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Perturbative vs. Non-Perturbative Scaling Violation in Quark Fragmentation*

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Abstract. We investigate the approach to scaling for non-perturbative quark fragmentation in the framework of the uncorrelated jet model. It is found that subasymptotic kinematic scale breaking is comparable in size to scaling violations from hard gluon emission a la QCD. Experimental data available at present do allow for such non-perturbative scale breaking effects.

1. Introduction

Considerable effort has been spent recently to test the experimental validity of Quantum Chromodynamics (QCD) [1]. In the phenomenological applications of QCD, one discriminates between soft and hard processes. So far, only hard processes can be predicted by the theory, using perturbation methods. The evaluation of soft processes on the other hand is plagued by infrared divergences and a large coupling constant, prohibiting a perturbative treatment [2]. In practice therefore, non-perturbative effects, wherever they matter, are treated in an ad hoc manner. In the case of quark fragmentation into hadrons, e.g., one generally thinks in terms of a quark cascade picture to describe the low p_T non-perturbative part of hadron distributions [3]. In general one follows Feynman and Field [3], who proposed a model of this type with built-in scaling. In this framework scaling violations are then considered to be due to hard gluon emission as obtained from Altarelli–Parisi like equations [4, 5].

It is obvious that the relevance of any such test of QCD rests heavily on the validity of one's prejudices about soft physics. If we accept the idea of asymptotic scaling in the non-perturbative quark fragmentation into (low p_T) hadrons, is it really reasonable to assume early scaling from the very beginning? In the case of current fragmentation studies in semiinclusive neutrino scattering at SPS energies [6], for example, where the available Q^2 range is $1\,\mathrm{GeV} \lesssim Q^2 \lesssim 100\,\mathrm{GeV}^2$, this turns out to be a crucial question for any QCD oriented analysis. Naively, one would expect that non-perturbative scaling is only reached, when longitudinal and transverse (with respect to original quark momentum) directions in the final state hadron configuration are clearly distinguished in the sense that $\langle p_{\|} \rangle \gg \langle p_T \rangle$. This condition is not even fulfilled for the kinematically favoured case of e^+e^- annihilation into hadrons, where Q^2 values as high as $900\,\mathrm{GeV}^2$ are nowadays accessible: first data taken at $Q^2 = 289\,\mathrm{GeV}^2$ by the TASSO detector at PETRA yield an average longitudinal (with respect to the jet axis) momentum of secondaries, which is still of the same order of magnitude as the average transverse momentum, $\langle p_{\|} \rangle \approx 2.7 \langle p_T \rangle = 920\,\mathrm{MeV}$ [7]. Indeed, subasymptotic scale breaking for the non-perturbative quark fragmentation in e^+e^- annihilation was predicted long ago to occur up to Q^2 values of around $400\,\mathrm{GeV}^2$ from an uncorrelated jet model study [8], that was based on input parameters from pp collisions available at that time.

In view of the general interest to extract information about leading order [6, 9–12] (and even next to leading order [13]!) QCD effects out of fragmentation studies we think it is necessary to study in more detail than before the problem of the size of subasymptotic scaling violations to be expected from the non-perturbative contribution. To be precise, we want to consider e^+e^- annihilation into pions, using a model with full kinematics but little dynamics. The uncorrelated jet model (UJM) [14], which is nothing else but a transversely limited phase space model, seems to be most appropriate for this purpose, since one would expect any realistic scaling model

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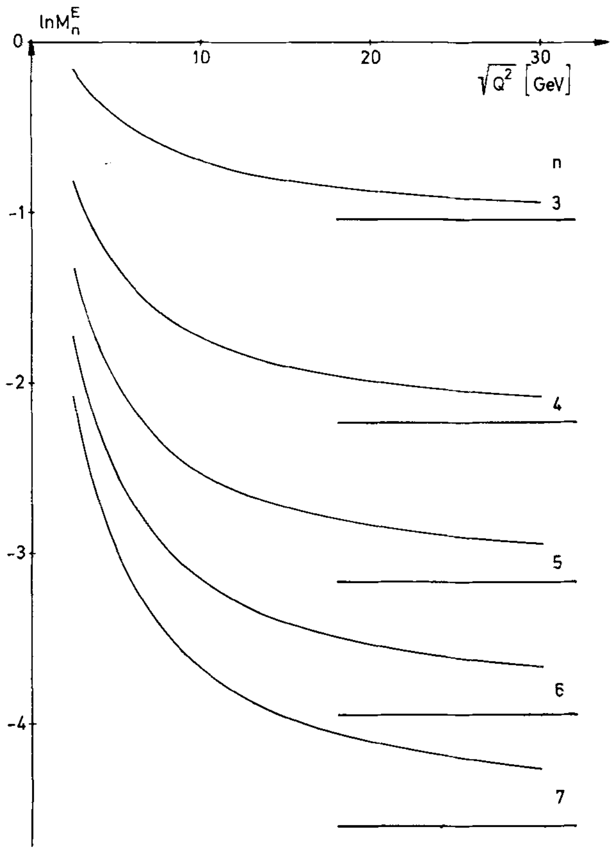

