

Color screening and deconfinement for bound states of heavy quarks [★]

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Abstract. We study the binding and deconfinement of heavy quarks in a thermal environment, using a non-relativistic confinement potential model with color screening. As a result, we obtain the dependence of the dissociation energies, the binding radii and the masses of heavy quark resonances (charmonium and bottomium states) on the color screening length r_D of the medium, and we determine for the different resonances those values of r_D below which no more binding is possible. Finally, we consider the implication of our results on resonance suppression as signal for deconfinement.

1 Introduction

Strongly interacting matter of sufficiently high density is predicted to undergo a transition to a state of deconfined quarks and gluons. Deconfinement occurs when color screening shields a given quark from the binding potential of any other quarks or antiquarks. Bound states of very heavy quarks, such as the J/ψ or the Y , have radii which are much smaller than those of the usual mesons and nucleons; hence they can survive in a deconfined medium until the temperature or density becomes so high that screening also prevents their tighter binding. The suppression of J/ψ

or ψ' production, however, appears to provide so far the only unambiguous signal for quark deconfinement [1]. Color screening and deconfinement for heavy quark resonances are therefore crucial for the experimental investigation of quark plasma formation.

In this paper, we want to study the onset of deconfinement for heavy quark bound states in the framework of a non-relativistic potential model for charmonium ($c\bar{c}$) and bottomium ($b\bar{b}$). The Hamiltonian of such a system is given by

$$H(r, T) = 2m - \frac{1}{m} \nabla^2 + V(r, T), \quad (1.1)$$

where m denotes the quark mass. For the interquark potential $V(r, T)$ we start from the Cornell form [2].

$$V(r, 0) = \sigma r - \frac{\alpha}{r}, \quad (1.2)$$

with $\sigma = 0.192 \text{ GeV}^2$ and $\alpha = 0.471$, as determined in a detailed recent analysis [3]; we also fix the quark masses at the values obtained there: $m_c = 1.320 \text{ GeV}$ and $m_b = 4.746 \text{ GeV}$. The $1/r$ term in (1.2) contains both transverse string motion [4] and the perturbative one-gluon exchange contribution [6]. In a thermodynamic environment of interacting light quarks and gluons, at temperature T , quark binding becomes modified by color screening. We parametrize this in the form

$$V(r, T) = (\sigma/\mu(T))(1 - e^{-\mu(T)r}) - (\alpha/r) e^{-\mu(T)r} \quad (1.3)$$

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where $\mu(T)=1/r_D(T)$ is the inverse screening length. The specific screening factor for the linear part of the potential is suggested by the Schwinger model [6]. For $\mu=0$, we recover the confining potential (2). For $\mu \neq 0$, the form (3) for the screened interquark potential satisfies

$$\lim_{r \rightarrow 0} r V(r, T) = -\alpha, \quad (1.4)$$

so that we have the expected $1/r$ behavior in the short distance limit. For large r ,

$$\lim_{r \rightarrow \infty} \frac{1}{r} \ln[\sigma/\mu - V(r, T)] = -\mu(T), \quad (1.5)$$

so that the range of the binding force decreases exponentially with the screening $\mu(T)$. Since the temperature dependence of the potential is completely contained in μ , we write from now on $V(r, \mu)$.

2 The semi-classical approximation

To obtain some feeling for the effect of screening on heavy quark bound states, we first look at the semi-classical approximation of (1.1). It is given by

$$E(r, \mu) = 2m + \frac{c}{mr^2} + V(r, \mu), \quad (2.1)$$

obtained from $\langle p^2 \rangle \langle r^2 \rangle = c$, where the uncertainty relation makes c of the order of unity; its precise value depends on the wave functions for the Hamiltonian (1). Minimizing $E(r, T)$ with respect to r gives us the temperature dependence of the lowest bound state radius r_0 . We equate $E(r_0, \mu=0)$ to the spin-averaged mass values [3]; thus we obtain $c=1.487$ (1.181) and $r_0=0.383$ fm (0.165 fm) for the $c\bar{c}$ ($b\bar{b}$) system. We note that the radii obtained by minimizing the semi-classical form (8) lie approximately 15% below the corresponding quantum-mechanical values [7] obtained by calculating the wave functions for (1.1) and from it the average radii.

We now increase the screening mass μ , keeping m , c , σ , and α fixed, and determine for each μ the value of r for which $E(r, \mu)$ has a minimum. For sufficiently small μ , such a minimum exists, because the decrease of the kinetic energy with r is overcome by the increase of the potential energy. Once the screening has become strong enough, however, this is no longer possible and $E(r, \mu)$ decreases monotonically with μ . Thus there is a largest $\mu = \mu_c$, above which bound states are impossible. We would like to know the value of μ_c and of the bound state radius at this point of "last binding" for both the $c\bar{c}$ and the $b\bar{b}$ system.

