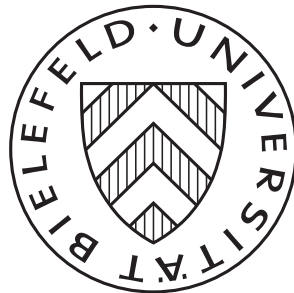


March 2013

On the desirability of tax coordination when countries compete in taxes and infrastructure

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Abstract

In our paper, we demonstrate that when countries compete in taxes and infrastructure, coordination through a uniform tax rate or a minimum rate does not necessarily create the welfare effects observed under pure tax competition. The divergence is even worse when the competing jurisdictions differ in institutional quality. If tax revenues are used to gauge the desirability of coordination, our model demonstrates that imposing a uniform tax rate is Pareto-inferior to the non-cooperative equilibrium when countries compete in taxes and infrastructure. This result is completely reversed under pure tax competition if the countries are sufficiently similar in size. If a minimum tax rate is set within the range of those resulting from the non-cooperative equilibrium, the low tax country will never be better off. Finally, the paper demonstrates that the potential social welfare gains from tax harmonization crucially depend on the degree of heterogeneity among the competing countries.

Keywords: Tax competition, infrastructure, tax coordination, tax revenue, social welfare

JEL classification: H21; H87; H73; F21; C72

*We appreciate valuable discussions with Robin Boadway, Raouf Boucekkine, Kai Konrad, Myrna Wooders, and Skerdilajda Zanaj. Of course, all mistakes and errors are our own.

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1 Introduction

The debate over corporate tax coordination among international jurisdictions remains unresolved. In particular, it has been argued that the member states of the European Union should coordinate tax policies¹ to avoid a “race to the bottom” that would undermine their modern welfare states (Baldwin and Krugman, 2004). For this purpose, in the 1990s, the Organization for Economic Cooperation and Development (OECD) launched a “harmful tax competition” initiative. In addition, the United Nations (UN) has called for the creation of an International Tax Organization, which would be specifically charged with curtailing tax competition.

These concerns are in keeping with the large tax competition literature (for systematic reviews, see Wilson, 1999; Wilson and Wildasin, 2004; Boadway and Tremblay, 2011). The main point is that independent governments engage in wasteful competition² over scarce capital through inefficiently low tax rates and public expenditure levels. Zodrow and Mieszkowski (1986) and Wilson (1986) have formally modeled this process.

The literature highlights two coordination devices designed to correct the inefficiencies³ caused by tax competition: tax harmonization and the imposition of a minimum tax rate. Tax harmonization is generally understood as a transition towards a common

¹The Ruding Report (1992) made several far-reaching harmonization proposals related to corporate taxation, including the imposition of an EU-wide minimum corporate tax rate (Haufler, 1999).

²Other existing studies concern welfare-improving tax competition. From the public choice perspective, Brennan and Buchanan (1980) argue that tax competition reduces the size of excessively large governments and improves welfare. Rouscher (1998) and Edwards & Keen (1996) formalize the notion by including tax competition in various Leviathan models. Another strand of the literature has been motivated by Tiebout (1956), who was the first to investigate the notion that competition between jurisdictions may promote efficiency if households are able to sort themselves into jurisdictions composed of individuals with similar preferences and receive public goods tailored to their incomes and preferences. However, this is not our focus.

³The existing literature suggests other ways of coordination. Wildasin (1989) suggest that central governments can provide regions with a ‘corrective subsidy’, while Boadway and Flatters (1982) discuss intergovernmental transfers when facing inefficiencies due to tax competition.

rate structure (Keen, 1987; Zissimos and Wooders, 2008).

More specifically, in the present paper, we define tax harmonization as the equalization of tax rates, which is consistent with the tax competition literature (see, for example, Kanbur and Keen, 1993; Baldwin and Krugman, 2004; Zissimos and Wooders, 2008) and common policy prescriptions⁴. The general conclusion of the classical literature is that appropriately selected uniform tax rates improve efficiency relative to tax competition. The reason is that an upward harmonization of capital tax rates can produce a Pareto improvement (Baldwin and Krugman, 2004). This conclusion also holds when the competing countries are asymmetric in size (Kanbur and Keen, 1993).⁵ Another type of coordination is the adoption of a minimum tax rate that allows some room for tax competition. An interesting result highlighted in the literature (see Keen and Konrad, 2012) is that the imposition of a minimum tax rate can be Pareto-improving for all partner countries.

Many authors argue that jurisdictions not only compete in taxes but also in infrastructure (for example, Hindriks et al., 2008; Zissimos and Wooders, 2008; Justman et al., 2002, and Pieretti and Zanaj, 2011).⁶ Moreover, recent empirical research (Hauptmeier et al., 2012) demonstrates that jurisdictions use independent and strategic business tax rates and public inputs to compete for capital. However, the existing literature on the desirability of tax coordination is primarily based on the assumption

⁴In 2003, the EU Council adopted a voluntary Code of Conduct to combat harmful tax competition, and more ambitious proposals for corporate tax harmonization have been advanced, including the introduction of a single EU corporate tax (Conconi et al., 2008).

⁵Kanbur and Keen (1993) show that there exists a critical level above which harmonization results in tax revenue exceeding, for each jurisdiction, that of the non-cooperative equilibrium. However, a uniform level between the Nash equilibrium rates is certain to harm the small country.

⁶Some authors (Keen and Marchand, 1997 and Fuest, 1995) consider the impact of public inputs on the production function of firms and thus account for the effect of infrastructure on internationally mobile capital. However, in these models tax rates and infrastructure expenditures are not independent variables. This results formally from the fact that tax rates and infrastructure expenditures are linked through a balanced budget. According to Wildasin (1991), equilibria in fiscal competition games with two instruments related via a budget constraint crucially depend on which instrument is set strategically. Consequently, if countries interact in taxes, expenditures are not a strategic variable.

that countries solely strategically compete in tax rates. However, an exception is Zissimos and Wooders (2008). These authors alternatively consider minimum taxation and tax harmonization as coordination devices. Governments are supposed to simultaneously set their tax levels subject to the constraints imposed by policy coordination for given levels of public goods fixed at the non-cooperative equilibrium. However, why would jurisdictions not adapt their infrastructure expenditures to the new environment caused by tax coordination when these expenditure levels are no longer their best choices? Maintaining a given level of infrastructure expenditures would not be rational. It would also drastically limit sovereign policy making, as many infrastructure expenditures primarily satisfy internal policy goals and are incidentally attractive to foreign investments. To account for this issue, we assume that tax coordination does not constrain infrastructure competition among sovereign jurisdictions. In other words, jurisdictions still compete in infrastructure, though they may coordinate their taxes.

