
CRITIQUES

Skill Automaticity in Mathematics Instruction: A Response to Gagné

IPKE WACHSMUTH, *Northern Illinois University
and Universität Osnabrück*

In his remarks on "Some Issues in the Psychology of Mathematics Instruction," Gagné (1983) has stimulated a renewed discussion of the cognitive phenomena involved when children learn mathematics and of the implications of cognitive learning theory for mathematics instruction. Gagné gives a three-phase performance model along with the following core message: Students should understand how to mathematize a concrete situation that is described verbally and how to validate a solution once it is obtained, but they need *not* understand how the solution is derived. Instead, the skills involved in the computation phase should be made automatic for the sake of optimal overall performance, and lots of practice should be devoted to automatizing such skills. Taken out of their context and inserted into the current climate in mathematics education, Gagné's statements are likely to be misunderstood, giving the wrong impression to teachers and perhaps causing researchers to reject his ideas.

The purpose of this critique is to make a more careful inquiry into Gagné's proposals and, at the same time, to set the discussion of skill automaticity in a new light. On the one hand, the inquiry may help reconcile Gagné's apparently unpopular theses with the views of mathematics educators that are best expressed in the term *learning with understanding*. On the other hand, the discussion raises some questions that seem to require further research attention.

THE QUESTION OF SKILL AND UNDERSTANDING

"A skill is what a learner should be able to *do*. Skills arise from concepts and principles and provide a foundation for the development of other concepts and principles. Conceptual thought is derived in part from the understanding attained as skills are developed" (Suydam & Dessart, 1980, p. 207). This view of skill posits intertwined dependencies between skills and conceptual knowledge. Resnick and Ford (1981) identify this issue as one of the

oldest concerns in the psychology of mathematics and claim that “instead of focusing on the *interaction* between computation and understanding, between practice and insight, psychologists and mathematics educators have been busy trying to demonstrate the superiority of one over the other. . . . The *relationships* between skill and understanding were never effectively elucidated” (p. 246; emphasis in original). Any attempt to analyze such relationships should certainly include other mathematical skills besides computation.

Modern cognitive psychology as represented, for example, in the work of Anderson (1980, p. 223) tends to categorize knowledge as *declarative* and *procedural*. Declarative knowledge comprises the facts that one knows, and procedural knowledge comprises the skills one knows how to perform. The declarative-procedural distinction graphically illustrates the two different aspects of any learning process. Declarative knowledge is stated in propositions a person constructs and can be verbally communicated; for example, $3 + 5 = 8$, $(a + b)^2 = a^2 + 2ab + b^2$, or “To solve quadratic equations, one method is to complete the square.” This type of knowledge is viewed as being stored in what is called long-term semantic memory. Skills, as procedural knowledge, are also stored in long-term (procedural) memory but as an ordered sequence of actions that are performed when recalled. The term *cognitive skill* (as opposed to motor skill) refers to the ability to perform various intellectual procedures. For example, writing out the steps necessary to solve any quadratic equation by completing the square is a cognitive skill. Even though the two types of knowledge are not entirely independent, they require very different instructional procedures.

The learning-with-understanding approach of contemporary mathematics education has led to the view that the learner should be put in the position of doing mathematics—for instance, solving a problem—by deriving it meaningfully from declarative knowledge. However, to a great extent it is procedural rather than declarative knowledge that governs skilled performance.

AUTOMATICITY OF SKILLS

In explaining his second hypothesis, Gagné (p. 15) refers to the limited capabilities of working memory, which he claims can be compensated for by automatizing skills. Anderson (1980, p. 226) lists three steps in skill learning: (a) the (declarative) *cognitive stage*, in which a description of the procedure is learned with the aim of understanding it; (b) the *associative stage*, in which a procedure is constructed from the declarative information, that is, a method for performing the actions is worked out; and (c) the *autonomous stage*, in which the skill becomes more and more rapid and automatic (by means of practice), and verbal mediation in the performance of the actions often disappears.

One may question—and this is Gagné’s concern—whether curriculum developers have not tended to ignore the importance of the third stage. Much emphasis has been put on the acquisition of factual (declarative) knowledge

in the belief that understanding a task will enable the learner to “figure it out.” Cost-effectiveness standards have been applied to the use of instructional time, and “too much” practice is to be avoided (NCTM, 1980, pp. 6–12). Further practice of a skill is to be left to problem-solving situations that require application of the skill. However, this approach may so space practice as to be counterproductive. Focusing attention on the execution of a nonautomatized procedure distracts attention from the problem-solving task. This distraction could displace trains of thought that have evolved in organizing the problem-solving process and thus could disturb the process.

