CONSTRUCTING OR COMPLETING
PHYSICAL GEOMETRY?
ON THE RELATION BETWEEN THEORY AND EVIDENCE
IN ACCOUNTS OF SPACE-TIME STRUCTURE*

MARTIN CARRIER†

Department of Philosophy
University of Konstanz

The aim of this paper is to discuss the relation between the observation basis and the theoretical principles of General Relativity. More specifically, this relation is analyzed with respect to constructive axiomatizations of the observation basis of space-time theories, on the one hand, and in attempts to complete them, on the other. The two approaches exclude one another so that a choice between them is necessary. I argue that the completeness approach is preferable for methodological reasons.

According to the operationalist point of view, a theory has to be constructed on the basis of available measuring procedures. The theory provides a sort of summary of the results obtained by the pertinent operations. Einstein's operational analysis of distant simultaneity (which played a prominent role in the genesis of Special Relativity) is usually regarded as the paradigm case of such an approach. If the available measuring devices are not sufficient to confer operational meaning to a notion (that is, to an observer-independent relation of simultaneity in this case), this notion lacks cognitive significance and has, accordingly, to be rejected. In other words, the operationalist view considers measurement procedures as the firm ground on which the theoretical edifice has to be erected.

*Received March 1988; revised July 1988.
†I wish to acknowledge the benefits of discussions with my colleague Claus Lämmerzahl (University of Konstanz) about the physical aspects of the problems addressed in this paper.

Copyright © 1990 by the Philosophy of Science Association.
Measurements are fundamental, theories are derivative in character.

An *axiomatization* of a theory selects certain propositions of that theory as basic. All other propositions are to be derived from this set of fundamental sentences. It is of great philosophical importance which propositions are chosen as basic. Following a terminology first introduced by Reichenbach in 1924, I distinguish between constructive and deductive axiomatizations. In a constructive axiomatization only those statements are accepted as fundamental that are immediately amenable to experimental control. A deductive axiomatization, by contrast, confers fundamental status to more abstract statements (such as variational principles) (compare Reichenbach 1924, pp. 2–3). Obviously, an operationalist approach is committed to the requirement of constructively axiomatizing theories.

It is one of the shortcomings of the operationalist position that theories built up in this way cannot adequately deal with the measuring processes themselves. If a theory is to be founded upon certain operations, the analysis of these operations cannot be founded in turn on that theory. Foundation presupposes linearity and reciprocal foundation, therefore, constitutes a sort of category mistake. Consequently, Einstein’s operational analysis of simultaneity relied on the availability of ideal clocks on whose functioning Special Relativity remained silent. This feature (and, accordingly, the operational approach from which it originates) was later regarded by Einstein as a serious methodological flaw. In 1949, Einstein observed that Special Relativity introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things, e.g., the electro-magnetic field, the material point etc. This in a certain sense is inconsistent; strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations (objects consisting of moving atomic configurations), not, as it were, as theoretically self-sufficient entities. (Einstein 1949, p. 59)

This is to say, rigid rods and atomic clocks on the one hand serve to establish the evidential basis of Special Relativity but they transcend, on the other, this theory’s domain of application. And Einstein’s complaint amounts to requiring that the laws of Special Relativity should be derivable from an atomic theory of the measuring devices. But the argument also supports the (somewhat different) postulate that only such processes as this theory can comprehensively deal with, should be regarded as constituting the observation basis of a theory in the first place. Accordingly, Special Relativity should rely on processes it can analyze by its own conceptual means. At any rate, both versions of this postulate advocate that a theory should be able to cope with the measuring devices needed to
connect its theoretical terms with experimental data. A theory which meets this requirement (a requirement that I will further elaborate below) is called complete. A complete theory manages to explain why certain empirical processes are reliable indicators for certain theoretical states specified within its framework.

In what follows I wish to focus attention on the relation between theory and evidence in physical geometry. The proper theory to turn to in this context is the General Theory of Relativity (henceforth abbreviated GTR). On its basis two levels of consideration should be distinguished. We have, on the one hand, the observation basis of GTR which consists of the geometrical relations between certain phenomena, that is, the metric structure. On the other hand, there is the explanatory level on which the metric is accounted for by recourse to its sources. Einstein’s field equations serve this latter purpose. There exist, correspondingly, two possible subjects of an axiomatization procedure; one may axiomatize the observation basis or the explanatory principles. I will restrict my discussion to the first subject. As indicated above there are two distinct approaches to such a problem. A constructive axiomatization of GTR’s observation basis seeks to specify fundamental experiments which can be used to measure the metric structure. A deductive axiomatization tries to clear up those theoretical principles on which the linkage of GTR’s theoretical terms to certain experiments actually rests. What is at issue in a deductive axiomatization is to elucidate the reasons why certain phenomena reliably measure the metric structure. The methodological idea of completeness obviously belongs to this deductive approach; it requires that all theoretical principles requisite here pertain to GTR itself.

Testing a hypothesis or theory requires that special instances of this hypothesis or theory are observed or experimentally realized. Such a test can only be regarded as serious if empirical results may arise that are incompatible with the hypothesis to be tested. It must not be the case that all the evidence an experiment can possibly supply is in accordance with this hypothesis. If this criterion (that will be cheerfully endorsed by Popperians) is applied to the test of complete theories, it seems dubious, at first sight at least, whether such theories can be seriously tested. If all the empirical instances of a theory are brought about by relying on this very theory itself, it is not apparent from the outset whether or how theory-laden data of this sort may possibly contradict the pertinent theory. I have dealt elsewhere with the general problem of testing complete theories (compare Carrier 1989) and so I want to restrict my discussion to an analysis of the bearing of (some aspects of) completeness on the possibility of unambiguously establishing the metric structure of space-time.

