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The (non-)robustness of influential cheap talk equilibria

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Abstract

Chakraborty and Harbaugh (2010) prove the existence of influential cheap talk equilibria in one sender one receiver games when the state is multidimensional and the preferences of the sender are state-independent. We show that only the babbling equilibrium survives the introduction of any small degree of uncertainty about the sender's preferences in the spirit of Harsanyi (1973). None of the influential equilibria are robust to this kind of uncertainty.

1 Introduction

This paper is concerned with the strategic information transmission (as first analyzed in Crawford and Sobel (1982)) between one informed sender and one uninformed receiver. The sender can attempt to communicate her information to the sender before the sender takes an action. The receiver would, ideally, like to make his choice of action dependent on the state of the

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world, but in a way, that differs from the sender's ideal choice of action. Thus, there is a conflict of interest. Communication is costless (termed "cheap" in the literature). Messages the sender transmits to the receiver have no intrinsic meaning, or no intrinsic meaning can be verified, and only possibly take on meaning (reveal information) in equilibrium.

One of the main findings of the cheap talk literature, started by Crawford and Sobel (1982), is that influential communication in one sender one receiver games is typically only possible if the conflict of interest is not too large.[3] This has been shown in the equilibrium characterization by Crawford and Sobel (1982) and expanded by Goltsman, Hörner, Pavlov, and Squintani (2009). If the conflict of interest is large, credible communication seemed only possible if messages are verifiable or costly (for a survey of this literature, see Sobel (2013)).

Chakraborty and Harbaugh (2010) propose and analyze a one sender one receiver game with a multi-dimensional state space with an extreme form of conflict of interest. The receiver is essentially as modelled in Crawford and Sobel (1982), but the informed sender actually does not at all care about the state itself:[4] The sender's preference is state-independent.

Surprisingly, and by a beautiful argument - which eventually allows the use of the Borsuk-Ulam theorem (a fundamental fixed-point theorem; see Section 5.2 for a version of that theorem) - Chakraborty and Harbaugh (2010) show that, in their model, influential cheap talk equilibria always exist.

To analyze games of incomplete information, such as those of the cheap talk literature, in addition to specifying players, strategies, and consequences (payoffs) to complete the model one has to make informational assumptions. The informational assumptions made in Chakraborty and Harbaugh (2010), as also in Crawford and Sobel (1982), are as follows. The utility functions of both sender and receiver are common knowledge, as is the receiver's "subjective" belief about the state.

[3]In an influential equilibrium the sender is able to influence the receiver's choice of action by the sender's choice of message.

[4]For *multiple sender* one receiver models Battaglini (2002) showed that a multi-dimensional state-space implies the existence of equilibria with full information revelation. For one sender one receiver models this is not typically true.

In fact Chakraborty and Harbaugh (2010) relax these informational assumptions in a robustness exercise in two different ways, and show, for each case, that the game so modified still exhibits influential equilibria. Both robustness exercises allow the sender to have possibly different utility functions. In both cases the sender knows her utility function and the receiver's subjective belief about the sender's utility function is common knowledge. In one specification this commonly known distribution has finite support with the number of positive probability utility functions less than the dimensionality of the state space. In the second specification this commonly known distribution places a sufficiently large atom on a single utility function.

As the state space is a compact subset of, at least, two-dimensional Euclidean space and as there are, in principle, an infinite number of possible utility functions the sender could have (even an infinite number of utility functions that are all very close to each other) we feel a different robustness check should also be undertaken. In this paper we assume that the receiver, while possibly having a good general idea about the sender's preferences does not believe that any particular utility function (out of the infinitely many possible ones) has positive probability. We call this the Chakraborty and Harbaugh (2010) model with *Harsanyi uncertainty*, as the uncertainty is very much as it is in the purification argument of Harsanyi (1973).[5] Completing this model by assuming that the receiver's subjective belief about the sender's utility function is common knowledge, we then find that this modified game has no influential equilibria. This result does not depend on the choice of the set of possible utility functions (as long as a belief without atom can be specified) nor on the exact shape of the distribution of these beliefs.