Fig. 1. $\sqrt{Q^2}$-dependence of the natural logarithm of moments from non-perturbative quark fragmentation with respect to the scaling variable $x = 2p_0/\sqrt{Q^2}$, see Eqs. (9, 10). The various n-values are listed. The horizontal lines indicate the asymptotic scaling limits of these moments

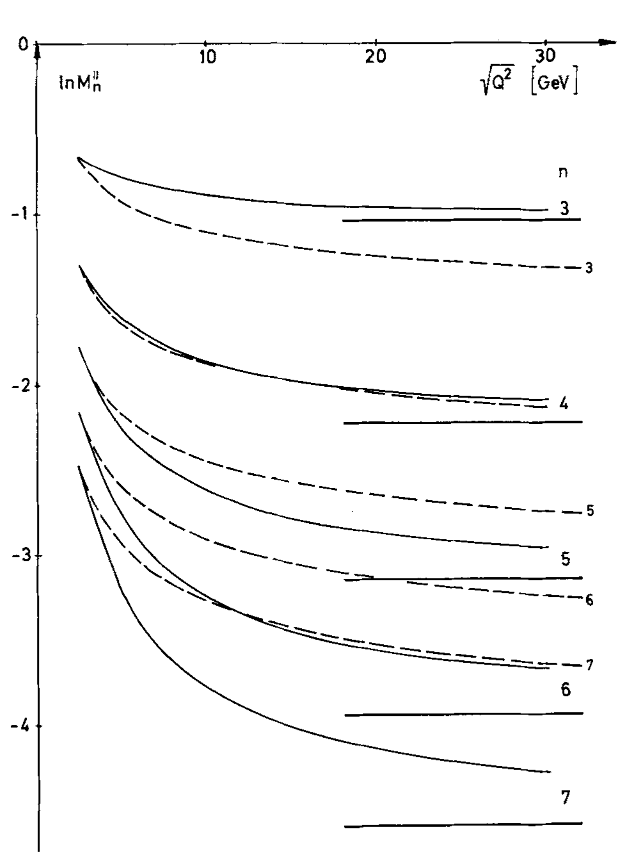

Fig. 2 Same as Fig. 1, but with respect to the variable $x_{\parallel} = 2p_{\parallel}/\sqrt{Q^2}$. The dashed curves represent the QCD predictions for nonsinglet moments, calculated with $\Lambda^2 = .5\,\text{GeV}^2$ [5]. Their Q^2-dependence is characteristic for the amount of scale breaking in QCD

to show such subasymptotic kinematical scale breaking effects.

In Sect. II we shortly remind the reader about the essentials of the uncorrelated jet model. Section III describes and discusses the results. The conclusions are given in Sect. IV.

2. The Uncorrelated Jet Model

We imagine the quark pair produced in e^+e^- annihilation to radiate pions[1] with limited transverse momentum according to the uncorrelated jet model [8]. The fully exclusive distribution for the production of N particles of four-momenta p_i in that model is given by

$$\Gamma_N \sim \frac{\nu^N}{N!}\delta^{(4)}\left(\sum_{j=1}^{N} p_j - q\right)\prod_{i=1}^{N}\frac{d^3p_i}{2p_{i0}}f(p_{iT}), \tag{1}$$

where q is the total four-momentum of the e^+e^--system and p_T is the momentum transverse to the jet axis. For simplicity we assumed the pions to be chargeless. The function $f(p_T)$ describes the transverse momentum cutoff and is normalized such that

