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Strong Coupling $1/N_c$ Expansion in the Gluonic Sector of Lattice QCD: The Next Order

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Abstract. Continuing our previous work we determined the gluonic vacuum state up to sixth order and the lowest states with external quark–antiquark and (unscreened) gluon–gluon sources up to fourth order in the strong coupling $1/N_c$ expansion on the lattice. Unlike previously, we used here the colour electric flux operators on the links to define the colour electric energy. Additional remarks concerning the screening of external gluon sources and the analytic continuation to $N_c = 3$ and zero lattice spacing are also included.

I. Introduction

The study of quantum chromodynamics with large number (N_c) of colours [1] seems to be helpful in understanding the general features of the hadronic spectrum. The resulting $1/N_c$ expansion (usually taken at $g^2 N_c$ fixed, with g the coupling constant) may even provide us with a powerful method for the calculation of hadron properties (for recent reviews see e.g. [2, 3]).

In [4] (hereafter referred to as I) we investigated a variant of the $1/N_c$ expansion, when g was kept fixed at $N_c \to \infty$, calculating various physical quantities in the lattice version of the theory [5, 6]. The resulting expansion is a sort of "strong coupling expansion". The series for physical quantities have, apart from an overall factor, the form

$$\sum_{i,j} C_{ij} (g^4 N_c)^{-i+a} N_c^{-j+b} \tag{1.1}$$

with some coefficients C_{ij} and some constants a,b. (Actually, in the quantities we calculated only even powers appeared: $i,j = $ even). The series are well converging if both N_c and $g^4 N_c$ are large. The lattice theory becomes trivial in the limit $N_c \to \infty$, g fixed (the zeroth order in the strong coupling $1/N_c$ expansion). The reason is that the colour magnetic term in the energy can be neglected compared to the colour electric one and the colour electric term can easily be diagonalized. This is an advantage as the zeroth order solution can easily be written down. The non-trivial dynamics comes in the higher order corrections in the expansion.

The present paper is a continuation of I. We determine here the next higher order corrections (6'th order in $1/N_c$ for the vacuum and 4'th order for the state with external sources). A change compared to I is that we return to the more conventional definition of the colour electric energy H_E in terms of the colour electric operators associated to links. This simplifies the higher order calculation as the number of required states is reduced. (The results of the lower orders given in I for the physical quantities are, however, essentially unchanged). Section II contains the calculation of the vacuum, Sect. III is devoted to the states with external quark and gluon sources, whereas Sect. IV contains summarizing and concluding remarks.

II. The Gluonic Vacuum

The notations in the present paper are the same as in I. That is: lattice points are denoted by "middle" letters: $i,j,k,l,\ldots$. The directions of the links are "late" letters $r,s,t,\ldots$. SU(N_c) indices are "early" letters: $b,c,d,\ldots$. A link is denoted by $l\mathbf{r}k = l\mathbf{r} = \mathbf{r}k$. (The lattice spacing is a). The plaquette boundary starting at the point k first in the direction r and then s is $k[rs]$. The "string operator" associated to the link $l\mathbf{r}k$ is $U[l\mathbf{r}k]$ and the one associated to the plaquette $k[rs]$ is $U[k[rs]]$. The SU(N_c)-trace of the latter is $A(k[rs])$.

The Hamilton-operator in the "temporal" gauge is [5–8]

$$H = H_E - H_A$$

$$H_E = \frac{g^2}{2a} \sum_{b,\mathbf{r}k} E_b[\mathbf{r}k]^2$$

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$$H_A = \frac{1}{ag^2}\sum_{k[rs]} A(k[rs]) \tag{2.1}$$

Here (unlike in I) the colour electric energy H_E is defined in terms of the colour electric flux operator $E_b[\mathbf{r}k]$ associated to the link $\mathbf{r}k$. The commutation relation between $U[l\,\mathbf{r}k]$ and $E_b[\mathbf{r}k]$ is:

$$[U[j\mathbf{s}i],\ E_b[l\mathbf{r}k]]$$
$$= \delta_{ik}\delta_{sr}\delta_{jl}\frac{\lambda_b}{2}U[j\mathbf{s}i] - \delta_{il}\delta_{-sr}\delta_{jk}U[j\mathbf{s}i]\frac{\lambda_b}{2} \tag{2.2}$$

(with λ_b the $SU(N_c)$ Gell–Mann matrix). Note that the colour electric flux associated to the point k, which was used in I, is defined by

$$E^{(k)}_{b,r} = \tfrac{1}{2}(E_b[\mathbf{r}k] - E_b[-\mathbf{r}k]) \tag{2.3}$$

The H_E defined in I and the one used here are not exactly equal. The main difference can, however, be absorbed in the suitable definitions of the length of arc of the curves on the lattice (see below).