The paper is organized as follows. We begin by restating the model of Chakraborty and Harbaugh (2010) and stating our modification to that model in Section 2. Section 3 demonstrates the main finding of Chakraborty and Harbaugh (2010), as well as the non-robustness to Harsanyi uncertainty of all influential equilibria, by means of the simplest possible example. The

[5]Harsanyi (1973) uses, what we here call, Harsanyi uncertainty to show that mixed equilibria, in which the players are indifferent between at least two pure strategies, can be thought of as pure strategy equilibria in the game played by, at least in the minds of the players, infinitely many possible "types". As explained in Section 3, the influential equilibria in Chakraborty and Harbaugh (2010) also rely on indifference. One way to state our result is that the influential equilibria in Chakraborty and Harbaugh (2010), even though they are actually in pure strategies, cannot be purified in the sense of Harsanyi (1973). Alternatively, one could also say that the influential equilibria in Chakraborty and Harbaugh (2010) are not *regular* in the sense of Harsanyi (1973).

main result of our paper is then stated and proven in Section 4. Section 5 concludes by providing a discussion of three related points. Section 5.1 shows by example that not all cheap-talk games suffer from this non-robustness. The example is a simple special case of Crawford and Sobel (1982) with almost common interest. Section 5.2 provides a theorem that states that, if the Harsanyi uncertainty in the Chakraborty and Harbaugh (2010) model is only about the receiver, then the game always has an influential equilibrium. Finally, Section 5.3 provides an argument that demonstrates that, even if the sender's preference is common knowledge, there may be higher order belief uncertainty (about the receiver's belief about the state), in the spirit of Bergemann and Morris (2005), that again implies the non-robustness of all influential equilibria.

2 The model

A sender (female) is privately informed about the realization of $\theta \in \Theta$, where Θ is a convex and compact subset of $\mathbb{R}^N$ with non-empty interior and $N \geq 2$. The sender can send a costless message m from a finite set of messages M to a receiver (male). The receiver observes the message and then takes an action in action space $\mathcal{A} = \Theta$. A sender strategy is thus a mapping from state space Θ to the set of messages M, while a receiver strategy is a mapping from message space M to action space Θ. The utility function of the receiver is given by $v(a, \theta) = -(a - \theta)^2$. This implies that, in any equilibrium, the receiver, "knowing" the sender's strategy, plays, as his best response, the (conditional) expectation of θ. The prior of the receiver is described by the distribution function F with full support on Θ. The utility of the sender is a function $u : \mathcal{A} \to \mathbb{R}$ that does not depend on the realization of the state variable θ.

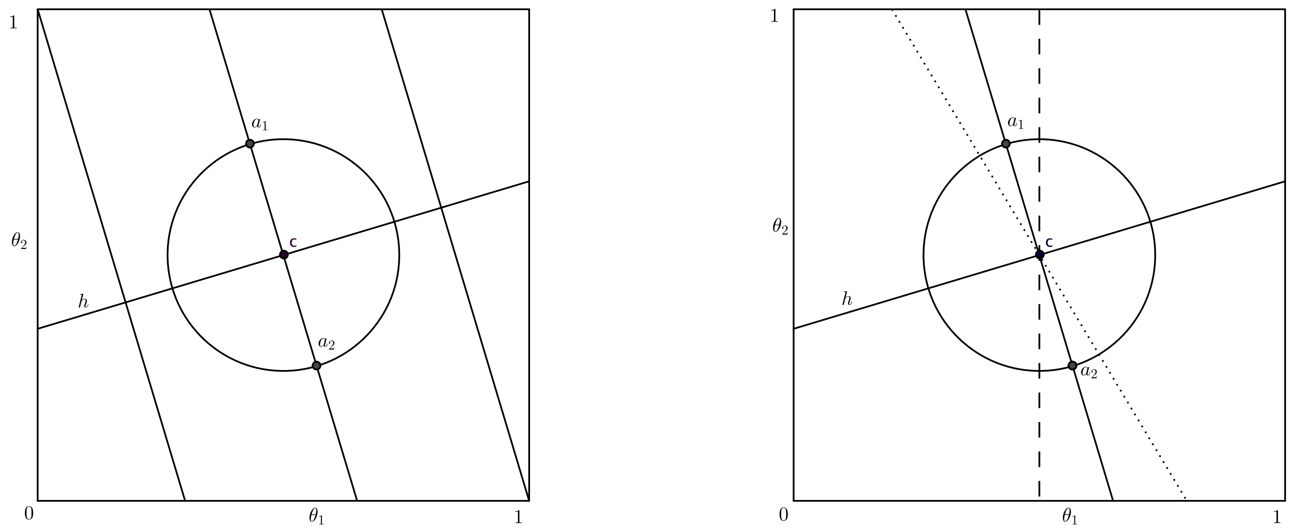
The equilibrium concept is Bayesian Nash. A Bayesian Nash equilibrium is termed *influential* if there are at least two messages (sent with positive probability according to F) which induce different actions.[6][7]