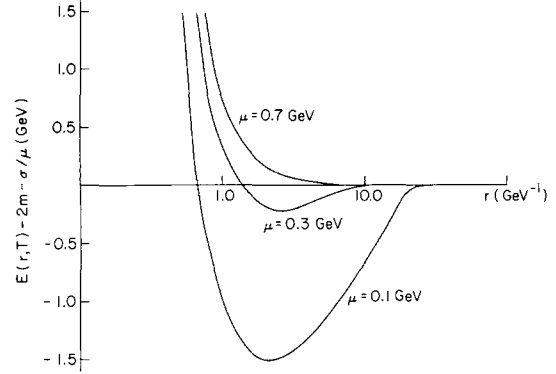


Fig. 1. Effective binding potential in the semi-classical approximation

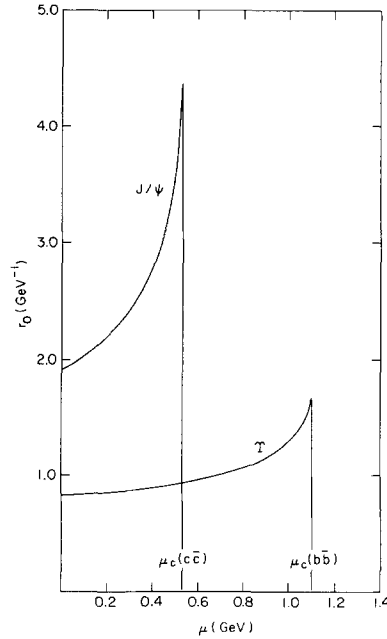


Fig. 2. Radii of the lowest $c\bar{c}$ and $b\bar{b}$ bound states in the semi-classical approximation

In Fig. 1 we illustrate the behavior of the effective binding potential $E(r, \mu) - 2m - \sigma/\mu$ for the $c\bar{c}$ system at several values of μ , and in Fig. 2 we show the μ -dependence of the lowest bound state radii for $c\bar{c}$ and $b\bar{b}$. The values of μ_c and of the corresponding bound state radii are given in Table 1. Also listed there are the corresponding screening lengths. We see from this that when μ has reached about 0.5 GeV ($r_D \simeq 0.4$ fm), all $c\bar{c}$ binding becomes impossible. At a higher temperature, corresponding to $\mu \simeq 1.1$ GeV ($r_D \simeq 0.2$ fm), the same happens to the $b\bar{b}$ system. In both cases, the radius at the point of "last binding" has increased to two or three times its value of $\mu=0$.

From these considerations, it is qualitatively quite clear what happens in the screening process. Let us now look in more detail at the full model defined by (1.1).

Table 1. Critical values for color screening parameters and binding radii in the semi-classical approximation

	$c\bar{c}$	$b\bar{b}$
μ_c [GeV]	0.53	1.10
$r_D = \mu_c^{-1}$ [fm]	0.38	0.18
$r_0(\mu_c)$ [fm]	0.87	0.33

3 The numerical evaluation of the Schroedinger equation

We want to consider the numerical solution of the eigenvalue equation

$$[H(r, \mu) - E_{n,l}(\mu)] \Phi_{n,l}(r, \mu) = 0, \quad (3.1)$$

where the eigenvalues are classified by the principal quantum number n and the orbital quantum number $l \leq (n-1)$. We shall here restrict ourselves to the first two radial excitations, corresponding to the (spin-averaged) J/ψ and Y for $n=1, l=0$, to the ψ' and Y' for $n=2, l=0$, and to the χ_c and χ_b for $n=2, l=1$. Solving (3.1) gives us the bound state masses as functions of μ , and with the wave functions $\Phi_{n,l}(r, \mu)$ we calculate the corresponding (r.m.s.) bound state radii.