What deserves special interest in this context is an argument put forth by Reichenbach in 1928, according to which the empirical determination
of the metric presupposes that certain distorting influences (the so-called universal forces) are known. The evaluation of these influences requires in turn that the metric has already been established. Both quantities form part of the same theory, namely, GTR and so this case presents us with the distinctive feature of completeness: the empirical determination of the metric essentially relies on the conceptual framework of GTR. But because the evaluation of both quantities mentioned are reciprocally dependent upon one another, serious testing appears to be ruled out. Reichenbach indeed drew this conclusion and affirmed that this untestability leads to the conventionality of the corresponding aspects of GTR.

To summarize these introductory remarks, it is the general aim of this paper to contribute to a clarification of the relation between theory and evidence both in constructive and deductive approaches. Concerning the latter ones I want to bring into focus the testability problems that may arise in complete theories. Eventual difficulties of this kind originate from reciprocal test dependencies and Reichenbach’s argument, as sketched above, constitutes the paradigm case of such a difficulty. So, in the first section, I will outline this argument and describe one of its more recent refinements. In the second section I will analyze (1) how constructive axiomatizations deal with the observation basis of a theory and expose (2) how the relation between theory and evidence is to be viewed on the basis of the completeness approach. The third section is devoted to an argument of Einstein’s which can be interpreted to the effect that GTR is, to a considerable extent, complete. It will become clear that, as a consequence, reciprocal test dependencies are of even greater importance than Reichenbach had supposed. In the fourth and final section I will analyze the impact of peculiarities of the reciprocal test dependencies upon the possibility of measuring the actually realized metric. I will discuss the example of the so-called Marzke-Wheeler clock so as to elucidate exactly what relation there is between theory and evidence in the completeness approach and compare it with the theory/evidence relation that arises in constructive axiomatizations. Eventually, I will argue for the thesis that completing our accounts of space-time structure is preferable to constructing them.

1. Universal Forces and the Empirical Underdetermination of Space-Time Structure. Fixing space-time structure amounts to measuring spatial lengths and time intervals. Length measurements are most conveniently performed by applying measuring rods. But not all rods are equally suited for this purpose; in order to obtain useful and consistent results one has to employ rigid rods. But how do we determine which rods are rigid? Rigidity can be achieved, firstly, by considering and correcting differential forces (as Reichenbach calls them). Differential forces act dif-
ferently on chemically distinct substances and because of these different effects they can easily be detected. Temperature is an example of a differential force. Temperature changes distort the length of measuring rods but the amount of distortion is dependent upon the material employed. The construction of gas or liquid thermometers makes use of the fact that the expanding effect of temperature increase is specific for each material. This is to say, differential influences can be evaluated experimentally and, accordingly, be corrected.

It is quite another thing if there exist distortions that act alike on all materials and for which there are no insulating walls. Forces possessing these properties are called universal by Reichenbach. It is not possible to evaluate directly the intensity (or even the presence) of such universal forces. Lengths can only be compared locally; so if two locally congruent rods are transported along different paths and are again found to be locally congruent, it cannot be ruled out empirically that during transport both rods change their lengths in a universal fashion. It is impossible to directly compare lengths at distinct locations. It is of no help to take recourse to optical methods because such methods rely on assumptions about light propagation. Paths of light rays, however, are also influenced by universal forces. Thus, optical procedures may suffer from the same distortions as transport experiments. Reichenbach concludes that distant congruence cannot be established empirically but has to be defined instead. A coordinative definition of rigidity is necessary and Reichenbach proposes to choose this definition in the following way: rigid bodies are solid bodies in which the influence of differential forces has been eliminated by corrections and all universal forces are set equal to zero. In other words, we decide that differentially corrected rods are rigid. That is, we decide that they retain their length during transport. Absence of universal forces is decreed and not detected.

The very same problem occurs with respect to the measurement of durations. As for spatial length determinations it cannot be empirically ruled out that time intervals are expanded or contracted by universal distortions that act on all clocks alike. Again, Reichenbach suggests that we should set such distortions equal to zero (compare Reichenbach 1928, pp. 113–119).

It is apparent from these examples that, in Reichenbach’s view, the need for a definitional or conventional choice is rooted in epistemic restrictions of testability. More specifically, comparing lengths at different locations, that is, evaluating the metric structure of space, presupposes that we know the influences of possible universal distortions. On the other

---

1For this sketch of Reichenbach’s ideas compare (Reichenbach 1928, pp. 10–24). Essential aspects of Reichenbach’s argument are anticipated in (Poincaré 1902, pp. 89–93).
hand, these influences can only be determined and their absence can only be verified, if we already know the metric structure. To put things more generally, if a hypothesis $H_1$ can exclusively be tested by relying on a hypothesis $H_2$, and if $H_2$ can only be checked by relying in turn on $H_1$, no empirical test of both hypotheses is feasible. Consequently, one of them has to be arbitrarily chosen and the other one has to be adjusted appropriately.