[6]Note that in any equilibrium the receiver will never want to randomize between two actions. His best response (for messages sent with positive probability) is always unique. In fact, for the purpose of this paper it is without loss of generality to restrict attention to pure strategies for both the sender and the receiver.

[7]Sobel (2013) differentiates between an influential and an informative equilibrium. In an influential equilibrium different messages induce different actions, while in an informative equilibrium different messages induce

Up to this point, the model we presented here is exactly the model introduced by Chakraborty and Harbaugh (2010). We now add uncertainty about the preferences of the sender in the following way to the model. There is a set of possible utility functions $\mathcal{U}$ for the sender. The sender is privately informed about her utility function $u \in \mathcal{U}$. The receiver has a prior belief given by distribution function ϕ, a distribution over the set $\mathcal{U}$ which has no atoms.[8] We call this extended model the Chakraborty and Harbaugh (2010) model with *Harsanyi-Uncertainty*, as the way we introduce uncertainty is essentially as in Harsanyi (1973), the "purification" paper.

3 The main example



(a) No uncertainty

(b) Uncertainty

Figure 1: Uncertainty vs. no uncertainty in the linear case

For our main example suppose that $\Theta = [0, 1]^2$ (i.e. $N = 2$) and that the sender's preferences are linear. That is, for any $a \in \Theta$, we have $u(a) = a_1 + xa_2$. The "indifference slope" x is known to the sender, but not known to the receiver. The receiver has a non-atomic prior ϕ over x in the interval $[x_0 - \epsilon, x_0 + \epsilon]$ for some fixed and commonly known $x_0 \in \mathbb{R}$ and $\epsilon > 0$. In terms of

different sender beliefs. In our context the two notions are identical.

[8]We assume the necessary technical assumptions on $\mathcal{U}$ are satisfied, such that a non-atomic distribution exists.

our general model we have $\mathcal{U} = \{u(a) = a_1 + xa_2 | x \in [x_0 - \epsilon, x_0 + \epsilon]\}$. Suppose, further, that the set of messages M consists of exactly two elements m_1 and m_2.

Consider first the case in which there is no uncertainty about the sender's preference. For such a case Chakraborty and Harbaugh (2010) show that there is an equilibrium of the following kind, as illustrated in Figure 1 (a). There is a hyperplane h that divides the state space Θ into two regions. In region 1 (say, above the hyperplane) the sender sends message m_1, which induces action a_1, while in the other remaining region 2 the sender sends message m_2 inducing action a_2. The two actions are simply (and necessarily in equilibrium) the updated expected state given the sender's strategy. Chakraborty and Harbaugh (2010) show, by a nice argument appealing to the Borsuk-Ulam theorem, that the hyperplane can be chosen (rotated around any arbitrary state c) such as to make the sender exactly indifferent between actions a_1 and a_2. Therefore, they show that an influential equilibrium exists.

Suppose now there is Harsanyi-Uncertainty about the slope of the indifference curve as modelled above. This case is illustrated in Figure 1 (b). Now consider the following strategy. The state space is divided into two regions (by, for instance, but not necessarily, a hyperplane). As before, the sender sends message m_1 in region 1 and message m_2 in region 2. It is now possible that there is a preference-type of the sender who is indifferent between the two induced actions a_1 and a_2. Note, however, that this is true for only exactly a single one of these preference-types of senders. All other preference-types have a strict preference for one or the other action. This means **all** other preference-types (and they have cumulative probability 1 in this model) will want to deviate to a strategy that involves sending one and the same message irrespective of the state. Thus, there is no such influential equilibrium in the model with Harsanyi-Uncertainty.