$$\pi\int_0^\infty f(p_T)p_T\,dp_T = 1\,. \tag{2}$$

The total phase-space volume is then given by

$$\begin{aligned}\Omega(q) &= \sum_{N=2}^{\infty}\frac{\nu^N}{N!}\int\prod_{i=1}^{N}\frac{d^3p_i}{2p_{i0}}f(p_{iT})\delta^{(4)}\left(\sum_{j=1}^{N}p_j - q\right)\\ &= \sum_{N=2}^{\infty}\frac{\nu^N}{N!}\Omega_N(q),\end{aligned} \tag{3}$$

and the normalized single-particle distribution in the c.m. system is

$$\frac{2p_0}{\sigma_{\text{tot}}}\frac{d^3\sigma}{d^3p} = \nu f(p_T)\frac{\Omega(Q-p)}{\Omega(Q)}, \tag{4}$$

with $Q = (\sqrt{s}, \mathbf{0})$; $Q^2 = s$.
For asymptotic energies the average multiplicity behaves as

$$\langle N\rangle \underset{s\to\infty}{\longrightarrow} \nu\ln\frac{s}{m^2} \tag{5}$$

1 It is clear that inclusion of heavier particles [15] would increase the nonasymptotic effects

and the inclusive single particle distribution reaches its scaling limit

$$\frac{1}{\sigma_{tot}}\frac{d^2\sigma}{dx\,dp_T^2}\approx\frac{\pi\nu}{x}f(p_T)(1-x)^{\nu-1}\equiv D_\infty(x,p_T),\qquad(6)$$

where $x=2p_0/\sqrt{s}$. The convergence to the scaling limit is, however, rather slow.

For finite energies ($\sqrt{s}\gtrsim 10$ GeV), a very accurate approximation for the normalized single particle spectrum was given by De Groot [16]

$$\frac{1}{\sigma_{tot}}\frac{d^2\sigma}{dx\,dp_T^2}\approx D_\infty(x,p_T)\cdot$$
$$\cdot(1-x)^{-\nu/\bar{n}(s)}\cdot\exp[-p_T^2(1-x)^{-\nu/\bar{n}(s)}/\langle p_T^2\rangle_A\bar{n}(s)],\qquad(7)$$

where $\langle p_T^2\rangle_A$ is the asymptotic average p_T^2, i.e.

$$\langle p_T^2\rangle_A=\pi\int_0^\infty f(p_T)p_T^3\,dp_T.\qquad(8)$$

The quantity $\bar{n}(s)$ approaches asymptotically the average multiplicity $\langle N\rangle$ and is given in Ref. 16. The correction factors in formula (7) show that the average p_T at finite energies will reach its asymptotic value only slowly.

In our actual calculations the transverse-momentum cutoff for the pions was taken to be

$$f(p_T)=\frac{\lambda^2}{\pi}\exp(-\lambda p_T).$$

The model depends only on the two parameters ν and λ. Their values were fixed by requiring $\langle p_T\rangle=336$ MeV and $\langle N\rangle=16.8$ at $\sqrt{s}=17$ GeV:

$$\lambda=4.5\ \text{GeV}^{-1},\quad \nu=4.65.$$

The phase-space volumes were then evaluated with the Fourier transform method [17] for $\sqrt{s}=2.5-30$ GeV and checked against formula (7) in the region $\sqrt{s}>10$ GeV.

3. Results and Discussion

Within the framework of QCD, scaling violations of fragmentation functions are expressed by the Q^2-evolution of their moments [5]

$$M_n(Q^2)=\int_0^1 dx\,x^{n-1}D(x,Q^2).\qquad(9)$$

In our model, the fragmentation function is given by

$$D(x,Q^2)=\frac{1}{\sigma_{tot}}\frac{d\sigma}{dx}(x,Q^2).\qquad(10)$$

At finite energies, the choice of the scaling variable x is not unique. Instead of choosing $x=2p_0/\sqrt{s}$, one might use $x_p=2p/\sqrt{s}$ or $x_\parallel=2p_\parallel/\sqrt{s}$. We have plotted in Figs. 1 and 2 the natural logarithm of the first nontrivial moments for the variables x and $x_\parallel$, respectively, and their asymptotic limits, which are

