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Coordination with independent private values: Why pedestrians sometimes bump into each other

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Abstract

Motivated by trying to better understand the norms that govern pedestrian traffic, I study symmetric two-player coordination games with independent private values. The strategies of "always pass on the left" and "always pass on the right" are always equilibria of this game. Some such games, however, also have other (pure strategy) equilibria with a positive likelihood of mis-coordination. Perhaps surprisingly, in some such games, these Pareto-inefficient equilibria, with a positive likelihood of mis-coordination, are the only evolutionarily stable equilibria of the game.

Keywords: incomplete information, continuously stable strategy, CSS, evolutionary stability, best-response dynamics

JEL codes: C72, C73, D82

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1 Introduction

When people walk across a busy square, shopping mall, train-station, or other busy places, they occasionally have to stall or make sharp maneuvers to avoid bumping into each other. On the other hand cars can drive down busy roads at high speed, passing each other at close quarters, without the fear of an accident.

In both cases, the two pedestrians or the two cars approaching each other, the game the two parties play seems to be a game of coordination. The most important objective seems to be to avoid bumping into each other.

In this paper I try to understand why there does not seem a clear norm for pedestrians such as "always pass other people on the right", while we do seem to have such a norm for cars. In fact, for cars we actually have a law, we do not only rely on a norm. (Young 1998, pp. 16-17), in his overview to his book, discusses how the behavior regarding "which side of the road to drive on" has evolved over time in Europe. Among other things he states that "... for the most part these norms were not codified as traffic laws until well into the nineteenth century." If there was a norm in place, why then was such a law introduced?

Young (1998) argues, and I agree, that these encounters are perfectly suited for an evolutionary game theoretic analysis (even beyond Nash equilibrium analysis). The reason for this is that the game is played over and over or, in the language of evolutionary game theory, "recurrently" (to distinguish it from the analysis of "repeated" games), by essentially always different people. Moreover any of a large population of individuals engage in such interactions. This gives all players the time to learn to play "well", while at the same time it makes little sense not to play myopically as the interactions in the future will be with very different individuals.

Suppose our model of such encounters, between pedestrians or cars, is the two-player coordination game with commonly known payoff structure given by the following matrix.

	L	R
L	1,1	0,0
R	0,0	1,1

The evolutionary analysis of such a game is straightforward. The mixed equilibrium is unstable, the two pure equilibria are stable. This is true under all evolutionary models I am aware of (see e.g. (Weibull 1995, Category II on p. 40 and p. 75) for a textbook treatment). In particular this means that the evolutionary stable play or "norm" is Pareto-optimal (and such that accidents do not happen).[1] If this is indeed the case, then the norm should really suffice and no law is required.

Possibly when pedestrians encounter each other in the process of crossing a busy square, these people, while their most important concern is to get passed each other, may yet have a slight preference for which side they would like to pass the other person on. Suppose for, instance, that a person after passing the opposed individual would then like to turn left for some reason (e.g. to enter a shop). This slight preference for one side over the other is very likely private information.

1.1 The content of this paper

In this paper, I therefore model this kind of interaction between pedestrians as a coordination game with independent private values. The game is such that, with probability one, players do not have a dominant strategy. The model is provided in Section 2. In Section 3 I find all symmetric Nash equilibria of this game in Proposition 1. The strategies "always pass on the left" and "always pass on the right" are always equilibria. Some games also have equilibria in which players use a cut-off strategy to decide whether to pass their opponent on the right or the left. These equilibria are Pareto-inferior to the aforementioned equilibria as shown in Proposition 2.

