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Fixed Prices, Rationing and Optimality

by

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1. Introduction

With the development of the theory of equilibria under rigid prices it has become part of economic folklore that fixed but non-Walrasian prices imply a welfare loss to the economy as a whole. When drawing such a conclusion in a competitive setting one usually relies on the first theorem of welfare economics and compares competitive equilibria with so called Drèze equilibria which seem to be the appropriate allocations for the comparison. A more precise formulation of such a statement would be that each rationing equilibrium can be Pareto improved by changing prices to the Walrasian ones. Whether such a conclusion is correct in general has not been shown in the literature. Of course, the question whether Walrasian prices lead to better allocations than others may also be asked in non-competitive models. The work by Benassy (1976) and Böhm/Lévine (1979) in the tradition of the theory with rationing or by Shapley (1976), Novshek and Sonnenschein (1978) and more recently by Hart (1980) could also be used to approach the welfare question. Although the results of this paper give no indication how the welfare question can be approached in these latter papers, there are some close relationships to the paper by Böhm/Maskin/Polemarchakis/Postlewaite (1983) and by Silvestre (1982).

For the competitive part of the theory there are by now a number of models and examples which show that Drèze equilibria may be very suboptimal for given fixed prices. It is well known that even the best Drèze equilibria can be Pareto improved at the given prices (Böhm and Müller (1977)).Although the literature on price constrained Pareto optimal allocations [Drèze/Müller (1980) and Balasko (1982)] indicates that, in general, these cannot be obtained as Drèze equilibria, a systematic analysis of the inefficiencies due to price rigidities cannot be made from these papers. Some aspects of such an analysis are revealed in a recent paper by Silvestre (1982) who addresses the question of wage and price negotiations in a fixed price model. Though the model used by Silvestre is similar to the one presented here, he attempts to give a characterization of the non-cooperative Nash equilibrium of a negotiation procedure rather than a general description of the welfare properties of the set of possible equilibria.

There are in principal two sources of suboptimality for Drèze equilibria which should be distinguished. The well known fact that there may exist a large set, possibly a continuum of Drèze equilibria for each fixed price system , stems from the fact that different rationing mechanisms yield different allocations. In such cases one could concentrate only on so called efficient rationing mechanisms, a concept which would have to be defined precisely and for which the price constrained Pareto optimal allocations supply the conceptual basis. The other source of suboptimality is to be found in the non-Walrasian prices themselves. This paper tries to give a general characterization of the second source of suboptimality by means of an example of an economy with three agents and three commodities assuming efficient rationing in all Drèze equilibria. Since the number of markets with rationing is at most two, it turns out that all Drèze equilibria are price constrained Pareto optimal allocations. The model represents a prototype economy à la Malinvaud. A careful interpretation of the results therefore gives some answers to questions of a macroeconomic nature, like e.g. 1) is higher employment always better or does it favor one group at the cast of another, 2) are unemployment states worse than inflationary states, or 3) does welfare increase if the number of markets with rationing decreases? The method to demonstrate the results will draw heavily on geometric properties well known from fix price theory and standard welfare economics. Mathematical calculations are avoided whenever a geometric argument suffices. Section 2 provides an example of an economy which yields the typical characteristics of a fix price model now well-known from the literature. In section 3 the general optimality properties for the example are derived using the qualitative properties of the example of section 2.

2. The Model

The economy consists of three agents, one producer, one consumer-worker and one consumer-capitalist. The capitalist receives all profits from production and workers earn labor income. Labor is used to produce an otherwise not available consumption good which is consumed by the worker and the capitalist. There is a third non-producible commodity in the economy owned by workers and capitalists. This third commodity corresponds to money in the model of Malinvaud and it will be called money here as well. The price of money will be constant and equal to one. However, no intertemporal aspect is attached to money in the example of this paper. The total stock of money M in the economy is positive and constant, where the worker owns $\bar{m}_L = \gamma M$ and the capitalist owns $\bar{m}_S = (1-\gamma)M$, with $0 < \gamma < 1$.

The producer is characterized by a strictly concave, strictly increasing production function which associates with an employment level ℓ the output level $x = \ell^{\alpha}$. The production function exhibits two properties used later in the analysis: under profit maximization 1.) the labor share in output is constant and equal to α and 2.) the wage bill decreases with higher wages and a constant output price. Given the price $p > 0$ of the consumption good and the wage rate $w > 0$, the producer maximizes profits.

Concerning the two distribution parameters α and γ of the economy it will be assumed that $\gamma \geqq \alpha$. The significance of this assumption will become clear in section 3 of the paper.

The worker's utility function $u_L(x_L, m_L, \ell)$ is assumed to be of the form

$$u_L(x_L, m_L, \ell) = x_L^{\beta} m_L^{1-\beta} - \delta \ell^2$$

where $\delta > 0$ and $\frac{1}{2} < \beta < 1$. u_L is strictly quasiconcave. Given a price-wage pair $(p,w) \gg 0$ and the initial money holdings $\bar{m}_L > 0$, the worker maximizes his utility.

The capitalist's utility function $u_S(x_S, m_S)$ is assumed to be of the form

$$u_S(x_S, m_S) = x_S^{1-\beta} m_S^{\beta}.$$

Given $\bar{m}_S > 0$ and any profit payment $\Pi \geqq 0$, the capitalist maximizes his utility.

The assumptions made so far guarantee the existence of a unique Walrasian equilibrium. This is summarized in the following proposition whose proof is given in the appendix.

Proposition 1: *For given parameters* $0 < \alpha \leqq \gamma < 1$, $\frac{1}{2} < \beta < 1$, $\delta > 0$, *there exists a unique price wage pair* $(p^*, w^*) \gg 0$ *such that notional demand and supply balance on all three markets.*

For the economy described so far rationing on the money market will be excluded. Hence, there remain three distinct equilibria under rationing: 1.) supply rationing on the labor and on the commodity market, 2.) demand rationing on both markets, and 3.) rationing of consumers on both markets. Following the terminology of Malinvaud the first category will be called Keynesian, denoted K, the second inflationary, denoted I and the third will be called classical, denoted C.

Keynesian states involve rationing of one agent on each market, i.e. the worker on the labor market and the producer on the commodity market. The capitalist is unaffected. Therefore, an inefficiency problem due to a particular rationing mechanism cannot arise if for a given pair (p,w) the level of employment is unique.

Inflationary and classical states involve demand rationing on the commodity market which affect both consumers. However, effective labor supply depends on the rationing level of the worker alone. Hence, the distribution of the rationing levels among consumers for a given total supply will influence the level of production in each demand rationing state and the optimality properties of the associated allocation. By the same argument, a particular pair (p,w) may imply different levels of employment and different allocations depending on the rationing mechanism. To avoid possible inefficiencies due to the rationing scheme, the mechanism chosen will have the following efficiency property: Given a price-wage pair (p,w) and an aggregate level of production X with its associated level of employment and distribution of wealth, then, the rationing levels (y_L, y_S) for the two consumers are chosen such that the resulting allocation $\left[(m_L, y_L), (m_S, y_S)\right]$ respects individual budgets and is Pareto optimal given X. In other words, given (p,w) the allocation is an equilibrium relative to some price p'. This is the same concept as in Drèze and Müller (1980) and in Balasko (1982). It is straight forward to show that the mechanism is well defined for all (p,w,X) and demand rationing. To see this, consider the Edgeworth box associated with X. Then, the unique rationing levels (y_S, y_L) are defined by the intersection of the budget line and the contract curve. This geometric characterization of the efficient rationing scheme will be used in section 3. It should be clear, that this property also prevails trivially in Keynesian states since both consumers pay the same price with no demand rationing.

With the imposition of efficient rationing it can now be shown that the example yields a typical three commodity fix price economy with the usual representation in (p,w)-space. The specific properties which form the basis for the analysis in the next section are summarized in the following propositions. The proofs are again given in the appendix.