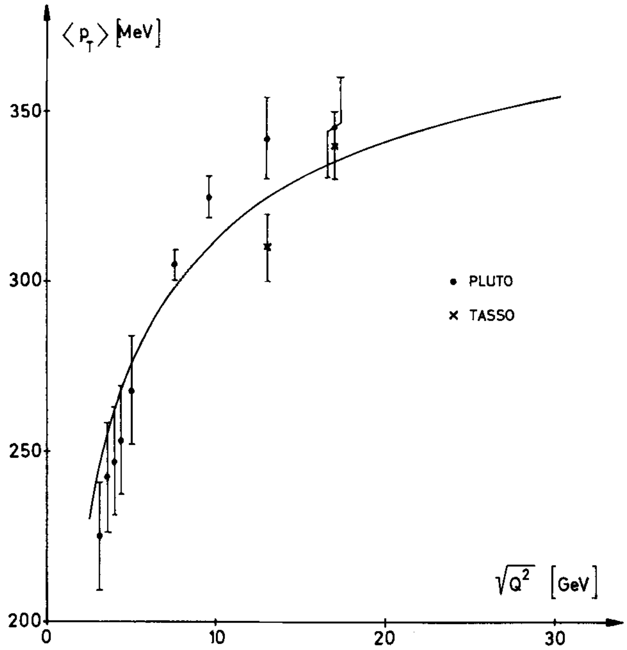

Fig. 3. Average transverse momentum of secondaries versus $\sqrt{Q^2}$ from the UJM. Experimental points are from PLUTO [18] and TASSO [7]

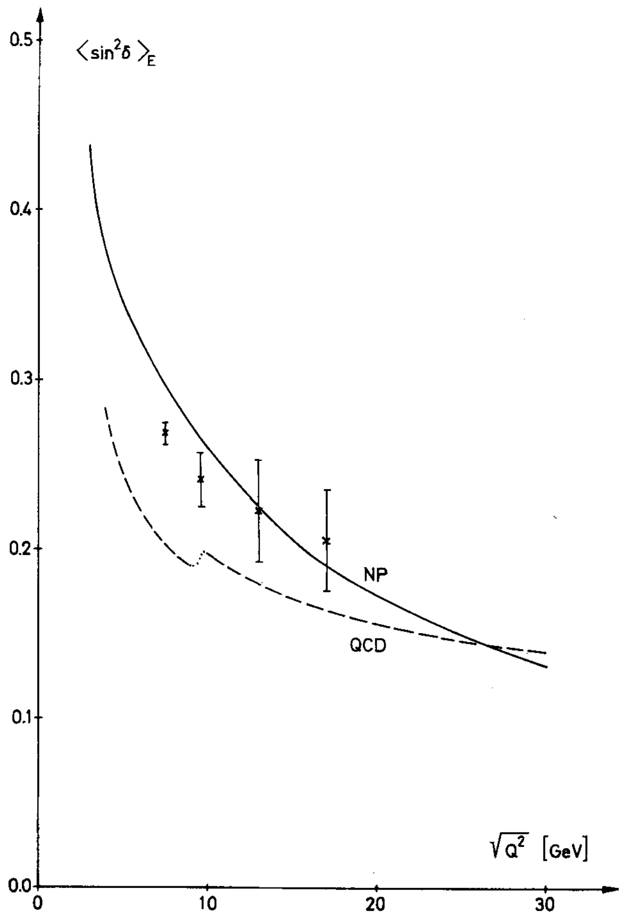

Fig. 4. $\langle\sin^2\delta\rangle_E$ from the UJM (solid line) and QCD (dashed line) with $\Lambda^2=.5$ GeV^2 and N_F varying from 4 to 5. The experimental points from PLUTO [20] refer to charged particles only