[1]Suppose the game is not one of pure coordination, but given, for instance, by payoff matrix

	L	R
L	4,4	0,2
R	2,0	3,3

Also in this game the mixed equilibrium is unstable, the two pure equilibria are stable, under any reasonable static or deterministically dynamic evolutionary process. The main point of Young (1993) and Kandori, Mailath, and Rob (1993) is that a stochastic model of evolution, with small but constantly occurring probabilistic mistakes or mutations, predicts that only one of the two pure equilibria will occur in the long-run. This *selection* is not driven by Pareto-optimality but by risk-dominance. In the example, this means the unique long-run stable equilibrium strategy is R. Note, nevertheless, that in the long-run stable equilibrium there is also no mis-coordination.

In Section 4 I first show that, unlike the pure coordination game with complete information the coordination game with independent private values is not a doubly-symmetric game (Proposition 3). This is of significance, as it is known that for doubly-symmetric games, evolution always leads to average-payoff improvements. This is the so-called *fundamental theorem of natural selection*, first stated by Fisher (1930), proven for the replicator dynamics in finite games by Losert and Akin (1983), and for games with a continuum of strategies by Oechssler and Riedel (2002). In the present context this fundamental theorem would predict the evolutionary stability of only the two strategies of "always pass on the left" and "always pass on the right". As the game at hand is not doubly-symmetric, however, the theorem does not necessarily hold, and, in fact I show that it does not hold. In Proposition 4 I show that there are coordination games with independent private values in which the only evolutionary stable strategies are Pareto-inferior and are such that a certain likelihood of mis-coordination is unavoidable.

1.2 Additional motivation

The present paper is heavily influenced and motivated by Goffman's (1971) "naturalistic" study of the "public order". Goffman (1971) is generally interested in the societal norms that govern human interaction in public places. In particular, in Chapter 1, "The individual as a unit", part II, "Vehicular Units", Goffman (1971) is concerned with "traffic codes" between all sorts of vehicular units (pedestrians, cars, bikes, ships, etc.).

Goffman's (1971) work is generally very amenable to an, especially evolutionary, game-theoretic analysis. The way Goffman (1971) thinks of a "norm", here e.g. a traffic code, is very similar to how an evolutionary game theorists would define it: as an evolutionary stable Nash equilibrium. For instance, (Goffman 1971, Preface, p.xx) states that "..., the rules of an order *are* necessarily such as to preclude the kind of activity that would have disrupted the mutual dealings, making it impractical to continue with them."[2] In my opinion this, in the language of game theory, means that no individual has an incentive to deviate from the norm. Goffman (1971) is also aware that there may be multiple equilibria (or norms): "However, it is also the case that the mutual dealings associated with any set of ground rules could probably be sustained with fewer rules or different ones, ...". He is aware that norms are

[2]See also (Goffman 1971, Preface, footnote 3, p.xxi) in which he demonstrates that he understands incentives in game theory very well.

not necessarily Pareto-efficient, as he continues the last sentence as follows: "..., that some of the rules which do apply produce more inconvenience than they are worth, ...".

Goffman's (1971) study is mostly descriptive. The present paper is an attempt to begin a complementary game-theoretic analysis that uses Goffman's (1971) observations and classifications and takes up his possible explanations. The present paper in some ways falls utterly short of the full analysis given in (Goffman 1971, Chapter 1.II), but, I believe, in other ways adds a formal precision that enables us to understand such norms even better.

The following quote probably allows to explain best where this paper falls short of, and where it improves upon Goffman (1971). (Goffman 1971, Chapter 1.II, p.6): "Take, for example, techniques that pedestrians employ in order to avoid bumping into one another. These seem of little significance. However, there are an appreciable number of such devices; they are *constantly* in use and cast a pattern on street behavior. Street traffic would be a shambles without them." I here do not study the "techniques that pedestrians employ in order to avoid bumping into one another", even though I feel this can be done by means of adding a cheap-talk communication phase prior to the actual coordination game. I certainly would like to analyse such a model in the future. What I do do here, however, is to ask myself, what norm would emerge if we did not have such an "appreciable number of such devices" at our disposal. I, thus, ask whether it would really be true that "[s]treet traffic would be a shambles without them". In other words, the present paper provides a, hopefully useful, benchmark result that could be the basis of further game-theoretic analysis.