Proposition 2: *Given the same assumptions on the parameters* α, β, γ, *and* δ *as in* Proposition 1, *for every* $(p,w) \gg 0$ *there exists a unique disequilibrium allocation, i.e. the* (p,w)*-plane can be partitioned into the three regions of classical unemployment* C, *of Keynesian unemployment* K, *and of repressed inflation* I.

Proposition 3: *If* $1/2 < \beta < 2/3$, *then the boundaries of the three regions are monotonic functions such that* $C \cap I$ *and* $C \cap K$ *are upward sloping and* $I \cap K$ *is downward sloping in* (p,w)*-space.*

Proposition 4: *The isoemployment curves in* (p,w)*-space are upward sloping in the two regions* C *and* K *and the highest employment level is reached at the Walrasian equilibrium*

WE. *For* $\frac{1}{2} < \beta \leqq \frac{2}{3}$ *the isoemployment curves in the* I-*region are downward sloping.*

3. Optimality

It is quite surprising that the properties given by Propositions 1 - 4 combined with some of the more specific assumptions of the example make it possible to derive some clear answers about the optimality properties of the fixed price allocations. Since most of these results can be derived using simple geometric arguments extensive calculations will be suppressed in the following analysis.

Consider the partitioning of the (p,w)-plane as given in Figure 1 and fix a particular employment level ℓ which is less than the Walrasian one represented by the curve (P,Q,R). Changes of prices and wages along (P,Q,R) imply changes of the wealth distribution among the two agents at constant levels of available total output and money. Hence, the reallocation along an isoemployment curve can be described by the changes in an Edgeworth box of fixed size (Figure 2). Let the point S denote the origin for the capitalist and L the origin for the worker. The curve [S,L] is the associated contract curve which is concave and lies above the diagonal since $\frac{1}{2} < \beta < 1$.