The mathematical vacuum is the state $|0\rangle$ which is annihilated by the electric flux operator:

$$E_b[\mathbf{r}k]|0\rangle = 0 \tag{2.4}$$

The physical states are created from the mathematical vacuum by the action of "gluon loop operators" $A(i \leftarrow i)$ belonging to the closed curve on the lattice $i \leftarrow i \equiv i\mathbf{r}_n i_n \mathbf{r}_{n-1} \ldots \mathbf{r}_1 i_1\ \mathbf{r}_0 i$. The action of the operator H_A is trivial in this basis: it creates additional gluon loops along the plaquettes. The action of H_E can be deduced, in the same way as in I, from the commutation relation (2.2). If (o.c.) denotes the "original configuration" of the gluon loops then the action of H_E gives

$$\frac{g^2}{2a}C_2(\Sigma\lambda)(\text{o.c.}) + \frac{g^2}{2a}\Sigma\delta\left(\text{s.c.} - \frac{1}{N_c}\text{o.c.}\right) \tag{2.5}$$

Here C_2 is the value of the quadratic Casimir-operator:

$$C_2 = \frac{N_c}{2} - \frac{1}{2N_c} \tag{2.6}$$

$\Sigma\lambda$ in (2.5) is the sum of the "length of arcs" λ of the curves in the configuration. λ is defined here simply as the number of links in the curve. The second term in (2.5) is present only if there are links belonging to more than one curve (or to one curve more than once). The sum is over these "common" links and over the different pairs of curves (if there are more than two curves on the link). For links with the same (opposite) orientation on the two curves $\delta = +1(-1)$. (s.c.) in (2.5) stands for the "switched over" configuration, when the common link is exchanged between the two curves (it is "switched over" from one to the other). For simple examples see Fig. 1.

The main effect of the different definition of H_E used here and in I is a difference in the natural "length of arc" of curves in the two cases. As a consequence, the lowest colour electric energy between two external sources is non-degenerate here if the sources are separated in the direction of some coordinate axis, whereas by the definition used in I it is non-degenerate in the case of diagonal separation (see I).

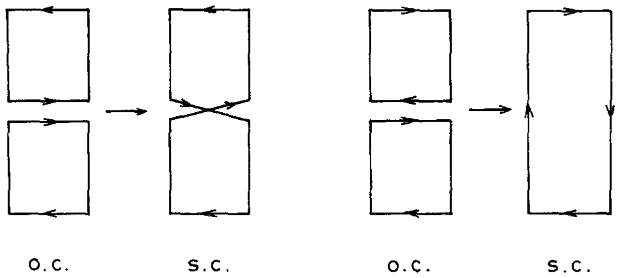

Fig. 1. Examples of the switching over of gluon loops due to the action of H_E in (2.5)

The states appropriate for the perturbative expansion are constructed in principle very similarly to I. The difference is that here, according to (2.5), common links of the plaquettes matter whereas in I the states were characterized by the number of common points on the plaquettes. Let the state with single plaquettes be

$$|1\rangle \equiv \sum_{k[rs]} A(k[rs])|0\rangle = ag^2 H_A|0\rangle \tag{2.7}$$

Acting on $|1\rangle$ by another ag^2H_A we obtain the states $|\bar{\mu}\rangle$ with two plaquettes:

$$ag^2 H_A|1\rangle$$
$$= \sum_{k_1[r_1s_1]}\sum_{k_2[r_2s_2]} A(k_1[r_1s_1])A(k_2[r_2s_2])|0\rangle = \sum_{\mu}|\bar{\mu}\rangle \tag{2.8}$$