It is important to note what is not at issue here. Reichenbach does not simply invent some strange but unobservable theoretical ornaments (namely, universal forces) in order to remove them by asserting that doing away with superfluous decorations requires a conventional choice. If something has been introduced arbitrarily, its subsequent removal also needs an arbitrary convention. True, we would say, but so what? In fact, however, distortions due to universal forces are empirically detectable (even if only in an indirect fashion) because such forces do not generally preserve coincidences. Inhomogeneities in the universal force field induce deformations in suitably constructed devices that can be observed in a straightforward manner (compare Reichenbach 1928, pp. 25–27). This means that Reichenbach’s conventionality argument is not identical to the traditional (and unsound) argument for the relativity of lengths, that is, multiplying the sizes of all bodies by the same factor would have no empirically detectable effects. Furthermore, there exists a serious candidate for such a universal force, namely, gravitation. Gravitational forces act alike on all materials and they cannot be screened. But if universal forces are sometimes coincidence-destroying and if, in addition, gravitational forces are of this kind, what can it mean that we should set them equal to zero? It means that the occurrence of universal deformations is not to be attributed to the action of a force but to a change of physical geometry (compare Reichenbach 1928, p. 27).

From a methodological point of view the essential aspect of this proposal is that, judged on its basis, Einstein’s approach to gravitational theory appears superior. One of the central characteristics of Einstein’s formulation of GTR is the local “geometrization” of gravitation. That is to say, freely falling particles are not acted upon by a force but follow an inertial path. Stated more generally, geometrization means that the variables that express the interaction between a particle and an external field are incorporated into the affine connection that characterizes the physical geometry. Geometrizing gravitation, accordingly, implies that gravitation is (at least locally) not to be interpreted as a force like, say, electromagnetic forces but as a manifestation of space-time structure. From this point of view, it is not the apple falling from a tree that is accelerated but the tree instead. The falling apple constantly remains in its natural state, that is, it is unaffected by any external influence. The tree, on the other hand,
is deviated from its straightest possible (geodesic) world-line by the action of cohesive forces. Thus, gravitation is conspicuously absent from this account of free fall (compare Synge 1960, pp. 132–133). But it seems to be at least of philosophical (or ontological) importance to ask whether or not this scheme represents an adequate way of putting the falling apple scenario. Accepting or rejecting universal forces is of considerable relevance for the philosophical interpretation of GTR if not for its empirical content. In deciding about universal forces we decide at the same time about the status of gravitation. We decide, that is, whether to regard it as an effect of physical geometry or of physical forces. I conclude that Reichenbach’s problem of universal forces is, in fact, non-trivial.

Reichenbach’s approach to physical geometry is flawed by some (more technical) shortcomings. It has been objected that Reichenbach’s separate treatment of isochronous clocks and rigid rods does not adequately reflect the essentially four-dimensional point of view of GTR. Secondly, as Synge’s analysis has revealed, reference to rigid rods is unnecessary; it is sufficient to make exclusive use of ideal (atomic) clocks and light rays. Synge refers to a photon reflected back and forth between two world-lines. If the time elapsed between emission and return of the photon (as measured by atomic clocks) turns out to be constant, both world-lines retain their distance and can thus be employed as a standard of rigidity (compare Synge 1960, p. 115). Thirdly, in the light of Einstein’s methodological criticism, as quoted in the introduction, rods and clocks are rather ill-suited for establishing the metric structure. It is methodologically preferable to base measurements of the metric on processes that lie within the scope of space-time theories. For these reasons, more recent attempts to establish space-time structure no longer rely on rods and clocks but instead on light rays and particle trajectories. The most fundamental and most ingenious approach of this kind was developed by Ehlers, Pirani, and Schild (henceforth EPS) in their (1972).

As EPS point out, the propagation of light can be used to determine the infinitesimal null-cone at each point of space-time. If a null-cone has been ascribed to each such point, space-time is endowed with a conformal structure in which the metric $g_{ik}$ is fixed up to conformal transformations ($g_{ik} = c^2(r,t)g_{ik}$). Moreover, the motions of freely falling particles can be used to obtain the four-geodesics of space-time. These motions fix the affine connection $\Gamma^j_{ik}$ of space-time up to projective (that is, geodesic-preserving) transformations. One further introduces a compatibility postulate to the effect that the null geodesics (light rays) of the conformal structure ought to be contained in the class of geodesics obtained from the projective structure. This expresses the requirement that particle trajectories always remain within the light cone. If this condition is met one arrives at a Weyl-structure. In such a structure a metric can be defined
which is unique up to linear transformation. This latter indeterminacy only reflects our freedom to choose a zero and a unit for measurements of space-time intervals. The Weyl metric thus obtained possesses, however, a peculiar property. In general, the lengths of intervals change during transport in a way that varies with the world-line along which the interval is moved. This means that two locally synchronized isochronous clocks, if transported along different world-lines to the same space-time location, will exhibit different ticking rates (second clock effect). EPS postulate that no such effects are found and this provides a necessary and sufficient condition for Weyl space-time to reduce to Riemann space-time (compare Ehlers, Pirani and Schild 1972; Ehlers and Schild 1973, pp. 120–122). Accordingly, the metric structure can be determined by recourse to light rays and particle trajectories.