4 The main result

We now state and prove the main theorem. In order to do so, we first define Condition (S), as stated in the online appendix of Chakraborty and Harbaugh (2010).

The set of possible utility functions $\mathcal{U}$ (that the sender might have, from the point of view of the receiver) satisfies *Condition (S)* if for any two actions a and a', if $u'(a) = u'(a')$ for $u' \in U$, then $u(a) \neq u(a^{'})$ for all $u \in \mathcal{U}, u \neq u^{'}$. For example, the linear preference model in our main example (Section 3) satisfies this property. More generally, Condition (S) holds for preferences whose indifference curves satisfy a single crossing property. The following theorem is the main result of this paper.

Theorem 1. *Consider a sender-receiver game as defined in Section 2. Suppose the set of possible utility functions for the sender, $\mathcal{U}$, satisfies Condition (S) and suppose that ϕ, the receiver's prior belief over $\mathcal{U}$, is non-atomic. Then there does not exist an influential equilibrium in this game.*

Proof. The proof is by contradiction. Suppose there exists an influential equilibrium. Hence, there exist messages m_1 and m_2 that are sent with positive probability (under F and ϕ) and induce different actions, $a_1 = E(\theta|m_1) \neq a_2 = E(\theta|m_2)$. In other words, for each message there is a set of senders (with positive probability under ϕ) that send this message in a set of states that also has positive probability under F. Action a_i, for $i \in \{1, 2\}$, is then the receiver's unique (and pure) best response to receiving message m_i (given the senders' strategies).

The strategy profile given is thus such that the receiver behaves optimally. We now turn to the (various types of) senders. In order for a sender to use message m_1 in some states and message m_2 in other states (and given the sender has state-independent preferences) the sender must be exactly indifferent between both induced actions a_1 and a_2. We thus must, at a minimum, have that there is a sender-type $u' \in \mathcal{U}$ such that $u'(a_1) = u'(a_2)$. But then Condition (S) implies that for all $u \in \mathcal{U}, u \neq u^{'}$, we have $u(a_1) \neq u(a_2)$. Given that distribution ϕ is non-atomic, the "event" $u \neq u'$ has probability one under ϕ. This means that a unit measure of senders has a strict preference to send only one of the two messages (over the other) *irrespective of the state.* This, in turn, implies that the receiver's best response to both messages must be the same. We thus arrive at a contradiction. □

Comments:

1. An example sketched in Figure 2 explains why a condition like Condition (S) is needed for the non-existence of an influential equilibrium. Take an interior point c and a hyperplane h which splits the state space in two halves. The indifference curves of the different sender

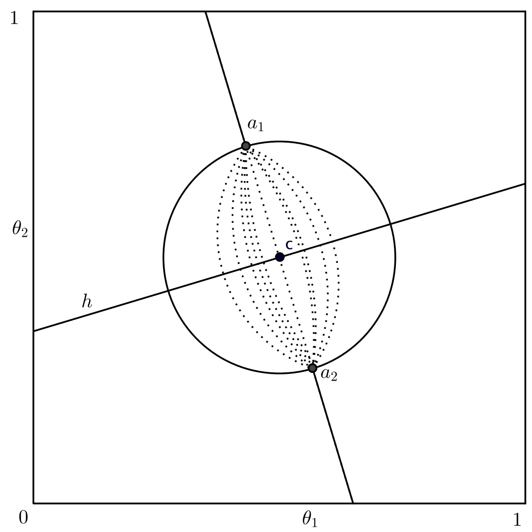


Figure 2: Existence of influential equilibrium despite Harsanyi-Uncertainty

types are the dotted lines.[9] Importantly all indifference curves intersect at two places (violating Condition (S)), which are exactly the best response actions a_1 and a_2 of the receiver to receiving message m_1 (state is above line h) and m_2 (state is below line h). Thus, there is an influential equilibrium. Condition (S) rules out such situations.

2. Nevertheless, it is straightforward to generalize Theorem 1 to a somewhat weaker condition than Condition (S): Say *Condition* (S$^{'}$) holds if for any two actions a and $a^{'}$, $P_\phi(u \in \mathcal{U} | u(a) \neq u(a^{'})) = 1$. The proof is the same.

