

Equation of state at non-zero baryon density based on lattice QCD

Short Communication

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Abstract: We employ the lattice QCD data on Taylor expansion coefficients to extend our previous parametrization of the equation of state to finite baryon density. When we take into account lattice spacing and quark mass dependence of the hadron masses, the Taylor coefficients at low temperature are equal to those of hadron resonance gas. Parametrized lattice equation of state can thus be smoothly connected to the hadron resonance gas equation of state at low temperatures.

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One of the methods to extend the lattice QCD calculations to non-zero chemical potential is Taylor expansion. In that approach pressure is Taylor expanded in chemical potentials, and the Taylor coefficients are calculated on the lattice at zero chemical potential. In this contribution we use the results of the most comprehensive lattice QCD analysis of the Taylor coefficients to date [1, 2] to construct a parametrisation of an equation of state (EoS) for finite baryon density. As in our earlier parametrisation of the EoS at zero chemical potential [3], we require that our parametrisation matches smoothly to the hadron

resonance gas (HRG) at low temperatures.

Taylor coefficients are derivatives of pressure, P , with respect to baryon and strangeness chemical potentials, μ_B and μ_S , respectively:

$$c_{ij}(T) = \frac{1}{i!j!} \frac{T^{i+j}}{T^4} \frac{\partial^i}{\partial \mu_B^i} \frac{\partial^j}{\partial \mu_S^j} P(T, \mu_B = 0, \mu_S = 0), \quad (1)$$

where T is temperature¹. As we discussed in [4], all the coefficients evaluated in Refs. [1, 2] are well below the

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¹ We use natural units where $c = \hbar = k_B = 1$ throughout the text.

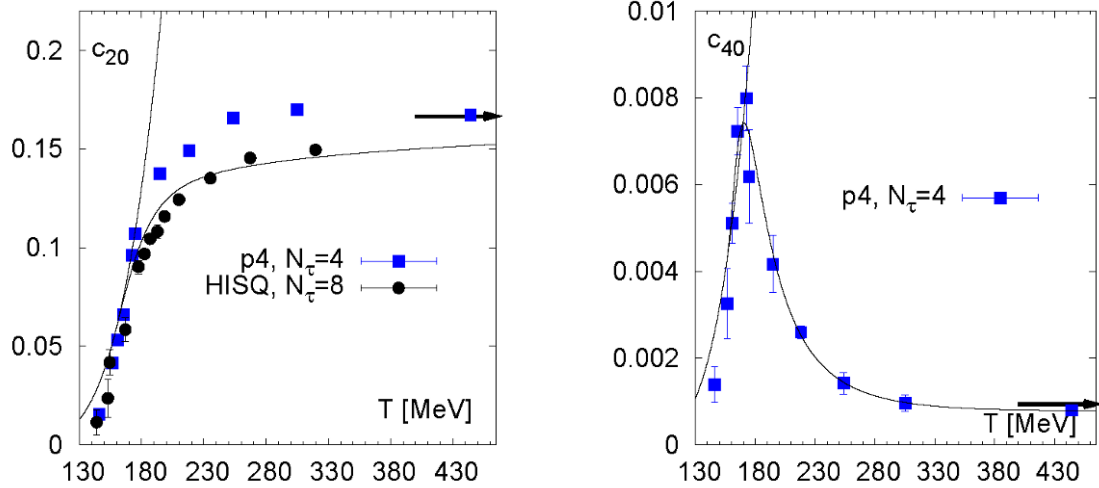


Figure 1. The parametrisation (solid line) and HRG value (dashed) of the c_{20} (left) and c_{40} (right) coefficients compared with the shifted p4 data (see the text). The recent lattice result for c_{20} with the HISQ action [6] is also shown. The arrows depict the Stefan-Boltzmann values.