As Glymour has made clear, Reichenbach's problem of universal forces can be translated into the framework of particle trajectories (compare Glymour 1977, p. 245). Consider the equation of an inertial motion, that is, the equation of a particle moving along the geodesic of an affine connection \( \Gamma^i_{jk} \). Written in generally covariant manner this equation assumes the form:

\[
\frac{d^2 x^i}{d\tau^2} + \Gamma^i_{jk} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau} = 0
\]  

\((*)\)

One is now at liberty to introduce a universal force \( F^i \), that is, a force acting on all particles independently of their properties. From this perspective, the same particle would not appear to follow a geodesic (that is, to be free from external influences) but to be subject to the action of \( F^i \). The particle is regarded as accelerated relative to a distinct connection \( \ast \Gamma^i_{jk} \). The corresponding equation of motion is:

\[
\frac{d^2 x^i}{d\tau^2} + \ast \Gamma^i_{jk} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau} = F^i
\]  

\((**)*\)

The expression for \( F^i \) is obtained by subtracting \((*)\) from \((**)\). Both equations describe the same motion, that is, it is impossible to tell them apart empirically. Although \( F^i \) thus exhibits the distinctive feature of a Reichenbachian universal force, the geometrical status of trajectory deflections effected by it is somewhat different from that of universal distortions of rigid bodies. Whereas the latter bear directly upon the metric the former only influence the connection. Connections do not primarily refer to lengths but to path structures (more precisely, to autoparallels) and they can be introduced in non-metrical manifolds.

It can be (and it actually has been) concluded that the EPS-scheme does not relieve us from the need to decide about the presence or absence of universal forces (compare Sklar 1977, p. 259). As previously mentioned, this decision concerns the question whether or not a particle that is only
subject to gravitational interaction is to be regarded as free. Setting universal forces equal to zero in this case amounts to contending that freely falling particles are not acted upon by any force. This is certainly a nontrivial allegation (and moreover one that at first has a counterintuitive ring). It should be noted, furthermore, that the empirical underdetermination of the values of universal forces arise, just as in the case of rigid body distortions, from a reciprocal test dependence between these values and the ensuing physical geometry.

2. Constructive Axiomatizations and Completeness. It is the general aim of both Reichenbach and EPS to establish the metrical structure of space-time on the basis of simple experiences. This means that both aim at providing a constructive axiomatization of the observation basis of GTR. More precisely, they seek to establish the evidential ground of GTR without making use of its conceptual apparatus. This amounts to determining the actually realized metric without relying on Einstein's field equations. In this vein, EPS declare:

[W]e attempt to derive the conformal, projective, affine, and metric structures of space-time from some qualitative . . . properties of the phenomena of light propagation and free fall that are strongly suggested by experience. (Ehlers, Pirani and Schild 1972, p. 65)

What is at issue is to construct the theory from scratch. In fact, the aim of EPS is even more ambitious than Reichenbach's. Reichenbach has no scruples to use complex things (such as material rods) for establishing the observation basis of GTR. Accordingly, he has to rely on other theories that supply the means necessary for differential corrections (such as thermic expansion). This is to say, Reichenbach permits that the whole of non-GTR physics (including quantum theory if atomic clocks are employed) enters as observation theory. Consequently, this approach is constructive only relative to presupposing the remainder of physical theory.

EPS, on the contrary, exclusively make use of phenomena that lie within the scope of the theory to be developed or axiomatized. This means that they refuse to apply any observation theory. After all, the only theory available to cope with these phenomena is the explanatory theory itself (GTR) and its application is forbidden by the rules of constructive axiomatization. Their primitive vocabulary only encompasses the concepts of event, light ray and freely falling particle. 'From scratch' here really means from scratch. The first question to emerge at this point is: is such an approach feasible at all? And the next one naturally follows: is it methodologically desirable?

Such an approach is, in fact, feasible: but not trivially so. And the way in which it is feasible sheds some light on the relation between theory and evidence. The main difficulty arising in this context is: how to deal
with distortions without having any theory at our disposal. We evidently cannot correct them, for corrections require physical laws. What we can do, however, is to put aside cases in which distortions are present. We can select pure or ideal cases. Let us cast a brief glance at how this selection can be performed.

We have to consider two general classes of distortions. Distortions of the first kind can be corrected by recourse to theories other than the one that is to be constructively axiomatized. Distortions of the second kind require for their correction the use of this very theory itself. The distinction between these two kinds of distortions is independent of the one previously mentioned between universal and differential distortions; in fact, these distinctions cross-classify one another. To make the difference between the two kinds of distortions more perspicuous let me first apply it to rigid rods and atomic clocks. As I have mentioned, the length of rods is influenced by thermic variations. Analogously, the presence of electromagnetic fields may change a body's dimensions. Correcting these distortions requires recourse to thermodynamic or electromagnetic theory so that, consequently, these influences constitute distortions of the first kind. Another situation arises for atomic clocks. If such instruments are deformed by tidal forces (forces that GTR accounts for) they no longer yield correct results (compare Misner, Thorne and Wheeler 1973, p. 396). In order to assess the precision of an atomic clock the intensity of tidal forces has to be known. But according to GTR, these forces originate from inhomogeneities of the metrical field. This implies that for atomic clock readings to be reliable the metric must be homogeneous. On the other hand, the metric is determined by means of just these atomic clock readings. Synge's previously mentioned procedure for establishing empirically the metric structure presupposes that atomic clocks measure proper time. But this is true only if the metric is sufficiently homogeneous. Otherwise, corrections are requisite that can only be performed on the basis of GTR itself. Influence due to tidal forces, consequently, constitute an example of distortions of the second kind.

