 = t_1 < \ldots < t_T$ to turn the second time series into vectors of the same dimensionality. This setting allows us to judge the effect of DTW as compared to the standard Euclidean distance, equipped with metric learning. Again, we investigate this setting with a full-rank and a low-rank adapted matrix with rank 3.

- **PCA-DTW-KNN:** For the dimension reduction experiment in section 3.6, we compare the low-rank representation found by low-rank LMNN with a dimensionality reduction by classical Principle Component Analysis (PCA) [32]. Thereby, PCA is applied directly to the vectorial sequence
entries from the training set. Afterwards, DTW is used for the major three principal components of the data entries, followed by a KNN classifier.

We use a 10-fold cross validations with 10 repetitions for evaluation. All experiments are carried out using the same cross validation partitions.

For evaluation and comparison of the proposed approach, we consider 3 different Mocap databases based on human motions. These data give rise to the following four different training sets:

### 3.2 CMU Mocap dataset:

We use the Human motion capture dataset from the CMU graphics laboratory [33]. The data is captured by Vicon infra-red cameras using 41 markers placed at different parts of the body, close to the joints (Fig.1). Afterward the images are augmented to 3D data, and transferred to kinematic information such as joint rotation / translation based on skeleton information. The result consists of 62 body features leading to 62 dimensional time series with a high correlation of their dimensions. We select two classification scenarios from the full data set to investigate different aspect of the proposed method.

- **Walking:** This dataset contains data from 7 different walking styles (normal, fast, slow, turn right, turn left, veer right and veer left) carried out by 4 different subjects. Totally, the dataset consists of 49 samples (7 samples per class) with 62 dimensional features.

- **Dance:** We use 35 samples of data related to two different styles of dancing: *Modern* and *Indian*; these are available from CMU graphics laboratory as participant subjects 5 and 94 [33]. Each class contains various dance performances related to subcategories of the class; as a result, the variations among the data within the classes produce overlap between the two classes, which makes the classification very challenging.

### 3.3 Cricket Umpire’s Signals

Cricket as a bat-and-ball team game is the world’s second most popular sport [34]. An umpire is a person in the game who makes decisions and announces different events going on on the cricket field. The umpire makes the communication using his arm movements. As an example, the event ‘No-Ball’ is signalled by holding one arm out at shoulder height to indicate that the ball is delivered along with player’s fault(s); in order to announce the start of the last hour of the play, the umpire taps his wrist and his watch [35].

For our classification experiment we use data provided in the approach [36]. This contains 12 umpire signals, each performed by four different persons in 3 to 4 repetitions. Data are captured via accelerometers on the umpire’s wrists while performing the signals [37]. Doing so, the data are recorded in a 3D spatial format (with X, Y and Z coordination) providing 6 features per time step for the classification task. The classification task is to distinguish between 12 classes.
3.4 Articulatory Words

People can have oral communication difficulties based on different reasons; one example is the treatment of larynx cancer via surgery, after which it is probable that the patient has considerable vocal impairments. One practical solution to facilitate this issue is to benefit from silent ‘speech’ recognition [38–40]: this technology uses facial data (such as lips and tongue movements) in order to recognise the person’s voiceless uttered words or phrases [39]. The required motion information can be captured by attaching Electromagnetic Articulograph (EMA) sensors to the person’s articulators such as lips and tongues [41]. The problem is to reliably classify the uttered word from this movement data.

For our experiments, we use the articulatory dataset from [38], which consists of EMA data related to 25 words uttered by different native English speakers. For the motion capture process, 12 EMA sensors are used to capture the 3D information (X, Y and Z positions) of various facial organs as time series data. As it is explained in [38], the sensors are placed at different parts of forehead, lips and tongues shaping up 36 available features, out of which we use 9 specific dimensions for our classification task collected from markers on the tongue tip, the upper lip and lower lip. More precisely, we used the 3D spatial data (X, Y and Z) related to the tip of tongue (T1), upper lip (UL) and lower lip (LL) which results in 9 features in total [42]. Hence the classification task can be
Table 1: Classification accuracy(%) for the proposed DTW-LMNN approach and its comparison to the regular Euclidean-LMNN and DTW-kNN methods for the four data sets. Variances are reported in parentheses, a paired t-test checks the hypothesis that DTW-KNN and DTW-LMNN do not differ.