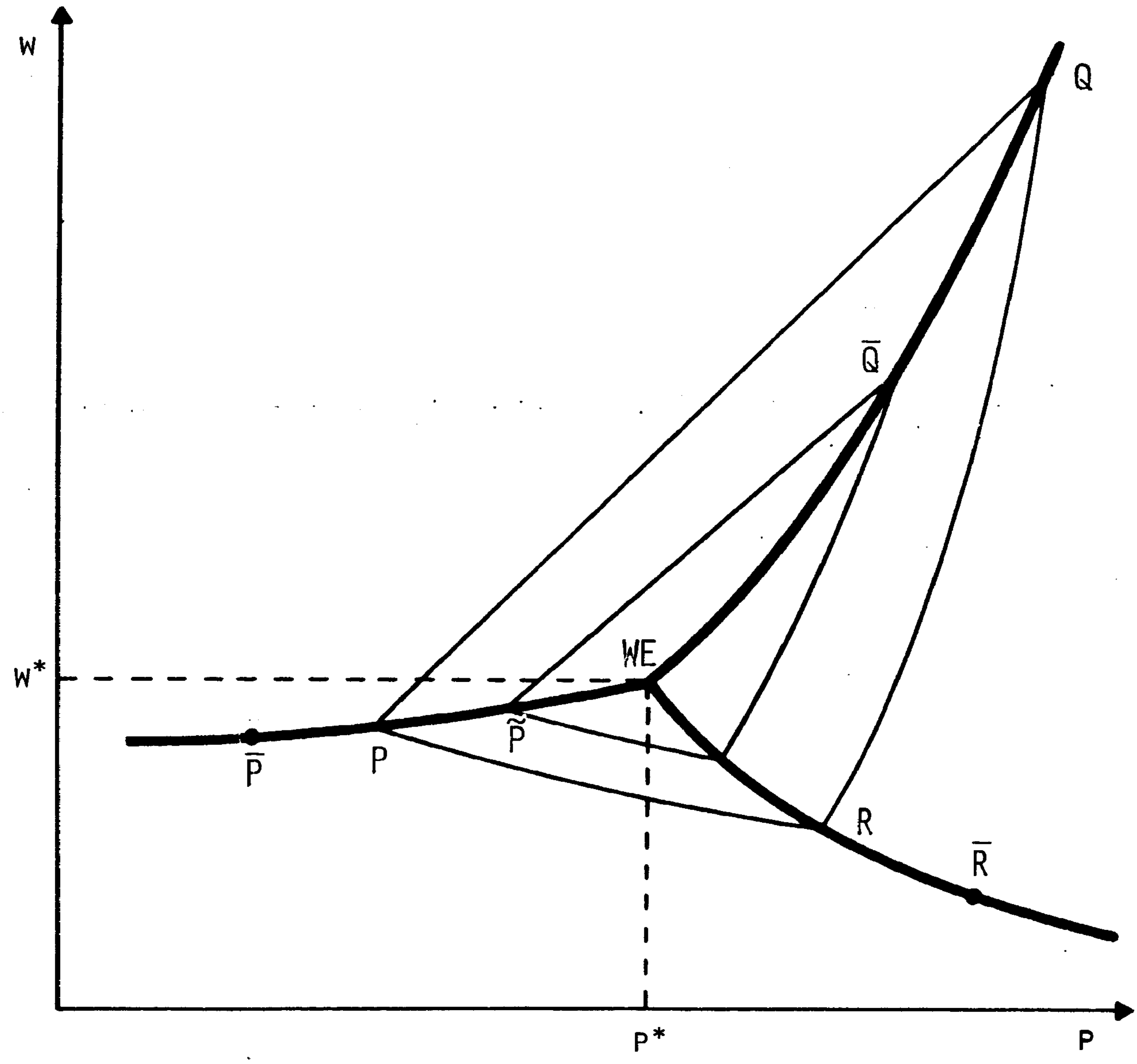

Figure 1

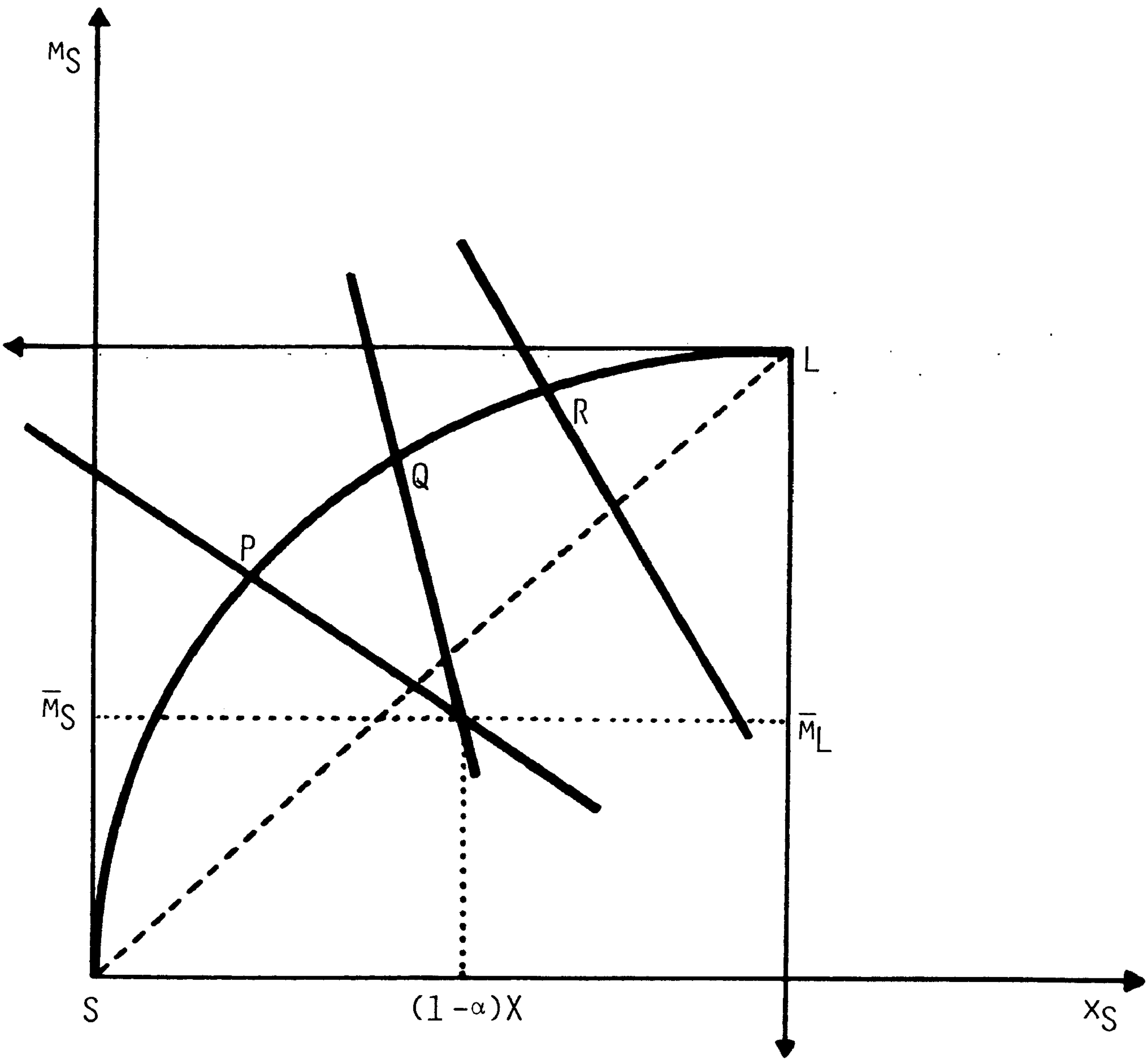

Figure 2

Now consider a movement from R to Q in the (p,w)-plane. Along the segment [R,Q] no demand rationing of consumers occurs and prices rise from R to Q. Therefore, the associated allocations must be on the contract curve [S,L] in Figure 2. Moreover, the homogeneity of the utility functions implies a movement away from L toward S, i.e. the points corresponding to R and Q in Figure 1 for Figure 2 are such that Q lies to the left and below R (see Figure 2). Thus the utility of the capitalist decreases and that of the worker increases. A description of these changes in utility space will become very helpful. Let $V_i(p,w)$, $i = L, S$, denote the continuous indirect utility function of the worker L and of the capitalist S given the rationing mechanism. Then, $V_L(Q) > V_L(R)$ and $V_S(Q) < V_S(R)$, i.e. the image of the segment [R,Q] in (p,w)-space of the map V defines a downward sloping segment also denoted [R,Q] in utility space (see Figure 3).

Next consider the movement along the isoemployment curve from Q to P in the classical region. Since prices and wages fall in the same proportion along [Q,P], profits and wages fall in the same proportion, i.e. nominal income of both consumers falls in the same proportion. Moreover, the proportional change of prices and wages imply that the consumption plans $(\bar{m}_S, (1-\alpha)X)$ and $(\bar{m}_L, \alpha X)$ remain in the budget sets of the two consumers, where X is the total production level. Since $\gamma \geq \alpha$, it follows that

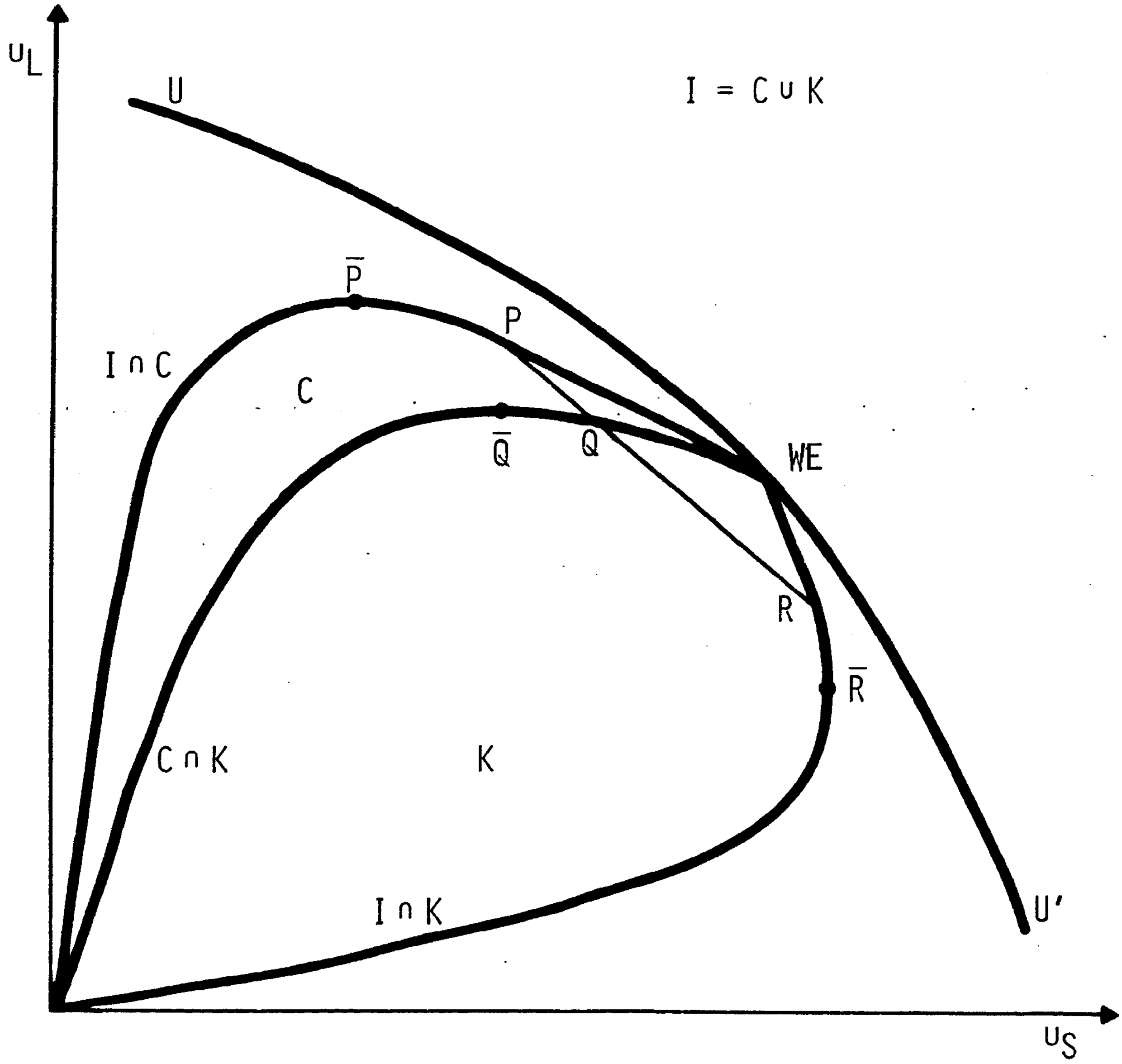

Figure 3

$$\frac{\overline{m}_S}{(1-\alpha)X} = \frac{(1-\gamma)M}{(1-\alpha)X} \leqq \frac{M}{X}.$$

Hence, the point $(\overline{m}_S,(1-\alpha)X)$ does not lie above the diagonal in Figure 2. Therefore, the decrease in prices and wages reduces the utility of the capitalist. Combined with the fact that the demand rationing in the classical region is efficient, this implies that the capitalist's utility decreases from Q to P and that of the worker increases. The corresponding points in Figure 2 and 3 are again denoted by P and Q and the movement from Q to P is monotonic. Furthermore, it is clear that the segment [P,R] in (p,w)-space is mapped onto the segment [P,R] in utility space.