In the autonomous stage of skill learning, a skill becomes automatized and autonomous in the sense that it does not require a verbal equivalent in declarative form. The efficiency of a skill increases as it gets more and more automatic, and the mental effort in performing the skill decreases; there is less need to monitor each action with care. Suydam and Dessart (1980) observe that mastered skills can be conducted at a “subconscious level.” As the demands on conscious resources diminish, a reduction in mental work load is achieved.

It has been shown (Anderson, 1980) that when a skill has been highly practiced, it ceases to interfere with other ongoing behavior; that is, different cognitive activities can be conducted simultaneously. Moreover, *automatizing* subskills before going on to higher skills can *contribute to learning* higher skills. The reason apparently lies in the relationship between attention and automaticity. (For an example from computer programming, see Anderson, 1980, pp. 249–252. As subcomponents in programming become more automatic, programmers can focus more attention on higher level problems.)

ROTE SKILLS OR MEANINGFUL SKILLS?

Given any skill, what makes the difference between its being (a) *rote* or (b) *meaningful*? We can further distinguish two cases: First, the skill in question is *not automatized*; second, it is *automatized*.

1a. A rote skill that has not been automatized requires the recall of one or more rules, which are then applied mechanically, without understanding. If part of a rule or knowledge of how to instantiate the variables is forgotten, there is no way to reconstruct it from other knowledge. (For instance, this often happens with the quadratic formula.)

1b. A skill derived meaningfully that has not been automatized makes reference to factual knowledge a learner has and uses it for a certain purpose. (For instance, a learner who knows how to represent numbers with Dienes blocks can use this knowledge to derive the subtraction algorithm, including “borrowing.”) Using a skill that has not been practiced to the point of recall can be viewed as a problem-solving process (Wachsmuth, 1982). Instead of manipulating symbols “concretely” (Gagné), the learner uses symbols “abstractly”; that is, the process actually deals with the referents of symbols.

2a and 2b. The idea of overpracticing a skill to the point of recall—whether or not it was acquired meaningfully—is the following: The *sequence of actions* necessary to do something is made to occur automatically—without having to derive or reconstruct any action from an understanding or recall of facts, purposes, and goals. In this process, one step triggers the next, and it is just the ability to perform the sequence of actions to achieve a goal that makes up the skill.

Children's ability to understand why and how a skill works may develop later than their ability to perform rote skills (e.g., see the discussion on fraction algorithms in Suydam, 1978, pp. 302–303). In fact, Bergeron and Herscovics (1982, p. 30) consider the acquisition of a procedure as a precursor of conceptual knowledge (of, e.g., addition); they speak of “procedural understanding.” The question is whether it would be advantageous if automatized rote skills preceded a “fuller” understanding of such skills.

When an automatic skill is called on, an action is not derived from an understanding of how the procedure works. It is rather guided by its own action scheme. This is somewhat like the difference between understanding the physics of bicycle riding and being able to ride a bicycle. The common view that practice in a motor skill should not be interfered with by thinking about it seems to have an equivalent in the acquisition of procedural mathematical knowledge. Gagné's first hypothesis (p. 15) about the teaching of computational skills (omitting reinforcement of incorrect performance plus teaching correct rules again) is further supported by Davis's (1978) advice to teachers: During drill sessions, it is best to emphasize remembering; don't explain. For the execution of a procedure, one apparently need not understand it.

Current curricula mix the two goals of learning to perform and learning to understand a skill. It is still controversial whether better understanding of a procedure itself gives rise to better performance. Although many mathematics educators would argue that it does, another issue seems to have been overlooked: Gagné's position is that an understanding of when to call on a procedure must be derived from Phases 1 and 3 in his model, *not* from understanding the procedure itself. Some understanding of a procedure, however, may be important for embedding it correctly in the context of a problem-solving task.

Two examples suggest why it might also be desirable that a student understand how a procedure works: (a) Having learned the subtraction algorithm for tasks like $238 - 190$, one would be greatly facilitated in *learning to extend* the algorithm to $2.38 - 1.9$ if one understood how it works. (b) If a rote skill has not been employed for a while, decay is possible; understanding the skill would allow one to *reconstruct* the procedure from declarative knowledge. Thus, it might well make sense to teach skills both rotely and meaningfully, perhaps in a spiral fashion.

Three questions emerge from the discussion thus far:

1. Should we not think of procedural and declarative knowledge as two related, but separate, issues of mathematics instruction?
2. Do there exist optimal mental ages for automatizing skills and for teaching children to understand what they do?
3. If the optimal age for automatizing skills is earlier than the optimal age for understanding skills, how should instruction proceed?