HRG results. This discrepancy can largely be explained by the lattice discretisation effects on hadron masses: When the hadron mass spectrum is modified accordingly (for details see [5]), the HRG model reproduces the lattice data, see Fig. 1 of Ref. [4]. Interestingly this change can be accounted for by shifting the modified HRG result of purely baryonic coefficients towards lower temperature by 30 MeV. The situation is similar for other Taylor expansion coefficients [5], although the strange coefficients might favour slightly smaller shift. Based on this finding and because the lattice data agree so well with the modified HRG we suggest that cutoff effects can be accounted for by shifting the lattice data by 30 MeV. We show the effect of such a shift in the left panel of Fig. 1, where we plot the HRG curve with physical masses (dashed line) and compare it with the lattice data, where all the points below 206 MeV temperature are shifted by 30 MeV, and the 209 MeV point by 15 MeV. For further confirmation of this procedure we also plot the recent HISQ result of c_{20} [6] in Fig. 1 (right): At low temperatures the shifted p4 data agree with the HISQ data.

We parametrise the shifted data as a function of temperature T using an inverse polynomial of four (c_{20}), five (c_{11} and c_{02}), or six (fourth and sixth order coefficients) terms:

$$c_{ij}(T) = \sum_{k=1}^m \frac{a_{kij}}{\hat{T}^{n_{kij}}} + c_{ij}^{\text{SB}}, \quad (2)$$

where c_{ij}^{SB} is the Stefan-Boltzmann value of the particular coefficient, a_{kij} are the parameters, and the powers n_{kij} are required to be integers between 1 and 42. $\hat{T} = (T - T_s)/R$ with scaling factors $T_s = 0$ and $R = 0.15$

GeV for the second order coefficients and $T_s = 0.1$ GeV and $R = 0.05$ GeV for all the other coefficients. We match this parametrisation to the HRG value at temperature $T_{\text{SW}} = 155$ MeV by requiring that the Taylor coefficient and its first, second, and third derivatives are continuous. Since the recent lattice data obtained using HISQ action [6] shows that the second order coefficients approach their Stefan-Boltzmann limits slowly, we require that their value is 95% of their Stefan-Boltzmann value at 800 MeV temperature. These constraints fix four (or five) of the parameters a_{kij} . The remaining parameters, are fixed by a χ^2 fit to the lattice data. As an example we show the parametrised c_{20} , c_{40} , and c_{60} in Figs. 1 and 2. Once the coefficients are known, pressure can be written as

$$\frac{P}{T^4} = \sum_{ij} c_{ij}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_S}{T}\right)^j, \quad (3)$$

and all the other thermodynamical quantities can be obtained from Eq. (3) by using the laws of thermodynamics. This kind of an expansion breaks down at large chemical potentials. However, at baryon densities of interest here, the contribution from coefficients of particular order is clearly below ($< 20\%$) the lower order contribution. Thus the approximation of the EoS in terms of fourth- and sixth order expansion looks reasonable. As pressure at $\mu_B = 0$, i.e. the coefficient c_{00} , we use our earlier parametrisation *s95p-v1* [3]. We describe the EoS in the right panel of Fig. 2 by showing the square of the speed of sound $c_s^2 = \partial P / \partial \epsilon|_{s/n_B}$ [7] on various isentropic curves with constant entropy per baryon s/n_B [8]. The curves at $s/n_B = 400, 65$, and 40 are relevant at collision energies $\sqrt{s_{\text{NN}}} = 200, 39$ and 17 GeV, respectively. At $s/n_B = 400$