Consider now a movement from WE to Q along $C \cap K$. From the proof of Proposition 3 one has

$$\left.\frac{dw}{dp}\right|_{C\cap K} = \frac{1}{\alpha}\,\frac{w}{p}, \text{ and } \left.\frac{d\ell}{dp}\right|_{C\cap K} = -\frac{1}{\alpha}\frac{\ell}{p}.$$

As a consequence one obtains that nominal wages and nominal profits are constant along $C \cap K$, i.e.

$$\left.\frac{dW}{dp}\right|_{C\cap K} = 0 \text{ and } \left.\frac{d\pi}{dp}\right|_{C\cap K} = 0.$$

Let $v_S(p,\pi)$ denote the usual indirect utility function of S. Then

$$V_S(p,w) = v_S(p,\Pi(p,w))$$

holds on $C \cap K$ where $\Pi(p,w)$ is the producer's unconstrained profit maximum. Differentiation yields

$$\left.\frac{dV_S}{dp}\right|_{C\cap K} = \frac{\partial v_S}{\partial p} + \left.\frac{\partial v_S}{\partial \pi}\frac{d\Pi}{dp}\right|_{C\cap K} = \frac{\partial v_S}{\partial p} < 0.$$

Therefore, $V_S(p^*,w^*) > V_S(p^Q,w^Q)$ for all Q on $C \cap K$. Proceeding in the same fashion for the worker, his indirect utility on $C \cap K$ is defined by

$$V_L(p,w) = v_L(p,W) - \delta\ell^2$$

where $\ell = z^*(p,w)$ and $W = \overline{m}_L + wz^*(p,w)$. Differentiation and Roy's identity yields

$$\left.\frac{dV_L}{dp}\right|_{C\cap K} = \frac{\partial v_L}{\partial p} + \left.\frac{\partial v_L}{\partial W}\frac{dW}{dp}\right|_{C\cap K} - 2\delta\ell \left.\frac{d\ell}{dp}\right|_{C\cap K}$$

$$= -x_L\frac{\partial v_L}{\partial W} + 2\delta\ell\frac{\ell^\alpha}{w}.$$

At (p^*,w^*) one has

$$\frac{\partial v_L}{\partial W} = \frac{1}{w}\frac{\partial v_L}{\partial \ell} = \frac{1}{w}2\delta\ell.$$

Therefore,

$$\left.\frac{dV_L}{dp}\right|_{C\cap K}(p^*,w^*) = \frac{1}{w}2\delta\ell[\ell^\alpha - x_L] > 0.$$

Hence, the utility of the worker increases along $C \cap K$ locally away from WE. However, for sufficiently high real wages V_L must decrease to zero. To see this consider an increasing unbounded sequence $(p^n,w^n) \in C \cap K$ and let $\ell^n = z^*(p^n,w^n)$. Clearly, $\ell^n \to 0$ and for all n one has

$$0 \leqq V_L(p^n,w^n) \leqq (\ell^n)^\beta(M)^{1-\beta} - \delta(\ell^n)^2.$$

Therefore, continuity of V_L implies

$$\lim V_L(p^n, w^n) = 0.$$

A similar argument for the shareholder shows that his utility must go to zero as well. Hence, there exists $\bar{Q}$ on $C \cap K$ which yields maximal utility to the worker and $V_L(p^{\bar{Q}}, w^{\bar{Q}}) > V_L(p^*, w^*)$.

Considering a movement from WE to R on $I \cap K$, one observes that the worker is unconstrained. Since $I \cap K$ is downward sloping, one has $p^R > p^*$ and $w^R < w^*$ for all R on $I \cap K$. Higher prices and lower wages imply a smaller budget set for the worker, so that $V_L(p^R, w^R) < V_L(p^*, w^*)$. For the capitalist one obtains

$$V_S(p,w) = v_S(p,\pi)$$

where $\pi = p[a(p,w)]^{\alpha} - wa(p,w)$ and $a(p,w)$ is the worker's unconstrained labor supply. Differentiating and using Roy's identity, the properties of $a(p,w)$ and $(dw/dp)_{I \cap K}$ from the proof of Proposition 3 yields

$$\left.\frac{dV_S}{dp}\right|_{I \cap K} = \frac{\partial v_S}{\partial \pi} \left\{ \frac{\beta \bar{m}_L}{p} + \left(2 - \frac{p}{w} \alpha \ell^{\alpha - 1}\right) \ell \left[\frac{\beta w}{p} - \left.\frac{dw}{dp}\right|_{I \cap K} \right] \right\}.$$

At (p^*,w^*) one has $(p^*/w^*)\alpha\ell^{\alpha-1} = 1$. Therefore,

$$\left.\frac{dV_S}{dp}\right|_{I \cap K} (p^*,w^*) > 0.$$

However, for $(p,w) \in I \cap K$ and p sufficiently large total output becomes small, so that the shareholder's utility will tend to zero as p tends to infinity. Hence, there exists $\overline{R} \in I \cap K$ which yields maximal utility to the shareholder and $V_S(p^{\overline{R}},w^{\overline{R}}) > V_S(p^*,w^*)$. Combining these results one obtains that a movement on $I \cap C$ away from the Walrasian equilibrium WE implies an increase of the worker's utility and a decrease of the capitalist's utility since

$$V_L(p^{\tilde{P}},w^{\tilde{P}}) > V_L(p^{\overline{Q}},w^{\overline{Q}}) > V_L(p^*,w^*)$$

and

$$V_S(p^{\tilde{P}},w^{\tilde{P}}) < V_S(p^{\overline{Q}},w^{\overline{Q}}) < V_S(p^*,w^*).$$

where $(p^{\tilde{P}},w^{\tilde{P}}) \in I \cap C$ and $w^{\tilde{P}}/p^{\tilde{P}} = w^{\overline{Q}}/p^{\overline{Q}}$. Therefore, both frontiers $I \cap C$ and $C \cap K$ of (p,w)-space are mapped onto two corresponding downward sloping lines in utility space near WE such that $I \cap C$ lies above $C \cap K$. On the other hand, as w/p tends to infinity on $I \cap C$ the utility of both consumers must tend to zero. Therefore, there exists $\overline{P} \in I \cap C$ which gives maximal utility to the worker.

To complete the utility analysis Figure 3 is now a straightforward matter. The three boundaries $I \cap C$, $I \cap K$, and $C \cap K$ meet at the Walrasian equilibrium WE which is a point on the overall utility frontier (U,U') of the economy. The classical region is the area between the two boundaries

$I \cap C$ and $C \cap K$, and the Keynesian region is the one between the boundaries $C \cap K$ and $I \cap K$. The inflationary region of the (p,w)-space is mapped onto the area $C \cup K$ in utility space. The line [P,Q,R] represents a constant level of employment where higher employment implies a shift to the right.

Figure 3 provides all of the necessary information to answer the questions raised in the introduction. The results are summarized in the following propositions, some of which may seem trivial by now, but they are listed for completeness.

Proposition 5: *The set of feasible utility allocations under rationing is a proper subset of the set of all utility allocations of the economy. No Rationing allocation belongs to the utility frontier of the economy.*

Proposition 6: *There exist rationing allocations which are not Pareto dominated by the Walrasian equilibrium.*

Proposition 6 implies in particular that it may be in the interest of a consumer to maintain a rationing situation with sticky prices rather than have a market clearing situation. Specifically, the worker prefers a large set of unemployment situations, classical as well as Keynesian, over the Walrasian equilibrium. On the other hand, the shareholder prefers a large set of inflationary and Keynesian situations over the Walrasian equilibrium. The points $\bar{P}$ and $\bar{R}$ correspond to the outcomes which one of the agents would achieve if he had monopoly power.

Proposition 7: *The frontier of the fix price utility allocations is defined by two of the boundaries whereas the third boundary lies in the interior of the set of utility allocations.*

As a consequence one obtains an answer to the question whether rationing on one market alone can be better than simultaneous rationing on two markets.

