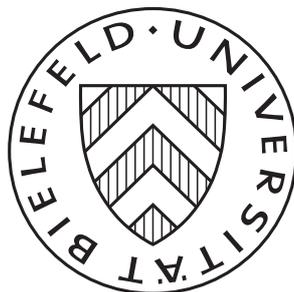


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## Procrastination and Projects

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Philipp Külpmann



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## Abstract

In this paper I analyze a dynamic moral hazard problem in teams with imperfect monitoring in continuous time. In the model, players are working together to achieve a breakthrough in a project while facing a deadline. The effort needed to achieve such a breakthrough is unknown but players have a common prior about its distribution. Each player is only able to observe their own effort, not the effort of others.

I characterize the optimal effort path for general distributions of breakthrough efforts and show that, in addition to free-riding, procrastination arises. Furthermore, in this model, procrastination is not a result of irrational behavior and is even present in the welfare-maximizing solution.

**Keywords:** Procrastination, Moral hazard in teams, Public good provision

**JEL codes:** C72, C73, D81, D82, D83, H41

*Never put off till to-morrow what you  
can do day after to-morrow just as well.*

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Mark Twain

## 1. Introduction

It already occurs when you are doing your homework in school, continues while you are writing a term paper in college and is probably still present when you have to do

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your taxes: procrastination. Many people delay working on unpleasant tasks, despite a deadline. And this phenomenon is not restricted to work you conduct on your own, sometimes it is even stronger when you work in a team.

Naturally this causes problems, not only for you, but for the whole team. This is especially the case when you are doing project work, i.e. working together towards a fixed goal after which your team will be terminated. In this paper I will focus on project work, as it is generally said to be more efficient and is frequently used in the workplace than individual work (Harvard Business School Press (2004)) and the classroom (Hutchinson (2001)).

This paper proposes a continuous-time model of working in projects, which explains procrastination not only in teams but also when working alone, not as a result of inefficiency or time-inconsistency but as an efficient, team-value maximizing consequence of a given deadline. The range of applications is quite broad, from large scale multi-national research projects to a single worker trying to write a report.

The main features of this model are:

- **public benefits**, which are realized upon completion of a project in the form of a lump-sum payment,
- **private costs**, which are assumed to be quadratic,
- an **unknown threshold for success**, with a commonly known distribution,<sup>1</sup>
- **unobservable efforts**, so only the player's own effort is known,
- and a **deadline**, after which the project cannot be completed anymore.

In the model the players exert effort over time until either the deadline is reached or the project is successful. While doing so, they only know that they have not been successful yet. Projects in this model are described by the assumed distribution of the breakthrough effort. This *breakthrough-effort distribution* can cover many different projects, e.g., projects in which only the current effort influences the probability of success or projects during which players learn about the quality of the project while trying to complete it. One simple example for a breakthrough-effort distribution is the uniform distribution on  $[\underline{e}, \bar{e}]$ . This means that the players think that the project needs effort between  $\underline{e}$  and  $\bar{e}$  to be completed. For examples of different types of projects and the corresponding breakthrough-effort distributions see Section 3.

I find that in the equilibrium there are three different effects at work: **free-riding**, which reduces the overall effort the more players are working on the project. The second effect is **encouragement**, which depends on the threshold distribution: Given a decreasing hazard rate my own effort encourages the other players to work less, while given an increasing hazard rate, my work encourages my coworkers to work more in the future. The last effect is **procrastination**, which causes players to work later rather

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<sup>1</sup>Which I will call *breakthrough-effort distribution*.

than earlier, even with the presence of a discount rate which lets players want to have a breakthrough as soon as possible.<sup>2</sup>

Free-riding is a common effect in moral hazard problems and already well understood. Encouragement also occurs frequently in the literature, however usually either only as a positive encouragement effect (for example in Georgiadis (2014)) or only as a negative encouragement effect (as in many bandit models, for example in Bonatti and Hörner (2011) it is the unnamed effect leading to procrastination in their model).<sup>3</sup> My model, however, can incorporate both effects and the occurrence of positive or negative encouragement depends in the type of the project (i.e. the breakthrough-effort distribution). The last effect, procrastination as a result of rational players, has, to the best of my knowledge, not been analyzed before in this context. In German there is a word for this effect that explains it even better than procrastination: “Torschlusspanik“<sup>4</sup>. In this model “Torschlusspanik“ or procrastination is caused by convex costs, which make it optimal for the players to spread their effort as evenly as possible, a deadline and uncertainty about the effort required for a breakthrough. When they start working on a project, the players have a belief about the threshold that includes very low effort levels and they are trying to find an optimal effort level given this belief. If they do not succeed at first, they realize that the effort level is not that low and that they have to update their beliefs about the threshold. Therefore, they also have to increase their effort level to reflect the updated beliefs. Hence, we can expect some level of procrastination even with a hyper-rational social planner trying to maximize social welfare. Close to the deadline, the effect of procrastination even outweighs any other effect, including encouragement and even strong discounting.

The paper is organized as follows: In the next section, I will give an overview of the relevant literature and how this work fits into it. Then I will explain the model in Section 2, followed by an explanation of the breakthrough-effort distribution and how it translates to different projects in Section 3. In Section 4, I derive the optimal effort for the non-cooperative and the welfare maximizing case. Additionally, I show that there is always procrastination in both cases. Then I will briefly discuss the effect of deadlines (Section 5.1), the model without discounting (Section 5.3) and how different types of projects react to changes in costs and changes in the quality of the project (Section 5.4). I will give some concluding remarks in Section 5.5.

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<sup>2</sup>Keep in mind that the discount rate does not only effect the costs but also the benefits. As the benefits are, by design, later than the costs and higher than the expected costs a discount rate lets players work earlier.

<sup>3</sup>The positive encouragement effect is very similar to strategic complementarity and the negative encouragement effect to strategic substitutability. For more detailed information see the Corollary in Section 4.

<sup>4</sup>“Torschlusspanik“, literally ”gate-closure panik“, roughly translates to eleventh-hour-panic and it means to stress out about something you have not yet accomplished shortly before a certain deadline.

## 1.1. Literature

This paper is related to different fields of the literature: Holmstrom (1982) started the game theoretic literature on *moral hazard in teams*, which was then expanded by Ma, Moore, and Turnbull (1988), Legros and Matthews (1993) and Winter (2004), to mention only a few important contributions. A common theme is the focus on free-rider problems due to shared rewards but costly private effort. This paper adds to this literature as it analyzes a dynamic moral hazard problem, in which players have very restricted information about the actions of others, which leads to free-riding and, procrastination.

In parts, this model is very similar to the literature on *strategic experimentation*, introduced by Karatzas (1984), Berry and Fristedt (1985) and Mandelbaum (1987), as it models the behavior of players who optimize their decisions while gathering information at the same time. In these games every player has to divide her time between a “safe” and a “risky” action (as in the arms of a two-armed bandit) with unknown but common payoffs. Bolton and Harris (1999) analyze a two-armed bandit problem with many players in which the arms yield payoffs which behave like a Brownian Motion, with different drifts for the safe and the risky arm. They analyze the stationary Markov equilibria and are able to identify free-rider and encouragement effects. Keller, Rady, and Cripps’s (2005) model of strategic experimentation, in which the risky arm yields a lump-sum with a certain intensity if the the risky arm is good and nothing if the risky arm is bad. In this model new information arrive as a Poisson process, as in most of the recent literature on bandit problems. Two examples for this literature are Klein and Rady (2011), where the risky arms are negatively correlated, and Klein (2013) who extended the model to three armed bandits.

My work is very closely related (and was inspired by) Bonatti and Hörner (2011).<sup>5</sup> They analyze a bandit model, similar to Keller, Rady, and Cripps (2005), in which efforts are private information and only outcomes are observable. After a success the game ends and payoffs are realized.

The model presented in this paper is a very particular model of strategic experimentation: Not only is the information a player gathers about the actions of the other players very restricted, but furthermore, players’ payoffs are perfectly correlated. However, models of strategic experimentation usually assume the news arrival to be a Poisson process, whereas my model has hardly any restriction on this news arrival process.

Bonatti and Hörner (2011) also exemplifies another strand of literature which this paper is related to: *dynamic contribution games*. Already suggested by Schelling (1960), these models analyze the dynamic contributions to public goods. Admati and Perry (1991) and Lockwood and Thomas (2002) are examples for games in which contributions are observable. In Georgiadis (2014), there is no uncertainty about the valuation of the

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<sup>5</sup>In fact, their benchmark model is a special case of my model with an incomplete exponential distribution and linear instead of quadratic costs. See Example 2 for more information.

public good but about how effort affects the provision of the public good. He assumes that effort affects the drift of a standard Brownian motion towards a (commonly known) threshold and is able to not only identify free-riding and encouragement effects, but also to show that the optimal contract only compensates on success. Although in my paper the uncertainty is about the threshold and not about the effect of effort, these two models are closely related when using increasing hazard rates as shown in Example 3. My paper introduces uncertainty about the effort needed to provide the public good. Therefore, players also have to incorporate information gathering into their decision process. Furthermore, I show that, due to the presence of a deadline, procrastination is optimal.