The most suitable quantity to observe the vanishing of bound states is the dissociation energy

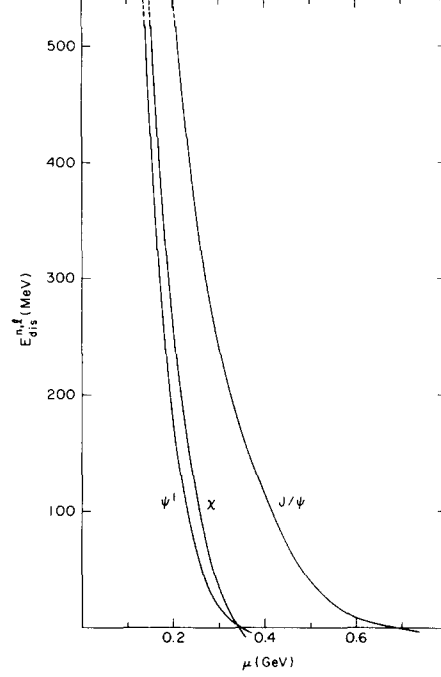
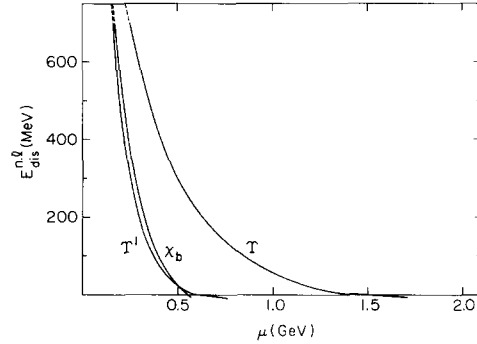
$$E_{\text{dis}}^{n,l}(\mu) \equiv 2m + \sigma/\mu - E_{n,l}(\mu); \quad (3.2)$$

it is positive for bound states and becomes negative for the continuum. Thus

$$E_{\text{dis}}^{n,l}(\mu_c) = 0 \quad (3.3)$$

defines the critical value of μ , beyond which there are no bound states of the given quantum numbers. In Figs. 3 and 4 we show our results for $E_{\text{dis}}(\mu)$ of the $c\bar{c}$ and $b\bar{b}$ states, respectively. The most important results are summarized in Table 2. In contrast to the semi-classical approximation, the quantum mechanical form appears to lead to diverging radii when $\mu \rightarrow \mu_c$; we show this in Figs. 5 and 6 for the $c\bar{c}$ and $b\bar{b}$ states, respectively. We further note that the masses of all bound states are only slightly effected by a change in μ . The masses of the $c\bar{c}$ bound states and those of the higher $b\bar{b}$ bound states decrease with μ , while the Y mass increases. This occurs because in general the positive string tension part of the potential dominates and is reduced as μ increases; only for the Y does the negative $1/r$ part give the main contribution.

In the form (1) of the finite temperature bound state problem, we have introduced the temperature dependence entirely through the screening factors.

**Fig. 3.** Dissociation energies for $c\bar{c}$ bound states**Fig. 4.** Dissociation energies for $b\bar{b}$ bound states

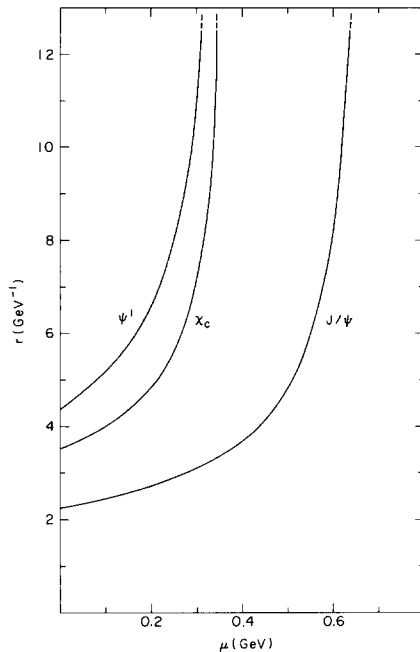
Statistical QCD will in general provide a temperature dependent potential $V(r, T)$, and this could of course also be parametrized in the form of a temperature dependent string tension, $\sigma(T)$, together with a screened $1/r$ term. We want to note, however, that our formulation is in fact equivalent to one with a temperature dependent string tension

$$\sigma(T) = \sigma(0) \left[\frac{1 - e^{-\mu(T)r}}{\mu(T)r} \right], \quad (3.4)$$

valid in some “confining” range of r values. With $r = r_{J/\psi}(\mu)$ as given in Fig. 5, we obtain the string tension shown in Fig. 7. We thus allow a non-vanishing, screened string tension term also in the deconfined phase. This should be thought of as a parametrization of non-perturbative features of the plasma close to the transition point. The detailed form of the potential