Quite the same features are present in the case of light rays and particle trajectories. The paths of charged particles are influenced by electromagnetic fields. Moreover, trajectories of particles with a fixed specific charge in a static electromagnetic field can also be geometrized, that is, they can be described as inertial motions with respect to a suitably chosen connection. In order to rule out such spurious geometrization we should expect that the application of electrodynamics is requisite to assure us of the neutrality of the particles or to supply the appropriate corrections. This, however, would vitiate EPS's claim of constructibility from scratch. In fact, EPS can overcome this difficulty by resorting to their compatibility postulate (as presented above). Only trajectories that are compatible
with the conformal structure obtained from the observation of light cones are accepted as four-geodesies. This condition removes charged particles from the scheme.

Another problem arises as regards distortions of the second kind. Particles that are spatially extended or that possess a spin or a gravitational multipole moment in no way follow a geodesic path. Furthermore, here again a spurious geometrization cannot be excluded outright. Consider a “frozen spin” scenario in which the spins of all particles at any space-time location are equally directed. In this case it is possible to geometrize the influence of the spin on the particle trajectories. In fact, it is always possible to geometrize any universal particle property. So if we do not want to follow this rather unilluminating strategy we have to correct for these influences. But in order to correct for them it is necessary to rely on GTR itself. More precisely, the influence of a particle’s extension of its trajectory is assessed by ascribing a spin and a gravitational quadrupole moment to the corresponding point mass (compare Stephani 1980, p. 93; Misner, Thorne and Wheeler 1973, pp. 477–478). The influence of the spin is determined by applying the condition of vanishing divergence of the energy-momentum tensor which is a consequence of Einstein’s field equations (compare Stephani 1980, pp. 93–94). Obviously, the treatment of these distortions necessarily rests on GTR theorems.\(^2\)

Again, the way out of these difficulties is not to correct for these distortions but rather to eliminate from consideration all cases spoiled this way. As Coleman and Korte have shown (relying on an earlier study of projective geometry by Ehlers and Schild) a practicable procedure to this effect is indeed available (compare Coleman and Korte 1980, pp. 1350–1351; Ehlers and Schild 1973, pp. 144–146). Coleman and Korte consider an inertial motion to be characterized by the fact that the corresponding four-trajectory is uniquely determined by the direction of this trajectory at any given event. In light of this characterization, gravitationally influenced particle motions come out as inertial because the uniqueness of four-trajectories is an immediate consequence of the Weak Principle of Equivalence (which states the equivalence of inertial and gravitational mass). This consideration implies that inertially moved particles that are released at infinitesimally near space-time points with four-directions differing only infinitesimally, remain infinitesimally near. In other words, geodesics (by virtue of being autoparallels) retain their direction under infinitesimal displacement.

Now it has to be shown that it is possible to check empirically and without relying on a metric whether, in a given class of particles, infin-

\(^2\)This peculiarity has been used to level the charge of circularity against the EPS-procedure. (Compare Grünbaum 1973, pp. 746–748 and Sklar 1977, pp. 259–260.)
itesimal parallellity is preserved. As Ehlers and Schild have made clear this can be achieved by exclusively invoking a projective geometrical construction which can be realized by particle trajectories. This construction reveals that the notion of infinitesimal nearness can be employed by relying on affine parameters (that is, on non-metrical quantities). For this purpose one has to make an affine parameter, describing non-metrically the size of the experimental set up, convergent to zero. In practice, this amounts to performing a limiting sequence of experiments steadily shrinking in size. This procedure supplies us with an empirical basis for singling out space-time geodesics, that is, it provides a means for endowing space-time with a projective structure. This means that the above characterization of inertial motion can indeed be put into experimental practice. One can successfully set aside trajectories influenced by distortions of the second kind (such as gravitational quadrupole moments) without recourse to the full-fledged theory. To obtain the full Weyl-structure one goes on as indicated above, that is, by requiring compatibility between the projective structure thus obtained and the conformal structure derived from the light cone.

From a systematic point of view, this construction makes use of the fact that neighboring geodesics remain parallel in flat space-time. But in the general case of curved space-time the flatness condition is only satisfied in tangent space. Accordingly, one tries to approach tangent space by constantly diminishing the size of the experiment. The results of this sequence should approach the limiting value that, strictly speaking, holds only in tangent space. It is essential to realize that this procedure can be carried through without invoking any metrical concepts so that linearity of construction can be accomplished.

It is important to recognize what the relation is between theory and evidence in constructive axiomatizations. Firstly, the theory in fact enters the scheme even if only in a hidden or indirect fashion. Why, after all, do we regard trajectories influenced by gravitational quadrupole moments as distorted whereas we consider paths determined by the action of gravitational monopoles as perturbation-free? Evidently, it is the explanatory theory (GTR) that induces and justifies this judgment. Theory enters constructive axiomatization by defining the ideal cases. It supplies criteria of adequacy for our choice of basic processes. Only because we already know GTR we can feign to be ignorant of it.