Proposition 8: *Every allocation which involves rationing on two markets is Pareto dominated by some allocation with rationing on one market alone.*

Proposition 9: *Every allocation which involves rationing on two markets can be improved by increasing the level of employment.*

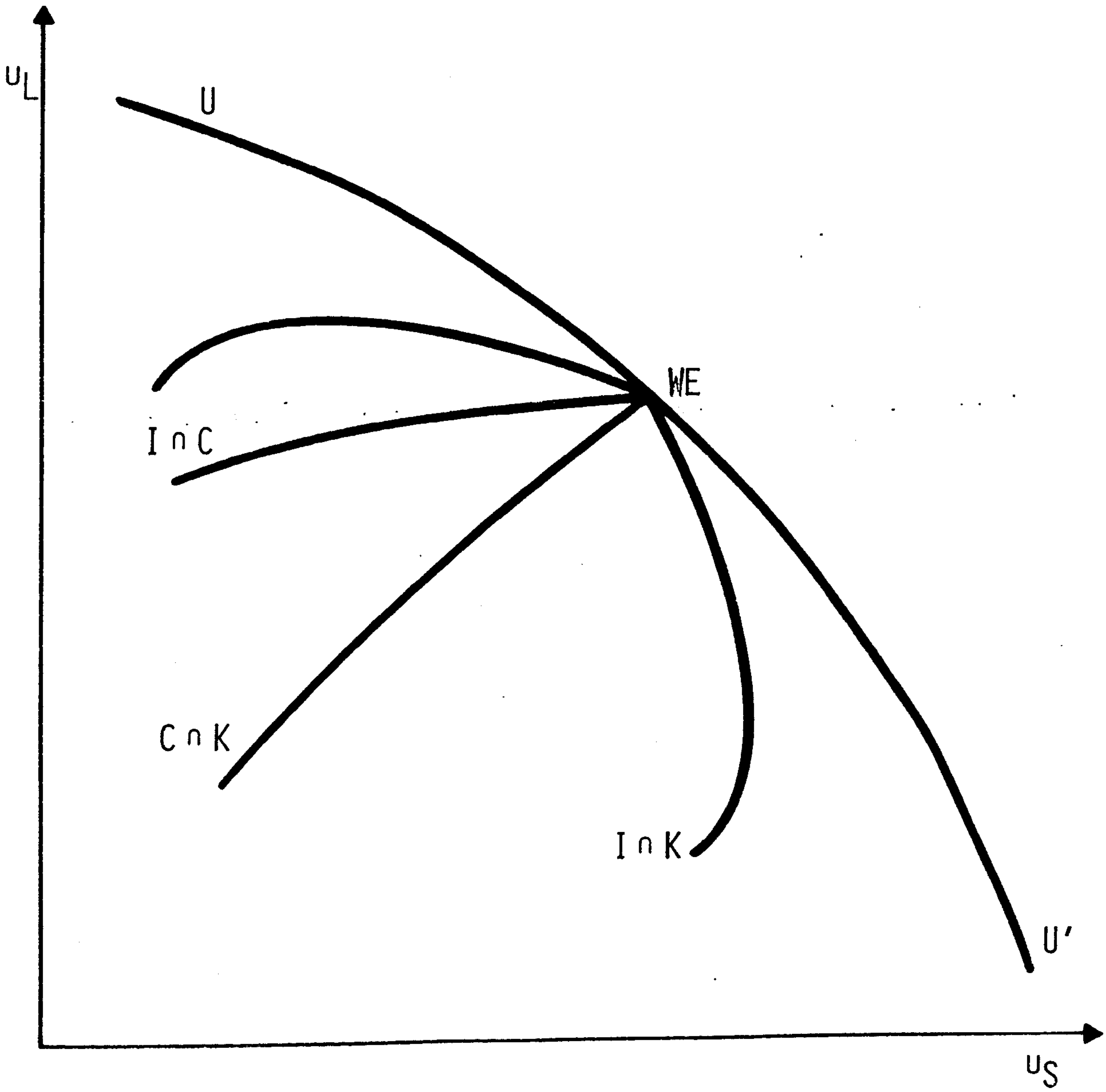

Figure 4

4. Conclusions

The analysis of the third section was carried out using only qualitative properties of the example constructed in section two. Hence small perturbations of the technology and of the consumer characteristics will not change the qualitative nature of the results, i.e. the propositions listed in section three will be true for an open set of economies. Nevertheless, it may be worthwhile to indicate some of the changes which may occur. Figure 4 and 5 indicate two possibilities.

In Figure 4 two of the boundaries of the three regions are in the interior of the utility possibility set under fixed prices. It is clear that this case will occur if the demand rationing mechanism is not efficient in the sense defined above. No general conclusion seems to be possible about the position and slopes of the boundaries in utility space.

Figure 5 describes a boundary case where some rationing situations yield Pareto optimal allocations. Whether these occur on some boundary or in the interior of some region is unclear. However, the type of rationing mechanism and the labor supply behavior of the worker may account for this fact. In either case of Figure 4 and 5, the propositions given in section three may have to be modified.

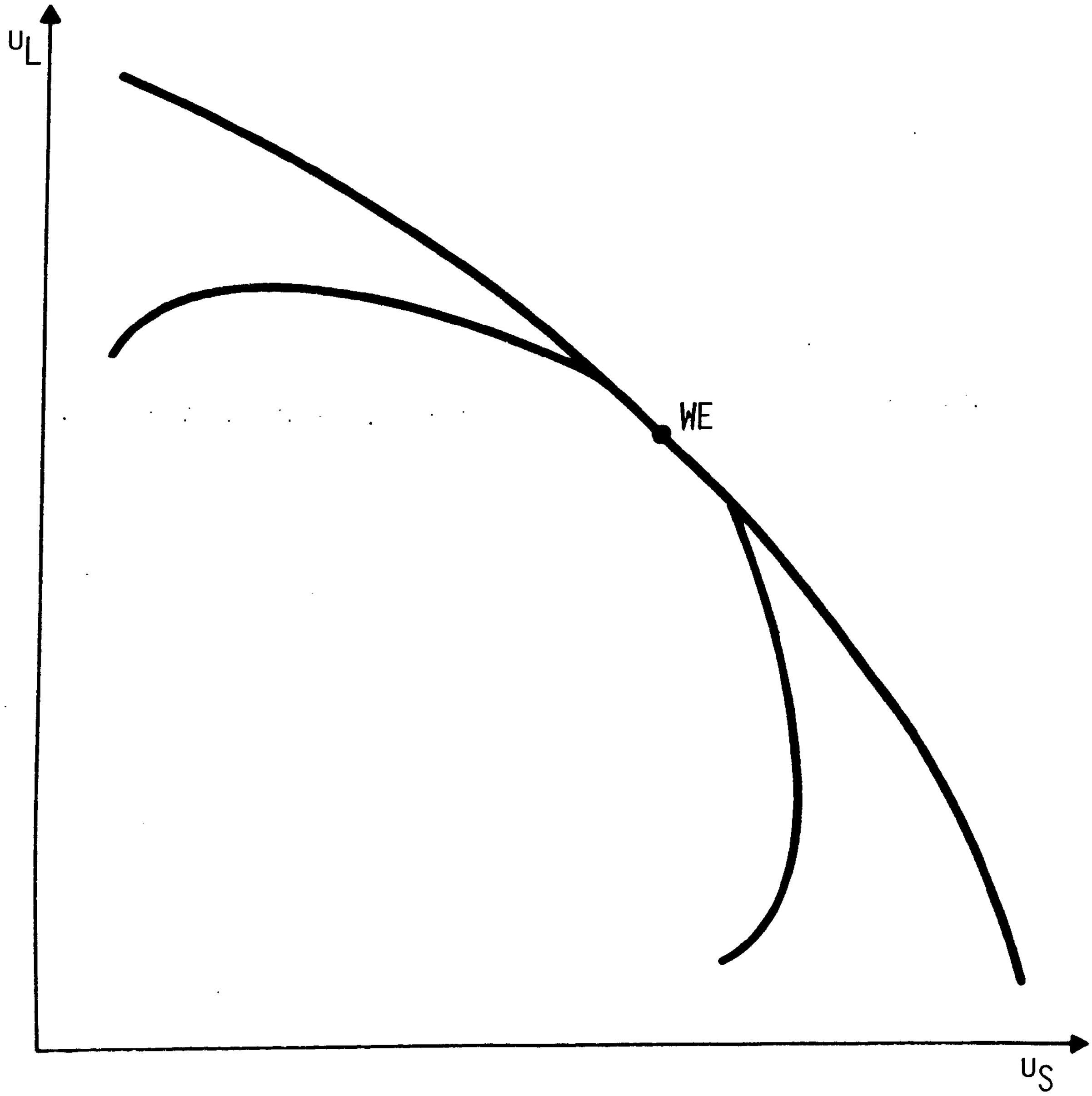

Figure 5

Appendix 1

Unconstrained maximization of the worker

$$\text{Max } x_L^{\beta} m_L^{1-\beta} - \delta \ell^2$$

subject to

$$p x_L + m_L - w\ell - \bar{m}_L = 0$$

yields the supply and demand functions

$$(1.1) \quad \ell^* = a(p,w) = \frac{1-\beta}{2\delta} \left(\frac{\beta}{1-\beta}\right)^{\beta} w p^{-\beta}$$