There is a huge literature on *procrastination in economics and psychology*. However, these usually attribute procrastination to self-control problems (O’Donoghue and Rabin (2001)) or time-inconsistencies like hyperbolic discounting (Laibson (1997)). Another explanation for procrastination is given by Akerlof (1991): According to him, procrastination is a consequence of “repeated errors of judgment due to unwarranted salience of some costs and benefits relative to others“ (Akerlof, 1991, p. 3).

The literature on procrastination in psychology is much more prominent than in economics but, like the economic literature, it almost exclusively focuses on some form of cognitive biased decisions (e.g., Wolters (2003) or Klingsieck (2015)).

This paper adds to this literature, as it models not only decision processes of a single person but also procrastination in teams, i.e. in a game-theoretic model. Furthermore, it provides an explanation for procrastination under rational and even welfare-maximizing behavior and therefore gives rise to completely different measures that should (or should not) be taken.

Bergemann and Hege (2005) use a very similar information structure to the one presented in this paper, but analyze a problem in discrete time with linear costs and a memoryless investment. Second, Khan and Stinchcombe (2015) analyze decision problems in which changes can occur at random times and require a costly reaction. They have identified situations in which delayed reaction is optimal, depending on the form of the hazard rate of the underlying changing probability distributions. The relationship of this model to the latter paper is mostly in the use of the hazard rate as a description of the projects players are working on.

To summarize, this paper contributes to the literature in two different ways: On the one hand, it provides a tractable model to analyze a very general class of dynamic contribution games in continuous time with many players and incomplete information about effort contribution. The model deals with very different types of projects: Projects in which the success probability decreases in effort already spent,<sup>6</sup> e.g. through learning about the quality of the project (which is very common in bandit models), investment projects similar to Georgiadis (2014) where the past effort increases the chance of success

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<sup>6</sup>Covered by decreasing hazard rates of the breakthrough effort distribution.

now and even projects in which past effort increases chance of success on some intervals and decreases on others.

Furthermore, my model can explain situations in which procrastination is not only rational (which was also observed in Bonatti and Hörner (2011))<sup>7</sup> but even welfare maximizing and arises without the assumption of time-inconsistencies or cognitive biased players. Furthermore, I can clearly distinguish free-riding and encouragement from procrastination.

## 2. The model

Consider  $n$  risk neutral players working together on a project in continuous time  $t \in [0, T]$ . Players can only observe their own past effort and whether the project was successful. After a success the players get a lump sum payment, normalized to 1, and the game ends. Every player  $i$  chooses at every point in time  $t$  whether to exert a costly effort  $u_i : [0, T] \rightarrow \mathbb{R}_+$  with quadratic instantaneous costs at  $t$ :  $cu_i(t)^2$ .

The utility function of player  $i$  is, given a breakthrough at time  $\bar{t}$ , therefore given by

$$\tilde{V}_i(u_i, \bar{t}) = re^{-r\bar{t}} - r \int_0^{\bar{t}} e^{rt} cu_i(t)^2 dt$$

with  $r$  being the common discount rate.

**Definition 1** (Effort Threshold). The project is successful in  $\bar{t}$  if the players have exerted enough effort, i.e. if

$$\bar{x} \leq \int_0^{\bar{t}} \sum_{\forall i} u_i(t) dt$$

**Remark.** *This definition implies the assumptions of symmetric, additively separable and linear effects of efforts and non-depreciation of effort.*

This threshold  $\bar{x}$  is drawn before the game and is unknown to all players. They have a common prior about its probability density function  $f$  and hence, about its cumulative distribution function  $F$ . This breakthrough effort distribution can be interpreted as the type of task or project (see Section 3).

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<sup>7</sup>However, what they call procrastination, is, in the terms of my model, an result of the (negative) encouragement effect, not the procrastination effect.

Due to Definition 1 we can define  $x(t)$  as the overall effort already spent up until  $t$ :  $x(t) := \int_0^t \sum_{\forall i} u_i(s) ds$  and  $u_{-i}(t) = \sum_{\forall j \neq i} u_j(t)$  as the effort of all players except  $i$  at a certain time  $t$  without loss of information.

From the definition of the game above, we can derive the expected utility for player  $i$ , given effort profile  $\{u_i, u_{-i}\}$ <sup>8</sup>

$$V_i(u_i(t), u_{-i}(t), x(t)) = r \int_0^T e^{-rt} (1 - F(x(t))) \left( \frac{f(x(t)) (\sum_{\forall j} u_j(t))}{1 - F(x(t))} - cu_i(t)^2 \right) dt \quad (1)$$

### 3. The breakthrough effort distribution

The most important characteristic of a project in this model is the “breakthrough-effort distribution” or, in other words, how much effort one has to spend for a certain chance of success, given the effort that has been spent by the team in the past. Therefore, this distribution describes how likely every possible effort threshold is at the present stage of the project. If the player thinks finding a cure for a disease costs around 100 billion man hours of research, she could assume for example some normal distribution around 100 billion. If I am certain I lost my keys in my apartment (again), but have no idea where they could be, assuming a uniform distribution over every place in my apartment seems reasonable.

In this section I am going to give examples of three basic classes of distributions and how they can be interpreted in the context of the model. The distributions will be denoted by their hazard rates  $h(x(t)) := \frac{f(x(t))}{1 - F(x(t))}$ , which basically describes the effect of past effort on the effectiveness of current effort.

**Example 1** (Constant hazard rate). *The first type of distribution has a constant hazard rate, i.e., the exponential distribution ( $F(x) = 1 - e^{-\lambda x}$  with a rate parameter  $\lambda > 0$ ). This distribution conveys the idea that the chance of success only depends on the current effort and past effort does not matter at all, for example if you are trying to push a boulder out of your way or trying to force a door open.*

**Example 2** (Decreasing hazard rate). *A variation of the exponential distribution is an incomplete exponential distribution<sup>9</sup> (i.e., an exponential distribution with a probability mass at infinity). Although technically a distribution with a decreasing hazard rate, the intuition is similar to the example of the memoryless distribution: The probability*

<sup>8</sup>For the derivation of the expected utility see Appendix A.1.

<sup>9</sup>Using this distribution in my model yields us a model very similar to the so-called good news bandit models. One example is Bonatti and Hörner’s (2011) benchmark model, the only difference being that I use quadratic instead of linear costs.

distribution itself is memoryless, however there is a chance of failure. As time proceeds, the expected probability of failure is updated and therefore increases in the effort already spent. A popular example for a decreasing hazard rate is a search model where you search at the most likely places first or investments into R&D: the more you invest without success, the higher is your belief that there is no solution to the problem. This class of effort distribution functions is probably the most important and will therefore receive special attention.

**Example 3** (Increasing hazard rate). *The last example is the class of increasing hazard rates (e.g., when the breakthrough effort is distributed uniformly on some interval). Possible applications are projects with a strong learning-by-doing effect and projects where the success in a certain period depends on the cumulative effort, not on current effort.<sup>10</sup> A simple example for this class is moving something heavy from A to B.*

For more examples I would like to refer to Section 2 in Khan and Stinchcombe (2015), who provide an overview about the meaning of success probability distributions, their hazard rates and their relations to different projects.

Although all examples in this paper will be from one of the three classes, the results also hold for general distributions.

## 4. Results

### 4.1. Non-Cooperative Solution

The best response of player  $i$  to the strategies of the other players  $u_{-i}(t)$  can be stated as the following optimal control problem (omitting the time index  $t$  from  $x(t)$  and  $u_i(t)$ ):

$$\max_{u_i, x} V_i = r \int_0^T e^{-rt} (1 - F(x)) \left( \frac{f(x)(u_i + u_{-i})}{1 - F(x)} - cu_i^2 \right) dt \quad (2)$$

with boundary conditions  $x(0) = 0$  for the cumulative effort at time 0.

The following technical assumption restricts our attention to the distributions for which there is neither a certain success nor a certain failure and there are no jumps in the breakthrough-effort distribution.

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<sup>10</sup>One example is Georgiadis (2014). In his model the uncertainty is about the effect of effort and not the threshold, but this is just a different way to model uncertainty about the relationship between effort spent and success. One can therefore generate a very similar model in the framework presented by choosing the appropriate breakthrough effort distribution, which would have an increasing hazard rate.

**Assumption 1.** The hazard rate  $h(x) := \frac{f(x)}{1-F(x)} > 0$  is continuous in  $x$  and bounded above for every finite  $x$ .

Given Assumption 1 on the hazard rate of the breakthrough-effort distribution we can find the symmetric equilibrium path

**Theorem 1** (Equilibrium Effort). There exists a unique symmetric equilibrium in which, on the equilibrium path,  $u$  (i.e., the individual effort of a player) evolves according to

$$\dot{u} = \frac{2n-1}{2}h(x)u^2 + ru - \frac{r}{2c}h(x)$$

and reaches  $u_T = \frac{1}{2c}h(x_T)$  at the deadline  $T$ .

To find this equilibrium effort path, I use the Pontryagin maximum principle to solve the optimal control problem given by Equation (2) and then use symmetry to find necessary conditions for the equilibrium effort. I then verify existence and uniqueness of this symmetric equilibrium and show sufficiency via a convexity argument. For the complete proof see Appendix A.2.