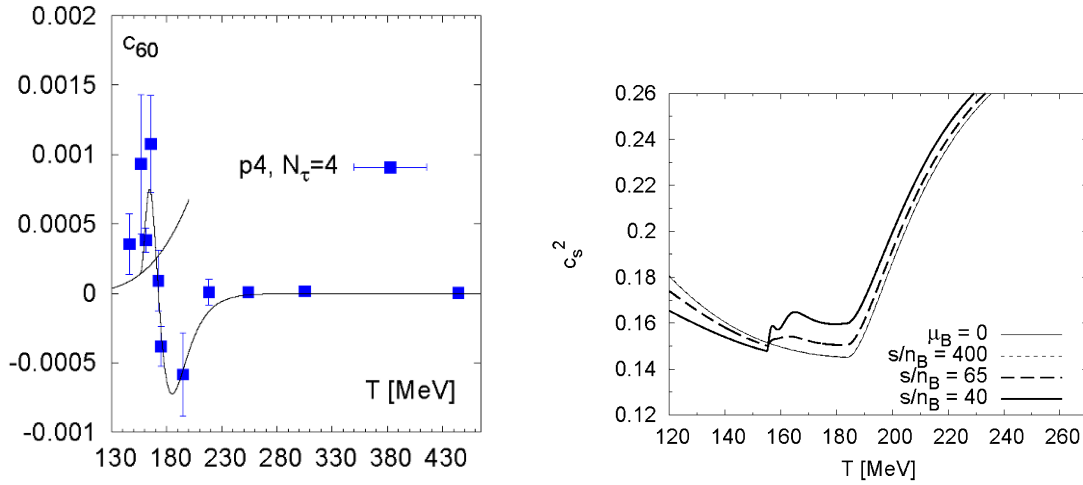


Figure 2. (Left) The parametrisation (solid line) and HRG value (dashed) of the c_{60} coefficient compared with the shifted p4 data (see the text). (Right) The square of the speed of sound c_s^2 as a function of temperature on various isentropic curves with constant entropy per baryon s/n_B (see the text).

(dotted line), the EoS is basically identical to the EoS at $\mu_B = 0$ (thin solid line). At larger baryon densities the effect of finite baryon density is no longer negligible. The larger the density, the stiffer the EoS above, and softer below the transition temperature.

Furthermore, additional structure begins to appear around the transition temperature with increasing density. We expect this structure to be an artifact of our fitting procedure: Our fit is too good and elevates errors to features leading to additional ripples in the speed of sound. Another interesting feature in the equation of state is the rapid change of the speed of sound around $T_{sw} = 155$ MeV and another change around $T \approx 185$ MeV. The latter has its origin in our baseline $\mu_B = 0$ EoS. It follows hadron resonance gas up to $T = 184$ MeV temperature causing a change in the speed of sound at that temperature. Nevertheless, when pressure is plotted as a function of energy density, these structures are hardly visible. Therefore we do not expect them to affect the buildup of flow and the evolution of the system, and consider our parametrisation a reasonable first attempt.

We have studied the effect of the EoS on flow by calculating elliptic flow in Pb+Pb collision at the full SPS collision energy ($\sqrt{s_{NN}} = 17$ GeV). Our results are similar to those we have shown earlier [4]: Even if in an ideal fluid calculation at RHIC energy proton $v_2(p_T)$ is sensitive to the order of phase transition [9], at SPS energy both proton and pion $v_2(p_T)$ are insensitive to it.

To summarise, we have shown that a temperature shift of 30 MeV is a good approximation of the discretisation effects in the lattice QCD data obtained using p4 action. We have constructed an equation of state for finite baryon

densities based on hadron resonance gas and lattice QCD data. At the full SPS energy ($\sqrt{s_{NN}} = 17$ GeV) the p_T -differential elliptic flow is almost insensitive to the equation of state. This is bad news for the experimental search of the critical point, since a change from a first order phase transition to a smooth crossover does not cause an observable change in the flow.

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References

- [1] C. Miao et al. [RBC-Bielefeld Collaboration], PoS LATTICE2008, 172 (2008)
- [2] M. Cheng et al., Phys. Rev. D 79, 074505 (2009)
- [3] P. Huovinen, P. Petreczky, Nucl. Phys. A 837, 26 (2010)
- [4] P. Huovinen, P. Petreczky, J. Phys. G 38, 124103 (2011)
- [5] P. Huovinen, P. Petreczky, J. Phys. Conf. Ser. 230, 012012 (2010)
- [6] A. Bazavov et al. [HotQCD Collaboration], J. Phys. Conf. Ser. 230, 012014 (2010)
- [7] D. Teaney, J. Lauret, E. V. Shuryak, arXiv:nucl-th/0110037
- [8] C. Schmidt, Eur. Phys. J. C 61, 537 (2009)
- [9] P. Huovinen, Nucl. Phys. A 761, 296 (2005)