3. Theorem 1 and the Condition (S$^{'}$) version of Theorem 1 give sufficient conditions for the non-existence of influential equilibria. It might be interesting to investigate necessary conditions for the non-existence.

4. Note that, in Theorem 1, the set of possible sender preferences $\mathcal{U}$, apart from the assumption that it admits a non-atomic distribution and satisfies Condition (S) or (S'), can be anything. Of course we have in mind that there is a modeler's choice of $u_0 \in \mathcal{U}$ (as, for instance, chosen by Chakraborty and Harbaugh (2010) as a good guess for the sender's preferences), and that all other possible $u \in \mathcal{U}$ are close to u_0. For instance, all $u \in \mathcal{U}$ are such that the maximal pointwise difference to u_0 is below some small positive real number

[9]One may think of a continuum of indifference curves between the left-most and the right-most curve.

ϵ. Theorem 1 implies that even if all $u \in \mathcal{U}$ are close to u_0 (and close to each other) the sender-receiver game with Harsanyi-Uncertainty does not have an influential equilibrium.

5. Note, finally, that if indeed all $u \in \mathcal{U}$ are ϵ-close to some $u_0 \in \mathcal{U}$ then any influential equilibrium of the game with sender preference u_0 and without uncertainty about the sender's preference, remains an ϵ-equilibrium of the sender-receiver game with Harsanyi-Uncertainty.

5 Discussion

5.1 Uncertainty about the bias in Crawford and Sobel

We have shown that the influential cheap talk equilibria of Chakraborty and Harbaugh (2010) do not survive the introduction of Harsanyi-Uncertainty about the type of the sender. In this section we show that this is not a general problem that all sender-receiver games suffer from. To see this we use a simple example in the spirit of Crawford and Sobel (1982) with a possibly biased sender in which information transmission can still happen despite uncertainty about this bias.

The state space is $\Theta = [-1, 1]$. The prior of the receiver is given by a distribution F (with density f) over Θ that is symmetric around zero.[10] The sender is privately informed about the realization of the state $\theta \in \Theta$ and can send a costless message $m \in M = \{H, L\}$ to the receiver. The receiver observes the message of the sender and takes an action $a \in \mathcal{A} = \Theta = [-1, 1]$. The sender has utility function $u(a, \theta, b) = -(\theta + b - a)^2$ and the receiver utility function $v(a, \theta) = -(\theta - a)^2$. Here, b denotes the sender's bias relative to the receiver. Recall that a denotes the action taken by the receiver, and θ the state. Suppose, first, it was common knowledge that the sender's bias is equal to zero. Thus, the game is one of complete common interest. This game has an influential equilibrium in which senders with state below zero send message L and senders with state above zero send message H. The receiver chooses actions

[10] The assumption of symmetry is not important for the result. It allows us, however, to dramatically simplify the equilibrium calculations.

which are equal to the conditional expectation of the state conditioning on the observed message and given the sender's strategy. For the case of a uniform prior F, for instance, the receiver chooses action $a_H = \frac{1}{2}$ upon receiving message H and $a_L = -\frac{1}{2}$ upon receiving message L.

We now introduce Harsanyi-Uncertainty about the bias into this example of a Crawford and Sobel (1982) sender-receiver game.[11] The sender knows her bias precisely, in addition to knowing the state. The receiver does neither know the true state nor the precise bias b. Instead, the receiver only has a prior ϕ (with density function φ) over an interval $[-\epsilon, +\epsilon]$ of possible biases of the sender for ϵ positive but small. The prior ϕ is assumed to be symmetric around 0 and orthogonal to the prior F over the state space.[12]