**Remark** (Asymmetric Equilibria). *In this paper I am not analyzing asymmetric equilibria, as, given the symmetric setting, restricting our attention to symmetric equilibria seems natural. In addition, it is clear that every asymmetric equilibrium is, in terms of welfare and as a consequence of the convex cost structure, inferior to the symmetric equilibrium, as can be seen in Lemma 1 in the following section.*

**Example 1** (Continued). *For the exponential distribution with rate parameter  $\lambda$ , the solution from Theorem 1 reduces to:*

$$\begin{aligned} \dot{u} &= \frac{2n-1}{2}\lambda u^2 + ru - \frac{r}{2c}\lambda \\ u_T &= \frac{1}{2c}\lambda \end{aligned} \tag{3}$$

*which has an explicit solution given in Appendix B. Given this solution, some observations about this class of distributions can already be made: Independent of the number of players (and the discount rate), the individual effort right before the deadline is always the same. Furthermore, we can see that the individual effort decreases in the number of players, despite the fact that the reward for completion for each player is independent of the number of players. Figure 1 shows an example of the equilibrium effort path for one and three players.*

**Remark** (Off-equilibrium behavior). *In the following I am going to focus my attention on the equilibrium behavior. However, let me briefly discuss the off-equilibrium behavior that arises if one player deviates from the equilibrium path. If she exerts less effort at time  $t$ , her continuation strategy after  $t$  depends on the hazard rate of the underlying breakthrough effort distribution:*

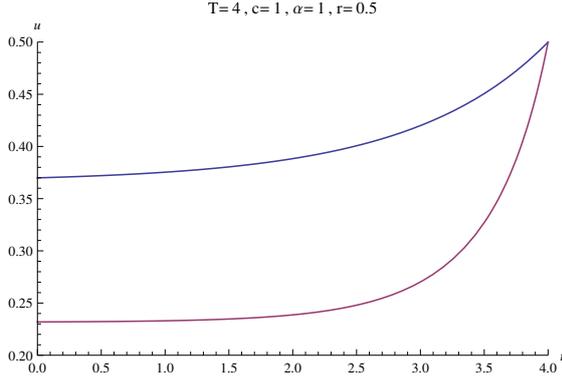


Figure 1: Equilibrium effort (constant hazard rate)

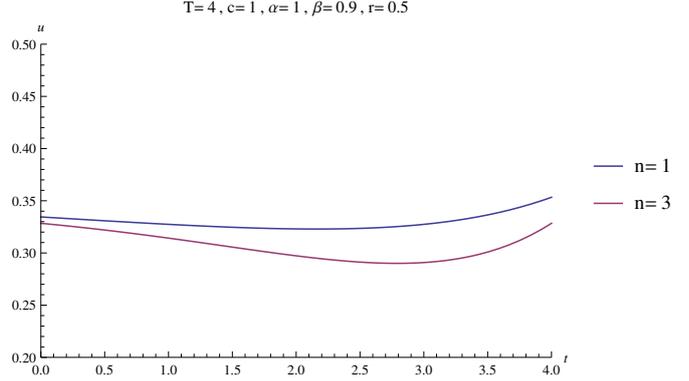


Figure 2: Equilibrium effort (decreasing hazard rate)

- Given a **constant hazard rate**, nothing changes for her. In this special case, behavior is independent of the past, hence she will immediately revert to the equilibrium effort.
- Given an **increasing hazard rate**, her belief about the probability of success is now lower than that of her collaborators. Therefore, she will also exert less effort in the future. Given a high enough slope of the hazard rate, this leads to divergence of her belief (and therefore effort) and the beliefs of the other players.
- Given a **decreasing hazard rate**, her belief about the probability of success is now higher than that of her collaborators. This leads to a higher effort until her belief coincides again with the belief of the other players, as soon as she has made up the effort she previously failed to exert. So, given enough time, in this case the player will revert to the symmetric equilibrium.

It is not surprising that the effort of players increases if they assume an increasing hazard rate. But what does the optimal effort for a decreasing hazard rate look like? From the example depicted in Figure 2 we already get a good idea of what the typical optimal effort path might look like:<sup>11</sup> At first, we have a decrease in effort. This is due to the encouragement effect: At first the (perceived) probability of success is high, but due to the decreasing hazard rate as more effort is invested, the success rate and therefore the effort level decreases. However, we can see that, after some time, the effort increases again. What might be the reason for this behavior? By investing early we discourage players to invest at every following point in time, so postpone investment to a later point in time. However, looking at the case of only one player in Figure 2 we see the same effect, albeit less pronounced. Therefore, as a single player is neither affected by the encouragement effect nor by free-riding we can clearly identify another effect: procrastination.

<sup>11</sup>In this example the required effort for a breakthrough is distributed according to the incomplete exponential distribution with rate  $\alpha$  and a mass point  $1 - \beta$  at  $\infty$ .

Is procrastination an effect we can observe in general, or was Figure 2 one of the examples where it is present? To analyze this problem, we first have to define procrastination: The Macmillan Dictionary defines to procrastinate as follows: “to delay doing something until later, usually something that you do not want to do“. As effort is, by definition, something that ”you do not want to do“, we only have to check for an increase in effort later, i.e., close to the deadline. Therefore, Definition 2 follows naturally:

**Definition 2** (Dominant Procrastination). Player  $i$  exhibits dominant procrastination if and only if  $\exists \delta > 0$  s.t.  $u_i$  is increasing on  $[T - \delta, T]$ .

I call this *dominant* procrastination, as, if we observe effort that behaves as defined in Definition 2, we not only know that there is procrastination, but that procrastination outweighs every opposing effect (e.g. the effect of strong discounting) near the deadline. As we cannot observe the effect procrastination directly, we therefore have to look for dominant procrastination.

Given Theorem 1, we can show that procrastination can be observed for almost every variation of the model:

**Theorem 2** (Procrastination). For every possible breakthrough-effort distribution that fulfills Assumption 1, players always procrastinate as defined in Definition 2.

For the complete proof I would like to refer to Appendix A.3. The interpretation, however, is clear: Whatever effects are at work during the first parts of the project, towards the end procrastination always dominates the other effects.

The best way to see ”pure“ procrastination in effect would be to look at an example, where free-riding and the encouragement effect are, artificially, removed. This is done in the following example (and Figure 1).

**Example 1** (Continued). *Let us now have another look at the case of constant hazard rates. This case is special, as current effort is not influenced by past effort at all. Therefore, we have no encouragement effect (only pure free-riding) and it is easy to see that, without a deadline (i.e.,  $T = \infty$ ), the effort would be constant<sup>12</sup>. However, if we introduce a deadline, this no longer true, and we see an increasing effort as depicted in Figure 1 and Equation (3).*

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<sup>12</sup>Just compare the problem at  $t_0 = 0$  and any other time  $t$ : The only differences between these two problems are the past time  $t$  and the effort already exerted  $x(t)$ . As  $t$  in the past has no influence on the best response now, time left is the same and, due to the special properties of the exponential distribution,  $x(t)$  has no effect on the beliefs about the success, the problems we are facing at  $t_0$  and  $t$  are the same. Therefore, assuming we also have a unique best response, the continuation strategies at  $t_0$  and  $t$  are the same for every  $t$ , i.e. players exert a constant effort, independently of  $t$ .

## 4.2. Welfare Maximizing Solution

So far, we have seen what happens in the non-cooperative equilibrium, but what would a social planner do?<sup>13</sup> To solve the problem of the social planner, we have to solve a problem similar to Equation (2). However, now we maximize the combined utility and therefore:

$$\max_{u_i} V_i = r \int_0^T e^{-rt} (1 - F(x)) \left( n \frac{f(x)(\sum_{\forall i} u_i)}{1 - F(x)} - \sum_{\forall i} cu_i^2 \right) dt \quad (4)$$

We can focus on the symmetric problem in which every player exerts  $\bar{u}(t)$  without loss of generality, as the following Lemma shows us:

**Lemma 1.** Every welfare-maximizing effort path has to be symmetric.

The intuition for Lemma 1 is as follows: Due to the assumptions of symmetric and additive-separable effects of efforts (Definition 1) and convex costs an equal distribution of the efforts exerted at every point in time results in the same probability of success but a lower sum of costs. For a short proof see Appendix A.4.

Therefore we get the social planners optimization problem

$$\max_{\bar{u}} V_i = r \int_0^T e^{-rt} (1 - F(x)) \left( \frac{f(x)(n^2\bar{u})}{1 - F(x)} - nc\bar{u} \right) dt \quad (5)$$

And its solution which is derived in a similar fashion to Theorem 1 in Appendix A.5.

**Theorem 3** (Welfare Maximizing Effort). The unique effort  $u$  that every player has to exert that maximizes the social planners problem (Equation (4)) evolves according to

$$\dot{u} = \frac{1}{2}h(x)u^2 + ru - \frac{nr}{2c}h(x)$$

and reaches  $u_T = \frac{n}{2c}h(x_T)$  at time  $T$ .