The purpose of this paper is to analyze the desirability of tax coordination when two heterogeneous jurisdictions compete for mobile entrepreneurs using taxes and infrastructure investments that improve firm productivity. These infrastructure investments may represent material or immaterial public goods such as laws and regulations protecting intellectual property and specifying accurate dispute resolution rules. We thus model two-dimensional strategic interactions within a game-theoretical approach. Furthermore, firms are assumed to be heterogeneous in their preferences or their ability to relocate abroad. The model also accounts for two real-world characteristics, asymmetries in both country size⁷ and institutional quality. Various authors have addressed the importance of size asymmetries in tax competition (Bucovetsky, 1991; Wilson, 1991; Kanbur and Keen, 1993), but the role of institutional differences across jurisdictions has been neglected. It is also generally recognized that a country's institutional environment impacts its economic performance. The reason is that

⁷Country size may be defined by population, area, or national income (Streeten, 1993). In this study, population, rather than area, is used to denote country size. More precisely, size is defined with respect to the number of capital owners residing in a country.

the quality of the existing institutional framework in which activity takes place significantly shapes the business environment⁸. Indeed, nations endowed with higher quality institutions provide firms with better auditing and judicial systems and offer more efficient property rights protection and enforcement⁹. In our model, we assume that a country's institutional environment translates into territory-specific productivity conditions that are shared by all firms located in a given jurisdiction, independent of their countries of origin.¹⁰

The main results may be summarized as follows. When tax revenue is used to gauge whether tax coordination dominates a non-cooperative equilibrium, the following results are obtained. If the jurisdictions decide to set uniform tax rates, coordination is Pareto-inferior to the non-cooperative equilibrium when countries compete in tax and non-tax instruments. By contrast, if jurisdictions only compete in taxes, our model indicates that tax harmonization can be Pareto-improving. Coordination consisting of the imposition of a lower bound on tax rates only increases the revenue of the high tax country if jurisdictions compete in taxes and infrastructure. In other words, the low tax country will never be better off, and the revenue loss increases in the weakness of its institutions. However, if inter-jurisdictional tax redistribution is feasible, it is conceivable that the country incurring a tax loss could be compensated if coordination increases joint revenue. We show, however, that for a range of minimum rate choices, compensation is not feasible. These results are at odds with the classical outcome that imposing an appropriate minimum rate improves the revenue of each country when jurisdictions simultaneously compete in taxes alone (see Keen and Konrad, 2012).¹¹

⁸La Porta et al. (2000) argue that countries have varying abilities to offer investors an attractive institutional environment. According to Acemoglu et al. (2001), institutions positively influence per capita GDP.

⁹Besley (1995) and Johnson et al. (2002) argue that strong property rights are attractive to investment.

¹⁰Hindriks et al. (2008) also consider a model with uneven productivity levels across two regions that compete in taxes and public inputs to attract capital. However, they do not consider tax coordination.

¹¹Our result is however consistent with Konrad (2009) who analyzes minimum tax coordination within a sequential tax game where firms compete à la Stackelberg. In this case, coordination drives down the revenue of the follower, which is the low tax country, and increases the leader's revenue.

The results differ when we consider the potential welfare gains from tax coordination. When the jurisdictions decide to set uniform tax rates, the profitability of coordination crucially depends on the degree of inter-country asymmetry. In particular, if countries are symmetric in population size but have very unequally developed institutions, tax harmonization will be less efficient than tax and infrastructure competition. However, this result is reversed if countries have equal institutional quality and are sufficiently similar in size. Finally, our model demonstrates that minimum tax coordination always increases social welfare.

The remainder of the paper is organized as follows. In section 2, we model tax and infrastructure competition between heterogeneous jurisdictions that attempt to attract imperfectly mobile firms. Section 3 analyzes the conditions under which tax harmonization is more desirable than tax and infrastructure competition. Section 4 examines the differences between minimum tax coordination and tax competition. Section 5 concludes.

2 The model

Consider two jurisdictions denoted h and f . The countries' populations are evenly distributed with unit density on a segment $[0, 1]$. Country h is assumed to be small in terms of total population, and its size is given by S with $0 < S < \frac{1}{2}$. It follows that the size of country f equals $\frac{1}{2} < 1 - S < 1$. Similar to Pieretti and Zanaj (2011), we assume that each individual owns one unit of capital and is simultaneously an entrepreneur and a worker. In other words, each member of the population corresponds to a one-person company¹². The entrepreneurs can relocate their activity abroad, but we assume (see Ogura, 2006) that they are heterogeneous in their preferences regarding relocation. The entrepreneurs are thus ranked according to their willingness to relocate

¹²It follows that the world population coincides with the population of firms. We could assume that each firm is run by more than one person, but this would unnecessarily complicate the model without providing further insights.

abroad¹³. The closer an individual is to the border separating countries h and f , the easier it is for her to relocate abroad. In other words, an entrepreneur of type $\alpha \in [0, 1]$ who moves abroad incurs a mobility cost equal to $|\alpha - S|$, which is the “distance” between the border S and an entrepreneur of type α .

Firms

Using one unit of capital, each individual living in country j ($j = h, f$) is able to produce $y_j = q_j + \bar{\theta}_j$ units of one final good. The parameter q_j ($j = h, f$) represents firm specific productivity, whereas $\bar{\theta}_j$ is the output fraction, which is country-specific. More precisely, we write $\bar{\theta}_j = \theta_j^0 + \theta_j$, where θ_j^0 is a state parameter describing the institutional environment in country j and θ_j is the level of infrastructure spending planned by the policy-makers in country j . In other words, the quality of institutions and infrastructure results from history and current decisions.

The focus of the paper is on how uneven institutional quality and infrastructure expenditures affect the welfare effects of tax competition. Therefore, we assume that firm-specific productivity is uniform across firms, which means that $q_j = q$ ($j = h, f$). For simplicity, we normalize $\theta_h^0 = 1$ and consider $\theta_f^0 = a\theta_h^0 = a \geq 0$, where ratio a reflects the difference in institutional quality between the two economies. The ratio can be equal to one (the two economies are equal in quality), larger than one (the small economy has poorer institutional quality) or smaller than one (the large economy has poorer institutional quality). Finally, we assume that the final goods are sold in a competitive market with a price normalized to one. The unit cost of production is assumed to be constant and normalized to zero.

A firm of type $x \in [0, S]$ with home country h is indifferent between producing at

¹³As in Ogura (2006), we assume that this population of entrepreneurs is heterogeneous in the degree of their attachment to the home country. The sources of this home bias can be different. For example, transferring activities abroad requires substantial information, which may differ across entrepreneurs. Another cause can be linked to material relocation costs, which can be specific to each firm.

home and in the foreign country f if

$$q_h + \bar{\theta}_h - t_h = q_f + \bar{\theta}_f - t_f - (S - x), \quad (1)$$

where t_h and t_f are source-based tax rates levied on capital in countries h and f , respectively.

Similarly, a firm of type $x \in [S, 1]$ located in country f is indifferent between investing at home and investing abroad if

$$q_f + \bar{\theta}_f - t_f = q_h + \bar{\theta}_h - t_h - (x - S). \quad (2)$$

The above two conditions yield

$$x = (1 - a) + (\theta_h - \theta_f) + (t_f - t_h) + S. \quad (3)$$

Note that if $x > S$, firms move from the large to the small country, while if $x < S$, firms move from the small country to its larger rival.