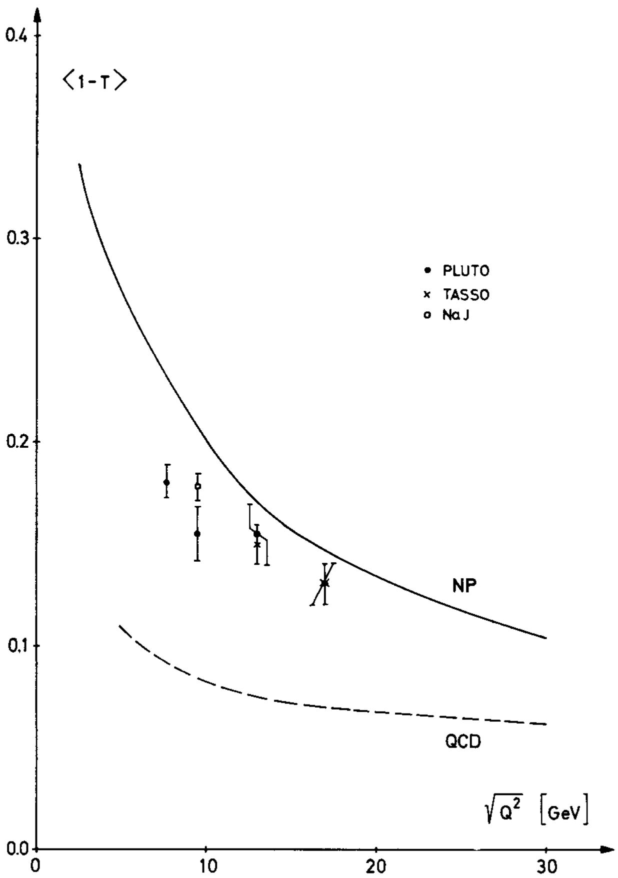

Fig. 5. Average thrust, plotted as $\langle 1-T\rangle$ versus $\sqrt{Q^2}$. Plotted are the UJM (Eq. (14), solid curve) and QCD predictions [21] (dashed curve). Experimental points are from PLUTO [20], TASSO [7], and the NaJ detector at DESY [20]

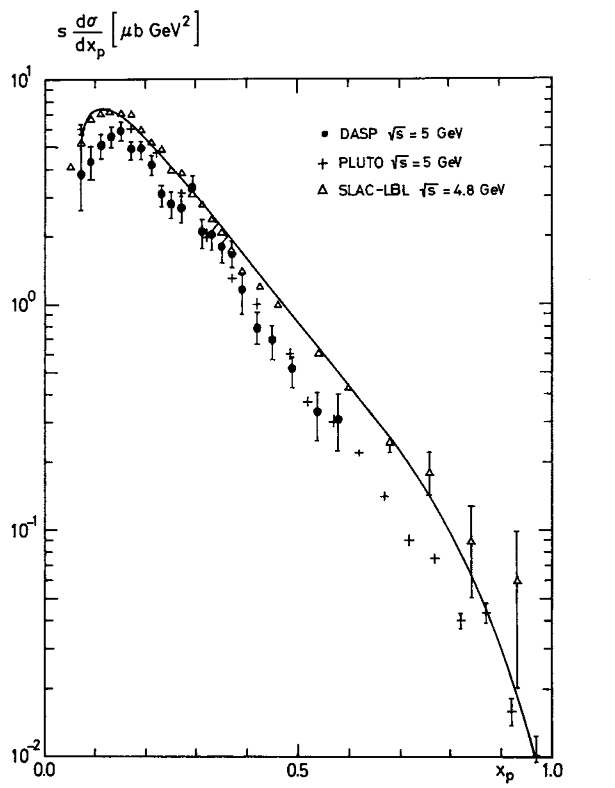

Fig. 6. Comparison of the *shape* of the (arbitrarily normalized) UJM prediction with charged hadron distributions observed around 5 GeV. The variable x_p is defined by $x_p = 2p/\sqrt{Q^2}$. Data as quoted in Ref. 7

obtained from Eq. (6). All moments show a substantial Q^2-dependence between $Q^2=(2.5\,\text{GeV})^2$ and $900\,\text{GeV}^2$. To get an idea about the scale breaking predicted by QCD, we show the Q^2-evolution of the moments of the non-singlet fragmentation function [5]. They were fixed at $Q_0^2=(2.5\,\text{GeV})^2$ to coincide with our model. The QCD variation turns out to be of the same order as our non-perturbative estimate! We convinced ourselves that the inclusion of singlet terms does not qualitatively change this situation. In view of the very late scaling found for the non-perturbative model, it therefore seems very hard to test perturbative QCD via moment analyses within this Q^2 region [6].