<table>
<thead>
<tr>
<th></th>
<th>Euclidean LMNN</th>
<th>DTW-KNN</th>
<th>DTW-LMNN</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking</td>
<td>92 (0.87)</td>
<td>95 (0.77)</td>
<td>100 (0)</td>
<td>–</td>
</tr>
<tr>
<td>Dance</td>
<td>80 (1.49)</td>
<td>77.5 (1.51)</td>
<td>90 (1.03)</td>
<td>0.01</td>
</tr>
<tr>
<td>Cricket Signals</td>
<td>95.56 (0.38)</td>
<td>99.44 (0.18)</td>
<td>100 (0)</td>
<td>–</td>
</tr>
<tr>
<td>Articulatory Words</td>
<td>97.30 (1.20)</td>
<td>98.61 (1.05)</td>
<td>99.06 (1.11)</td>
<td>&lt; 0.01</td>
</tr>
</tbody>
</table>

characterised as categorising 25 classes of different words using 575 sample of data which are represented based on 9 motion-based features.

Table 2: Accuracy of the KNN classifier used for low-rank data / distance representations as obtained by PCA, low-rank Euclidean LMNN, and low-rank LMNN with DTW, choosing rank 3. A paired t-test evaluates the hypothesis that the results of a PCA projection together with DTW and LMNN are the same as low-rank DTW-LMNN learning.

<table>
<thead>
<tr>
<th></th>
<th>low-rank Euclidean LMNN</th>
<th>PCA with DTW-KNN</th>
<th>low-rank DTW-LMNN</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking</td>
<td>86.6 (1.10)</td>
<td>96 (1.08)</td>
<td>98.8 (1.80)</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>Dance</td>
<td>75 (1.52)</td>
<td>76 (1.51)</td>
<td>95 (0.80)</td>
<td>0.012</td>
</tr>
<tr>
<td>Cricket Signals</td>
<td>96.11 (0.46)</td>
<td>99.44 (0.18)</td>
<td>100 (0)</td>
<td>–</td>
</tr>
<tr>
<td>Articulatory Words</td>
<td>98.60 (0.14)</td>
<td>94.24 (0.25)</td>
<td>99.12 (0.17)</td>
<td>&lt; 0.01</td>
</tr>
</tbody>
</table>
3.5 Classification Accuracy

In this section, we study the proposed methods as concerns their classification accuracies. We compare the classification accuracy of the DTW-LMNN algorithm, Euc-LMNN, and DTW-KNN. We accompany the resulting accuracies by the variance, and by the p-value from a paired t-test testing the hypothesis that plain DTW and the metric adaptation technique DTW-LMNN yield the same result [43]. All results from the classification task are displayed in Table 1.

According to the classification results, DTW-LMNN outperforms the Euclidean version of the algorithm for all data sets. This observation supports the expectation that DTW constitutes a suitable dissimilarity measure for motion data due to its ability to account for different motion durations. From another point of view, metric adjustment enables an improvement of the classification accuracy for all cases. Interestingly, it causes a slight superiority of the Euclidean metric as compared to DTW without metric adjustment for the dance dataset. For all settings, metric adjustment leads to an improvement of the classification in connection to DTW.

3.6 low-rank matrix representation

We study the dimensionality reduction performance of DTW-LMNN using the datasets introduced in section 3, in order to obtain a compressed representation of the data for the classification tasks. As discussed in section 2 we can use a low-
rank matrix $\mathbf{M}$ or $\mathbf{L}$, corresponding to low-rank constraints in the optimisation problem (Eq. 7). Apart from a compressed representation, this can lead to a significant increase in the time performance of the kNN classification in the low-dimensional projection space [44]. We use a rank 3 matrix $\mathbf{L}$ corresponding to a projection into the space $\mathbb{R}^3$. For comparison, we also investigate the effect of a rank restriction for the Euclidean version of LMNN, and we investigate the result of classical PCA for dimensionality reduction of the data before classification. The results of these low-rank classification pipelines are reported in Table 2.

As reported in Table 2, low rank DTW-LMNN preserves the good classification accuracy of DTW-LMNN for the reported data sets. In contrast, PCA does not achieve the same accuracy, nor does Euclidean LMNN. Interestingly, rank restrictions improve the classification accuracy for DTW-LMNN for the Dance data set. Conversely, PCA reduces the classification accuracy for the Articulatory words data set. Hence projection directions which are learned by LMNN optimisation can enhance the discriminative aspects of DTW alignment.
for a low-rank matrix representation. As an interesting point in the results, the DTW-LMNN algorithm managed to classify the Cricket dataset with 100% accuracy while obtaining a compressed representation as well.