Governments

We now assume that countries attempt to attract companies by competing in taxes and public infrastructure that enhance private productivity. Jurisdictions h and f are thus able to influence the productivity parameter θ_j ($j = h, f$) of the firms located within their respective boundaries. As in Hindriks et al. (2008) and Pieretti and Zanaj (2011), we assume that one additional unit of the public good produces one additional unit of the private good. It follows that θ_j also represents the amount of the public good supplied by jurisdiction j ($j = h, f$). The cost of providing this public good in each country j is given by the quadratic cost function $C(\theta_j) = \frac{1}{2}\theta_j^2$. Each jurisdiction j ($j = h, f$) is assumed to maximize its total tax revenue¹⁴, net of public expenditures, by selecting the appropriate tax rate t_j and infrastructure level θ_j . The governments' objective functions are given by

$$B_h = t_h x - \frac{1}{2}\theta_h^2, \quad B_f = t_f(1 - x) - \frac{1}{2}\theta_f^2. \quad (4)$$

¹⁴For a similar assumption, see Kanbur and Keen (1993), Zissimos and Wooders (2008) or Pieretti and Zanaj (2011). By so doing, we do not assume that jurisdictions are self-interested governments. We simply assume that collected taxes are used to finance public goods in the interest of their populations.

We now assume that the two jurisdictions wish to attract productive capital by competing in taxes and infrastructure. To this end, we consider a two-stage game¹⁵. First, the governments non-cooperatively select infrastructure levels. Then, they set the tax rates. Finally, firms decide where to locate their production processes. We solve the game by backward induction.

Beginning from the second stage, each government maximizes its objective with respect to its tax rate while taking its rival's rate as given. The first order conditions¹⁶ yield the following unique equilibrium tax rates

$$\begin{aligned} t_h &= \frac{1 - a + (1 + S) - \theta_f + \theta_h}{3}, \\ t_f &= \frac{a - 1 + (2 - S) + \theta_f - \theta_h}{3}. \end{aligned} \quad (5)$$

It follows that the number of companies located in countries h and f are, respectively, x and $1 - x$, with

$$x = \frac{1 - a + (1 + S) + \theta_h - \theta_f}{3}.$$

After substituting the above tax rates into the jurisdictions' objective functions, we can solve for stage 1 of the game, where the two governments compete in public infrastructure θ_h and θ_f . It is simple to verify that the objective function B_j ($j = h, f$) is strictly concave in θ_j ($j = h, f$). The first order conditions thus lead to the unique equilibrium expenditures

$$\theta_h^* = \frac{2}{15}(4 - 3a + 3S), \quad \theta_f^* = \frac{2}{15}(1 + 3a - 3S). \quad (6)$$

Introducing (6) into (5) yields the equilibrium values

$$t_h^* = \frac{3}{2}\theta_h, \quad t_f^* = \frac{3}{2}\theta_f. \quad (7)$$

¹⁵The choice of sequentiality follows from the rule that the most irreversible decision must be made first.

¹⁶The second order conditions can be easily verified.

Therefore, the strategy-tuple $(\theta_h^*, \theta_f^*, t_h^*, t_f^*)$ is a unique subgame perfect Nash equilibrium.

Equation (7) shows that the country that taxes more than its rival also provides more public infrastructure.

The number of firms located in equilibrium in country h is given by

$$x^* = \frac{1}{5} (4 - 3a + 3S). \quad (8)$$

It is straightforward to show that $x^* \in [0, 1]$ and $\theta_j^* \geq 0 (j = h, f)$ if $a \in (S - \frac{1}{3}, S + \frac{4}{3})$.

The tax differential between the large and small countries equals

$$t_f^* - t_h^* = \frac{3}{2} (\theta_f^* - \theta_h^*) = \frac{3}{5} (a - \bar{a}), \quad \text{where } \bar{a} = \frac{1}{2} + S. \quad (9)$$

According to (9), it follows that $t_f^* > t_h^*$ if $a \in (\bar{a}, S + \frac{4}{3})$, and $t_h^* > t_f^*$ if $a \in (S - \frac{1}{3}, \bar{a})$. The intuition underlying equation (9) is best understood if we suppose that countries would share the same institutional environment ($a = 1$). In this case, we obtain the standard result that the smallest country sets the smallest tax rate. If we now assume that the small country has the best institutional environment, it will be able to increase its tax rate. Consequently, competition will equalize the tax rates across jurisdictions if the small country's institutional quality is high enough. This is precisely the case when $a = \bar{a}$. In other words, \bar{a} is the level of institutional disparity that exactly compensates for the effect of asymmetric size on inter-jurisdictional tax differences. If countries are equal in size ($S = \frac{1}{2}$), the level of \bar{a} equals 1.

But if country h becomes smaller relative to f (S decreases), the compensation level of institutional quality of country h has to increase relative to f . Therefore, \bar{a} decreases with S . Finally we can say that the larger the gap $|a - \bar{a}|$, the more the competing jurisdictions differentiate themselves.

The equilibrium tax revenues of both countries are

$$B_h^* = \frac{7}{225} (4 - 3a + 3S)^2 \quad \text{and} \quad B_f^* = \frac{7}{225} (1 + 3a - 3S)^2. \quad (10)$$

Therefore, the joint tax revenue is $B^* = B_h^* + B_f^*$. As in Zissimos and Wooders (2008), we define efficiency as the maximum level of surplus available to all individuals in the two economies

$$W(x) = (\pi_h + \pi_f) + (B_h + B_f) - \int_0^{|x^T - S|} y dy. \quad (11)$$

The two terms in the brackets include, respectively, the total firms' profits¹⁷ and total tax revenues. The last term is the relocation cost faced by relocating companies. After simplification, the (joint) social welfare W^* resulting from inter-jurisdictional competition equals

$$W^* = \left[q + (1 + \theta_h^*) x^* + (a + \theta_f^*) (1 - x^*) - \frac{(\theta_h^*)^2}{2} - \frac{(\theta_f^*)^2}{2} \right] - \int_0^{|x^* - S|} y dy. \quad (12)$$

The sum of the terms included in brackets is the global output and the second term is the total mobility cost of firms.

Plugging the equilibrium values of θ_h^* , θ_f^* , and x^* into (12) yields

$$W^* = q + \frac{1}{450} [333a^2 - 18a(37S + 6) + 18S(6S + 31) + 352]. \quad (13)$$

It is convenient to show that $W^* > 0$ for all $a \in (S - \frac{1}{3}, S + \frac{4}{3})$ with $S \in (0, \frac{1}{2})$.