$$(1.2) \quad x_L^* = \beta \, \frac{\bar{m}_L + wa(p,w)}{p}$$

$$(1.3) \quad m_L^* = (1-\beta)(\bar{m}_L + wa(p,w)) \; .$$

Unconstrained maximization of the capitalist

$$\text{Max } x_S^{1-\beta} m_S^{\beta}$$

subject to

$$p x_S + m_S - \pi - \bar{m}_S = 0$$

yields the demand functions

$$(1.4) \quad x_S^* = (1-\beta) \frac{\bar{m}_S + \pi}{p}$$

$$(1.5) \quad m_S^* = \beta(\bar{m}_S + \pi) .$$

Unconstrained profit maximization of the producer yields the supply function

$$(1.6) \quad y^* = \left(\alpha \frac{p}{w}\right)^{\frac{\alpha}{1-\alpha}} \quad ,$$

the labor demand function

$$(1.7) \quad z^* = (\alpha \frac{p}{w})^{\frac{1}{1-\alpha}}$$

and the profit function

$$(1.8) \quad \pi^* = \Pi(p,w) = (1 - \alpha)p(\alpha \frac{p}{w})^{\frac{\alpha}{1-\alpha}} \quad .$$

Proof of Proposition 1

Define excess demand on the labor market as

$$(1.9) \quad L(p,w) = (\alpha \frac{p}{w})^{\frac{1}{1-\alpha}} - Awp^{-\beta}$$

$$\text{where} \quad A = \frac{1-\beta}{2\delta}\left(\frac{\beta}{1-\beta}\right)^{\beta} \quad .$$

For all $p > 0$, $L(p,w)$ is strictly decreasing in w, and $L \to +\infty$ for $w \to 0$ and $L \to -\infty$ for $w \to +\infty$. Similarly, for all $w > 0$, $L(p,w)$ is strictly increasing in p, and $L \to +\infty$ for $p \to +\infty$ and $L \to -\infty$ for $p \to 0$. Hence, for each $p > 0$, there exists a unique $w > 0$ such that $L(p,w) = 0$. The equilibrium pairs of (p,w) for the labor market are given by

$$(1.10) \quad w = \left[\frac{1}{A}\,\alpha^{\frac{1}{1-\alpha}}\right]^{\frac{1-\alpha}{2-\alpha}} p^{\frac{1+\beta(1-\alpha)}{2-\alpha}} \quad .$$

Since

$$0 < \frac{1+\beta(1-\alpha)}{2-\alpha} < \frac{1+(1-\alpha)}{2-\alpha} = 1,$$

(1.10) defines a strictly increasing and concave function.

Define the excess demand for money as

$$(1.11)\quad \tilde{M}(p,w) = (1-\beta)(\bar{m}_L + wa(p,w)) + \beta(\bar{m}_S + \Pi(p,w)) - M$$

$$= M\left[\gamma(1-2\beta) + \beta - 1\right] + (1-\beta)Aw^2p^{-\beta} + \beta\Pi(p,w).$$

Using (1.8), (1.10) this reduces to a function in p alone of the form

$$(1.12)\quad \tilde{\tilde{M}}(p) = M\left[\gamma(1-2\beta) + \beta - 1\right] + Ap^{\frac{2-\alpha\beta}{2-\alpha}} + Bp^{\frac{2-\alpha-(1+\beta(1-\alpha))\alpha}{(1-\alpha)(2-\alpha)}}$$

where A and B are positive constants. Since $1 > \beta > \frac{1}{2}$ one finds that

$$M\left[\gamma(1-2\beta) + \beta - 1\right] < M(\beta - 1) < 0.$$

Moreover, the two exponents of p are positive, i.e.

$$\frac{2-\alpha\beta}{2-\alpha} > 0$$

and

$$\frac{1}{1-\alpha}\left[1 - \frac{\alpha(1+\beta(1-\alpha))}{(2-\alpha)}\right] > \frac{1}{1-\alpha}\left[1 - \frac{\alpha(2-\alpha)}{2-\alpha}\right] > 0.$$

Therefore, $\tilde{\tilde{M}}(p)$ is strictly increasing in p, $\tilde{\tilde{M}}(0) < 0$ and $\tilde{\tilde{M}}(p) \to +\infty$ as $p \to \infty$. Hence, there exists a unique $\bar{p}$ such that $\tilde{\tilde{M}}(p\ ) = 0$. Substituting into (1.10) yields the unique Walrasian equilibrium. QED

Note that the condition $\alpha \leqq \gamma$ was not used in the proof.

Appendix 2

Keynesian states involve supply rationing only. Define

$$(2.1) \quad c_u(p,w,\ell) = \beta \frac{\overline{m}_L + w\ell}{p}$$

and let

$$(2.2) \quad k(p,w,\ell) = \ell^{\alpha} - (1-\beta)\frac{\overline{m}_S + \pi(\ell)}{p}$$
$$= -(1-\beta)\frac{\overline{m}_S}{p} + \beta\ell^{\alpha} + (1-\beta)\frac{w}{p}\ell$$

denote net supply to the worker after satisfying the capitalist's demand. An employment level $\tilde{\ell}$ such that

$$(2.3) \quad c_u(p,w,\tilde{\ell}) - k(p,w,\tilde{\ell}) = 0 \qquad \tilde{\ell} < \ell^* \quad \tilde{\ell} < z^*$$

is a Keynesian state. It is obvious that $\tilde{\ell}$ is unique.

Effective labor supply of the worker under demand rationing is given by the solution of

$$\text{Max } x_L^{\beta} m_L^{1-\beta} - \delta\ell^2$$

subject to $px_L + m_L - w\ell - \overline{m}_L = 0$

$$x_L \leqq y_L < x_L^* = \beta\frac{\overline{m}_L + wa(p,w)}{p} .$$

This yields as a relation between y_L and ℓ

$$(2.4) \quad y_L = \lambda(p,w,\ell) = \frac{A(\overline{m}_L + w\ell)\ell^{1/\beta}}{1 + pA\ell^{1/\beta}}$$

where $A = \left[\frac{2\delta}{(1-\beta)w}\right]^{1/\beta}$.