Now let us compare the welfare maximizing solution to the (non-cooperative) equilibrium effort for the case of the constant hazard rate:

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<sup>13</sup>We are assuming here that the social planner has some kind of utilitarian utility function, which, given the perfect symmetry of the players is not a very strong assumption.

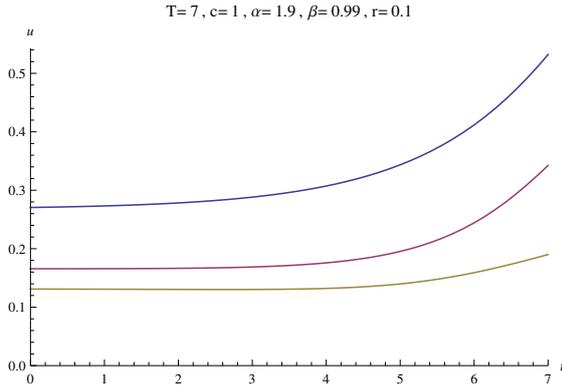


Figure 3: Nash equilibrium effort

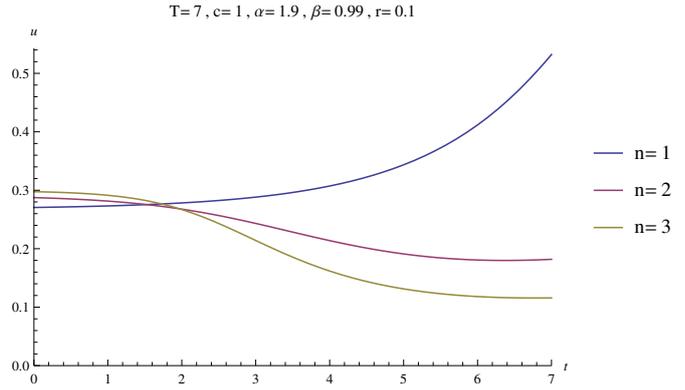


Figure 4: Welfare maximizing effort

**Example 1** (Continued). *If we have constant hazard rates, we can directly compare the non-cooperative equilibrium and the socially optimal effort. It turns out that in this case the socially optimal effort is always larger than the equilibrium effort. This can be checked by simply calculating the difference between the welfare-maximizing effort and the equilibrium effort, as stated in Appendix B.*

Given this example, one might suspect that the welfare maximizing solution is to always exert more effort than in the equilibrium. While this can be observed with an increasing or a constant hazard rate, it is not true for decreasing hazard rates, as can be seen in the following example in Figure 3 and Figure 4 (which uses the incomplete exponential distribution with rate  $\alpha$  and failure rate  $1 - \beta$ ). Here we can see that the welfare-maximizing effort starts off being higher but, due to the decreasing belief in the success of the project, decreases much faster than in the equilibrium.

The following proposition shows that we can expect a rational social planner to procrastinate.

**Theorem 4** (Procrastination of the Social Planner). The welfare-maximizing behavior always leads to dominant procrastination as defined in Definition 2, independent of the breakthrough-effort distribution.

The proof Theorem 4 is analogous to the proof of Theorem 2.

Now that we have shown that procrastination is also present in the social planner's solution, we can derive conclusions about the three effects mentioned in the introduction.

**Corollary** (Effects). From Theorems 1, 2, 3 and 4, we can differentiate three different effects and their causes.

- **Free-riding:** As this effect is probably one of the best analyzed and understood effects in games, I will only point out behavior that is special to this model: If

the number of players increases, the individual effort decreases but the cumulative effort does not necessarily decrease. This is due to the assumption of fixed benefits for every player, which leads to a higher sum of possible benefits from a breakthrough. However, comparing the equilibrium efforts and welfare maximizing efforts for different players shows us free-riding in this model, as can be seen in Appendix C and Example 1.

- **Encouragement effect:** This effect can be described as: My actions affect the efficiency of effort for every player in the future and therefore their choice of effort. The direction in which my effort affects the effort of others depends on the hazard rate. For an increasing hazard rate, the effect is called encouragement effect<sup>14</sup> for a good reason: Every effort I spend now increases the efficiency and therefore the effort of everyone in the future. Clearly, this also leads to higher efforts now. However, with a decreasing hazard rate, this effect is a negative encouragement or discouragement effect: If I spend much effort now and we do not succeed, we have a lower belief about the chance of succeeding in the future and therefore we will work less. This leads to less effort, especially in the earlier periods. However, this does not quite explain procrastination. In the case of a constant hazard rate, we would expect no encouragement effect, only free-riding, but even there we can observe more effort at the end. Therefore, we know there is a third effect at work:
- **Procrastination:** We have seen that even in the welfare maximizing case without memory (i.e. the exponential distribution), more effort is invested close to the deadline. However, it is not present in Bonatti and Hörner (2011), a very similar model with linear instead of quadratic costs.<sup>15</sup> Therefore, we can safely assume that this effect is only present when we have convex costs, a deadline and when there is uncertainty about the amount of work we have to put into a project to succeed. Section 5.3 shows us that a higher discount rate can dampen this effect, but we know from Theorem 2 that it can never eliminate it completely. As this effect is also present in the welfare maximizing case we can conclude: Procrastination is (to a certain extent) not only commonly observed in reality, but might also be rational and even welfare maximizing.

**Remark** (Strategic complements/substitutes and the encouragement effect). *As already mentioned in the introduction, the positive encouragement effect (due to an increasing hazard rate of the breakthrough-effort distribution) is very similar to strategic complementarity: My effort now is a strategic complement to all players' future efforts. In the same sense, the negative encouragement effect is similar to a strategic substitute for future efforts.*

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<sup>14</sup>For example by Georgiadis (2014), where past efforts always have a positive effect on the effectiveness of effort, or in terms of this paper: projects always have a breakthrough effort distribution with an increasing hazard rate.

<sup>15</sup>In their model, the welfare-maximizing effort is as follows: As the chance of success decreases in the invested effort, players invest the maximal amount of effort until the marginal benefits of effort are lower than the marginal costs. After that point is reached, no effort is invested anymore.

*if that is the case, why can't one just say that with increasing hazard rates, current efforts and future efforts are strategic complements, and with decreasing hazard rates they are strategic substitutes? The problem is that current efforts of different players are substitutes (and therefore strategic substitutes), which is the source for free-riding. Hence, we cannot clearly say if, given an increasing hazard rate, efforts are strategic substitutes or complements. However, for decreasing (and constant) hazard rates, we know that the efforts of all players at all times are strategic substitutes.*

## 5. Discussion

### 5.1. Deadlines

We have already seen that deadlines induce procrastination, i.e. an accumulation of effort shortly before the deadline. But is it possible to improve welfare by a deadline? Given that I only consider rational individuals, one would not expect a deadline to be beneficial if the hazard rate of the breakthrough-effort distribution is constant or even increasing. Now, Bonatti and Hörner (2011) have shown that, in their setup (i.e., with a decreasing hazard rate and linear costs), there is almost always a deadline that improves welfare.

However, with quadratic costs, I was not able to identify any situation in which deadlines improve welfare and simulations suggest that the welfare maximizing deadline is always the least restrictive (i.e. the deadline which allows the most time to complete the task) one. This might be because the smoothing effect of convex costs, which makes it optimal to spread costs over time as evenly as possible, is severely restricted by deadlines. In Figure 5 you can see that the efforts behave as expected: Shorter deadlines lead to overall higher efforts and, given a short enough deadline, we can even prevent the decrease of effort early on. However the effect of a (shorter) deadline on the utility (and therefore the welfare) is, at least in my simulations, always negative, as shown in the example in Figure 6.

The question whether deadlines can improve welfare is therefore still open. However, my simulations suggest that the procrastination effect in this model prevents deadlines from ever being beneficial.