3 Harmonization versus tax competition

We now assume that the two countries cooperatively select uniform tax rates, for a given level of infrastructure expenditures. Therefore, they only compete in infrastructure. We further assume that the uniform tax rate is designed to maximize either global

¹⁷The profit in country j ($j = h, f$) is $\pi_j = (q + \bar{\theta}_j - t_j)x_j$.

tax revenue or global social welfare. The two cases will be considered successively. Then, we analyze the conditions under which harmonization is desirable, successively applying the tax revenue and social welfare perspectives.

3.1 Tax harmonization

We define the uniform tax rate as follows:

$$t_h = t_f = t, \quad t \geq 0.$$

Therefore, the number of firms that locate in the small country is given by

$$x = (1 - a) + (\theta_h - \theta_f) + S.$$

We first solve the infrastructure game. Each jurisdiction selects a level of public infrastructure θ_j by maximizing its revenue for a given tax rate t .

In equilibrium, we obtain

$$\theta_h^u = \theta_f^u = t.$$

It follows that

$$x^u = (1 - a) + S.$$

If institutional quality is higher in the small country ($1 > a$) it attracts $x^u - S$ firms from its large rival. Otherwise, ($1 < a$), $S - x^u$ firms leave the small country. Because $x^u \in [0, 1]$, we impose $a \in [S, 1 + S]$.

The tax revenues of countries h and f resulting from infrastructure competition for a given uniform tax rate is as follows

$$B_h^u = t((1 - a) + S) - \frac{1}{2}t^2 \quad \text{and} \quad B_f^u = t(a - S) - \frac{1}{2}t^2. \quad (14)$$

Joint tax revenue becomes

$$B^u(t) = B_h^u + B_f^u = t(1 - t), \quad (15)$$

where $B^u(t)$ is positive if $t \in (0, 1)$.

The aggregate social welfare resulting from infrastructure competition with uniform tax rates is

$$W^u = q + [a^2 - (1 + S)a + (1 + S + t(1 - t))] - \frac{1}{2} (1 - a)^2. \quad (16)$$

Given that $t \in (0, 1)$ and $S \in (0, \frac{1}{2})$, W^u is always positive.

We are now able to calculate the harmonized tax rate. First consider the case where the jurisdictions agree on a uniform rate that maximizes joint tax revenue. It is easy to see that $\bar{t} = \arg \max B^u(t) = \frac{1}{2}$. It follows that $\bar{B}^u = B^u(\bar{t}) = \frac{1}{4}$, $B_h^u(\bar{t}) = \frac{1}{8}(4S - 4a + 3)$ and $B_f^u(\bar{t}) = \frac{1}{8}(4a - 4S - 1)$. If tax harmonization is intended to maximize global social welfare we show that $t^s = \arg \max W(t) = \frac{1}{2}$. The resulting maximum social welfare equals $W(t^s) = \frac{1}{4}[4S(1 - a) + 2a^2 + 3]$.

3.2 Comparing tax revenues

In this section we analyze the desirability of tax harmonization with respect to tax revenues. Comparing tax revenues resulting from tax and infrastructure competition with the maximum revenue resulting from tax harmonization shows that $B_h^* > B_h^u(\bar{t})$ and $B_f^* > B_f^u(\bar{t})$ for all $a \in [S, 1 + S]$ and all $S \in (0, \frac{1}{2})$. In other words, if the common rate equals \bar{t} , tax harmonization does not make both countries better off. The intuition underlying this result can be explained as follows. As in (Hindriks et al., 2008), our model implies that the more jurisdictions improve their attractiveness by investing in infrastructure in the current period, the fiercer tax competition will be in the second stage. The competing jurisdictions anticipate this effect in the first stage and thus underinvest in infrastructure relative to the tax harmonization scenario. This last case reduces to a one-stage game without strategic interaction between taxes and public investments. As a result, tax revenue net of infrastructure expenditures is lower under tax harmonization option.

The above finding is at odds with classical results, according to which tax harmo-

nization dominates pure tax competition if the uniform tax rate is sufficiently high (see for example, Kanbur and Keen, 1993; Baldwin and Krugman, 2004 and Boadway and Tremblay, 2011). Our model leads to a similar conclusion (see Appendix A) if we restrict ourselves to pure tax competition with symmetric institutional quality by setting $a = 1$ and $\theta_h = \theta_f = 0$. Indeed, in that case, tax harmonization generates more revenue than tax competition for both jurisdictions provided that the two countries are not excessively asymmetric with respect to size. However, if revenue transfers are feasible, both countries are always better off under an appropriate common rate.

We can now state the following proposition

Proposition 1 *Moving from tax and infrastructure competition to tax harmonization decreases the tax revenues of all competing countries. However, if the countries compete in taxes only, harmonization can be Pareto-improving in tax revenue.*

3.3 Comparing social welfare

Now we use social welfare to gauge the desirability of tax harmonization. To this end, consider the difference

$$W^* - W(t^s) = Aa^2 + Ba + C,$$

where $A = \frac{6}{25}$, $B = -\frac{6}{25}(1 + 2S)$ and $C = \frac{1}{900}(216S + 216S^2 + 29)$. It is straightforward to show that $W^* < W(t^s)$ if $a \in (a_1, a_2)$, where¹⁸ $a_1 = \bar{a} - \frac{5}{36}\sqrt{6}$ and $a_2 = \bar{a} + \frac{5}{36}\sqrt{6}$ and $W^* > W(t^s)$ if $a \in (S, a_1)$ or $a \in (a_2, 1 + S)$. It follows that harmonization dominates tax competition as long as the difference in institutional quality across countries is not too distant from \bar{a} . Recall that the larger the gap between a and \bar{a} , the more the competing jurisdictions are differentiated from a tax perspective. In the same vein, countries will be considered similar from a tax perspective if $a = \bar{a}$.

To explain in detail what happens, we can decompose the welfare difference $W^* - W(t^s)$ in the following manner

¹⁸It can be shown that a_1 and a_2 respectively satisfy the conditions $S < a_1$ and $a_2 < 1 + S$.

$$W^* - W(t^s) = \Delta B + \Delta \pi, \quad (17)$$

where $\Delta B = (B_h^* + B_f^*) - [B_h(t^s) + B_f(t^s)]$ and $\Delta \pi = (\pi_h^* + \pi_f^*) - [\pi_h(t^s) + \pi_f(t^s)]$. From the previous section, we know that the movement from interjurisdictional competition to harmonization decreases net joint tax revenue ($\Delta B > 0$ for all a). However, it can readily be shown that the same change of regime increases joint profits (net of moving costs) ($\Delta \pi < 0$ for all a). However, the opposite signs of ΔB and $\Delta \pi$ have a common cause. Indeed, inter-state competition generates more tax revenue than harmonization but less infrastructure expenditures. This benefits the governments and, by the same token, hurts the private economy. Which of the two effects will dominate depends¹⁹ on the value of a . Indeed, it is convenient to show that $\Delta B + \Delta \pi < 0$ if $a \in (a_1, a_2)$ and $\Delta B + \Delta \pi > 0$ if $a \notin (a_1, a_2)$. To explain the intuition underlying this result, note that ΔB increases more rapidly than $-\Delta \pi$ when a moves away from \bar{a} (see Figure 1). When a deviates from \bar{a} to a greater extent, the competing jurisdictions become increasingly different and tax competition becomes less intense. Consequently, joint tax revenue increases at a faster pace than infrastructure expenditures. Less intense interjurisdictional competition makes taxpayers more captive but decreases the importance of infrastructure attractiveness. At the governmental level, tax receipts increase more rapidly than infrastructure expenditures, and from the firms' perspective, the productivity induced by public expenditures grows more slowly than tax payments. Two cases can then be considered.