2 The Model

There are two players, artificially termed row and column player. Players can choose one of two actions, L and R. Players cannot condition their behavior on their role. Each player has a privately known "value" or "type", denoted by u for the row player and by v for the column player. The two values u and v are independently drawn from a common distribution on the interval $[0, 1]$ with distribution function F. For convenience we shall assume that F has full support and admits a continuous density function, denoted by f. In particular, F has no atoms. For any realized type pair the players play the following coordination game given by the following payoff matrix.

	L	R
L	1-u,1-v	0,0
R	0,0	u,v

This game could, thus, be called a *symmetric two-player coordination game with independent private values.* A pure strategy is any measurable function $\sigma : [0,1] \rightarrow \{L, R\}$. The set of measurable pure strategies shall be denoted by Σ. Let Δ denote the set of all mixed strategies, i.e. all probability distributions over elements in Σ. A special class of pure strategies of interest in the present analysis is the set of cut-off strategies. In fact, if we deal with cut-off strategies, it is without loss of generality to consider only those cut-off strategies, denoted σ_x for some $x \in [0,1]$, that have the property that $\sigma_x(u) = R$ if $u \geq x$ and $\sigma_x(u) = L$ otherwise.[3] When dealing with a cut-off strategy $\sigma_x \in \Sigma$ within the context of only all cut-off strategies I shall abuse notation to denote this strategy simply by its cut-off x.

For any two mixed strategies $\mu, \mu' \in \Delta$ denote by $\pi(\mu, \mu')$ the resulting ex-ante expected payoff for the player playing μ when her opponent plays μ'. If it is clear from the context I abuse notation slightly and simply write $\pi(x, y)$ when dealing with pure cut-off strategies σ_x and σ_y for $x, y \in [0,1]$.

It is useful to consider some special classes of coordination games with independent private values, depending on properties of the commonly known distribution of values F. A game shall be called *left-biased* if F is first order stochastically dominated by the uniform distribution (i.e. $F(x) \geq x$ for all $x \in [0,1]$). This means that the distribution tends to produce low values. A game is *right-biased* if, conversely, the uniform distribution first order stochastically dominates F (i.e. $F(x) \leq x$ for all $x \in [0,1]$). A game is *action-symmetric* if $f(x) = f(1-x)$ for all $x \in [0,1]$ (i.e. the distribution is symmetric around $\frac{1}{2}$). A game is *extreme-biased* if F is a mean-preserving spread of the uniform distribution. That is, the mean value under F is $\frac{1}{2}$ (as in the uniform distribution) and $\int_0^x F(t)dt \geq \int_0^x tdt$. Finally, a game is *center-biased* if F second order stochastically dominates the uniform distribution. That is, the mean value under F is again $\frac{1}{2}$ and now $\int_0^x F(t)dt \leq \int_0^x tdt$.

[3] Cut-off strategies in which L is played for values above the cut-off and R for values below the cut-off play no special role in the analysis. We could, also consider cut-off strategies in which we replace the weak inequality in the definition of the cut-off strategy (i.e. in $u \geq x$) by a strict one. This would, however, change behavior only with probability zero given the no-atom assumption for F. Any mixture of these two pure cut-off strategies, one with a weak the other with a strict inequality for the same cut-off x, is also behaviorally equivalent with probability one to the pure cut-off strategy.

There are, of course, games that are in none of these classes.