We shall now compute an influential equilibrium that is close to the equilibrium without bias uncertainty given above. Suppose that the receiver plays action a_H if he observes message H and action a_L if he observes message L with (without loss of generality) $a_L < a_H$. Then the behavior of the sender must be as follows. If the state θ is below a cut-off of $q(b)$, which depends on the sender's bias, then she sends message L, otherwise she sends message H. The cut-off must be such that the sender with bias b and state equal to this cut-off $q(b)$ is indifferent between the two messages. This consideration leads to $q(b) = \frac{a_H + a_L}{2} - b$. The symmetry in the two distributions implies that $a_L = -a_H$. This in turn implies that the cutoff is $q(b) = -b$ and independent of the two actions.[13] It then remains to calculate the equilibrium action a_H. It is given by the conditional expectation, from the sender's point of view, of the state $q(b)$ given that message H is sent, i.e. given that $\theta > q(b)$. For ϵ small enough, this can be expressed as the double-integral

$$2 \int_{b=-\epsilon}^{\epsilon} \int_{\theta=-b}^{1} \theta f(\theta) \varphi(b) d\theta db,$$

where the 2 is the reciprocal of the probability that $\theta > q(b)$ (derived from the symmetry in the two distributions). For the special case of two uniform distributions for F and ϕ we obtain $a_H = \frac{1}{2} - \frac{\epsilon^2}{6}$ and $a_L = -a_H$. Thus, except for the receiver's actions being just slightly closer to

[11]Papers with uncertainty about the bias in the cheap talk literature include Morgan and Stocken (2003) and Li and Madarász (2008). We are not aware of a paper that introduces uncertainty about the bias in a way similar to ours.

[12]In other words the receiver's joint prior about state and bias is the product of the two marginal priors. Bias and state are, in the receiver's view, independently drawn.

[13]Without symmetry in the distributions this would not be true, and calculations would be more cumbersome.

the center than under the case without bias-uncertainty, the equilibrium has hardly changed. In particular, as ϵ tends to zero the influential equilibrium of the game with bias-uncertainty converges to the original influential equilibrium of the game without bias-uncertainty. To see this not only for the double-uniform prior case, note that, generally, the condition $\theta > q(b) = -b$, as ϵ tends to zero, tends to the condition $\theta > 0$, which is the condition employed in the model without bias uncertainty.

5.2 Different receiver types

Suppose there is no Harsanyi-Uncertainty about the preferences of the sender. That is, as in Chakraborty and Harbaugh (2010), there is only one type of sender with state-independent utility function $u : \mathcal{A} \to \mathbb{R}$, where $\mathcal{A} = \Theta$ and Θ a convex and compact subset of $\mathbb{R}^N$ with $N \geq 2$. Instead, there are possibly infinitely many different receiver types in terms of the receiver's subjective belief F over the state space Θ. That is, there is a set $\mathcal{F}$ of distributions over the state space. Each receiver privately knows his distribution F. The sender is not informed about the receiver's prior, but holds her own prior ψ over the set $\mathcal{F}$. This prior ψ is commonly known and can be a continuous distribution or can have atoms, or can even be a finite distribution.

Theorem 2. *Consider a sender-receiver game as defined in Section 2 with the information structure as given in Section 5.2. Then this game has an influential equilibrium.*

Proof. The proof follows the existence result of Chakraborty and Harbaugh. Fix an arbitrary $c \in \text{int}(\Theta)$ which exists as Θ is nonempty. Let $h_{s,c}$ be the hyperplane through c with "orientation" $s \in \mathbb{S}^{N-1}$. The orientation is orthogonal to the hyperplane and has (Euclidean) length 1. Thus, $\mathbb{S}^{N-1}$ is the unit sphere in $\mathbb{R}^N$. The hyperplane splits (essentially partitions) the state space into two nonempty regions $\mathbf{R}^1(h_{s,c})$ and $\mathbf{R}^2(h_{s,c})$. The expert sends message m_1 if $\theta \in \mathbf{R}^1$ and m_2 if $\theta \in \mathbf{R}^2$. Receiver type F best responds to the sender's strategy by choosing optimal action $a_i^F(h_{s,c}) \in \mathbf{R}^i(h_{s,c})$ upon receiving message m_i (for $i \in \{1, 2\}$).

The sender, with given fixed prior ψ, computes, for $i \in \{1, 2\}$, her expected utility $u_i(h_{s,c}) = \mathbb{E}_\psi \left[a_i^F(h_{s,c}) \right]$. For a fixed interior point c, each $u_i(h_{s,c})$ is a continuous function in $s \in \mathbb{S}^{N-1}$. For opposite orientations $s, -s \in \mathbb{S}^{N-1}$, we have $\mathbf{R}^1(h_{s,c}) = \mathbf{R}^2(h_{-s,c})$ and $\mathbf{R}^2(h_{s,c}) = \mathbf{R}^1(h_{-s,c})$

implying $u_1(h_{s,c}) = u_2(h_{-s,c})$ and $u_1(h_{-s,c}) = u_2(h_{s,c})$.