a) When $a \in (a_1, a_2)$, the competing jurisdictions are not too different and tax harmonization is the most preferable option from the social perspective. Tax payers are then moderately captive relative to the importance of infrastructure attractiveness. As a result (see Figure 1), the relative gain induced by tax and infrastructure competition is not sufficiently high to compensate for the benefit of tax harmonization ($\Delta B < -\Delta \pi$).

b) When the institutional gap between the competing countries is sufficiently large,

¹⁹More exactly, we have $\Delta B = \frac{14}{25}a^2 + -\frac{14}{25}(1 + 2S)a + \frac{1}{900}(504S + 504S^2 + 251)$ and $\Delta \pi = -\frac{8}{25}a^2 + \frac{8}{25}a(1 + 2S) - \frac{1}{150}(48S + 48S^2 + 37)$.

i.e., $a \in (S, a_1) \cup (a_2, 1 + S)$, tax harmonization is no more the most efficient option. Tax competition has become less intense, and taxing captive firms is relatively more beneficial than providing infrastructure. Accordingly, the relative benefit of tax and infrastructure competition (ΔB) increases to such an extent that it exceeds (see Figure 1) the benefit of harmonization ($-\Delta\pi$).

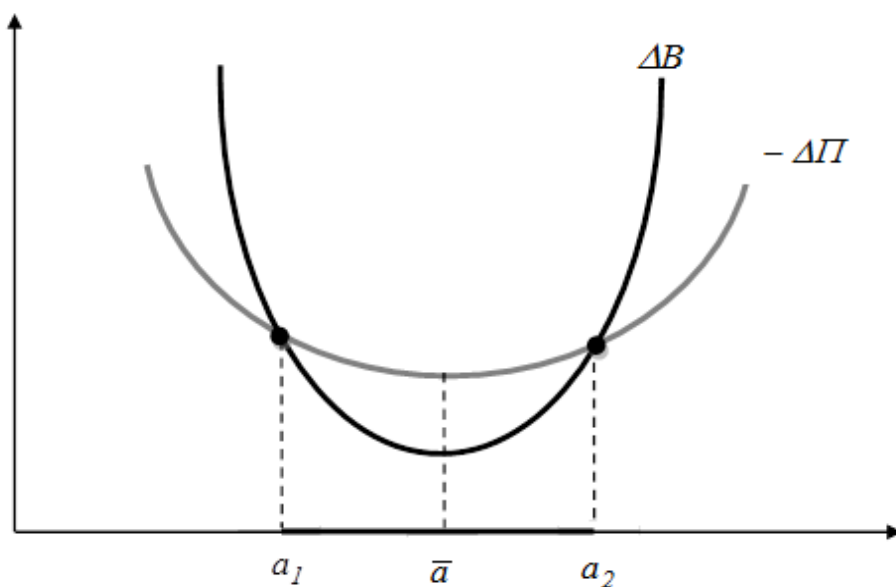


Figure 1

As a corollary to the above analysis, if countries are symmetric in size and have equally developed institutions²⁰, tax harmonization always dominates tax and infrastructure competition. However, if we consider asymmetric size while still supposing uniform institutional development, the result can be reversed. Indeed, tax and infrastructure competition is more efficient than tax harmonization ($W^* > W(t^s)$) if the size S of the small country is smaller than $\hat{S} = \frac{1}{2} - \frac{5}{36}\sqrt{6}$. This last finding does not appear if we restrict ourselves to pure tax competition. Indeed, our model shows (see Appendix A) that moving from tax competition to tax harmonization always improves social

²⁰This assumption is generally taken for granted in the tax competition literature.

welfare if $\theta_h = \theta_f = 0$ and $a = 1$.

The following proposition concludes

Proposition 2

- (a) *If countries have equally developed institutions and are symmetric in size, tax harmonization is more efficient than tax and infrastructure competition. This result is, however, reversed if the countries' sizes are sufficiently asymmetric.*
- (b) *If countries have unequal institutional quality, harmonization can be less efficient than tax and infrastructure competition. This result arises if the competing jurisdictions are sufficiently differentiated.*

4 Minimum tax versus tax competition

We now assume that the jurisdictions agree on a minimum tax rate τ which is in between the tax rates resulting from tax and infrastructure competition. This option has been analyzed by some authors (see, for example, Kanbur and Keen, 1993). We showed above that $a > \bar{a}$ implies $t_h^* < t_f^*$. Thus, the minimum tax rate τ will be $\tau > t_h^*$. However, when $a < \bar{a}$ we have $t_h^* > t_f^*$ and thus $\tau > t_f^*$. In the following, we only analyze in detail the case where $t_h^* < t_f^*$, as the conclusions we derive still hold for the alternative case.

4.1 Competition with a minimum tax rate

We now assume that the jurisdictions first compete in infrastructure expenditures and then in tax rates, which are bounded from below. As we assume that $a > \bar{a}$, the non-cooperative tax rate of the small country will be the lower bound ($\tau > t_h^*$). We know that the objective function $B_h(t_h)$ is concave in t_h . Consequently, the small country chooses its best tax rate, which is $t_h^o(\tau) = \tau$. If the common lower bound τ is higher

than t_f^* , the large country will also set $t_f^o(\tau) = \tau$, and we recover the case of harmonization. Thus, we assume that $t_h^* < \tau < t_f^*$. The large country then chooses the tax rate $t_f^o[t_h^o(\tau)]$ that is its best response to $t_h^o(\tau)$. Solving the game backwardly, we first analyze tax competition for a given level of infrastructure expenditures and then consider infrastructure competition. The solution of the game yields the following subgame perfect equilibrium values

$$\theta_h^o = \frac{\tau}{2}, \quad \theta_f^o = a - S + \frac{\tau}{2}, \quad (18)$$

$$t_h^o = \tau, \quad t_f^o = a - S + \frac{\tau}{2}. \quad (19)$$

The share of firms that locate in the small country is $x^o = S - a - \frac{1}{2}\tau + 1$. As $x \in [0, 1]$, we impose $\tau < \tau_m = 2(1 - a + S)$, which requires that $a < 1 + S$. Furthermore, to guarantee that $\tau_m > t_h^*$, we impose $a < \frac{6}{7} + S$. Therefore, in the sequel we assume that $\tau \in [t_h^*, \min\{t_f^*, \tau_m\}]$ and $a \in (\frac{1}{2} + S, \frac{6}{7} + S)$.