3 Symmetric Equilibrium

Lemma 1 *Let $\mu \in \Delta$ be an arbitrary mixed strategy. Its unique best response, denoted $\mathcal{B}(\sigma)$, must be a cut-off strategy σ_x with cut-off $x \in [0, 1]$.*[4]

Proof: Let $\mu \in \Delta$ be an arbitrary mixed strategy played, w.l.o.g. by the column player. Let p_μ denote the induced probability of playing action L. Then the row-player's payoff, if she is of type u, from playing action R is given by $(1 - p_\mu)u$, and from playing action L is given by $p_\mu(1 - u)$. She weakly prefers R over L if and only if $u \geq p_\mu$. QED

Proposition 1 *A strategy is a symmetric Nash equilibrium strategy if and only if it is a cut-off strategy σ_x with cut-off $x \in [0, 1]$ satisfying $F(x) = x$.*

Proof: By Lemma 1 any symmetric equilibrium strategy must be a cut-off strategy. Denote the cut-off by $x \in [0, 1]$. Suppose the column player uses this cut-off strategy. The row player's unique best response is then to choose a cut-off strategy with cut-off $F(x)$ as she strictly prefers R if $u > F(x)$, is indifferent between R and L if $u = F(x)$, and strictly prefers L otherwise. QED

Note that this game, thus, has no equilibrium other than in pure strategies. Note, furthermore, that in the proof of this proposition we also identified the best response function against cut-off strategies, identified by cut-off $x \in [0, 1]$, which is given by $\mathcal{B}(x) = F(x)$.

Corollary 1 *For any distribution F the coordination game with independent private values has at least two symmetric equilibrium strategies. These are cut-off strategies with cut-offs 0 and 1 and correspond to the strategies of always playing R and always playing L, respectively. Left-biased games and right-biased games have no other equilibria.*

[4] In principle, there is an infinite number of mixed best-responses and one other pure best response, but all these are with probability one behaviorally equivalent to the stated cut-off strategy. See also Footnote 3.

An action-symmetric game must have $F(x) = 1 - F(1-x)$ for all $x \in [0,1]$ and, in particular $F(\frac{1}{2}) = \frac{1}{2}$. This implies that any action-symmetric there has at least one other symmetric equilibrium strategy with cut-off $x = \frac{1}{2}$.

A very special example is given by the uniform distribution. In this case every cut-off strategy is a symmetric equilibrium strategy.

Proposition 2 *Any symmetric equilibrium strategy with a cut-off strictly between* 0 *and* 1 *is ex-ante expected payoff dominated by at least one of the extreme cut-off equilibrium strategies (i.e. with cut-off* 0 *or* 1*).*

Proof: Let $x \in (0,1)$ be the cut-off for a symmetric equilibrium strategy. The ex-ante expected payoff in this equilibrium can be written as

$$\pi(x,x) = F(x)\int_0^x (1-u)f(u)du + (1-F(x))\int_x^1 uf(u)du,$$

or equivalently as

$$\pi(x,x) = (F(x))^2 \mathbb{E}[1-u|u<x] + (1-F(x))^2 \mathbb{E}[u|u \geq x].$$

The expected payoff for the cut-off strategy equilibrium with cut-off $x = 0$ is given by

$$\pi(0,0) = \int_0^1 uf(u)du = \mathbb{E}[u],$$

which can also be written as

$$\pi(0,0) = F(x)\mathbb{E}[u|u<x] + (1-F(x))\mathbb{E}[u|u \geq x].$$

Analogously, the expected payoff for the cut-off strategy equilibrium with cut-off $x = 1$ is given by

$$\pi(1,1) = \int_0^1 (1-u)f(u)du = \mathbb{E}[1-u],$$

which can also be written as

$$\pi(1,1) = F(x)\mathbb{E}[1-u|u<x] + (1-F(x))\mathbb{E}[1-u|u \geq x].$$

Now consider the convex combination of $\pi(0,0)$ and $\pi(1,1)$ with weights $(1-F(x))$ on the former and $F(x)$ on the latter. It is immediate that this convex combination strictly exceeds $\pi(x,x)$. Therefore, at least one of the two payoffs, $\pi(0,0)$ or $\pi(1,1)$, must be strictly higher than $\pi(x,x)$. QED