The index $\mu = 0, \pm 1, \pm 4$ gives the number of common links on the two plaquettes. (The sign of μ specifies the relative direction of the common links). To the state $|\bar{\mu}\rangle$ belong $\xi(\mu)$ terms in the double sum in (2.8). If P is the number of different plaquettes in the cube of periodicity:

$$P = \sum_{k[rs]} 1 \tag{2.9}$$

then it is not difficult to see that we have

$$\xi(0) = P(P-26)$$
$$\xi(\pm 1) = 12P$$
$$\xi(\pm 4) = P \tag{2.10}$$

The action of the colour electric energy operator H_E on the zero-, one-, and two-plaquette states is, according to (2.5)

$$H_E|0\rangle = 0$$
$$H_E|1\rangle = g^2a^{-1}2C_2|1\rangle$$
$$H_E|\bar{\mu}\rangle = g^2a^{-1}4C_2|\bar{\mu}\rangle + \frac{g^2\mu}{2a}\left[|\underline{\mu}\rangle - \frac{1}{N_c}|\bar{\mu}\rangle\right] \tag{2.11}$$

Here $|\underline{\mu}\rangle$ denotes (for $\mu = \pm 1, \pm 4$) the two-plaquette state with "switched over" plaquettes belonging to $|\bar{\mu}\rangle$. (The case $\mu = \pm 1$ is illustrated in Fig. 1). For $\mu = -4$ we obviously have

$$|\underline{-4}\rangle = \sum_{k[rs]} \mathrm{Tr}(1)|0\rangle = PN_c|0\rangle \tag{2.12}$$

therefore this does not give a new state. In the other cases ($\mu = \pm 1, 4$) the action of H_E on $|\underline{\mu}\rangle$ is:

$$H_E|\underline{\mu}\rangle = g^2 a^{-1}(4-\delta_{-1\mu})C_2|\underline{\mu}\rangle + \frac{g^2\mu}{2a}(1-\delta_{-1\mu}) \cdot \left[|\bar{\mu}\rangle - \frac{1}{N_c}|\underline{\mu}\rangle\right] \tag{2.13}$$

The states introduced up to now are not orthonormal. The scalar products can easily be calculated using the results in Sect. III of I. Knowing the scalar products one can introduce the appropriate orthonormal states in the following way:

$$\begin{aligned}
|0) &\equiv |0\rangle \\
|1) &\equiv \frac{1}{\sqrt{P}}|1\rangle \\
|\bar{0}) &\equiv \frac{1}{\sqrt{2P^2-52P}}|\bar{0}\rangle \\
|\bar{4}) &\equiv \frac{1}{\sqrt{2P}}|\bar{4}\rangle \\
|\underline{4}) &\equiv \frac{1}{\sqrt{2P}}|\underline{4}\rangle \\
|\overline{-4}) &\equiv \frac{1}{\sqrt{2P}}[|-\bar{4}\rangle - P|0\rangle] \\
|\overline{\pm 1}) &\equiv \frac{1}{\sqrt{24P}}|\overline{\pm 1}\rangle \\
|\underline{\pm 1}) &\equiv \frac{1}{\sqrt{24P(1-N_c^{-2})}}\left[|\underline{\pm 1}\rangle - \frac{1}{N_c}|\overline{\pm 1}\rangle\right]
\end{aligned} \tag{2.14}$$

In order to go to the 6th order of the perturbation expansion in $1/N_c$ we also need the three-plaquette states. These are given by a straightforward generalization of the two plaquette case. If the number of common links between the three different pairs of the three plaquettes are, respectively, λ, μ, ν then we have, as a generalization of (2.8)

$$ag^2 H_A|\bar{\lambda}\rangle = \sum_{\mu,\nu}|\lambda,\mu,\nu\rangle \tag{2.15}$$

The state $|\lambda,\mu,\nu\rangle$ is obviously symmetric under the permutation of indices λ, μ, ν. The possible, different values of λ, μ, ν are the following:

$$\begin{aligned}
\lambda,\mu,\nu = {}& 0,0,0;\ 1,0,0;\ -1,0,0;\ 4,0,0;\ -4,0,0;\\
& 4,1,1;\ 4,-1,-1;\ 4,4,4;\ 4,-4,-4;\\
& -4,1,-1;\ 1,1,0;\ 1,-1,0;\ -1,-1,0;\\
& 1,1,1;\ 1,-1,-1;\ 1,1,-1;\ -1,-1,-1.
\end{aligned} \tag{2.16}$$

The orthonormalization of these states and the calculation of matrix elements do not involve any new difficulty.