The tax revenue of the small and the large countries are, respectively, $B_h^o = (1 - a + S)\tau - \frac{5}{8}\tau^2$, and $B_f^o = \frac{1}{8}(2a - 2S + \tau)^2$.

The joint tax revenue becomes

$$\begin{aligned} B^o &= B_h^o + B_f^o \\ &= \frac{1}{2}[(a - S)^2 + (2 - a + S)\tau - \tau^2]. \end{aligned} \quad (20)$$

The equilibrium social welfare resulting from the above equilibrium is

$$W^o = q + a^2 - 2aS + \frac{1}{8}[4S(2 + S) + (4 - 3\tau)\tau + 4]. \quad (21)$$

4.2 Comparing tax revenue and social welfare

Tax revenue

We first analyze whether tax coordination, by imposing a minimum tax rate, increases the tax revenues of the competing countries. To this end, we compare for each

country j ($j = h, f$) the difference $B_j^* - B_j^o$. In Appendix B (claims 1 and 2), we show that for $a > \bar{a}$ we obtain $B_h^* > B_h^o$ and $B_f^o > B_f^*$. In other words, imposing a lower bound on tax rates does not unanimously improve the revenues of both coordinating countries. Indeed, it appears that the lower tax country²¹ loses tax revenue by moving from a non-cooperative equilibrium to minimum tax coordination. Consequently, accounting for the fact that countries can, in addition to tax competition, also compete simultaneously in infrastructure qualifies a classical result (see Kanbur and Keen, 1993) according to which imposing a minimum tax rate Pareto-improves the countries' tax revenues (see Appendix A).

However, if coordination improves joint revenue, the winner could possibly compensate the loser and each country could thus be made better off. Therefore let us analyze whether a joint revenue improvement ($B^o > B^*$) is possible. In Appendix B (claim 3), we show that $B^* > B^o$ if and only if $\tau \in (t_h^*, \min\{\underline{\tau}, t_f^*\})$ which is only possible for $a \in (\bar{a}, a_m)$ where $a_m = \frac{5\sqrt{3}-3}{9} + S$. In other words, for certain minimum rate choices, there is no room for compensation if the institutional quality of the high tax country is sufficiently high. The following figure illustrates the just described conditions. If the minimum tax rate is included in the grey area, we have $B^* > B^o$ and $B^o > B^*$ in the yellow area. Note that the figure merges the case where $a > \bar{a}$ with the one where $a < \bar{a}$ for which we did not provide calculations because the two cases are symmetric.

²¹Note that the lower tax country is the small one if $a > \bar{a}$, but it will be the large country if $a < \bar{a}$.

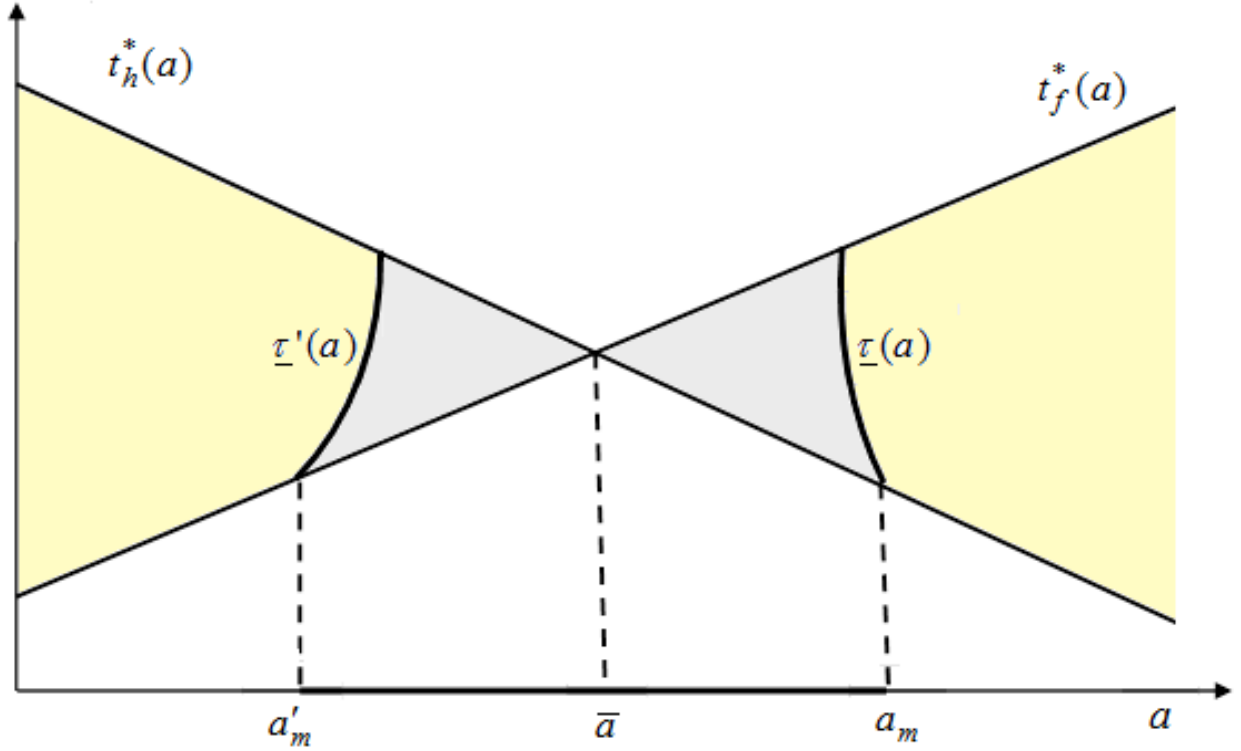


Figure 2

We just showed that, for a given level of institutional quality, moving from a non-cooperative situation to the imposition of a minimum tax rate reduces the revenue of the low tax country. Does this loss increase with reduced institutional quality? In other words, when the losing country is h , does the difference $B_h^* - B_h^o$ increase with a ? It is straightforward to show²² that $\frac{\partial}{\partial a} (B_h^* - B_h^o) > 0$. Therefore, the more the low tax country is institutionally underdeveloped, the more it will lose by moving from competition to minimum tax coordination. If countries are equal in size ($S = \frac{1}{2}$), we observe that the country with the less developed institutions²³ will lose under the imposition of a minimum tax rate.

²²Indeed, we show that $\frac{\partial}{\partial a} (B_h^* - B_h^o) = \frac{14}{25}[(a - S) - \frac{4}{3}] + \tau > \frac{14}{25}(x - \frac{4}{3}) + t_h^*$, with $x = a - S$ and $x \in (\frac{1}{2}, \frac{6}{7})$. As $t_h^* = \frac{4-3x}{5}$, we can write $\frac{\partial}{\partial a} (B_h^* - B_h^o) = \frac{4}{75} - \frac{1}{25}x$. This last expression is decreasing in x . Therefore, $\frac{\partial}{\partial a} (B_h^* - B_h^o) > 0$ for all $x \in (\frac{1}{2}, \frac{6}{7})$ as it is positive for the highest value of x .