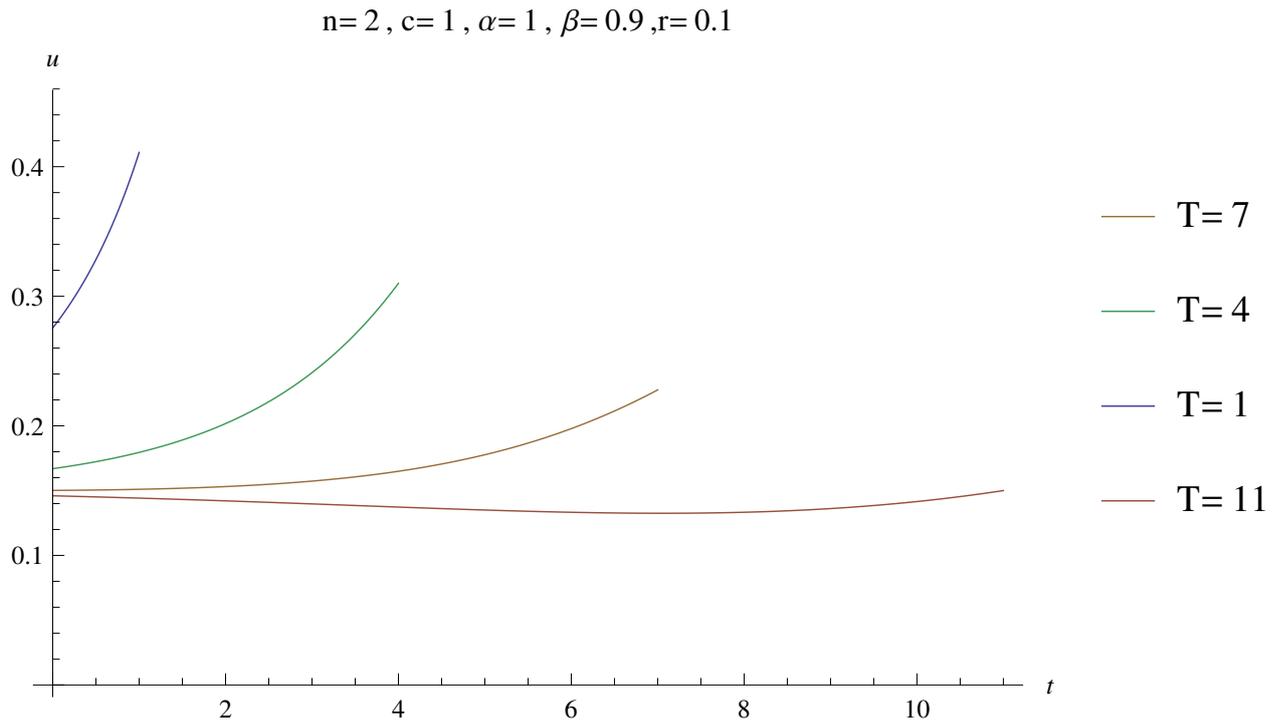


Figure 5: Effect of deadlines on efforts

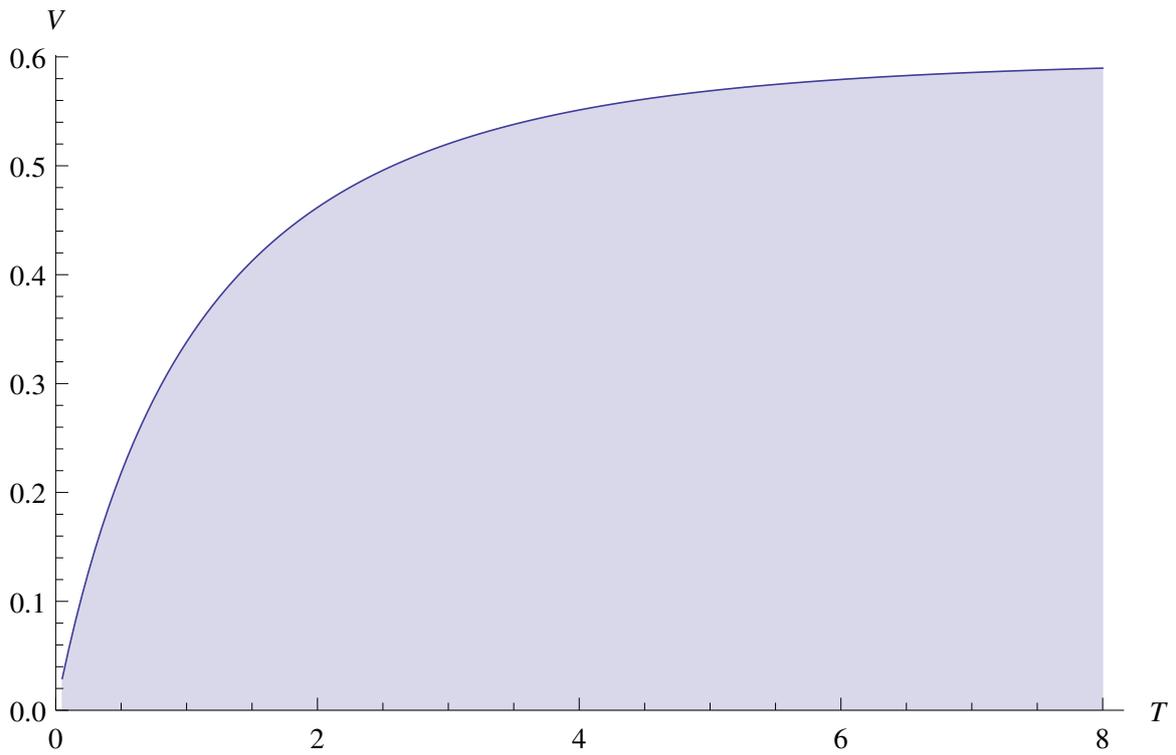


Figure 6: Effect of deadlines on welfare

## 5.2. Full Information

One might expect that procrastination is also optimal in the full information case, i.e. when the players (or the social planner) already know the effort threshold  $\bar{x}$ . The following simple example shows that this is not true, if the costs or the threshold are sufficiently low or if the discount rate is sufficiently high:

**Example 4.** *Assume that the costs are low enough, s.t. (abusing the notation of  $c(\cdot)$ ):  $c\left(\frac{\bar{x}}{n}\right) \leq 1 - e^{-rT}$ . Then the losses due to procrastination until the end  $T$  are higher than the highest possible costs that can occur in the symmetric equilibrium.*

*Let us compare the utility from getting the work done at  $t = 0$ :  $V_0 := 1 - c\left(\frac{\bar{x}}{n}\right)$  and from getting the work done at  $t = T$ :  $V_T := e^{-rT} - c_T$  where  $c_T$  are some non-negative costs. Then we know that:*

$$V_0 = 1 - c\left(\frac{\bar{x}}{n}\right) > e^{-rT} - c_T = V_T$$

*Completing the project at  $t = 0$  is strictly better (but not necessarily optimal). Therefore, working until the end can never be optimal.*

Solving the problem with full information<sup>16</sup> shows that the optimal solution is to distribute the required effort between 0 and some time  $t^* \leq T$ , such that the marginal costs are the same at every point in time. I.e., due to the hazard rate, they are increasing until  $t^*$  and zero afterwards.

## 5.3. Patient Players

So far, I have only considered the problem in which players are impatient. For patient players ( $r = 0$ ), the solution from Theorem 1 simplifies to

$$\begin{aligned} \dot{u} &= \frac{2n-1}{2} h(x) u^2 \\ u_T &= \frac{1}{2c} h(x_T) \end{aligned}$$

As we know that  $h(x)$  and  $u$  are always positive, the following Proposition 1 is an obvious result:

**Proposition 1.** The equilibrium effort of patient players (i.e.  $r = 0$ ) is increasing everywhere, concave for decreasing hazard rates and convex for increasing hazard rates.

---

<sup>16</sup>For example by finding the best path  $u^*(t)$  for the problem  $\max_{u^*} e^{-rt^*} - \int_0^{t^*} e^{-rt} c(u^*(t)) dt$  s.t.  $\int_0^{t^*} c(u^*(t)) dt = \bar{x}$ , for every  $t^*$ , and then finding the best  $t^*$ .

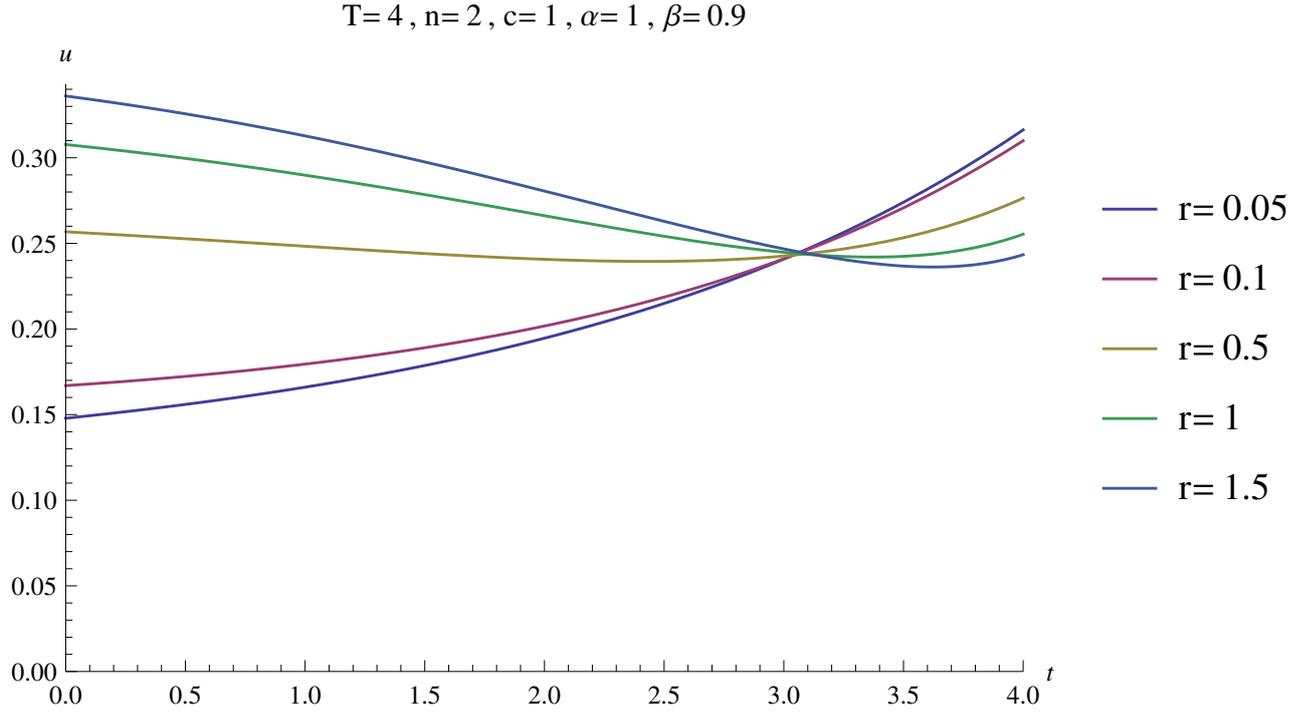


Figure 7: Equilibrium effort for different discount rates

It is not surprising that, without an incentive to work early, we observe even more procrastination, i.e. effort is shifted to the end. A very nice example for this phenomenon can be seen in Figure 7, where equilibrium efforts paths of different discount rates  $r$  are shown. Furthermore, we can see that in our model decreasing efforts, which are for example observed in Figure 2 and in Bonatti and Hörner (2011) are, only possible if impatience is strong enough to counteract procrastination.