$\lambda(p,w,\ell)$ is strictly positive for all $(p,w,\ell) \gg 0$. Moreover,

$$\frac{\partial\lambda}{\partial\ell}(p,w,\ell) > 0 \qquad \text{all } (p,w,\ell) \gg 0$$

$$(2.5) \quad \frac{\partial^2\lambda}{\partial\ell^2}(p,w,\ell) > 0$$

$$\frac{\partial\lambda}{\partial\ell}(p,w,0) = 0 \qquad \text{all } (p,w) \gg 0.$$

Efficient demand rationing at (p,w,ℓ) means that the two rationing levels (y_L,y_S) satisfy $y_L + y_S = \ell^\alpha$ and that the allocation $((y_L,m_L),\ (y_S,m_S))$ is on the contract curve and respects individual budgets. Let

$$(2.6) \quad x_L - \tilde{\beta}\,\frac{\ell^\alpha - x_L}{M - m_L}\,m_L = 0 \qquad \tilde{\beta} = \left(\frac{\beta}{1-\beta}\right)^2$$

denote the contract curve for given ℓ.(2.6) defines a strictly concave and strictly increasing function in x_L since $1 > \beta > \frac{1}{2}$. The net supply to the worker after satisfying the capitalist's demand under efficient rationing is defined by using (2.6) and the budget constraint by

$$y_L(M + (\tilde{\beta} - 1)m_L) - \tilde{\beta}\ell^\alpha m_L = 0$$

$$(2.7)$$

$$py_L + m_L - \bar{m}_L - w\ell = 0$$

which defines y_L and m_L uniquely. For all (p,w,ℓ) such that $\alpha p\ell^{\alpha-1} \geq w$ define the net supply function to the worker as the solution of (2.7) by

$$(2.8)\qquad y_L = h(p,w,\ell)\ .$$

It is straightforward to show that $h(p,w,0) = 0$, $\partial h/\partial\ell(p,w,\ell) > 0$, $\partial h/\partial\ell(p,w,0) = +\infty$ and $h(p,w,\ell)/\ell$ is a decreasing function. Hence, $h(p,w,\ell)$ is strictly concave and increasing in ℓ. Moreover, since the capitalist is rationed along $h(p,w,\ell)$, one has

$$(2.9)\qquad \begin{aligned} &c_u(p,w,\ell) > h(p,w,\ell) > k(p,w,\ell) \qquad 0 \leqq \ell < \tilde{\ell} \\ &h(p,w,\tilde{\ell}) - k(p,w,\tilde{\ell}) = 0. \end{aligned}$$

An inflationary state $\bar{\ell} > 0$ is a solution of

$$h(p,w,\bar{\ell}) - \lambda(p,w,\bar{\ell}) = 0\ . \qquad \begin{aligned} &\bar{\ell} < \tilde{\ell} \\ &\bar{\ell} < \ell^* \\ &\bar{\ell} < z^* \end{aligned}$$

Since h is concave and λ is convex $\bar{\ell}$ is unique.

Proof of Proposition 2

Let $\tilde{\ell} > 0$ denote the unique solution of (2.1) and (2.2). Given $(p,w) \gg 0$, then $(\tilde{\ell},z^*,\ell^*)$ is strictly positive. Moreover, any disequilibrium state defines an employment level $0 < \ell \leqq \text{Min}\ \{\tilde{\ell},z^*,\ell^*\}$, (see Figure A1). The following seven mutually exclusive relative positions of $(\tilde{\ell},z^*,\ell^*)$ yield uniquely the seven possible disequilibrium states WE, I, C, K, $I \cap C$, $I \cap K$, and $C \cap K$ and their associated employment level $\bar{\ell}$.

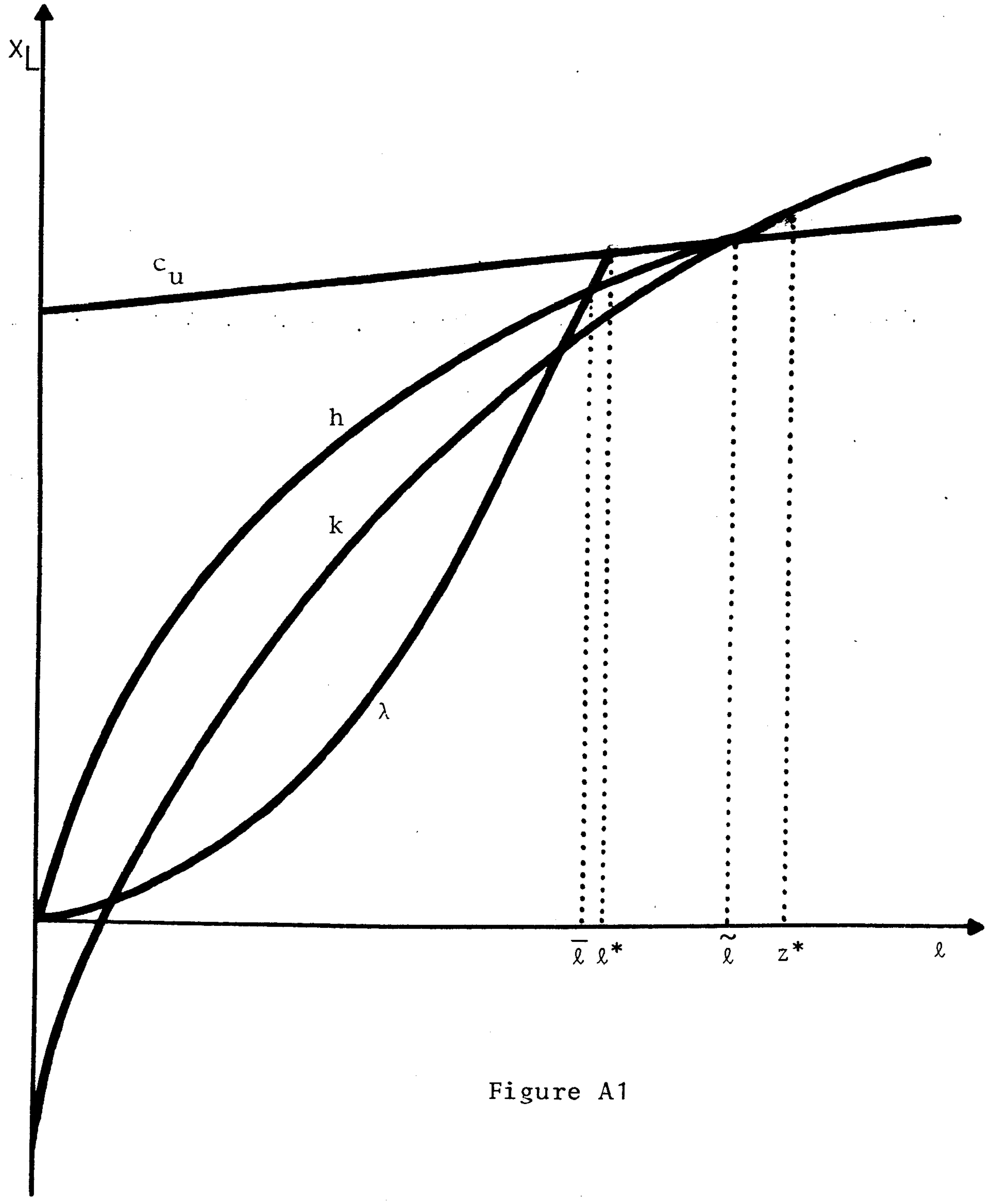

Figure A1

Suppose $z^* < \tilde{\ell}$ and $z^* < \ell^*$. $\tilde{\ell} > \ell^*$ implies that there exists a unique $\ell^* > \overset{=}{\ell} > 0$ such that $h(p,w,\overset{=}{\ell}) = \lambda(p,w,\overset{=}{\ell})$.

1.) $z^* < \overset{=}{\ell}$ or $\ell^* \geqq \tilde{\ell}$ implies $\bar{\ell} = z^*$, therefore $(p,w) \in C$.

2.) $z^* = \overset{=}{\ell}$ implies $\bar{\ell} = z^* = \overset{=}{\ell}$, therefore $(p,w) \in I \cap C$.

3.) $z^* > \overset{=}{\ell}$ implies $\bar{\ell} = \overset{=}{\ell}$, therefore $(p,w) \in I$.

Now assume:

4.) $z^* > \tilde{\ell}$ and $\ell^* > \tilde{\ell}$. This implies $\bar{\ell} = \tilde{\ell}$. Therefore $(p,w) \in K$,

5.) $z^* > \tilde{\ell} = \ell^*$ which implies $\bar{\ell} = \tilde{\ell}$. Therefore $(p,w) \in I \cap K$.