Consider the difference between the two utilities: $\Delta(\cdot, c) : \mathbb{S}^{N-1} \to \mathbb{R}$, where $\Delta(s, c) = u_1(h_{s,c})) - u_2(h_{s,c}))$. The property that $\Delta(s, c) = -\Delta(-s, c)$ makes this a (continuous) odd map in s. The Borsuk-Ulam theorem[14] then implies that there is a $s^* \in \mathbb{S}^{N-1}$ such that $\Delta(s^*) = 0$. Thus, there exists for every interior c an orientation $s^* \in \mathbb{S}^{N-1}$ such that $u_1(h_{s^*,c})) - u_2(h_{s^*,c})) = 0$. Thus, we have found an influential cheap talk equilibrium. □

5.3 Higher-order belief uncertainty

In the spirit of Bergemann and Morris (2005) and the so-termed "Wilson-doctrine" one could ask how robust the influential equilibria of Chakraborty and Harbaugh (2010) are to higher-order belief uncertainty. The previous subsection demonstrates that the influential equilibria of Chakraborty and Harbaugh (2010) are robust to uncertainty that the sender might have about the receiver's belief. In this small section we argue that essentially any higher-order belief uncertainty with a continuum of sender-types will again remove all influential equilibria of Chakraborty and Harbaugh (2010).

The argument is not more complex than our argument, presented in Section 4, to show that the influential equilibria of Chakraborty and Harbaugh (2010) are non-robust to Harsanyi-Uncertainty. It does, however, require a bit more notation. Let $\theta \in \Theta$ be the state, privately known to the sender. Let $F \in \mathcal{F}$ be the subjective belief of the receiver about the state, privately known to the receiver. Let u be the sender's utility function, commonly known to sender and receiver. Let $\psi \in \Psi$ be the subjective belief of the sender about the receiver's subjective belief, privately known by the sender. Let, finally, μ be the belief of the receiver about the sender's private belief ψ, commonly known to sender and receiver.

This model shares with the original Chakraborty and Harbaugh (2010) model and the model of Section 5.2 that there is common knowledge of the sender's utility function. It differs from the Chakraborty and Harbaugh (2010) model, but still agrees with the model of Section 5.2,

[14] The Borsuk-Ulam theorem implies that all continuous odd functions $f : \mathbb{S}^{N-1} \to \mathbb{R}$ have a zero, i.e. there exists s^* such that $f(s^*) = 0$.

in so far as the receiver's belief about the state is not common knowledge. It differs from the model of Section 5.2 in so far as the sender's belief about the receiver's belief is not common knowledge. Thus, there are again, as in the model of Section 2 and unlike the model of Section 5.2, multiple types of sender.

Now suppose that there is an influential equilibrium with at least two used messages m_1 and m_2. Suppose each message m_i induces optimal receiver actions a_i^F (different for different receiver beliefs F). The sender evaluates the expected utility of these actions according to her private belief $\psi \in \Psi$ about the distribution over the receiver's private belief F by $\mathbb{E}_\psi u(a_i^F)$. Suppose further that the commonly known belief of the receiver, μ, over the private beliefs of the sender is non-atomic and the set Ψ satisfies some condition like Condition (S). Then, if one sender-type ψ is indifferent between the two messages, i.e. $\mathbb{E}_\psi u(a_1^F) = \mathbb{E}_\psi u(a_2^F)$, no other sender-type ψ' is indifferent. That is, for all $\psi' \in \Psi$ with $\psi' \neq \psi$ we have that $\mathbb{E}_{\psi'} u(a_1^F) \neq \mathbb{E}_{\psi'} u(a_2^F)$. By the same argument as in the proof of Theorem 1 almost all sender-types will want to deviate from the proposed strategy. Thus, this game (with higher-order belief uncertainty as described here) has no influential equilibria.

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