The slow approach to asymptotia of the UJM can be seen as well in the p_T-distribution of secondaries. Figure 3 shows $\langle p_T\rangle$ as a function of $\sqrt{Q^2}$. We observe that the asymptotic limit

$$\langle p_T\rangle_\infty = \frac{2}{\lambda} = 444\ \text{MeV} \tag{11}$$

is by far not reached at $\sqrt{Q^2}=30\,\text{GeV}$, while the region below 10 GeV exhibits clearly the opening of phase-space. In the region between 10 and 30 GeV, on the other hand, which corresponds roughly to ISR energies (if leading particle effects are subtracted), the value of $\langle p_T\rangle$ lies in the range $335\pm 20\,\text{MeV}$. As far as the experimental points [7, 18] in Fig. 3 are concerned, it should be remembered that the observed events at $\sqrt{Q^2}\lesssim 5\,\text{GeV}$ are very close to isotropy and that the sphericity minimization procedure used to determine $\langle p_T\rangle$ experimentally tends to underestimate the average p_T.

A popular quantity [11] to determine the QCD jet profile is the energy-weighted mean

$$\langle \sin^2\delta\rangle_E \equiv \frac{1}{\sqrt{Q^2}}\frac{1}{\sigma_{\text{tot}}}\int d^3p\,\frac{p_T^2}{p^2}p_0\frac{d^3\sigma}{d^3p}. \tag{12}$$

As can be seen in Fig. 4, the non-perturbative model dominates the perturbative QCD effects

$$\langle \sin^2\delta\rangle_{E,\text{QCD}} = \frac{24}{(33-2N_f)\ln Q^2/\Lambda^2} \tag{13}$$

with $\Lambda^2=0.5\,\text{GeV}^2$. The same conclusion has been reached by Steiner [19], who assumed a specific form of angular scaling for the energy flow distri-

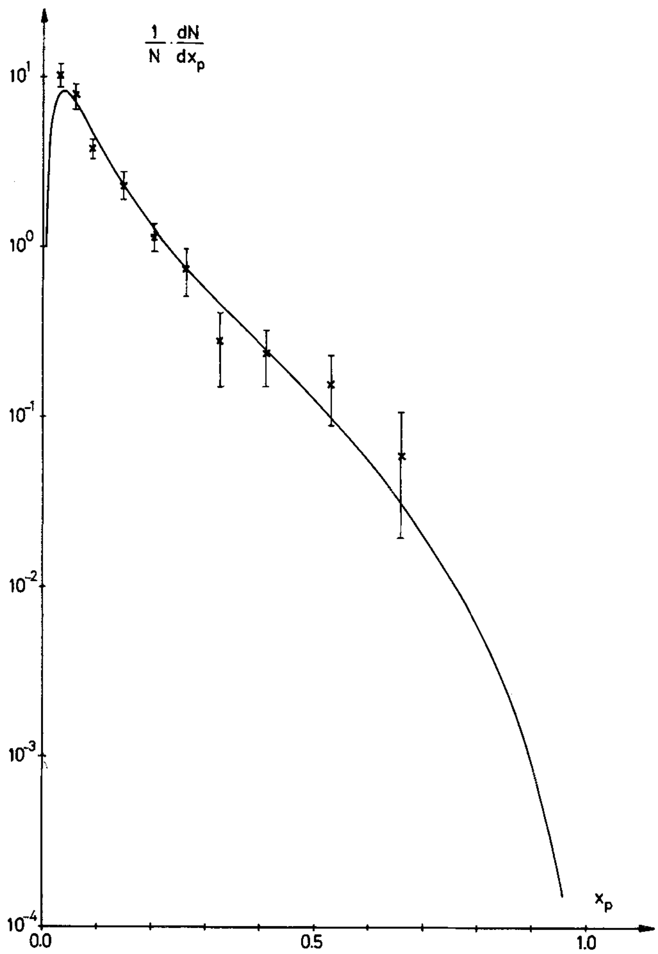

Fig. 7. One particle spectrum, $1/N dN/dx_p$ from the UJM at 17 GeV. Data points refer to charged particles measured at TASSO [7]. x_p as in Fig. 6