²³Note that low institutional development is generally associated with economic underdevelopment.

Social welfare

In Appendix B (claim 4), we show that moving from a non-cooperative equilibrium to minimum tax coordination always increases social welfare.

The following proposition can now be stated

Proposition 3

- (a) Moving from tax and infrastructure competition to minimum tax coordination has opposite effects on the jurisdictions' tax revenues. The high tax country's revenue is improved, while the low tax country is made worse off.*
- (b) If the institutional quality of the high tax country is not sufficiently high, there is no scope for compensating the loser, even if a compensation mechanism exists.*
- (c) Moving from tax and infrastructure competition to minimum tax coordination always increases social welfare.*

5 Conclusion

The purpose of the paper is to investigate whether tax coordination is desirable when countries compete in taxes and infrastructure. To address this question, we develop a model where governments strategically select tax rates and the level of public expenditures to maximize net tax revenues. In addition to asymmetric size, the model incorporates asymmetric institutional quality, which has been largely ignored in the tax competition literature. The desirability of tax coordination is then separately analyzed through its impact on tax revenue and social welfare.

Our results are in stark contrast to the findings of the pure tax competition literature. This is particularly relevant for policy issues because the belief that tax competition generally causes the "erosion of national tax bases" may prove erroneous if countries compete in tax and non-tax instruments. Indeed, in our two-country model

we show that a uniform tax causes a tax loss to each country and that imposing a minimum tax rate only hurts the low tax jurisdiction. These results are however strongly contrasted if jurisdictions only compete in taxes.

It is also worth noting that asymmetries in size and institutional quality play an important role in gauging the desirability of tax coordination. For example, we show that tax harmonization is less efficient than tax and infrastructure competition if the competing countries are equal in size but have very different levels of institutional development.

If we focus on international institutional disparities, which often reflect uneven economic development, our model demonstrates that tax coordination is harmful to the less developed jurisdiction which is also the low tax one.²⁴ This begs the following question. Can tax and infrastructure competition be a way for lagging countries to catch-up in terms of economic development? Future research could address this question by employing a dynamic version of our model. This would allow to investigate under which conditions tax and infrastructure competition could, in the long run, promote convergence across unequally developed countries.

²⁴Assuming that the countries are of equal size.

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A Pure tax competition

In the case of pure tax competition with symmetric institutional quality ($a = 1$), we have $\theta_h = \theta_f = 0$. Solving the tax game yields the equilibrium rates of countries h and f , which are respectively $t_h^T = \frac{1}{3}(1 + S)$ and $t_f^T = \frac{1}{3}(2 - S)$. The corresponding countries' tax revenues are $B_h^T = \frac{1}{9}(S + 1)^2 = (x^T)^2$ and $B_f^T = \frac{1}{9}(2 - S)^2 = (1 - x^T)^2$. The joint tax income is thus $B^T = \frac{1}{9}(2S^2 - 2S + 5)$.

A.1 Tax harmonization

The impact on tax revenues

If both countries opt for tax harmonization, the uniform tax rate can equal any value $t^u \in [0, 1]$. As a result, $x^u = S$ companies will be located in the small country and $1 - x^u$ in the large economy. The tax revenues of the two countries are then respectively $B_h^u = t^u S$ and $B_f^u = t^u(1 - S)$. The joint maximal revenue is $B^u = B_h^u + B_f^u = t^u$. It is now convenient to show that $B^u > B^T$, if $t \in [\frac{5}{9}, 1]$ for all $S \in (0, \frac{1}{2})$. It implies that if the unified tax rate is higher than $\frac{5}{9}$, tax harmonization generates higher total tax revenue than pure tax competition. This is consistent with the tax competition literature (see for example, Kanbur and Keen, 1993; Baldwin and Krugman, 2004 and Boadway and Tremblay, 2011).

We now consider each country individually. For the large country we can easily show that $B_f^u > B_f^T$ for $S \in (0, \frac{1}{2})$ and $t \in [\frac{(2-S)^2}{9(1-S)}, 1]$. In the same way we can show for the small country that $B_h^u > B_h^T$ for all $S \in (\frac{7}{2} - \frac{3}{2}\sqrt{5}, \frac{1}{2})$ and $t \in [\frac{(1+S)^2}{9S}, 1]$. In other words, if the competing economies are not too uneven in size, the presence of a uniform tax rate, which is high enough, leads to a Pareto-improvement in tax revenue. Moreover, each country can be made better off for any $S \in (0, \frac{1}{2})$, by imposing a uniform tax rate $t \in [\frac{5}{9}, 1]$, if inter-jurisdictional revenue redistribution is feasible.

The impact on social welfare

If tax rates are the same across jurisdictions, the social welfare equals $W^u = q$. The aggregate welfare resulting from pure tax competition is $W^T = q - \frac{1}{18}(2S - 1)^2$. Consequently we get $W^T - W^* = -\frac{1}{2}(t_f^T - t_h^T)^2 < 0$. Moving from tax competition to tax harmonization is thus welfare improving.

A.2 Minimum tax

The impact on tax revenues

We assume that the tax rates set by the jurisdictions are now bounded from below by τ such that $\tau \in (t_h^T, t_f^T)$. In that we follow Kanbur and Keen (1993). The small country will set $\tilde{t}_h = \tau$ since it is its best choice. The large country chooses its best reply $\tilde{t}_f = \frac{\tau}{2} + \frac{1-S}{2}$. It follows $\tilde{x} = \frac{1}{2}(1 + S - \tau)$. The tax income for each country is respectively $\tilde{B}_h = \frac{1}{2}\tau(1 + S - \tau)$ and $\tilde{B}_f = \frac{1}{4}(\tau - S + 1)^2$. The aggregate tax income is then $\tilde{B} = \frac{1}{4}(S^2 - 2S + 4\tau - \tau^2 + 1)$.

It is then easy to check that for $\tau > t_h^T$ we have $\tilde{B}_h > B_h^T$ and $\tilde{B}_f > B_f^T$. It follows that imposing a minimum tax rate to the competing jurisdictions is a Pareto-improvement in tax revenue. This result is reminiscent of Kanbur and Keen (1993).

The impact on social welfare

The social welfare resulting from a minimum tax bound $\tau \in (t_h^T, t_f^T)$ equals $\tilde{W} = q - \frac{1}{2}(\frac{1}{2} - \frac{1}{2}\tau - \frac{1}{2}S)^2$. Hence, $\tilde{W} - W^T = \frac{1}{72}(S - 3\tau + 1)(7S + 3\tau - 5)$. Since $\tau \in (t_h^T, t_f^T)$ it is straightforward to show that $W(\tau) > W^T$. Consequently, a minimum tax which lying between the non-cooperative equilibrium tax rates is welfare improving. This result is in line with Kanbur and Keen (1993).