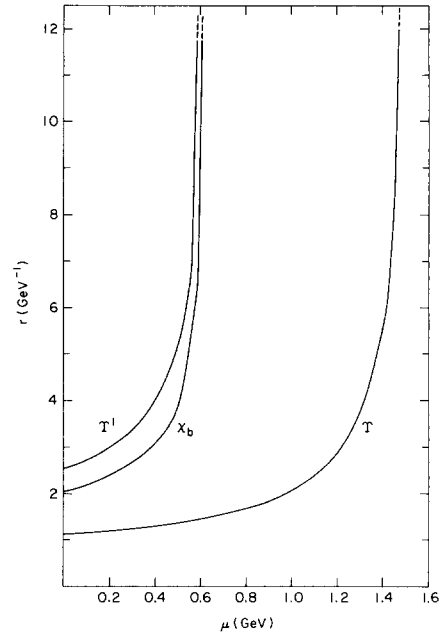
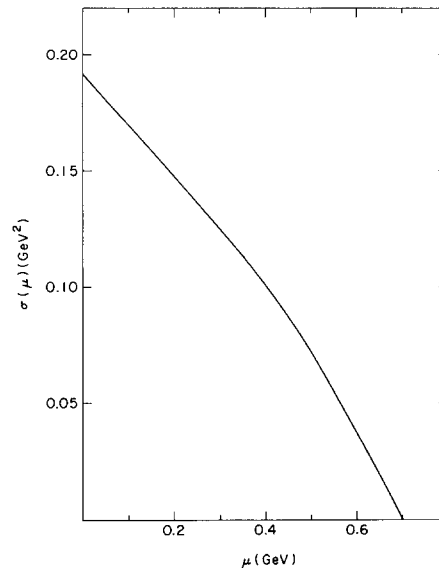
Table 2. Parameters for $c\bar{c}$ and $b\bar{b}$ bound states at $\mu=0$ and $\mu=\mu_c$

		$c\bar{c}$	$b\bar{b}$
$n=1, l=0$ ($J/\psi, \Upsilon$)	μ [GeV]	0	0
	r [GeV^{-1}]	2.263	1.130
	r [fm]	0.453	0.226
	M [GeV]	3.070	9.445
	μ_c [GeV]	0.699	1.565
	$M(\mu_c)$ [GeV]	2.915	9.615
$n=2, l=0$ (ψ', Υ')	μ [GeV]	0	0
	r [GeV^{-1}]	4.373	2.546
	r [fm]	0.875	0.509
	M [GeV]	3.698	10.004
	μ_c [GeV]	0.357	0.671
	$M(\mu_c)$ [GeV]	3.177	9.778
$n=2, l=1$ (χ_c, χ_b)	μ [GeV]	0	0
	r [GeV^{-1}]	3.478	2.039
	r [fm]	0.696	0.408
	M [GeV]	3.500	9.897
	μ_c [GeV]	0.342	0.558
	$M(\mu_c)$ [GeV]	3.198	9.829

**Fig. 5.** Radii for $c\bar{c}$ bound states

is, of course, at present unknown, but may emerge from future lattice studies.

Finally, we want to emphasize that the critical μ values which we have obtained here correspond to the disappearance of the bound states under consideration in a “static” world. Actually, the thermal motion of the constituents in the medium will through scattering certainly lead to an earlier dissociation. A quantitative study of such an effect is, however, still

**Fig. 6.** Radii for $b\bar{b}$ bound states**Fig. 7.** Effective string tension at $r(\mu)=r_{J/\psi}(\mu)$

lacking, and other mechanisms to shift the dissociation point have been considered as well [8].

4 The temperature-dependence of the screening mass

Up to now, we have studied the binding and deconfinement of a heavy quark system as function of the screening mass μ . To apply these considerations to actual physical situations, we need to know the specific dependence of $\mu(T)$ on T . If nuclear collisions pro-

duce strongly interacting matter, then it is the temperature, not μ , which can be empirically determined.

At $T=0$, we have $\mu=0$ only in a world without light quarks. In the presence of light quarks, the binding of any quark-antiquark system is broken when its binding energy exceeds that needed for the spontaneous creation of a $q\bar{q}$ state out of the vacuum. Hence $\mu(T=0)\neq 0$. We expect the corresponding vacuum screening length to be of the order of one fermi, and this is in fact confirmed by lattice studies [6, 9].

For a $c\bar{c}$ system at $T=0$, vacuum screening implies a breakup when

$$c\bar{c} \rightarrow c\bar{q} + \bar{c}q \quad (4.1)$$

becomes energetically favorable, with $q=u$ or d . Analogous reasoning applies to the $b\bar{b}$ case. In Table 3, we list for the bound states here considered the dissociation energies at $T=0$, together with the corresponding μ -values obtained in Sect. 3. For the J/ψ ,

$$E_{\text{dis}}(T=0) \equiv 2m_D - m_{J/\psi}, \quad (4.2)$$

and similarly for the other states; we again use spin-averaged mass values. In Fig. 8 the functional behavior is illustrated for the J/ψ and the Y . We see that the results in Table 3 are indeed in reasonably good agreement with $\mu(T=0) \simeq 0.2$ GeV, or a screening length of one fermi.