4 Evolutionary Stability

4.1 Preliminary Thoughts

The pure coordination game with complete information is a doubly symmetric game, that is both players always get the same payoff. For such games, and for most models of evolutionary adaptation, the so-called *fundamental theorem of natural selection* holds. See Losert and Akin (1983) or (Weibull 1995, pp. 109-110) for a textbook treatment. This theorem says that as evolution unfolds the average payoff always increases. For the pure coordination game with complete information given by, for instance,

	L	R
L	1,1	0,0
R	0,0	1,1

this implies that the mixed strategy equilibrium is not evolutionarily stable under essentially any model of evolution as, among all symmetric strategy profiles the mixed equilibrium provides the lowest possible payoff. If we inject a proportion of L-strategists to the mixed equilibrium and start evolution from there, by the fundamental theorem we cannot go back to the mixed equilibrium.

Oechssler and Riedel (2002) show, among other things, that this *fundamental theorem of natural selection* also holds for games with a continuum of strategies if such a game is doubly symmetric.

Proposition 3 *The coordination game with independent private values is not doubly symmetric.*

Proof: Suppose that the row player plays σ_0 (i.e. always R), while the column player plays $\sigma_{\frac{1}{2}}$. The payoff to the row player is given by $\pi(0, \frac{1}{2}) = \left(1 - F(\frac{1}{2})\right) \mathbb{E}[u]$, while the payoff to the column player is given by $\pi(\frac{1}{2}, 0) = \left(1 - F(\frac{1}{2})\right) \mathbb{E}[u|u \geq \frac{1}{2}]$, which is strictly greater than $\left(1 - F(\frac{1}{2})\right) \mathbb{E}[u]$. QED

Therefore, the stated sufficient condition (that the game is doubly symmetric) for the *fundamental theorem of natural selection* is not satisfied and further analysis of evolutionary stability is warranted.

4.2 Stability

For evolutionary stability I focus on Lyapunov (or asymptotically) stable equilibria in cut-off strategies under the continuous-time best-response dynamics.[5]

In the light of Lemma 1 it seems innocuous to restrict attention to cut-off strategies (as they are the only possible best responses. The best response dynamics is, thus, given by $\dot{x} = \mathcal{B}(x) - x = F(x) - x$.

A cut-off strategy x is then stationary under the dynamics if and only $\dot{x} = 0$, i.e. if and only if $x = F(x)$, or in other words if and only if x is an equilibrium strategy.

Definition 1 *A cut-off strategy x is Lyapunov stable under $\dot{x} = F(x) - x$ if and only if it is stationary $x = F(x)$ (i.e. an equilibrium) and there is an $\epsilon > 0$ such for all $y \in (x, x + \epsilon)$ $F(y) \leq y$ (i.e. $\dot{y} \leq 0$) and for all $y \in (x - \epsilon, x)$ $F(y) \geq y$ (i.e. $\dot{y} \geq 0$). It is asymptotically stable if we replace the weak inequalities by strict ones.*[6]

While I find it more convenient to work with this definition directly it is perhaps useful to the reader to think in terms of derivatives of the vector field $V(x) = F(x) - x$. A necessary condition for Lyapunov (and asymptotic) stability is $V'(x) \leq 0$ (i.e. $F'(x) = f(x) \leq 1$). A sufficient condition for Lyapunov (and asymptotic) stability is $V'(x) < 0$ (i.e. $F'(x) = f(x) < 1$). If $V'(x) = 0$ the equilibrium is *hyperbolic* and many things are possible. See e.g. (Hofbauer and Sigmund 1998, Chapter 3.1).

Note that the necessary condition for Lyapunov (and asymptotic) stability is also a necessary condition for an equilibrium being a continuously stable strategies (CSS), as defined by Eshel and Motro (1981). Also the sufficient condition for Lyapunov (and asymptotic) stability is also a sufficient condition for an equilibrium being a CSS.