**Example 1** (Continued). *For the exponential distribution with rate parameter  $\lambda$  and  $r = 0$ , the solution from Theorem 1 reduces to:*

$$x(t) = \frac{2\lambda(\log(2c + (2n - 1)T) - \log(2c + (2n - 1)(T - t)))}{2n - 1}$$

*which is clearly increasing in  $t$ .*

## 5.4. Effect of Costs and Project Quality on Different Projects

One might expect that costs and the general quality of a project (i.e., the chance of success), might have very simple effects on the efforts of the players: Higher costs should lead to lower efforts, while higher probabilities of success should lead to higher efforts.

$$T=8, n=2, \alpha=1, \beta=0.9, r=0.1$$

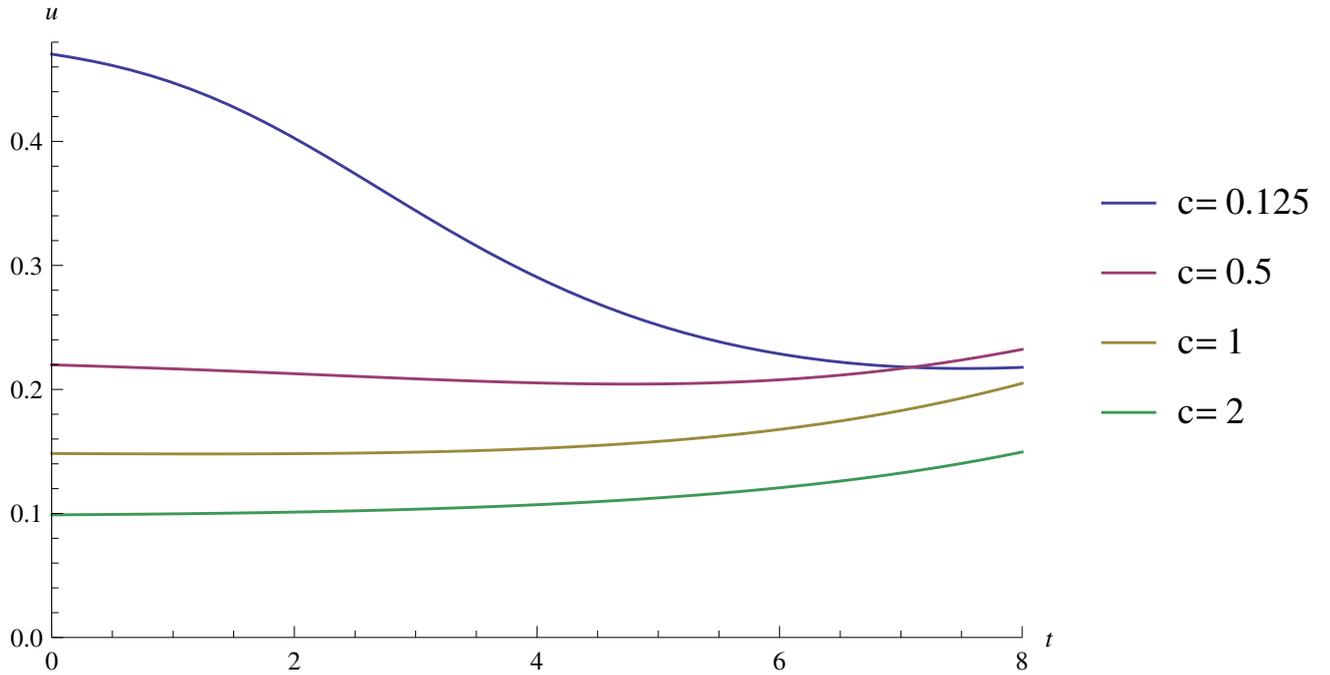


Figure 8: Equilibrium effort paths for different costs

While this intuition is true for the constant and increasing hazard rates, this effect is not that simple if we have a project that is described by a decreasing hazard rate, for example if there is a chance of failure. In this case, we might observe that due to the negative encouragement effect, the instantaneous effort starts out being higher but declines faster and finally is even lower than in the case with higher costs (Figure 8) or a better quality project (Figure 9). In the latter case we can even observe that it is possible that the cumulative effort is lower, due to faster updating of the failure probability.

## 5.5. Concluding Remarks

I have analyzed a problem of a team working together on a project where the individual team members are unable to observe each others' efforts and have only a rough idea about the amount of effort that will be needed to complete the project. This leads to free-riding and encouragement, as well as to procrastination. The procrastination effect analyzed here is not a result of inefficient behavior but a necessary consequence of the deadline and convex costs, given the information structure. Although different types of projects lead to very different behavior, procrastination always occurs as long as a deadline is present. The encouragement effect on the other hand has very different effects, depending on the type of project. In this model, Procrastination is so prominent

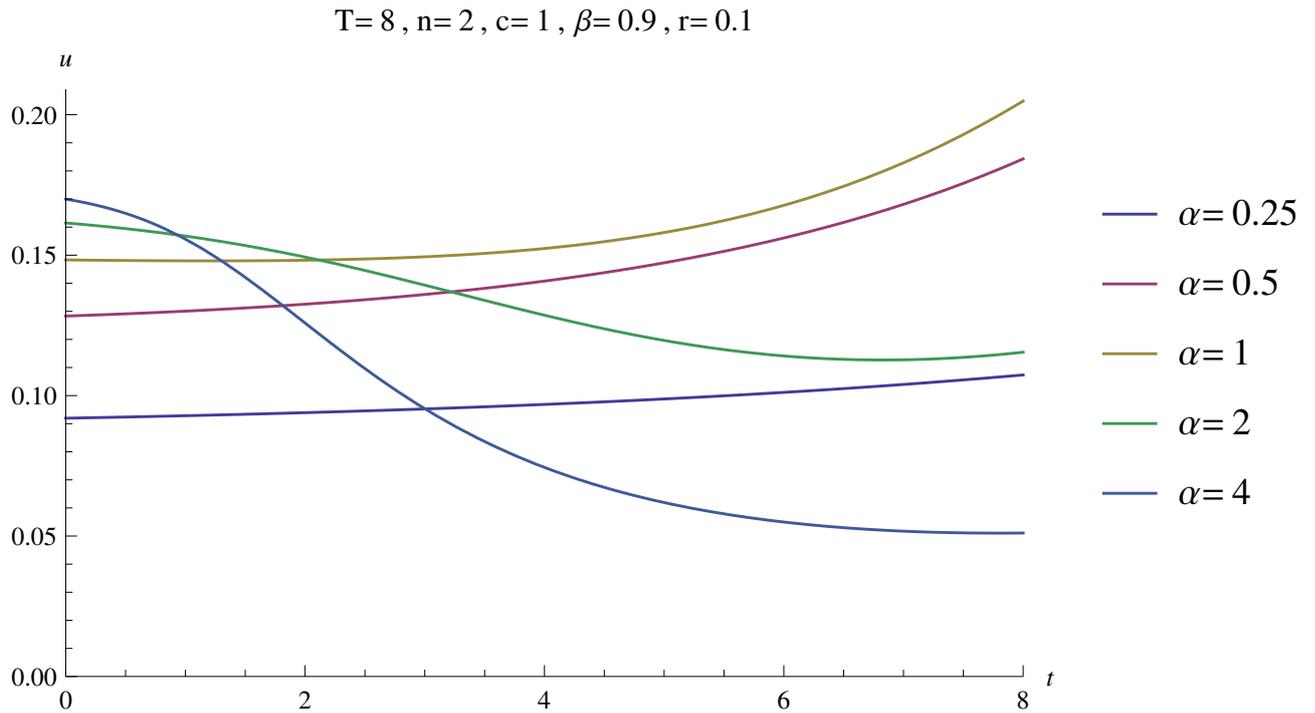


Figure 9: Equilibrium effort paths for different project qualities

that close to the deadline, it is stronger than every other effect.

