

GLUON THERMODYNAMICS NEAR THE CONTINUUM LIMIT

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The critical parameters of the SU(3) deconfinement phase transition are determined on lattices of sizes $12^3 \times 5$ and $16^3 \times 6$ points which allows an analysis close to the continuum limit. We find a drastic improvement in the scaling behaviour of the critical temperature measured in units of Λ_L : on the $16^3 \times 6$ lattice we find a value of $T_c/\Lambda_L = 65.5 \pm 1$ at $\beta = 5.93 \pm 0.01$.

1. Introduction. Strong evidence has been found for the existence of a first order deconfining phase transition in pure gauge lattice QCD [1-3]. In order to translate the values for the critical temperature and the latent heat from the lattice units to the physical ones, one needs an independent measurement of a physical dimensionful quantity on the lattice.

Typically, one takes the value of the slope of a linearly rising q-q potential, i.e. the string tension. In principle the critical temperature (T_c) and the string tension can be measured at different values of the coupling constant ($\beta = 6/g^2$) provided that the measurement is performed in a region of β where both quantities follow the scaling law implied by the asymptotic freedom:

$$a\Lambda_L = \exp\left[-\frac{4}{33}\pi^2\beta + \frac{51}{121}\ln\left(\frac{8}{33}\pi^2\beta\right)\right]. \quad (1)$$

While the old measurement of the string tension showed a rough agreement with such a law [4], the latest more precise results show drastic deviations for values of β between $\beta = 5.4$ and $\beta = 6$ [2,3]. Therefore, in order to take advantage from the knowledge of the string tension closer to the continuum limit, one has to push the measurements of T_c to the corresponding values of β . Actually, the values of T_c at β larger than 5.7 may serve for a more accurate analysis

of the scaling violations themselves.

The analysis of the deconfinement transition at large β involves the use of lattices with a sizeable time direction: in this paper we present our results for lattices of $12^3 \times 5$ and $16^3 \times 6$ points. We will discuss the implications of our results for the thermodynamics of those systems and their relevance for the analysis of the scaling behaviour of lattice QCD.

2. Gluon thermodynamics. The dynamics of lattice QCD at finite temperature can be obtained from the following partition function:

$$Z_\beta = \int \prod_{x,\mu} dU_{x,\mu} \exp\left(-\beta \sum_p \left(1 - \frac{1}{3} \text{Re Tr } U_p\right)\right), \quad (2)$$

where the $U_{x,\mu}$ are the usual link variables on an asymmetric lattice of size $N_\sigma^3 \times N_\tau$ [7] and U_p is the product of links around a plaquette. The temperature of the system is related to the finite extent of the system in the time direction:

$$Ta = 1/N_\tau, \quad (3)$$

where a is the lattice spacing.

Characteristic of a first order phase transition is the presence of discontinuities in thermodynamical quantities as the energy density which is related to the

existence of a latent heat. At the critical temperature two different phases can coexist and they can be monitored by the value of the natural order parameter of the transition, the expectation value of the thermal loop defined as:

$$L = \left\langle \text{Tr} \prod_{x_4=1}^{N_\tau} U(\mathbf{x}, x_4, \hat{4}) \right\rangle, \quad (4)$$

which is related to the free energy F_q of a static quark:

$$L \sim \exp(-F_q/T). \quad (5)$$

In the confined phase one expects $L = 0$, while a nonzero value is expected in the deconfined one: for a first order phase transition L has a discontinuity at the transition point. Similarly, the discontinuity in the energy density leads to a latent heat $\Delta\epsilon$ which can be expressed in terms of the space-like (P_σ) and time-like (P_τ) average plaquette ^{#1}. In fact in the most general case where different coupling constants are defined for the space and the time plaquettes, the energy density reads ^{#2}:

$$\epsilon a^4 = 18 [g^{-2}(P_\sigma - P_\tau) + c'_\sigma(P - P_\sigma) + c'_\tau(P - P_\tau)], \quad (6)$$

where c'_σ, c'_τ are related to the derivative of the couplings with respect to the lattice spacing in the time direction and P is the corresponding average plaquette for a symmetric lattice. The latent heat is given by the difference of the energy density in the two phases.

For an accurate estimate of the latent heat $\Delta\epsilon$ one would need the knowledge of the space-like and the time-like plaquettes measured at the critical temperature both in the confined and in the deconfined phase. Given the large amount of computer time required by the analysis of the large lattices we are considering, we could not search for the value of the critical temperature defined through the value of β where the two phases coexist. For a fixed lattice size we scan a discrete set of values of β which allows us to establish a small window where the phase transition occurs.

As a consequence, our estimate of the latent heat will be only an approximate one:

$$\Delta\epsilon/T^4 = 3N_\tau [\beta^+(P_\sigma^+ - P_\tau^+) - \beta^-(P_\sigma^- - P_\tau^-)], \quad (7)$$

where the plus (minus) indicates the value taken at the larger (smaller) β . The actual size of the terms related to the coefficients c'_σ and c'_τ in eq. (6) is known to be of the order of ten percent what makes our approximate estimate of eq. (7) rather good.