B Claims and their proofs

Claim 1. *With $\tau \in [t_h^*, t_f^*]$, we always have $B_h^* > B_h^o$.*

Proof. We recall that the tax revenue of country h resulting from tax and infrastructure competition is $B_h^* = \frac{7}{225} (3S - 3a + 4)^2$ with $S + \frac{1}{2} < a < 1 + S < S + \frac{4}{3}$. The non-cooperative equilibrium tax rates are $t_h^* = \frac{4-3a+3S}{5}$ and $t_f^* = \frac{1+3a-3S}{5}$. In addition, $B_h^o = (1 - a + S)\tau - \frac{5}{8}\tau^2$ is positive only if $0 < \tau < \frac{8(1-a+S)}{5}$. It is easy to check that B_h^o reaches its maximum at $\hat{\tau} = \frac{4(1-a+S)}{5}$. Furthermore, B_h^o is decreasing in τ for $\tau \in [\hat{\tau}, t_f^*]$. Since $\hat{\tau} - t_h^* = -\frac{a-S}{5} < 0$, it follows that B_h^o decreases in τ for $\tau \in [t_h^*, t_f^*]$ and reaches its maximum at t_h^* . Therefore, to prove the claim, we only need to show that $B_h^* > B_h^o(t_h^*)$. It is straightforward to show that $B_h^o(t_h^*) = t_h^* \frac{4-5a+5S}{8}$. Thus, $B_h^* - B_h^o(t_h^*) = \frac{t_h^*}{360} (44 + 57a - 57S) > 0$. That finishes the proof.

Claim 2. *There is $B_f^* < B_f^o$ for $\tau \in [t_h^*, t_f^*]$.*

Proof. We know that $B_f^o = \frac{1}{8}(2a - 2S + \tau)^2$ and $B_f^* = \frac{7}{225} (3S - 3a - 1)^2$. Given that B_f^o is increasing in τ for $\tau \in [t_h^*, t_f^*]$, the claim is proved if the inequality $B_f^o > B_f^*$ holds for the minimum value of B_f^o which equals $B_f^o(t_h^*) = \frac{1}{8} \left(\frac{4+7a-7S}{5} \right)^2$. After straightforward calculations, we get $B_f^o(t_h^*) - B_f^* = \frac{1}{25.72} [88 + 56 \times 3(a - S) - 63(a - S)^2]$ with $S + \frac{1}{2} < a < 1 + S$, which implies, $\frac{1}{2} < a - S < 1$.

If we set $x = a - S$, we can write $B_f^o(t_h^*) - B_f^*$ as a second order polynomial in x , which equals $f(x) = -63x^2 + 56 \times 3x + 88$ with $\frac{1}{2} < x < 1$. The function $f(x)$ is concave and it is easy to check that it is positively signed for $x \in (\frac{1}{2}, 1)$. Consequently, $B_f^o(t_h^*) - B_f^* > 0$ for all $a \in (\frac{1}{2} + S, 1 + S)$ and $B_f^* < B_f^o$ for $\tau \in [t_h^*, t_f^*]$.

Claim 3. *If $a \in \left(\frac{1}{2} + S, \frac{5\sqrt{3}-3}{9} + S \right)$ and $\tau \in (t_h^*, \underline{\tau})$ it follows that $B^* > B^o$.*

Proof. Set $x = a - S$ and let $\underline{\tau} = 1 - \frac{1}{2}x - \sqrt{\frac{13x^2}{100} + \frac{3x}{25} - \frac{13}{225}}$ and $\bar{\tau} = 1 - \frac{1}{2}x + \sqrt{\frac{13x^2}{100} + \frac{3x}{25} - \frac{13}{225}}$ be the solutions of $\Psi(\tau) = B^* - B^o = 0$. The function $\Psi(\tau)$ is negative for $\tau \in (\underline{\tau}, \bar{\tau})$, since it is convex in τ . It can further be checked that $\underline{\tau} > t_h^*$ if $\frac{1}{2} < x < \frac{5\sqrt{3}-3}{9}$ ($< \frac{6}{7}$) and that $t_f^* < \bar{\tau}$ if $\frac{1}{2} < x < \frac{6}{7}$. It follows that $\Psi(\tau) > 0$ for $\tau \in (t_h^*, \underline{\tau})$ which is only possible for $x \in \left(\frac{1}{2}, \frac{5\sqrt{3}-3}{9} \right)$, or for $a \in \left(\frac{1}{2} + S, \frac{5\sqrt{3}-3}{9} + S \right)$.

Claim 4. *$W^o > W^*$ for $\tau \in [t_h^*, \min\{\tau_m, t_f^*\}]$ with $\tau_m = 2(1 - a + S)$ and $\frac{1}{2} + S < a <$*

$\frac{6}{7} + S$.

Proof. It is convenient to show that W^o is strictly concave in τ and reaches its maximum at $\tau^+ = \frac{2}{3}$. Thus, the minimum of W^o can only be attained at one of the two boundaries, t_h^* or $\min\{t_f^*, \tau_m\}$. Now we determine the minimum value of W^o . First, it is easy to show that $W^o(t_h^*) - W^o(t_f^*) = -\frac{t_f^* - t_h^*}{8} < 0$ and so $W^o(t_f^*) > W^o(t_h^*)$. It follows that $W^o(t_h^*) = \inf W^o$ if $t_f^* < \tau_m$. If $\tau_m < t_f^*$, we consider the cases $\tau_m < \tau^+$ and $\tau^+ < \tau_m < t_f^*$. Because W^o is strictly concave and $\tau_m > t_h^*$, we must have $W^o(\tau_m) > W^o(t_h^*)$ if $\tau_m < \tau^+$ and since $W^o(t_f^*) > W^o(t_h^*)$, we must have $W^o(t_h^*) < W^o(t_f^*) < W^o(\tau_m)$, if $\tau^+ < \tau_m < t_f^*$. In any case, W^o reaches its minimum at t_h^* .

We now prove that the minimum value of W^o is above W^* . For that purpose it is sufficient to compare $W^o(t_h^*)$ with W^* . Direct calculation leads to $W^o(t_h^*) - W^* = \frac{1}{2} \left(\frac{x^2}{4} + \frac{3x}{5} - \frac{11}{45} \right)$, $\forall x \in [\frac{1}{2}, \frac{6}{7}]$ with $x = a - S$. It is easy to see that the polynomial $f(x) = \frac{x^2}{4} + \frac{3x}{5} - \frac{11}{45}$ is convex in x and reaches its minimum at $\bar{x} = -\frac{6}{5}$. The function $f(x)$ is thus increasing in $(-\frac{6}{5}, \infty)$. Noticing that $f(0) = -\frac{11}{45}$ and $f(\frac{1}{2}) = \frac{1}{8} > 0$ it follows that $f(x) > 0$ for $x \in [\frac{1}{2}, \frac{6}{7}]$. In other words $W^o(t_h^*) > W^*$, and thus we prove that $W^o > W^*$ for all $\tau \in [t_h^*, \min\{t_f^*, \tau_m\}]$ and $\frac{1}{2} + S < a < \frac{6}{7} + S$. We finish the proof.