3.7 Regularised Relevance Profile:

In this section we investigate the resulting relevance profiles for two of the introduced datasets for the metrics obtained by DTW-LMNN. We restrict the analysis to DTW-LMNN due to its superior performance for all data sets. Further, we investigate only two of the four datasets in order to focus on the interesting effects of the matrix regularisation. The data sets Articulatory Words and Cricket Signals are of minor interest for this section due to their comparably low-dimensionality (9 and 6 sensors only, without major correlations). On the contrary, the two human body datasets (Dance and Walking) have a high number of features (62) with considerable correlations among the joints. Hence we can expect interesting effects when regularising the learned matrix.

Matrix regularisation has different effects: (I) It enables a valid interpretation of the feature relevance profile since it avoids spurious relevance peaks and random effects due to data correlations – we will evaluate this effect by an inspection of the simplicity and variance of the relevance profile within a cross validation. (II) It suggests ways to reduce the data dimensionality by eliminating the most irrelevant features according to the found relevance profile. We will investigate this effect by evaluating the classification performance if the feature dimensions are iteratively deleted according to their relevance.
3.7.1 Dance dataset

For the dance dataset, we calculate the relevance value of features as $\text{diag}(L'L)$ based on the transformation matrix $L$ which is obtained via DTW-LMNN, see section 3.5. For the graphical display, we normalise the profiles to the range $[0,1]$. Since $L$ is different for different cross-validation partitions, we report the average and variance of each diagonal value. The resulting relevance profile without regularisation is displayed in Fig. 4-bottom. The total variance of this profile is 4.47.

In comparison, we regularize the matrix $L$ according to Eq. 12. Thereby, the eigenvectors $\vec{u}^i$ of the vectors of the distance matrix $D$ for the eigenvalue $0$ are determined based on the training set. We report the result for the choice of 12 effective dimensions of the eigenvectors, Fig. 2 shows the corresponding eigenvalue profile. The resulting regularized profile is shown in Fig. 3-top. Obviously, much fewer features are singled out as relevant. Further, the variance of the profile is reduced to 2.86.

We test the ability of metric learning to induce a feature selection by sorting the input dimensions (features) according to their relevance values in each of the two relevance profiles in Fig. 4. Then we select the important features according to this order and report the resulting classification accuracy on the test set. Doing so, the classification accuracy as displayed in Fig. 5 results. Interestingly, both relevance profiles enable a deletion of a large number of sensors without a reduction of the classification accuracy. This feature reduction
can be iterated until only 9 features are left for the regularised profile, and 26 features for the non regularised one. Hence regularisation greatly enhances the feature selection ability of the technique. The resulting 9 features are displayed based on the skeleton information in Fig. 6. The semantic meaning of these features is reported in Table. 3.

According to Fig. 4, the regularisation matrix has positive effects on the relevance profile. While it retains the classification performance at its highest rate (see Fig. 3), it reduces redundancy in the profile and produces a sparse representation for the relevance values of inputs (features). In addition, based on the variance measure for the relevance profile over different cross-validation partitions, the regularised profile has less variance and thus is more reliable than the normal one.

As a semantic interpretation, it can be concluded from Fig. 6 that hands and feet are both important discriminative features for this dancing task. From another point of view, this is a difficult task because each class has different subcategories within itself which account for overlaps with other class; hence the combination of both (hand and foot) is required to distinguish between the two dance categories. Furthermore, as another interesting semantic interpretation, only the data related to one side of the body (right side) is necessary to achieve the highest classification performance. This coincides with the fact that dancing is typically a symmetric whole body movement in which symmetry can be found between the left and the right sides of body.

\subsection{3.7.2 Walking dataset}

We repeat this experimental setting for the walking dataset. This leads to 14 effective dimensions selected from the regularisation matrix. The obtained regularised profile can be seen in Fig. 7. The total variances of the relevance profiles before and after the regularisation are 10.7 and 2.51, respectively. Again, similar to the dance dataset, the most relevant features and the resulting classification accuracies are displayed in Fig. 8. The most important joints are listed in Tab. 4.