In order to get a reasonable precision on the value of β where the transition occurs we scanned with steps of $\Delta\beta = 0.03$ on the $12^3 \times 5$ lattice and of $\Delta\beta = 0.02$ on the $16^3 \times 6$ which roughly corresponds to a temperature window of $2\Lambda_L$ units in both cases. We give the values of the real part of the thermal loop modulo a $Z(3)$ rotation averaged over fifty sweeps at the values of β which determine the "critical window" in fig. 1 for the $12^3 \times 5$ lattice and in fig. 2 to the $16^3 \times 6$: in total we run for 2000 sweeps at each β

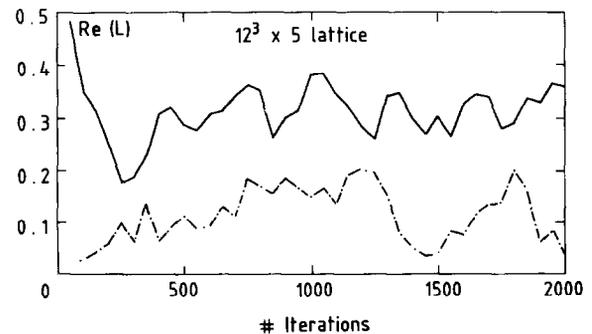


Fig. 1. The real part of the thermal Wilson line on a $12^3 \times 5$ lattice versus the number of Monte Carlo sweeps for $\beta = 5.82$ (solid curve) and $\beta = 5.79$ (broken curve).

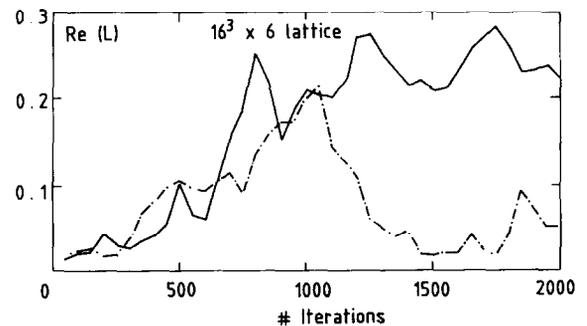


Fig. 2. The real part of the thermal Wilson line on a $16^3 \times 6$ lattice versus the number of Monte Carlo sweeps for $\beta = 5.94$ (solid curve) and $\beta = 5.92$ (broken curve).

^{#1} A plaquette is said to be time-like if it contains two time-like links and space-like otherwise.

^{#2} For details on the finite temperature formalism on euclidean see ref. [7].

Table 1

Summary of Monte Carlo results for the average plaquette (P), the difference of space- and time-like plaquettes (ΔP) and the real part of the thermal Wilson line [$\text{Re}(L)$]. The number in brackets indicates the error in the last digit.

β	N_τ	P	ΔP	$\text{Re}(L)$
5.79	5	0.4336(3)	0.00008(6)	0.11(2)
5.82	5	0.4274(1)	0.00039(6)	0.32(2)
5.92	6	0.4155(2)	0.00001(4)	0.06(2)
5.94	6	0.41250(8)	0.00014(4)	0.238(8)

value. The values of the plaquette, of the difference between space and time plaquettes and of $\text{Re}(L)$ averaged over the last 1500 sweeps for $N_\tau = 5$ and over the last 1000 sweeps for $N_\tau = 6$ are reported in table 1. The critical temperature can be calculated in terms of Λ_L units assuming the validity of eq. (1); we get:

$$T_c/\Lambda_L = 68.5 \pm 1 \quad (N_\tau = 5),$$

$$T_c/\Lambda_L = 65.5 \pm 1 \quad (N_\tau = 6). \quad (8)$$

The corresponding values for the latent heat are:

$$\Delta\epsilon/T_c^4 = 3.1 \pm 0.4 \quad (N_\tau = 5),$$

$$\Delta\epsilon/T_c^4 = 2.7 \pm 0.7 \quad (N_\tau = 6), \quad (9)$$

where a correction factor has been included because of the finite size of our lattice [8]. More explicitly the deviation from the behaviour of an ideal gas in an infinite volume is calculated to amount to a factor 1.37 and 1.23 for lattice sizes of $12^3 \times 5$ and $16^3 \times 6$ respectively: one divides by these factors the naive values which can be obtained from the table 1 to-

gether with eq. (6) to obtain the values reported in eq. (8). The comparison with preexisting results is summarized in the first four columns of table 2: the fact that the dimensionless ratio T_c/Λ_L does not stay constant confirms that previous calculations were not performed in a region where the asymptotic scaling of eq. (1) is valid. Most of the violations occur between $\beta = 5.5$ and $\beta = 5.8$ while our results between 5.8 and 5.9 seem to indicate an approach to the scaling regime. As far as the values of $\Delta\epsilon/(T_c)^4$ are concerned, one should keep in mind that, besides the statistical errors quoted in eqs. (8), one has also an additional systematic error coming from the finite lattice size which is not accounted by the correction factor that we calculated. For, on a lattice with finite space dimensions the sharpness of the transition between the two phases is smoothed because of phase flips even only slightly below or above the critical temperature. We believe that this effect tends to reduce the measured value of the latent heat with respect to the one obtained with infinite space dimensions.