Proposition 4 *Every coordination game with independent private values (they differ in terms of distribution F) has a Lyapunov stable strategy. More-*

[5]For the continuous-time best-response dynamics see e.g. Gilboa and Matsui (1991), Matsui (1992), and Hofbauer (1995). A discrete time version is given by Moulin (1984).

[6]This can be shown to follow from general definitions of Lyapunov and asymptotic stability, as given, for instance, in (Weibull 1995, Definition 6.5).

over if $f(0) > 1$ and $f(1) > 1$ then any Lyapunov stable strategy is a Pareto-dominated symmetric equilibrium (with interior cut-off $x \in (0,1)$).

Proof: Consider first cut-off strategy 0. There are two cases. Case 1: Suppose that there is an $\epsilon > 0$ such that $F(y) \le y$ for all $y < \epsilon$. Then strategy 0 is Lyapunov stable. Case 2: Otherwise strategy 0 is not Lyapunov stable and there is an $\epsilon > 0$ such that $F(y) > y$ for all $y \in (0, \epsilon)$. Now consider cut-off strategy 1. Case 2a: Suppose there is an $\epsilon < 1$ such that $F(y) \ge y$ for all $y > \epsilon$. Then strategy 1 is Lyapunov stable. Case 2b: Suppose not. Then strategy 1 is not Lyapunov stable and there is an $\epsilon < 1$ such that $F(y) < y$ for all $y \in (\epsilon, 1)$. Now as F is a continuous function and maps the interval $[0, 1]$ onto itself, and as F is above the 45 degree line for some low x and below that line for some high x, there must be a point $x \in (0, 1)$ such that $F(x) = x$ with the desired property that there is an $\epsilon > 0$ such for all $y \in (x, x + \epsilon)$ $F(y) \le y$ and for all $y \in (x - \epsilon, x)$ $F(y) \ge y$. This proves existence. To prove the final claim of the Proposition note that if $f(0) > 1$ and $f(1) > 1$ we are in case 2b and any Lyapunov stable cut-off strategy x must be interior (i.e. $\in (0, 1)$) and, by Proposition 2, payoff-inferior to one of the 0 and 1 cut-off equilibria. QED

Corollary 2 *For every left-biased game the 1 cut-off (always play L) is Lyapunov stable. For every right-biased game the 0 cut-off (always play R) is Lyapunov stable. For every extreme-biased game an interior $x \in (0, 1)$ cut-off strategy is Lyapunov stable. For every center-biased game the two extreme cut-offs 0 and 1 are Lyapunov stable.*

Proof: A left-biased game has the property that $F(x) \ge x$ for all $x \in [0, 1]$. Thus, $\dot{x} = F(x) - x \ge 0$ for all $x \in [0, 1]$ and, thus, the best-response dynamics can only increase x. The cut-off strategy 1 is then Lyapunov stable. Similarly a right-biased game has the property that $F(x) \ge x$ for all $x \in [0, 1]$ and $\dot{x} = F(x) - x \le 0$ for all $x \in [0, 1]$ and, the best-response dynamics can only decrease x. The cut-off strategy 0 is then Lyapunov stable. An extreme-biased game has the property that $\int_0^x F(t)dt \ge \int_0^x tdt$ for all $x \in [0, 1]$. In particular, let $x_0 = \inf\{x | F(x) \ne x\}$. Note that x_0 could be 0. Then, there is an $\epsilon > 0$ such that for all $y \in (x, x + \epsilon)$ we must have $F(y) > y$. Thus, the cut-off strategy 0 is Lyapunov stable. An analogous argument can be made for cut-off strategy 1. Finally, for a center-biased game we must have that $\int_0^x F(t)dt \le \int_0^x tdt$ for all $x \in [0, 1]$. In particular, let $x_0 = \inf\{x | F(x) \ne x\}$. Note that x_0 could be 0. Then, there is an $\epsilon > 0$ such that for all $y \in (x, x+\epsilon)$ we must have $F(y) < y$. Thus, by the case 2b in the proof of Proposition 4 we have the existence of a Lyapunov stable interior cut-off strategy. QED

Note that for an appropriate "strict" notion of the various classes of games, no strategy other than those identified in the above corollary would be Lyapunov stable in the respective cases.[7] Moreover, the Lyapunov stable strategies identified in the corollary would then be asymptotically stable.