This paper opens up a lot of questions for further research, examples being an analysis of the optimal compensation scheme for different projects, or whether choosing deadlines is an efficient tool for a social planner or a principal who is only interested in a breakthrough. Another interesting question is the effect of team size, which is briefly touched upon in Appendix C.

## A. Proofs

### A.1. Derivation of the Expected Utility

We know that the breakthrough effort is drawn from a distribution with the probability distribution function  $f$  and the cumulative distribution function  $F$  and that  $x(t)$  is by Definition 1

$$x(t) = \int_0^t u_i(s) + u_{-i}(s) ds$$

with  $u_{-i}(s) = \sum_{\forall j \neq i} u_j(s)$ . The breakthrough time  $\bar{t}$  is the first time enough effort (i.e., the breakthrough effort) is accumulated:

$$\bar{t} = \inf\{t \geq 0 | x(t) \geq \bar{x}\}.$$

The expected utility is then, given breakthrough time  $\bar{t}$

$$\tilde{V}_i(u_i, \bar{t}) = re^{-r\bar{t}} - r \int_0^{\bar{t}} e^{rt} c(u_i(t)) dt.$$

Therefore we know that the payoff part of the expected utility is equal to the distribution of  $\bar{t}$ :  $f(\bar{t})$ . As we know that the CDF  $F(\bar{t})$  of  $\bar{t}$  is

$$F_{\bar{t}}(t) = P[t \geq \bar{t}] = P[x(t) \geq \bar{x}] = F(x(t)) \Rightarrow f_{\bar{t}}(t) = f(x(t)) (u_i(t) + u_{-i}(t)),$$

so the expected payoff is

$$\mathbb{E}_{\bar{t}} [re^{-r\bar{t}}] = r \int_0^T e^{-rt} f(x(t)) (u_i(t) + u_{-i}(t)) dt. \quad (6)$$

For the expected costs  $r \int_0^{\bar{t}} e^{rt} c(u_i(t)) dt$ , we have to distinguish between two cases: One in which the project is successful (i.e.,  $\bar{t} < T$ ) and we pay until  $\bar{t}$  and one in which it is unsuccessful ( $\bar{t} \geq T$ ) and we only pay until  $T$ . As we know that  $1 - F_{\bar{t}}(\infty) = P[\bar{t} = \infty] = P[\bar{x} > x(\infty)] = 1 - F(x(\infty))$  we get:

$$\begin{aligned}
& \int_0^\infty \int_0^{\min\{t,T\}} e^{-rs} c(u(s)) ds dF(x(t)) + (1 - F(x(\infty))) \int_0^T e^{-rs} c(u(s)) ds \\
&= \int_0^\infty \int_0^\infty \mathbb{1}_{s<t} \mathbb{1}_{s<T} e^{-rs} c(u(s)) ds dF(x(t)) + (1 - F(x(\infty))) \int_0^T e^{-rs} c(u(s)) ds \\
&= \int_0^\infty \int_0^\infty \mathbb{1}_{s<t} \mathbb{1}_{s<T} e^{-rs} c(u(s)) dF(x(t)) ds + (1 - F(x(\infty))) \int_0^T e^{-rs} c(u(s)) ds \\
&= \int_0^\infty \mathbb{1}_{s<T} e^{-rs} c(u(s)) \int_0^\infty \mathbb{1}_{s<t} dF(x(t)) ds + (1 - F(x(\infty))) \int_0^T e^{-rs} c(u(s)) ds \\
&= \int_0^\infty \mathbb{1}_{s<T} e^{-rs} c(u(s)) (1 - F(x(s))) ds \\
&= \int_0^T e^{-rs} c(u(s)) (1 - F(x(s))) ds. \tag{7}
\end{aligned}$$

If we add the expected payoff (Equation (6)) and the expected costs (Equation (7)), we get the expected utility, as stated in Equation (1):

$$V_i = r \int_0^T e^{-rt} (1 - F(x(t))) \left( \frac{f(x(t)) (\sum_{j \neq i} u_j(t))}{1 - F(x(t))} - c(u_i(t)) \right) dt.$$

## A.2. Theorem 1 (Optimal Effort)

### Candidate Solution

Finding the best response  $u_i$  of some player  $i$  to the strategies of the other players  $u_{-i}$  in the problem stated in equation (Equation (2)) is a discounted optimal control problem of the following form

$$\begin{aligned}
H(x, u, \lambda, t) &= f(x) (u_i + \sum_{j \neq i} u_j) - c u_i^2 (1 - F(x)) + \lambda (u_i + u_{-i}) \\
\dot{x} &= u_i + \sum_{j \neq i} u_j \\
x(0) &= 0 \\
\lambda(T) &= 0 \\
u_i, x &\in \mathbb{R}_+ \quad \forall i
\end{aligned}$$

Using the Pontryagin maximum principle (Pontryagin, Boltyanskii, and Gamkrelidze (1962)) in the version of Kamien and Schwartz (2012), we know that the necessary conditions for a maximum are

$$\frac{\partial H}{\partial u} = 0 \quad (8)$$

$$\frac{\partial H}{\partial x} = r\lambda - \dot{\lambda} \quad (9)$$

$$\frac{\partial H}{\partial \lambda} = \dot{x} \quad (10)$$

$$\lambda(T)x(T) = 0 \quad \Rightarrow \quad \lambda(T) = 0 \quad (11)$$

with the Hamiltonian (Equation (8)), the equation of motion for the state variable (Equation (9)), the equation of motion for the costate variable (Equation (10)) and the transversality condition (Equation (11)) for  $x(T)$  being free. In addition we can see that the optimal control does not depend on the  $u_j$ s of the other players but only on the sum  $u_{-i} := \sum_{j \neq i} u_j$ . Therefore we get

$$\begin{aligned} u_i &= \frac{f(x) + \lambda}{2c(1 - F(x))} \\ \dot{\lambda} &= r\lambda - f'(x)(u_i + u_{-i}) - cu_i^2 f(x) \\ \dot{x} &= u_i + u_{-i} \\ x(0) &= 0, \quad \lambda(T) = 0 \end{aligned}$$

From here on, we only consider symmetric equilibria, therefore we can replace  $u_{-i}$  by  $(n - 1)u_i$ . Hence, the necessary conditions for a best response are:

$$\begin{aligned} u_i &= \frac{f(x) + \lambda}{2c(1 - F(x))} \\ \dot{\lambda} &= r\lambda - f'(x)nu_i - cu_i^2 f(x) \\ \dot{x} &= nu_i \\ x(0) &= 0, \quad \lambda(T) = 0. \end{aligned}$$

Using  $u_i$  we get

$$\begin{aligned} \dot{\lambda} &= r\lambda - f'(x)n \left( \frac{f(x) + \lambda}{2c(1 - F(x))} \right) - c \left( \frac{f(x) + \lambda}{2c(1 - F(x))} \right)^2 f(x) \\ \dot{x} &= n \left( \frac{f(x) + \lambda}{2c(1 - F(x))} \right) \\ x(0) &= 0, \quad \lambda(T) = 0. \end{aligned}$$

So, the equation of motion for the costate and its time derivative are

$$\begin{aligned}\lambda &= \frac{2c}{n} (1 - F(x)) \dot{x} - f(x) \\ \dot{\lambda} &= \frac{2c}{n} (1 - F(x)) \ddot{x} - \frac{2c}{n} f(x) \dot{x}^2 - f'(x) \dot{x}\end{aligned}$$

Using these, we get the boundary value problem:

$$\begin{aligned}(1 - F(x)) \ddot{x} &= -\frac{nr}{2c} f(x) - \frac{1}{2n} f(x) \dot{x}^2 + f(x) \dot{x}^2 + r (1 - F(x)) \dot{x} \\ x(0) &= 0, \\ \lambda(T) &= \frac{2c}{n} (1 - F(x_T)) \dot{x}_T - f(x_T) = 0\end{aligned}$$

Introducing the hazard rate  $h(x) := \frac{f(x)}{1-F(x)}$ , we have necessary conditions for Equation (2)

$$\begin{aligned}\ddot{x} &= -\frac{rn}{2c} h(x) + \frac{2n-1}{2n} h(x) \dot{x}^2 + r \dot{x} \\ x(0) &= 0, \\ \dot{x}_T &= \frac{n}{2c} h(x_T)\end{aligned}\tag{12}$$

Or, in terms of the individual effort:

$$\begin{aligned}\dot{u} &= -\frac{r}{2c} h(x) + \frac{2n-1}{2} h(x) u^2 + ru \\ u_T &= \frac{1}{2c} h(x_T),\end{aligned}\tag{13}$$

which is a non-linear boundary value problem of the second order.

## Existence, Uniqueness and Sufficiency of the Solution

Now that we have necessary conditions for the equilibrium effort, we still have to check for existence of the solution initial value problem and therefore the Nash equilibrium and if the necessary conditions are sufficient.

Checking for existence and uniqueness first gives us the following

**Proposition 2** (Existence and Uniqueness). A solution to the initial value problem from Equation (12) (and therefore also for Equation (13)) exists and is unique.