Secondly, the constructive axiomatization does not exhaust the whole observation basis of GTR. After all, GTR can in fact cope with trajectories distorted by gravitational multipole moments; it can do more with those cases than simply put them aside. By relying on the full-fledged theory it is indeed possible to establish the metric structure even if only distorted cases are available. That is to say, we can empirically explore
the prevailing physical geometry by exclusive recourse to the motion of particles endowed with a gravitational quadrupole moment. So distortions actually belong to the observational basis of GTR. A constructive axiomatization, however, necessarily leaves distortions out of consideration; it cannot specify any principles linking GTR's theoretical terms to empirical indicators which are available in these distorted cases. A constructive axiomatization is, therefore, always incomplete and, accordingly, incomplete. In the case of the EPS-construction there is, in addition, incompleteness of a different kind. As previously mentioned EPS can achieve the transition from Weyl space-time to Riemann space-time only by postulating that second clock effects are absent. The validity of this postulate can only be checked by transcending the original EPS-scheme (I will dwell on this aspect later). This means that EPS do not exactly axiomatize the observation basis of GTR but rather, they axiomatize a larger class of metric structures. This class includes Riemannian and non-Riemannian metrics, and the EPS construction gives no reason for preferring the first ones. But this implies that the EPS scheme fails to determine unambiguously the empirical indicators of GTR theoretical states.

In fact, the first feature just outlined implies the second one. Because the explanatory theory is only permitted to supply criteria of adequacy and is prevented from doing more than definitional work, the axiomatization fails to deal with cases other than ideal. The extremely restricted application of the theory entails the incompleteness of the pertinent axiomatization. Selecting pure instances makes constructive axiomatizations possible but at the same time unsatisfactory. And in order to achieve a more comprehensive treatment of the observational basis it is necessary to allow for a less restricted role of theoretical considerations.

It is apparent from the foregoing analysis that a comprehensive account of the evidential basis of GTR can only be given by invoking GTR itself. But what here appears as a defect of constructive axiomatization and, accordingly, as a methodological vice may also be regarded as a methodological virtue. As Feigl stated in 1950, a theory ought to be able to cope with its own observation basis. If this is achieved by a theory it constitutes a veritable Copernican turn which consists in the derivation, with corrections coming from the theoretical scheme, of the peculiarities of the very basis of confirmation. . . . As long as a science has not attained a very high level of explanation the process of indication may not be deductible from theoretical premises. . . . [T]his corresponds to the theory of acids and bases in chemistry at a time when the process involved in the well known litmus paper test was not itself logically derivable [from chemical theory]. . . . The behavior of such thermometric substances
as alcohol or mercury was not theoretically deducible until the kinetic (molecular) theory of heat put these indicator processes on a par with countless other thermodynamic processes as interpreted on the micro-level. . . . The Copernican turn then consists in relating the observer to the observed, the indicator to the indicated,—not epistemically,—but so to speak cosmologically. What epistemically must be looked at as the confirmation bases of the hypothetical construction, will in the fullfledged theory be given a place within the cosmos of which the theory treats. (Feigl 1950, pp. 40–41; partially emphasized in the original)

In view of the similar requirement by Einstein, as quoted in the introduction, I will call this peculiarity Einstein-Feigl-completeness (henceforth EFC). It should be realized what EFC amounts to. It is not sufficient that a theory be capable of specifying criteria of adequacy in order for certain processes to qualify as empirical indicators for certain theoretical states assumed within this theory. The theory must moreover be able (1) to explain why these indicator processes constitute reliable measurement procedures and (2) to account for possible distortions. In other words, the theory must provide the means for a theoretical analysis of the procedures associated with its own theoretical terms including possibly necessary corrections.

What we encounter here is a conflict between two methodological intuitions. Constructive axiomatizations follow the ideal of founding the theory upon measurements. EFC, on the other hand, expresses the goal of a most parsimonious theoretical explanation of measurements. Both aims cannot be pursued simultaneously. As the above discussion sought to make clear EFC can only be achieved if we relinquish the ideal of constructive axiomatization, and vice versa. Attempting to complete our accounts of space-time structure presupposes a commitment to the deductive approach. So a decision between EFC and constructive axiomatization is necessary but such a decision is not necessarily to be made arbitrarily. Let me, therefore, now ask the same questions concerning EFC that I asked previously with respect to constructive axiomatization: can EFC be reached at all and, provided that the answer is affirmative, is EFC a methodologically desirable goal?

3. Differential Forces and the Empirical Underdetermination of Space-Time Structure. EFC requires of a theory that it provide a basis for the corrections one has to employ in properly applying its theoretical terms to experience. It may appear dubious that such self-corrections or corrections of the second kind can unambiguously be carried out because, as described above, self-corrections may issue in a situation in which two
quantities cannot be evaluated independently of one another. It is tempting to suppose that the ensuing circularity imposes serious restrictions on the testability of the pertinent theory. Let me bring into focus the problems arising here by turning to an argument put forth by Einstein. Against Reichenbach’s account of physical geometry, Einstein argues that the metric structure is not uniquely fixed even after the influence of universal forces has been excluded. In a fictitious dialog between Reichenbach and Poincaré, Einstein, disguised as Poincaré, objects to Reichenbach:

In gaining the real definition [of the rigid body] improved by yourself [by means of differential corrections] you have made use of physical laws, the formulation of which presupposes (in this case) Euclidean geometry. The verification, of which you have spoken, refers, therefore, not merely to geometry but to the entire system of physical laws which constitutes its foundation. An examination of geometry by itself is consequently not thinkable. (Einstein 1949, p. 677)