Thus, there are coordination game with independent private values with the property that in any Lyapunov stable strategy (one necessarily exists) a certain degree of mis-coordination is unavoidable. This is true, even though, the strategies "always play R" and "always play L" are also equilibrium strategies. These are, however, not evolutionarily stable in these cases.

Remark 1 *For the special case of F the uniform distribution, every cut-off $x \in [0,1]$ is Lyapunov stable.*

Remark 2 *We think of the game being played recurrently by, in each stage, randomly chosen pairs of individuals from a large population of individuals. A Lyapunov stable strategy, in the present context, can then be interpreted to require stability of an incumbent strategy with respect to all individuals in the population playing another cut-off strategy, where the cut-off is close to the cut-off of the incumbent strategy. This is not necessarily the only kind of "small" perturbation one would like to consider. One could also consider, more in the spirit of Maynard Smith and Price's (1973) notion of an evolutionary stable strategy (ESS), a perturbation in which a small fraction of individuals play any strategy. See Oechssler and Riedel (2002) for a full treatment of this issue. In the present context any such small perturbation (where small is in the sense of the weak topology, see Oechssler and Riedel (2002)), must be such that the probability of a randomly selected opponent playing R is close to what it is in the incumbent strategy. The best response to such a perturbation must then, by Lemma 1, be a cut-off strategy, which is then also close to the incumbent strategy. Thus, any Lyapunov stable strategy I here identify is also Lyapunov stable with respect to any small perturbation, where "small" is "small in the weak topology".*

Remark 3 *There are many other models of evolution I could have employed here. Not all provide exactly the same result. For instance, Oechssler and Riedel's (2002) notion of evolutionary robustness, an extension of ESS to continuous strategy games and the weak topology, has as a necessary condition for*

[7] A strict left-biased game, for instance, would be defined as satisfying $F(x) > x$ for all $x \in (0,1)$.

stability that the strategy is a neighborhood invader strategy (NIS) as defined by Apaloo (1997). See also Oechssler and Riedel (2002) for an argument that a strategy is a NIS if and only if it is Lyapunov stable under a version of the replicator dynamics (of, originally, Taylor and Jonker (1978) for finite strategy games, and Bomze (1990), Oechssler and Riedel (2001), and Oechssler and Riedel (2002) for the case of a continuum of strategies). A necessary condition for a strategy to be a NIS is that $\frac{d}{dx}\mathcal{B}(x) = F'(x) = f(x) \leq \frac{1}{2}$ *(see also Oechssler and Riedel (2002)). This is obviously more restrictive than being Lyapunov stable under the best-response dynamics. Note that we cannot guarantee existence of a NIS in coordination games with independent private values, as the uniform distribution example demonstrates. This means, that under some dynamics and with occasional perturbations we do not expect any stable social norm to emerge at all.*

Remark 4 *Another model of evolution one could look at is the discrete-time best response dynamics. In this case there is the* additional *(to the CSS condition) requirement for stability that the best-reply function, at the equilibrium, has a derivative that exceeds* -1 *(see Moulin (1984)). Given that the derivative of the best-reply is the density* f*, this condition is automatically satisfied.*

Remark 5 *A norm without mis-coordination is certainly observationally distinguishable from a norm based on interior cut-off strategies, as the latter produces occasional mis-coordination while the former does not. At first thought, one might think that a situation in which there is a stable norm, which, however, is a cut-off strategy, may not be observationally different from a situation in which there is no stable norm at all. This is not necessarily true, however. If we looked at the frequency of left and right action choices over time (perhaps computing moving averages), in the first case we would expect this frequency to roughly stay the same, while in the second case we could observe that this frequency fluctuates.*