The physical vacuum $|v\rangle$ is the physical state with lowest energy. To zeroth order in $1/N_c$ it is coinciding with the mathematical vacuum $|0\rangle$ which has zero energy. In higher orders of $1/N_c$ $|v\rangle$ gets components containing any number of gluon loops. The standard time independent perturbation theory gives for the energy density ε_v in the physical vacuum (measured relative to $|0\rangle$):

$$\varepsilon_v = -6g^2 N_c a^{-4}\left(\frac{1}{N_c^2 g^8} + \frac{1}{N_c^4 g^8} + \frac{1}{N_c^6 g^8} + \frac{11}{4N_c^6 g^{16}} + \frac{126}{N_c^6 g^{24}} + \dots\right) \tag{2.17}$$

In spite of the different definitions of H_E this coincides with (4.17) in I up to the order $1/N_c^4$ in the brackets and apart from an overall factor 2. (The overall factor 2 here and in (2.18) can also be corrected by multiplying H_E in I by a factor 2. If the definition of λ in I is also multiplied by 2 then all the physical quantities calculated in I are unchanged up to the order calculated there).

The vacuum expectation value of the gluon field squared (or of the Yang–Mills Lagrangian density) up to the order $1/N_c^6$ turns out to be:

$$\begin{aligned}
\langle v|L_{YM}(0)|v\rangle &= -6g^2 N_c a^{-4}\left(g^{-4} - \frac{3}{N_c^2 g^8} - \frac{3}{N_c^4 g^8} - \frac{3}{N_c^6 g^8} - \frac{77}{4N_c^6 g^{16}} + \frac{126}{N_c^6 g^{24}} + \dots\right) \\
&= -6g^{-2}a^{-4}\left(3 - g^{-4} - \frac{1}{9}g^{-4} - \frac{1}{81}g^{-4} - \frac{77}{972}g^{-12} + \frac{14}{27}g^{-20} + \dots\right)
\end{aligned} \tag{2.18}$$

The second line here corresponds to $N_c = 3$. This expression will be compared with experimental data together with the slope of the $q\bar{q}$ potential in the next section.

III. External Sources and the Potential Energy

In this section we determine the lowest energy states of the gluon field in the presence of external sources giving, by definition, the potential energy between the sources. In order to have a non-degenerate lowest eigen-value of the $N_c \to \infty$ ("unperturbed") Hamiltonian we now put the external sources on the lattice in such a way that they are separated in some of the directions of the links (say, in the x-direction). The "distance" of the external sources (equal to the number of links between them) will be denoted by $\lambda = L$.

Let us first consider the case of a pair of quark-antiquark sources, when the states contain a single open "string" starting from the quark and ending on the antiquark (in addition to the arbitrary closed gluon loops). In the state with the lowest colour electric energy (and hence the lowest unperturbed energy for $N_c \to \infty$) there are no closed gluon loops and the string is a straight line connecting the sources. Let us denote this state by $|G\rangle$. Its colour electric energy is given, according to (2.5), by

$$H_E|G\rangle = \frac{g^2}{2a} L C_2 |G\rangle \tag{3.1}$$

In higher order of the $1/N_c$ expansion the lowest energy state get components containing additional gluon loops and/or a string with different shape. The simplest such states are obtained by the addition of single gluon loops along the plaquettes:

$$ag^2 H_A|G\rangle = \sum_{k[rs]} A(k[rs])|G\rangle = \sum_{\mu} |G\mu\rangle \tag{3.2}$$

Here the index μ takes on the values 0, ± 1. It gives the number (and relative direction) of common links between the open string and the plaquette. In order to calculate the potential energy up to fourth order we need also the switched over states with one plaquette: $|G\underline{\mu}\rangle$ (for $\mu = \pm 1$) and the states with two additional plaquettes $|G\lambda, \mu; \nu\rangle$ defined by

$$ag^2 H_A|G\lambda\rangle = \sum_{\mu,\nu} |G\lambda, \mu; \nu\rangle \tag{3.3}$$

The values λ and μ give the number of common links between the string and the plaquettes and ν is the number of common links between the two plaquettes. We have, therefore, the symmetry $|G\lambda, \mu; \nu\rangle = |G\mu, \lambda; \nu\rangle$.