Similar to the results for the dance dataset, regularisation of the learned metric results in a sparse representation of the relevance profile and a reduced variance. Furthermore, according to Fig. 8, a classification accuracy of 100% can be achieved while choosing fewer features (7 features for the regularised profile instead of 25 for the standard one).

Based on the observations from Fig. 9, for this dataset (and this classification

Table 3: Total variance in the regularised and non-regularised relevance profiles along with the feature selection results for the Dance dataset.

<table>
<thead>
<tr>
<th>Total variance of profile (before regularization): 4.47</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total variance of regularized profile: 2.86</td>
</tr>
<tr>
<td>Selected Joints and related feature number:</td>
</tr>
<tr>
<td>rhumerus(27,28,29), rthumb(36), rfemur(49,50,51), rfoot(53) and root(6)</td>
</tr>
</tbody>
</table>
task), hands are more important than feet. In addition, as the classes are very similar (all of them are connected to walking), the classification algorithm also needs some information about both sides of the body in order to carry out the classification task with a perfect result. We tested this hypothesis by using

Table 4: Total variance of the regularised and non-regularised relevance profiles and selected features for the Walking dataset.

<table>
<thead>
<tr>
<th>Description</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total variance of plain profile</td>
<td>10.70</td>
</tr>
<tr>
<td>Total variance of regularized profile</td>
<td>2.51</td>
</tr>
<tr>
<td>Selected Joints and feature number:</td>
<td></td>
</tr>
<tr>
<td>root(5), lhumerus(40,41), rhand(33), rthumb(36), lowerneck(18), lthumb(48)</td>
<td></td>
</tr>
</tbody>
</table>
Figure 8: Classification performance of walking dataset based on the selected features according to the regularised profile

$Lhand$ instead of $Rhand$ or deleting $Rthumb$ (since we already have $Lthumb$); but in both cases the performance decreased (around 3 to 4%) showing that those selected features are all necessary even though they are symmetrical in the skeleton structure.

According to the results given in section 3.7, after applying the regularisation matrix, both datasets showed improved results. The regularisation enhances the reliability of the relevance profiles, corresponding to a smaller variance. In addition, regularisation enables a more efficient feature selection strategy.

4 Conclusions and Future Work

In this paper we introduced a distance based extension (DTW-LMNN) to the popular metric learning method LMNN in a very generic way, opening up the possibility to also transfer auxiliary concepts such as metric regularisation. This enables us to benefit from the DTW dissimilarity measure which is particularly suited to the analysis of Mocap data. While dealing with multi-dimensional motion data such as human movements, the component-wise dissimilarity values achieved by the DTW measure can be treated as different features to obtain a semi-vectorial representation for the dissimilarities among the data. Having that, the representation can be combined with the LMNN framework in order to efficiently adapt the feature ranking and correlation according to the classification tasks at hand. The mentioned dissimilarity representation for the motion data also conserves the convexity of the optimisation problem Eq.3.
The strong results achieved with DTW-LMNN show that augmenting the LMNN approach with the DTW dissimilarity measure can significantly improve the metric learning performance. For both, classification and dimensionality reduction tasks, the proposed approach outperforms the Euclidean version of LMNN as well as the DTW based K-nearest neighbour method. Therefore, it can be concluded that DTW-LMNN can benefit from the strength of the DTW dissimilarity metric and LMNN metric learning together. According to our encouraging results across diverse motion based benchmarks, the DTW-LMNN framework offers a suitable discriminative method to achieve high performance classifications and compressed representations dealing with motion based datasets.

As another contribution of this paper, we devised a way to transfer the concept of metric regularisation to alignment based representations; this concept has recently been proposed for the vectorial case [26]. According to the results in section 3.7, this regularisation step is a crucial prerequisite for a valid interpretation of the relevance profile. To that end, we managed to use the dissimilarity-based information in order to remove the highly correlated dimensions related to the null space contributions. According to the results, the dissimilarity-based regularisation brings significant effects to the relevance profile, further, it increases the semantic interpretability of the resulting discriminative models. It is important to mention that the proposed regularisation step can be applied to any other dissimilarity-based metric framework as well.

Relying on these promising results achieved by the proposed DTW-LMNN
framework, there is considerable potential for future research in dissimilarity-based metric learning: the principle can be transferred to other metric learners which are not linked to KNN. In addition to that, an encouraging research line could be to investigate more advanced regularisation techniques to achieve a further enriched relevance profile, such as proposed in [25].

5 Acknowledgment

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