6.) $\ell^* > \tilde{\ell} = z^*$ which implies $\bar{\ell} = \tilde{\ell}$. Therefore $(p,w) \in C \cap K$.

Finally,

7.) $z^* = \ell^* = \tilde{\ell}$ implies $(p,w) \in WE$.

QED.

Appendix 3

Proof of Proposition 3: The boundary $C \cap K$ is defined by

$$(3.1) \qquad \beta\bar{m}_L + \beta w z^* - p z^{*\alpha} + (1-\beta)\bar{m}_S + (1-\beta)\Pi(p,w) = 0.$$

Substituting the unrestricted labor demand and profit functions from Appendix 1 and solving, yields

$$(3.2) \qquad w\Big|_{C\cap K} = \alpha\left[\frac{\beta\bar{m}_L + (1-\beta)\bar{m}_S}{\left[\beta + \alpha(1-2\beta)\right]}\right]^{\frac{\alpha-1}{\alpha}} p^{1/\alpha}.$$

Since $\frac{1}{2} < \beta < \frac{2}{3}$ one has

$$\beta + \alpha(1-2\beta) > \beta + 1 - 2\beta = 1 - \beta > 0.$$

Hence the boundary is well defined and

$$(3.3) \qquad \frac{dw}{dp}\Big|_{C\cap K} = \frac{1}{\alpha}\frac{w}{p}.$$

The boundary $I \cap K$ is defined by

$$(3.4) \qquad \begin{aligned} &\beta\bar{m}_L + (1-\beta)\bar{m}_S + w\ell[2\beta - 1] - \beta p\ell^{\alpha} = 0 \\ &\ell = a(p,w) \end{aligned}$$

Differentiating (3.4) and using (1.1) yields

$$(3.5) \qquad \frac{dw}{dp}\Big|_{I\cap K} = \frac{\beta\left[(2\beta-1)\frac{w}{p}\ell + (1-\alpha\beta)\ell^{\alpha}\right]}{\ell\left[2(2\beta-1) - \frac{p}{w}\alpha\beta\ell^{\alpha-1}\right]}$$

The numerator is positive since $\beta > \frac{1}{2}$. Since $\frac{p}{w}\alpha\ell^{\alpha-1} \geqq 1$

along $I \cap K$ and $\frac{1}{2} < \beta < \frac{2}{3}$ one has for the denominator

$$2(2\beta - 1) - \frac{p}{w}\,\alpha\beta\ell^{\alpha-1} \leqq 3\beta - 2 < 0.$$

Hence (3.5) is negative.

The boundary $I \cap C$ is defined by

$$(3.6) \qquad \begin{aligned} \lambda(p,w,\ell) - h(p,w,\ell) &= 0 \\ \ell - z^{*}(p,w) &= 0 \,. \end{aligned}$$

Define $\hat{x} = y_L/m_L$. Then (3.6) can be written as

$$(p\hat{x} + 1)\left[\hat{x}M - \tilde{\beta}\ell^{\alpha}\right] + \hat{x}(\tilde{\beta} - 1)(\bar{m}_L + w\ell) = 0$$

$$(3.7) \qquad \hat{x} = \left[\frac{2\delta}{w(1-\beta)}\right]^{\frac{1}{\beta}} w^{-\frac{1}{\beta}}\, \ell^{\frac{1}{\beta}}$$

$$\ell = \left(\alpha\,\frac{p}{w}\right)^{\frac{1}{1-\alpha}}$$

where $\tilde{\beta} = \beta^2/(1 - \beta)^2$.

Differentiating (3.7) yields

$$(3.8) \qquad \left.\frac{dw}{dp}\right|_{I \cap C} = \frac{w}{p} \cdot \frac{1 - \alpha\beta + p\hat{x}\left[\frac{\alpha(2\beta - 1) - \beta^2}{\beta} + (1 + \beta(1 - \alpha))\frac{M\hat{x}}{\tilde{\beta}\ell^{\alpha}}\right]}{2 - \alpha - \alpha\beta + p\hat{x}\left[\frac{\alpha(\alpha(2\beta - 1) - \beta^2)}{\beta} + (2 - \alpha)\frac{M\hat{x}}{\tilde{\beta}\ell^{\alpha}}\right]}$$

Efficient demand rationing implies that

$$\frac{\hat{x}M}{\ell^{\alpha}} > 1 \text{ and } p\hat{x} < \frac{\beta}{1 - \beta}\,.$$

One obtains for the numerator

$$p\hat{x}\left[\frac{1-\alpha\beta}{p\hat{x}} + \frac{\alpha(2\beta-1)-\beta^2}{} + (1+\beta(1-\alpha))\,\frac{M\hat{x}}{\tilde{\beta}\ell^{\alpha}}\right]$$

$$> \frac{p\hat{x}}{\beta^2}\left[\alpha\beta(3\beta-2)+1-2\beta^2\right] > \frac{p\hat{x}}{\beta^2}\left[3\beta^2-2\beta+1-2\beta^2\right]$$

$$= \frac{p\hat{x}}{\beta^2}(1-\beta)^2 > 0.$$

Similarly one obtains for the denominator

$$\left[p\hat{x}\;\frac{2-\alpha-\alpha\beta}{p\hat{x}} + \frac{\alpha(\alpha(2\beta-1)-\beta^2)}{\beta} + (2-\alpha)\,\frac{M\hat{x}}{\tilde{\beta}\ell^{\alpha}}\right]$$

$$> \frac{p\hat{x}}{\beta^2}\left[(1-\beta)(2+\alpha\beta)-\alpha+\alpha^2\beta(2\beta-1)\right]$$

$$> \frac{p\hat{x}}{\beta^2}\left[(1-\beta)(2+\beta)-1\right] = \frac{p\hat{x}}{\beta^2}(1-\beta)^2 > 0\;.$$

Therefore, the denominator is positive and

$$\left.\frac{dw}{dp}\right|_{I\cap C} > 0\;.$$

QED

Appendix 4

Proof of Proposition 4: For $(p,w) \in C$ employment is constant if and only if w/p is constant.

For $(p,w) \in K$ one has from (2.3)

$$\beta(\bar{m}_L + w\ell) + (1-\beta)\bar{m}_S - p\beta\ell^{\alpha} - (1-\beta)w\ell = 0 .$$

Hence

$$(4.1) \qquad \left.\frac{dw}{dp}\right|_{\substack{K \\ \ell=\text{const}}} = \frac{\beta\ell^{\alpha}}{(2\beta-1)\ell} > 0 .$$

For the isoemployment curves in I, it suffices to differentiate the first two equations of (3.7) keeping ℓ constant.

$$(4.2) \qquad \left.\frac{dw}{dp}\right|_{\substack{I \\ \ell=\text{const}}} = \frac{\beta w\hat{x}[\hat{x}M - \tilde{\beta}\ell^{\alpha}]}{\tilde{\beta}\ell^{\alpha} + p\hat{x}^2M - \beta(\tilde{\beta}-1)\hat{x}w\ell}$$

The numerator is negative. Since $p\alpha\ell^{\alpha} \geq w\ell$ and $(1-\beta)p\hat{x} < \beta$ one obtains for the denominator

$$\tilde{\beta}\ell^{\alpha} - \beta(\tilde{\beta}-1)\hat{x}w\ell + p\hat{x}^2M$$

$$\geq \frac{w\ell}{p}\tilde{\beta}\left[\frac{1}{\alpha} - \frac{(2\beta-1)}{\beta}p\hat{x}\right] > \frac{w\ell}{p}\tilde{\beta}\left[\frac{1}{\alpha} - \frac{2\beta-1}{1-\beta}\right] > 0$$

for $\frac{1}{2} < \beta < \frac{2}{3}$. Therefore,

$$\left.\frac{dw}{dp}\right|_{\substack{I \\ \ell=\text{const}}} < 0 .$$

QED

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