3. Scaling violations. The scaling violations for the string tension in units of Λ_L follow a path similar to the one of the critical temperature: actually, some of the recent measurements at $\beta = 6$ compared to the results at $\beta = 5.4$ and $\beta = 5.7$ indicate even larger deviations from the asymptotic scaling law eq. (1). This results in a variation of the ratio $T_c/\sigma^{1/2}$ which, with the present data, ranges between 0.55 and 0.7 over the region from $\beta = 5.5$ to $\beta = 6$. This corresponds to a variation of T_c in physical units between 220 and 280 MeV and of $\Delta\epsilon$ between 1 and 2 GeV/fm³. A great source of uncertainty in these numbers comes from the determination of the string tension from Wilson loop expectation values. In particular, sublead-

Table 2

Critical parameter of the deconfinement phase transition obtained on lattices with different extent in time direction (N_τ). The different entries are explained in the text. The data in column 2, 3 and 4 for $N_\tau = 2, 3, 4$ are taken from T. Celik et al. quoted in ref. [3].

N_τ	$6/g^2$	Δ/T_c^4	T_c/Λ_L	T_c/Λ_{eff}	T_c/Λ_{eff}
2	5.11 ± 0.01	3.65 ± 0.15	78 ± 1	—	—
3	5.55 ± 0.01	3.9 ± 0.2	86 ± 1	17.5	65
4	5.70 ± 0.01	3.7 ± 0.5	76 ± 1	18.6	63
5	5.79 – 5.82	3.1 ± 0.4	68.5 ± 1	18.6	60
6	5.92 – 5.94	2.7 ± 0.7	65.5 ± 1	18.8	60

ing terms in the behaviour of the q - q potential at medium distances may influence the extraction of the term which is linearly increasing. The determination of the critical temperature seems to us affected by systematic errors which are under better control and allow for a cleaner analysis of scaling violations. These appear to be sufficiently small in the range between $\beta = 5.8$ and $\beta = 5.9$ to be explained by the present estimates of the perturbative corrections of order $1/\beta$ to the asymptotic behaviour of eq. (1) [9]; those for β between 5.4 and 5.8 seem too large to support such an explanation.

Indeed nonperturbative phenomena play an important role in the same β region where they give rise to the peak in the plaquette-plaquette correlation. On the other hand, there is a kind of universal character in the pattern of scaling violations in units of Λ_L which suggests that they can be simultaneously reduced by a suitable redefinition of β . A possible choice has been already used in the past for the nonlinear sigma model and consists in defining an effective β in terms of its perturbative relation to the average plaquette [10]:

$$\beta_{\text{eff}} = 2/(1 - \frac{1}{3} \langle \text{Tr } U_p \rangle). \quad (10)$$

With this definition the new β will follow the fluctuations of the average plaquette in the crossover region. The new ratios in terms of Λ_{eff} defined by the eq. (1) by replacing β by β_{eff} are given in the fifth column of table 2. The scaling violations are now down to a perturbative size over the whole range of β .

A standard explanation of the peak of the plaquette-plaquette correlation in the crossover region relies on the analysis of the phase structure of the theory in the enlarged coupling constant space which includes the coupling related to a term in the action containing the plaquette made of links in the adjoint representation (β_a). The behaviour in the crossover region is then explained as due to the influence of a first order phase transition line which ends near to the $\beta_a = 0$ axis. Using a "universality" hypothesis for different lattice actions and some arguments based on the loop equations. Makeenko and Policarpov [11] have found an effective β which should reduce the scaling violations for the Wilson action. The scaling in this variable corresponds to the naive scaling of eq. (1) along a line in the (β, β_a) plane which moves further away from the first order transition line with

respect to the $\beta_a = 0$ line. By using their variable we get the results shown in the last column of table 2. The calculation of ref. [11] can be further improved by the knowledge of higher order ($1/N$) corrections evaluated in ref. [12].

The improvement found by using an effective β value suggests that the naive scaling of eq. (1) might be more precocious if the simulations were performed directly in the (β, β_a) plane. However, the most important result for us remains the sizeable reduction of the scaling violations close to $\beta = 6$ for the Wilson action. This strongly indicates that our calculations are sufficiently near the continuum limit to make us confident on the validity of our estimates of the critical parameters of pure gauge lattice QCD.

The updating procedure used for our Monte Carlo program was the heat bath method of Cabibbo and Marinari [13]. The total amount of computer time spent was about 180 CPU hr on the CDC 875.

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