The potential energy is the energy of the lowest state with quark sources minus the vacuum energy (2.17). With $r \equiv aL$ we have, up to fourth order in $1/N_c$:

$$V_q(r) = \frac{rg^2 N_c}{a^2}\left(\frac{1}{4} - \frac{1}{4N_c^2} - \frac{3}{N_c^4 g^8} - \frac{312}{N_c^4 g^{16}} + \dots\right) + \frac{g^2 N_c}{a}\left(\frac{304}{N_c^4 g^{16}} + \dots\right) \tag{3.4}$$

For $N_c = 3$ the slope of the potential is therefore

$$\frac{1}{A^2} = \frac{g^2}{2a^2}\left(\frac{4}{3} - \frac{2}{9g^8} - \frac{208}{9g^{16}} + \dots\right) \cong \frac{g^2}{2a^2}(1.333 - 0.222 g^{-8} - 23.11 g^{-16} + \dots) \tag{3.5}$$

It is interesting to compare this with the strong coupling expansion at $N_c = 3$ (see [9]);

$$\frac{1}{A^2} = \frac{g^2}{2a^2}(1.333 - 0.2876 g^{-8} - 0.2990 g^{-12} - 0.2034 g^{-16} - 0.09815 g^{-20} + \dots) \tag{3.6}$$

The main difference is that in (3.5) only powers of g^{-8} appear. Besides, the coefficient of g^{-16} is much larger there. The agreement between the two expansions can, in principle, be better if higher orders of $1/N_c$ are also included (see the general form of the series given by (1.1)). The agreement for finite lattice spacing a is, however, not so essential because an analytic continuation to $a \to 0$ ($g \to 0$) is also necessary.

From the experimental numbers quoted in I: $A = 2.34\,\text{GeV}^{-1}$ and $\langle v|L_{YM}(0)|v\rangle g^2 = -0.12\,\text{GeV}^4$ one can determine g and the corresponding a. Combining (3.5) and (2.18) up to fourth order (the first three terms in the brackets) we obtain the following equations ($x \equiv g^{-4}$):

$$a^4 = (150 - 55.6x)\,\text{GeV}^{-4}$$
$$0 = 13.7 - 150x + 51.1x^2 - 475x^4 + 79.2x^6 + 4120x^8 \tag{3.7}$$

The second equation has two real roots ($x_1 = 0.0941$ and $x_2 = 0.659$) giving the two sets of values ($\alpha_s \equiv g^2/(4\pi)$):

$$g_1 = 1.81, \quad \alpha_{s1} = 0.26, \quad a_1 = 0.68\,\text{fermi}$$
$$g_2 = 1.11, \quad \alpha_{s2} = 0.098, \quad a_2 = 0.64\,\text{fermi} \tag{3.8}$$

The first set is almost the same (in fact, within experimental errors the same) as (5.10) in I. The only essential influence of the fourth order terms is that it brings the second value closer to the first one. The first values are certainly reasonable. For instance, at $Q^2 \cong 870\,\text{GeV}^2$ the PETRA e^+e^- data on three jets give $\alpha_s(Q^2) \cong 0.22-0.23$ (see [10]).

The above calculation of the potential between a quark and an antiquark can also be repeated for external gluon (i.e. colour octet) sources (see I). For octet sources, however, the question of screening versus potential energy arises. (The same problem for the quark sources is, of course, also there because of the existence of light quarks. It did not appear here only because of the neglect of quarks in the Hamiltonian altogether.) In the case of a pair of external octet sources there are in fact, two sorts of states. In one case there are two oppositely oriented open strings connecting the sources. Such states can be called "unscreened". The other possibility is that the two strings start and end on the same source (see Fig. 2). These can be called, by definition, "screened" states. It is clear that the lowest energy screened state has a constant energy, independently from the distance of sources, whereas the energy of the lowest unscreened state rises linearly with the distance similarly to the case of quark external sources. For finite N_c the unscreened states are unstable with respect to the decay into screened states. For $N_c \to \infty$, however, the colour magnetic term in the energy is negligible, therefore the strings cannot change their shape and the unscreened states are also stable. In other words, the screening of the colour octet sources