When T is increased, vacuum screening will continue to dominate the long distance behavior for heavy quark bound states, until at $T=T_c$, the ‘‘physical’’ screening due to the presence of light on-shell

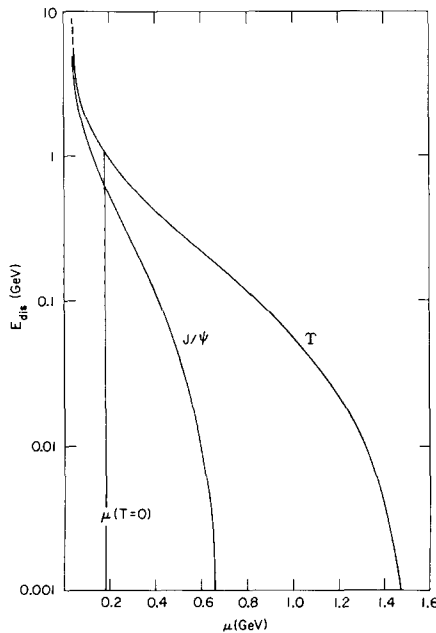


Fig. 8. Dissociation energies for J/ψ and Y ; the line $\mu(T=0)$ indicates the vacuum screening limit at $T=0$

Table 3. Dissociation energies at $T=0$ and vacuum screening masses

State	E_{dis} [GeV]	$\mu_c(T=0)$ [GeV]
J/ψ	0.67	0.18
ψ'	0.05	0.26
χ_c	0.32	0.18
Y	1.09	0.18
Y'	0.52	0.19
χ_b	0.66	0.18

Table 4. Screening masses (in GeV) at different temperatures

Method	Reference	T/T_c		
		1	1.5	2
Perturbation theory, $N_f=0$	11, 13	0.33	0.46	0.59
Lattice SU(3)	11	0.7	0.75	1.0
Lattice SU(3) with $N_f=3$ dyn. quarks	12	0.61	1.40	2.34

quarks takes over. Above T_c , $\mu(T)$ will increase ($r_D(T)$ decrease) according to the temperature dependence of the color charge density in statistical QCD.

The quantitative study of $\mu(T)$, both above and below T_c , has been the subject of considerable interest for some time [10, 11]. Unfortunately, at the present there still is quite a bit of diversity in the results. Lattice studies with dynamical quarks are presently underway [12] and may provide clarification soon.

For the time being, all we can do is list for some temperatures those μ -values which have so far emerged from lattice and perturbation theory studies. We show in Table 4 the μ values at $T/T_c=1, 1.5$ and 2. Perturbation theory for N_f light quark flavors gives in leading order [10]

$$\mu^2(T)/T^2 = (1 + N_f/6) g^2(T), \quad (4.3)$$

where $g^2(T)$ is the temperature-dependent running coupling constant. A recent study [13] suggests for $N_f=0$ to the form

$$g^2(T) = \frac{24\pi^2}{33 \ln(19T/A_{\text{MS}})}, \quad (4.4)$$

as relevant temperature-dependent coupling for the static electric screening mass. However, it should be pointed out that there is still a great amount of ambiguity in the definition of $g^2(T)$ (see for instance, the discussion in [14]). From lattice evaluations of pure SU(3) gauge theory one has [15]

$$T_c/A_{\text{MS}} = 1.78 \pm 0.03. \quad (4.5)$$

Combining these results gives the perturbation theory values shown in Table 4. We have in Table 4 always taken $T_c=200$ MeV.*

All values for $\mu(T)$ obtained so far imply that both ψ' and χ_c disappear essentially at T_c . The J/ψ is expected to vanish there if one takes the lattice results seriously; perturbation theory would require $T/T_c \simeq 2$.

5 Conclusions

We have studied the effect of color screening on the binding of heavy quark states. With increasing temperature, the energy E_{dis} necessary to break up such bound states decreases; for each bound state there is a critical screening mass μ_c , at which $E_{\text{dis}}=0$, so that the state becomes dissociated. We have determined the μ_c values for the main $c\bar{c}$ and $b\bar{b}$ bound states. Given a relation between screening mass μ and temperature – provided either by lattice studies or by perturbation theory – we can then estimate the temperature of the medium at which each state becomes unbound. The actual dissociation is expected to be shifted to lower temperatures by the kinetic motion of the constituents.

* Pure SU(3) gauge theory gives, together with our string tension value, $T_c=254$ MeV, while lattice calculations of hadron masses tend to give values around or below 200 MeV

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