*Proof.* As  $u_i : [0, T] \rightarrow \mathbb{R}_+$  is continuous and maps from a compact space to a metric space, we know that it is bounded. Therefore,  $\frac{r}{2c}h(x) + \frac{2n-1}{2n}h(x)u^2 - ru$  is Lipschitz-continuous in  $u$  and  $t$ . Thus (by Picard-Lindelöf), we know that a unique solution to the initial value problem for  $u_i$  exists.

As  $x(t) = \int_0^t nu(s) ds$  and  $x_0 = 0$  and  $u$  exists and is unique,  $x(t)$  also exists uniquely.  $\square$

Now we know that the candidate solution from Equation (13) exists and is unique, we have to show that it is in fact the maximum:

**Proposition 3** (Maximum). The candidate equilibrium strategy  $u_i$  as defined in Equation (13) is the solution to the maximization problem in Equation (2) and therefore maximizes  $V_i$ .

*Proof.* Given Assumption 1 and Proposition 2 we know that  $u_i$  exists, is unique and continuous and we know that  $V_i$  is continuous in  $u_i$ . Furthermore, we know that (abusing the notation of  $u_i = c$  as  $u_i(t) = c \quad \forall t \in [0, T]$ ):

$$\begin{aligned} u_i = 0 &\Rightarrow V_i = 0 \\ \exists \varepsilon > 0 : u_i = \varepsilon &\Rightarrow V_i > 0 \\ \lim_{u_i \rightarrow \infty} V_i &= -\infty \end{aligned}$$

Therefore  $V_i$  is concave in  $u_i$  and, as it is also continuous in  $u_i$ , the necessary conditions for a maximum from Equation (13) are sufficient.  $\square$

Therefore we have established the necessary and sufficient conditions for an equilibrium and have shown that, given Assumption 1, this equilibrium always exists uniquely, as stated in Theorem 1.

### A.3. Theorem 2 (Procrastination)

To proof Theorem 2, we use continuity of  $x$  to show that the negative part of  $\dot{u}$  vanishes near the deadline and is therefore strictly positive.

*Proof.* As  $h(x_t)$  is continuous in  $x$ , it is also continuous  $t$ . We also know from Theorem 1 that  $u(t)$  is continuous in  $t$  and that it satisfies

$$\dot{u} = \frac{2n-1}{2}h(x)u^2 - r(u - \frac{1}{2c}h(x)), \quad u_T = \frac{1}{2c}h(x_T)$$

As  $u_t$  and  $h(x_t)$  are continuous in  $t$ , we know that:

$$\Rightarrow \lim_{t \rightarrow T} \left( u - \frac{1}{2c} h(x) \right) \rightarrow \left( u_T - \frac{1}{2c} h(x_T) \right) = 0$$

Now define  $\varepsilon(t) = \frac{2n-1}{2r} h(x_t) u_t^2$ . Then we know,

$$\begin{aligned} \exists \delta > 0 : \hat{t} := T - \delta &\Rightarrow \left( u_t - \frac{1}{2c} h(x_t) \right) < \varepsilon(\hat{t}) = \frac{2n-1}{2r} h(x_{\hat{t}}) u_{\hat{t}}^2 \\ \Rightarrow \dot{u}_{\hat{t}} &= \frac{2n-1}{2} h(x_{\hat{t}}) u^2 - r \left( u_{\hat{t}} - \frac{1}{2c} h(x_{\hat{t}}) \right) > 0. \end{aligned}$$

□

Therefore, we know that there is always an interval  $[\hat{t}, T]$  in which the effort is strictly increasing.

#### A.4. Lemma 1 (Asymmetric Equilibria)

*Proof.* Assume there is an asymmetric equilibrium that is welfare maximizing. Then  $\exists i, t, j : u_i(t) > u_j(t)$ . However, it would be possible to improve welfare by setting a new  $u_i^*(t)$  and  $u_j^*(t)$  as follows:  $u_i^*(t) = u_j^*(t) = \frac{u_i(t) + u_j(t)}{2}$  as this does not change the overall effort (and therefore the chance of success) but reduces, due to the quadratic costs, the combined expected costs of the project. Therefore, an asymmetric equilibrium can never be welfare maximizing. □

#### A.5. Theorem 3 (Social Planner)

Applying similar methods as in Appendix A.2, we get the welfare maximizing cumulative effort:

$$\begin{aligned} \ddot{x} &= -\frac{rn^2}{2c} h(x) + \frac{1}{2} h(x) \dot{x}^2 + r\dot{x} \\ x(0) &= 0, \\ \dot{x}_T &= \frac{n^2}{2c} h(x_T) \end{aligned}$$

Or, in terms of instantaneous effort  $u$ :

$$\dot{u} = \frac{1}{2} h(x) u^2 + ru - \frac{nr}{2c} h(x)$$

which reaches  $u_T = \frac{n}{2c} h(x_T)$  at time  $T$ .

**Remark.** *It is easy to see that the properties derived in Proposition 2 and Proposition 3 for the optimal effort in the game also apply to the solution of the social planner's problem (Theorem 3).*

## B. Constant hazard rate

For the exponential distribution with rate parameter  $\lambda$ , the solution from Theorem 1 evolves according to

$$\begin{aligned}\dot{u} &= \frac{2n-1}{2}\lambda u^2 + ru - \frac{r}{2c}\lambda \\ u_T &= \frac{1}{2c}\lambda,\end{aligned}$$

which has the following explicit solution:

$$u(t) = \frac{\lambda \left( e^{\frac{(t-T)\sqrt{cr(cr+\lambda^2(2n-1))}}{c}} + 1 \right) \left( \lambda^2(2n-1) + cr - \sqrt{cr(cr+\lambda^2(2n-1))} \right) + 2\sqrt{cr(cr+\lambda^2(2n-1))}}{2c(cr+\lambda^2(2n-1)) \left( \left( 1 - \frac{2cr+\lambda^2(2n-1)}{2\sqrt{cr(cr+\lambda^2(2n-1))}} \right) e^{\frac{(t-T)\sqrt{cr(cr+\lambda^2(2n-1))}}{c}} + \frac{2cr+\lambda^2(2n-1)}{2\sqrt{cr(cr+\lambda^2(2n-1))}} + 1 \right)}.$$

The welfare maximizing effort has the following explicit solution

$$u(t) = \frac{\lambda n e^{-T\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}} \left( \left( r - \sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)} \right) e^{t\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}} - \left( \sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)} + r \right) e^{T\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}} \right)}{\lambda^2 n \left( e^{(t-T)\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}} - 1 \right) + 2cr \left( e^{(t-T)\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}} - 1 \right) - 2\sqrt{cr(cr+\lambda^2 n)} \left( e^{(t-T)\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}} + 1 \right)}.$$

Calculations show that the non-cooperative equilibrium effort is always lower than the welfare maximizing effort.

## C. The Effect of the Number of Players

So far I have assumed a fixed number of players. What happens if the number of players changes? Assuming increasing hazard rates, the effect is pretty clear: we have stronger free-riding but in turn have more overall effort which, due to the encouragement effect, leads to higher efforts by the players. With a constant hazard rate, we only have free-riding, so every individual will work less but thanks to quadratic costs and the fact that the payout does not depend on the number of players, we can expect higher overall efforts. As the encouragement effect only depends on the cumulative effort, we can also

expect the same effect with a decreasing hazard rate, which can be observed in the example of an incomplete exponential distribution in Figure 10 and Figure 11. Here we can see the lower individual efforts and higher cumulative efforts for higher number of players.

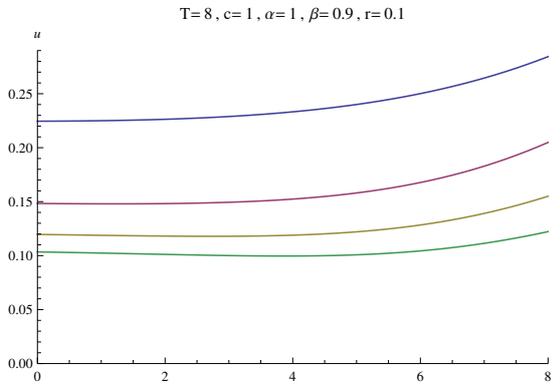


Figure 10: Effort for different numbers of players

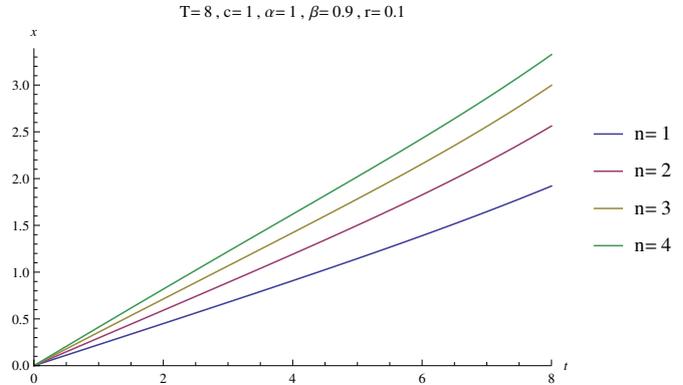


Figure 11: Cumulative effort for different numbers of players

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