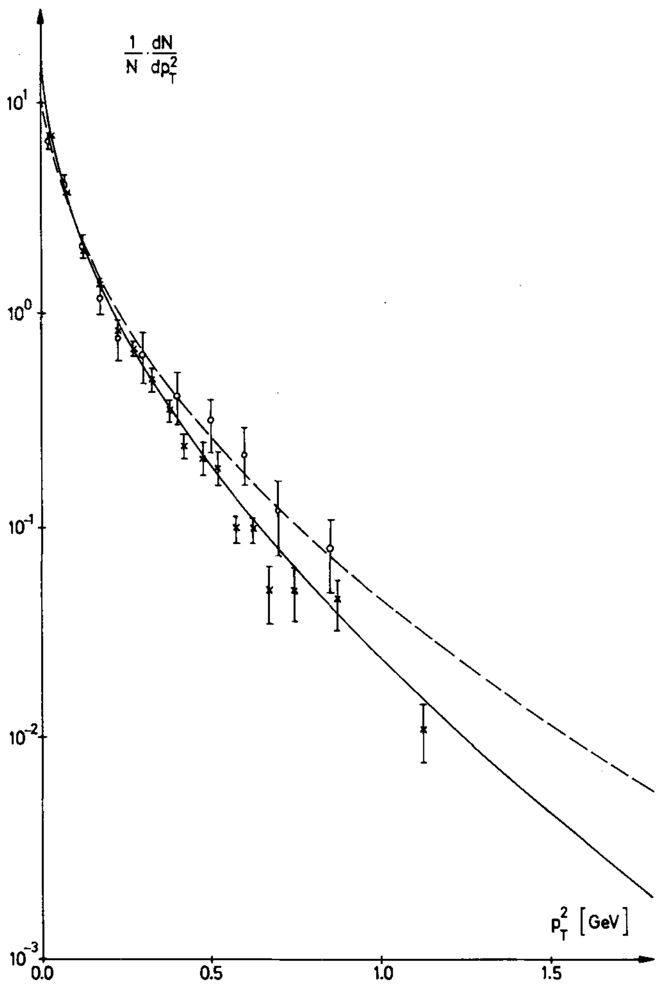

Fig. 8. Transverse distribution, $1/N dN/dp_T^2$, versus p_T^2 at $\sqrt{Q^2} =$ 17 GeV (– – –) and 7.5 GeV (———). Data points refer to charged secondaries measured at 17 GeV (o) and 7.7 GeV (x) by PLUTO [20]

bution. The data points from the PLUTO group [20] contained in Fig. 4 refer to charged tracks only. They are in reasonable agreement with our curve.

The chance to observe QCD effects from average thrust is even worse (see Fig. 5). To a good approximation $\langle T \rangle$ can be easily calculated from inclusive observables

$$\langle T \rangle \approx \frac{\langle |p_\parallel| \rangle \langle N \rangle}{\sqrt{Q^2}}. \tag{14}$$

The QCD prediction for $\langle 1 - T \rangle$ in Fig. 5 was taken from Ref. 21. In contrast to the optimistic expectation expressed in this reference, we have no hope that measurements of $\langle T \rangle$ will reveal QCD effects below 30 GeV.

So far we have only considered mean values. One might think that our trivial two-parameter model is too simple to explain the inclusive distributions as well and therefore should not be seriously discussed. However, as can be seen from Figs. 6, 7 and 8, both the x_p- and p_T^2-distributions are in embarrassing agreement with experiment.

4. Conclusions

We conclude that perturbative QCD and subasymptotic non-perturbative scale breaking effects at present storage ring energies are expected to be of the same order of magnitude and similar structure. Hence it will be difficult to separate them from each other. However, the precision of available secondary hadron spectra measured from e^+e^- annihilation is not at all sufficient to detect any scaling violation.

Furthermore, it was demonstrated that QCD infrared-safe global quantities like $\langle 1 - T \rangle$ and $\langle \sin^2 \delta \rangle_E$, contrary to earlier hopes [10, 11], at presently accessible storage ring energies are not sensitive enough to extract information about hard gluon bremsstrahlung.

It is obvious that such non-perturbative scale breaking effects of at least similar size are present in quark fragmentation from deep inelastic collisions at SPS and FNAL and will substantially affect the proposed tests of factorization breaking from next to leading order corrections [13].

We do not claim that the uncorrelated jet model is the ultimate wisdom to understand jet physics but it is certainly sensible and legitimate to use it in order to sharpen ones eyes for the dynamics of non-perturbative jet development. A prejudice for cascade models with precocious scaling on the other hand might be very misleading.

The hope remains that more detailed and more accurate data will provide us with unambiguous signals in favour of QCD.

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