To see more clearly what is at issue let us first remove reference to Euclidean geometry (which stems from Einstein’s choice of Poincaré as his spokesman). With this generalization, Einstein’s argument is to be read as follows. Metrical concepts are already involved in the differential correction laws such as the accounts of thermic expansion or of mechanical deformations. Because of this peculiarity the application of correction laws presupposes that the metric is known. On the other hand, the metric can only be determined by recourse to these correction laws. For this reason one is always free to cling to one’s pet geometry in the face of whatever empirical outcome.³

It should be noted that Einstein does not buttress his argument on common Duhem conventionality. For Duhem conventionality, as it is usually conceived, concerns the methodological intertwining of hypotheses that are logically independent of each other. That is, in order to test a hypothesis $H_1$ one needs to employ some auxiliary suppositions $H_2, H_3$ etc., but to check $H_2$ and $H_3$ it is not requisite to apply $H_1$ in turn (although some other hypotheses might be necessary). By contrast, Einstein’s argument amounts to the contention that explanatory hypothesis and auxiliary assumptions are not independent of each other; instead each presupposes the other one. That is, testing $H_1$ requires reference to $H_2$ and checking $H_2$ necessarily makes use of $H_1$. So we have a situation that strongly resembles the one encountered in our discussion of universal forces. Thus Einstein sets forth a differential analog of Reichenbach conventionality.

³Here I follow Grünbaum’s reading of Einstein’s objection. (Compare Grünbaum 1960, p. 80; Grünbaum 1973, pp. 131–135.)
The distinctive marks of EFC are present in this scenario. Differential corrections rely in part on physical geometry itself; in the case under discussion differential corrections are corrections of the second kind. This means that GTR is Einstein-Feigl-complete to a greater extent than it may have appeared at the outset. Furthermore, EFC and Reichenbach conventionality (or its analogs) seem to be intimately linked to each other. EFC does not necessarily entail reciprocal test dependencies, to be sure, but, as our case reveals, the former may indeed engender the latter. So it is tempting to assume that here, as for universal forces, the differential perturbational influences cannot be conclusively evaluated by empirical means and cannot, consequently, be corrected. But this feature seems to imply considerable impediments to tests of physical geometry and this is exactly the conclusion Einstein draws from his argument. Accordingly, one gets the impression that EFC may go along with test restrictions and the latter may appear to be too high a price for the former.

Einstein sets forth his conventionality argument exclusively with respect to rigid rods, that is, as regards the need for correcting the length of the rods encountered in experience. One may ask, therefore, whether this testability problem can be solved by turning to particle trajectories and light rays which also constitute a well-suited means for establishing the metric structure. In fact, neither deformations due to temperature variations nor to elastic stresses play any role in this method. Nevertheless, an analogous difficulty occurs here that is based—just as for rigid rods—on the need to correct differential distortions. In properly applying the trajectory method one has to correct electromagnetic influences on the particle’s trajectory. With \( F_{bc} \) denoting the electromagnetic field-strength tensor one gets for the electromagnetic force on a particle with charge \( q \) in a gravitational field:

\[
   f^a = q g^{ab} F_{bc}(dx^c/d\tau) \quad \text{(compare Weinberg 1972, p. 125)}
\]

(***)

Obviously, to employ this generalized version of Lorentz’s force law for correcting perturbational influences owing to the presence of electromagnetic fields, the metric has to be known. On the other hand, the metric can in general only be obtained by applying the above correction law. This shows that the same mutual dependence between the metric and external perturbations (or between knowledge of the gravitational potential and knowledge of the intensity of distorting fields) appears in the rigid-rod procedure and in the trajectory method alike. Accordingly, Einstein’s analog to Reichenbach conventionality seems to be of general importance in attempts to establish space-time structure.

Is there any way to overcome this difficulty? As regards Einstein’s original version of the argument, Grünbaum has developed a method of successive approximations to dissolve the apparent correction circularity.
One starts with an arbitrary geometry in the correction laws and determines physical geometry on its basis. The geometry obtained, however, does not generally coincide with the one used for the corrections. So in a second step, one employs the improved version of geometry to carry out the corrections and then repeats the whole procedure until an agreement is reached between the geometry entering the corrections and the geometry obtained by performing measurements with the accordingly corrected rods (compare Grünbaum 1973, p. 145). If this procedure of reciprocal adaptations indeed yields convergent results, the geometry that finally emerges is independent of the one used at the start and it is, furthermore, identical to the one obtained by exploring the coincidence behavior of transported rods in perturbation-free regions of space-time. So we can reasonably consider the resulting geometry as the geometry of space-time. Unfortunately, however, Grünbaum’s ingenious method is not generally applicable for there exist circumstances (among them all non-constantly curved three-spaces) under which the procedure does not supply convergent results (compare Grünbaum 1973, pp. 145–146, 808). Consequently, it does not provide a general way out of the difficulties expounded by Einstein.

Concerning the trajectory method the situation is even worse. We are in a better starting position, to be sure, but this initial lead is of no help. Our advantage consists in the fact that the class of possible correction metrics is more restricted than in the case of rigid-body measurements. For the metric is already fixed by the light cones up to conformal transformations, and particle trajectories should only bear the burden of singling out a metric from within this limited set. Nevertheless, the burden is still too heavy. The problem arising here is that we have, in addition, no metric-independent access to an evaluation of charges. But, as equation (****) reveals, we need such an evaluation for even starting a procedure of successive approximations (that proved to be partially successful in the rigid-body case). Here we cannot even tentatively calculate a correcting Lorentz-force that could provide a basis for a process of reciprocal adaptations. My conclusion is that there seems to be no generally applicable method for solving Einstein’s problem of differential correction circularity.