Remark 6 *This is about the assumption that the support of values is exactly* $[0,1]$ *making the game such that with probability* 1 *no player has a dominant strategy. Suppose, first that the support is a proper subinterval of* $[0,1]$ *bounded away from* 0 *and* 1*. Then the extreme cut-off equilibria* 0 *and* 1 *are stable (as* $\frac{d}{dx}\mathcal{B}(x) = 0$*). This does not imply, however, that there are no stable interior cut-off equilibria. If the distribution has support on a smaller interval, but is also extreme-biased and, let us assume for convenience action-symmetric (i.e. values close to* $\frac{1}{2}$ *are relatively unlikely), then the* $\frac{1}{2}$ *equilibrium is also stable. In such cases one could complement* *the deterministic stability analysis by studying a stochastic model of evolution similar to e.g. Foster and Young (1990), Young (1993), and Kandori, Mailath, and Rob (1993). My conjecture, based on my reading of this literature, here would be that the stochastic stability of the various Lyapunov stable cut-off strategies would then depend on the size of the basin of attraction of the various Lyapunov stable strategies. Such a model would then, again, predict for some games that the only stochastically stable strategy is an interior cut-off strategy.*

Remark 7 *Suppose, now that the support of the distribution F extends beyond the interval $[0, 1]$ on both sides. Then a contagion argument, not unlike the argument in the global games literature (started by Carlsson and Van Damme (1993) and Morris and Shin (1998)), implies that the extreme cut-off strategies 0 and 1 are not equilibria of the game. In this case, a certain degree of mis-coordination is unavoidable in any equilibrium. Note, however, that, depending on the distribution, there could be equilibria very close to the extreme cut-off equilibria. Note, furthermore, that equilibria close to the extreme cut-off equilibria would (generically) be Lyapunov stable. As in the previous remark, this does not mean that there cold not also be a stable and "more interior" cut-off equilibrium. Again, in this case one could study a stochastic model of evolution which, one conjectures, would show that the size of the basin of attraction determines the stochastic stability of the equilibrium.*

Remark 8 *I now ask the question why there are laws governing* driving *but no law (or at least fewer laws) governing* walking. *One could probably argue that for driving all the payoffs in the game have to be multiplied by a large factor. This would not change incentives. Possibly one could argue that the distribution of values for the case of driving is typically more center-biased compared to the case of walking. This, however, would imply that no law governing driving is needed as, in this case, the only stable norms are the extreme cut-off strategies which are also Pareto-optimal. The fact that there is a law for driving, and not just a norm, suggests that individual drivers would, in the absence of a law, be sometimes tempted not to follow the norm after all (such behavior might then, in turn, lead to the gradual collapse of the norm). It is, thus, possible that the driving game, at least in some cases (or places) is not that different from the walking-game. The fact that there is no law for walking is then best explained by the fact that walking accidents are not very costly and no individual would like to press charges on the basis of a law (as this is also costly) if there was one. Finally, I would like to explain how a law could help to stabilize an extreme cut-off equilibrium. Suppose the* *law states that one should walk on the right and that it has the effect that in the event of mis-coordination (an accident) the person walking on the left is clearly the guilty party and can be penalized. The game is, thus, as before with one change:*

	L	*R*
L	*1-u,1-v*	*-c,0*
R	*0,-c*	*u,v*

Best responses are still in cut-off strategies, but the best response function is now given by $\mathcal{B}(x) = \max\{0, F(x)(1+c) - c\}$. *Note that for x close to zero the best-response function is flat and equal to* 0. *Thus, at the* 0*-cut-off the derivative of the best-response is equal to zero and, thus,* 0 *is now an asymptotically stable strategy for any distribution function F.*

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