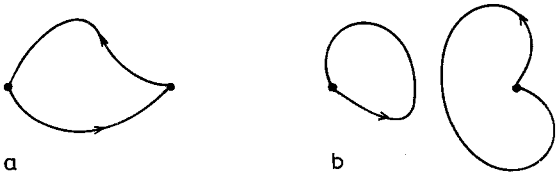

Fig. 2. "Unscreened a and "screened" b states for a pair of external gluon sources

is suppressed in the $N_c \to \infty$, g = fixed limit similarly to the screening of the quark sources (Actually, the effect of the screened gluon-gluon source states comes in the strong coupling $1/N_c$ expansion of the lowest energy unscreened state only in the order $1/N_c^{10}$).

This gives the possibility to define in the strong coupling $1/N_c$ expansion a gluon-gluon potential energy by the energy of the lowest unscreened state with external gluon sources. This is, in fact, a very similar procedure to the definition of the quark-antiquark potential in the limit when all the quark masses are taken to infinity. Nevertheless, in reality there is presumably a difference between the importance of the screening of colour triplet and octet sources. Namely, gluon pair creation is expected to be stronger than quark pair creation because of the stronger colour charge. (Such a difference can be manifested, for instance, as a difference between gluon and quark jets in hard collisions). Although there is one thing which can help, namely, that in low energy situations there are no open channels where the creation can lead to. (Confinement means also a non-zero mass threshold in multigluon channels!) In such cases it may be reasonable to introduce a gluon confinement potential similarly to the quark confinement potential.

In spite of these reservations we give here the fourth order result also for the gluon-gluon potential:

$$V_g(r) = \frac{rg^2 N_c}{a^2}\left(\frac{1}{2} - \frac{6}{N_c^4 g^8} - \frac{312}{N_c^4 g^{16}} + \dots\right) + \frac{g^2 N_c}{a}\left(\frac{304}{N_c^4 g^{16}} + \dots\right) \qquad (3.9)$$

The ratio of the slopes of the gluon-gluon to quark-antiquark potentials is, for $N_c = 3$:

$$\lim_{r\to\infty} \frac{V_g(r)}{V_q(r)} = \frac{1 - \frac{4}{27g^8} - \frac{208}{27g^{16}} + \dots}{\frac{4}{9} - \frac{2}{27g^8} - \frac{208}{27g^{16}} + \dots} \qquad (3.10)$$

Taking $g = g_1$ from (3.8) this turns out to be slightly (1.001-times) larger than the $0(1/N_c^2)$ value 9/4.

IV. Concluding Remarks

The next order corrections to [1] (sixth order in the case of (2.17) and (2.18) and fourth order in (3.4) and (3.9)) are rather small if we take the value g_1 in (3.8) of the coupling constant g determined from the measured values of the vacuum expectation value of the gluon field squared and the slope of the $q\bar{q}$ potential. The typical order of magnitude of the corrections is only a few percent! Therefore it seems probable that the higher orders influence the physical quantities calculated on the lattice only very little.

The investigation of higher orders is, however, important if we want to connect the lattice theory to the continuous theory (in order to restore e.g. Lorentz invariance). This requires an analytic continuation to $g \to 0$, $a \to 0$ where the series are divergent. A possible way to do this (as shown by examples in statistical physics) is to use Padé-approximants [9]. In the context of the strong coupling $1/N_c$ expansion the analytic continuation in N_c from large values to the physical value $N_c = 3$ is also needed. A simultaneous analytic continuation in N_c and g, e.g. by two variable Padé approximants seems to be the most economic and perhaps also the best procedure. This would give an independent possibility to reach the continuum theory besides the continuation in g of the usual strong coupling expansion at $N_c = 3$. The advantage of the analytic continuation in two variables (over the subsequent continuation in N_c and g) is that there is a hope to overcome also the anomalies in N_c (for fixed g) noticed by de Wit and 't Hooft [11] (see also I). The number of terms in the strong coupling $1/N_c$ expansion calculated up to now does not seem to suffice yet for this purpose. The determination of the next higher orders (two or perhaps four more powers of $1/N_c$) does not seem, however, to represent any insurmountable difficulty.

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