VALIDATING THE SOLUTION

Gagné notes that students may trust in a result obtained by an algorithm without inspecting the solution in a meaningful way to reveal a faulty computation. This point addresses the different levels at which a person can deal with mathematical symbols. The first is the *syntactic* level, at which one manipulates symbols (here numerals) according to certain rules as “concrete” objects of thought, totally removed from their meanings (as numbers). This is the normal situation when an algorithm is used routinely. The second is the *semantic* level, which Gagné calls “abstract” (p. 13), at which one deals with the symbols by referring back to their meanings.

In a deeper sense the topic of validating addresses the *interaction* between the two levels, which must function well if mathematics is to be employed meaningfully. Davis and McKnight (1980) report a phenomenon they encountered with third and fourth graders who could use the standard subtraction algorithm correctly, in general, but almost always made the error of “skipping over intermediate zeros” in certain subtraction problems. These children could, on request, use Dienes blocks to represent numbers and explain trading. But that latent knowledge did not alert them in the algorithmic situation. The two levels at which one can deal with a computation—syntactic and semantic—appeared to coexist unconnected in their minds.

A major question raised by Davis and McKnight is whether the phenomenon is “essentially inevitable for most children of this age [third and fourth graders], or whether it is a consequence of an excessively ‘algorithmic’ (and meaningless) program of instruction” (p. 75). Gagné’s position on ways of verifying solutions is that “it would be desirable if [such ways] were deliberately taught” (p. 13). The actual problem can be expressed in two questions:

4. How can children be taught ways of verifying solutions, in the sense that their algorithmic behavior in situations governed by syntactical rules is monitored by their semantic knowledge?
5. Can such an interaction of syntactical and semantical levels of information processing be taught at all to children of certain grade levels?

As one of the ten areas of basic mathematical skills listed in the position paper on basic mathematical skills of the National Council of Supervisors of Mathematics (1977), “Alertness to the Reasonableness of Results” is viewed

as an essential skill. Apparently, alertness comprises the automatic *generation of warnings* or cautions in the course of a task performance. So, if a curriculum that develops both understanding and skilled performance is the objective, research efforts will have to be devoted to the following question:

6. How can warnings be automatized so as to achieve a reliable, alert interplay between skill and understanding in mathematics?

Gagné's view that the cognitive capability required for students in Grades K–12 is a set of automatized rules (p. 17) is apparently only part of the story. The rules must also be capable of being carried out meaningfully and must be accompanied by alertness skills that permit one to "switch to the meaningful mode" when necessary.

REFERENCES

- Anderson, J. R. *Cognitive psychology and its implications*. San Francisco: Freeman, 1980.
- Bergeron, J. C., & Herscovics, N. A constructivist model of understanding. In S. Wagner (Ed.), *Proceedings of the Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Athens: University of Georgia, Department of Mathematics Education, 1982.
- Davis, E. J. Suggestions for teaching the basic facts of arithmetic. In Suydam, M. N., & Reys, R. E. (Eds.), *Developing computational skills*. Reston, VA: National Council of Teachers of Mathematics, 1978.
- Davis, R. B., & McKnight, C. The influence of semantic content on algorithmic behavior. *Journal of Mathematical Behavior*, 1980, 3 (1), 39–87.
- Gagné, R. M. Some issues in the psychology of mathematics instruction. *Journal for Research in Mathematics Education*, 1983, 14, 7–18.
- National Council of Supervisors of Mathematics. Position paper on basic skills. *Arithmetic Teacher*, 1977, 25 (1), 19–22.
- National Council of Teachers of Mathematics. *An agenda for action: Recommendations for school mathematics of the 1980s*. Reston, VA: Author, 1980.
- Resnick, L. B., & Ford, W. W. *The psychology of mathematics for instruction*. Hillsdale, NJ: Erlbaum, 1981.
- Suydam, M. N. Review of recent research related to the concepts of fractions and of ratio. In E. Cohors-Fresenborg & I. Wachsmuth (Eds.), *Proceedings of the Second International Conference for the Psychology of Mathematics Education*. Osnabrück: University of Osnabrück, Department of Mathematics, 1978.
- Suydam, M. N., & Dessart, D. J. Skill learning. In R. J. Shumway (Ed.), *Research in mathematics education*. Reston, VA: National Council of Teachers of Mathematics, 1980.
- Wachsmuth, I. Letter to the editor. *Journal of Mathematical Behavior*, 1982, 3(2), 188–192.

[Received November 1982]