It should be noted, however, that for practical purposes this problem is only of minor importance. For, as explained in section 2, we can, in fact, identify the undistorted cases in a non-circular way; derive the prevailing metric from them; and insert this metric into the correction laws.⁴

⁴One might assume that a metric-independent evaluation of charges is feasible by recourse to Maxwell’s equations. But these equations exhibit the same metric-dependence as Lorentz’s force law.
But this expedient would obviously fail if there existed only charged particles in the world. So if one refers to the general case there remains an empirical underdetermination of the metric due to differential circularity. Because methodology is a matter of principle it has to be admitted that EFC may entail non-trivial test problems.

4. EFC and Measurement. In order to explore the relation between theory and evidence in Einstein-Feigl-complete theories a bit more systematically, let me briefly examine another example, namely, the Marzke-Wheeler clock. This examination is intended to bring into focus how a comprehensive account of measurement procedures can be achieved. This discussion paves the way for addressing the problem of the methodological desirability of the approaches mentioned above to an account of spacetime structure. It may have been noted that I asked twice about this desirability (once as regards constructive axiomatizations and again with respect to EFC) without, as yet, attempting an answer. So it is high time to turn to the question whether it is preferable to construct or to complete physical geometry.

The Marzke-Wheeler clock is explicitly designed to show that GTR is, in fact, a complete theory. Its authors set themselves the goal of establishing that

\[ \text{general relativity, in and by itself, provides its own means for defining intervals of space and time. . . . (Marzke and Wheeler 1964, p. 62)} \]

---

5An analogous procedure is also available for differential distortions of rigid bodies (compare Grünbaum 1973, pp. 140–141).

6Both credit Bohr and Rosenfeld (1933) with the first formulation of the completeness condition (compare Marzke and Wheeler 1964, p. 48) but this is historically unsound. First and foremost, Einstein formulated the completeness requirement as early as 1920 in a remark made in a discussion. As he said: "It is a logical weakness of the theory of relativity in its present state that it has to introduce measuring rods and clocks separately instead of being able to construct them as solutions of differential equations" (Einstein 1920, p. 662; my translation). Secondly, Bohr and Rosenfeld only presented a very limited version of the completeness condition. In fact, it is hard to see how Bohr, as the originator of the Copenhagen Interpretation, could have advocated a condition of this kind. After all, it is one of the fundamental tenets of the Copenhagen Interpretation that the results of quantum mechanical measurements have to be described by means of classical physics. Quantum mechanics as interpreted in the Copenhagen fashion is necessarily Einstein-Feigl-incomplete. Accordingly, Bohr and Rosenfeld declare in the Copenhagen vein that "all measurements of physical quantities, by definition, must be a matter of the application of classical concepts" (Bohr and Rosenfeld 1933, p. 387). They discuss, to be sure, in a second step the influence of quantum field theoretical effects on the measurability of pertinent quantities (and this discussion presumably constitutes the basis for the historical thesis put forward by Marzke and Wheeler) but they restrict this discussion to the limitations of measurability brought about by quantum effects. Accordingly, they do not at all suggest that a theory ought to account for the means to establish its own observation basis.
And they try to achieve this aim by elaborating Synge's procedure described in section 2. This procedure makes use of a photon reflected back and forth between two world-lines of unchanging distance; and that the distance remains unchanged is detected by means of an atomic clock. Obviously, Synge's scenario can only serve as a basis for establishing the EFC of GTR if reference to atomic clocks has been removed. This requires that world-lines of equal distance can be singled out without relying on any time measurements. For this purpose Marzke and Wheeler determine rigidity by recourse to parallel straight lines in flat space-time. They propose a construction in projective geometry that invokes Pappus' theorem to show that by making use of light rays and straight particle trajectories the parallel to any given world-line can be specified, provided that the flatness condition is satisfied (compare Marzke and Wheeler 1964, pp. 50–52). Reversing now Synge's approach, the standard time interval is defined by the time elapsed between emission of a photon and its return to its spatial origin after reflection at a parallel world-line. This means that Synge defines rigidity by recourse to a standard time interval that is in turn fixed by reference to an atomic clock. Marzke and Wheeler define the standard time interval by recourse to parallelism and parallelism is obtained by means of a construction in projective geometry.

Generalizing this procedure to curved space-time requires the multiple application of the Marzke-Wheeler clock along several points of a four-geodesic. By this means a transport of the standard interval along a time-like geodesic can be effected so that the length of arbitrary intervals can be compared. It must be secured, however, that in each single application space-time is, in fact, flat. With this proviso, the Marzke-Wheeler clock allows for a detection of second clock effects (compare Marzke and Wheeler 1964, pp. 56–60). Accordingly, the clock constitutes a suitable means for completing the EPS-scheme which, as mentioned in section 2, could only postulate and not establish the absence of such effects (and the corresponding transition to Riemannian space-time).

The important aspect of this clock construction in the context of our discussion is that flatness has to be presupposed for its orderly functioning. If the curvature is non-negligible within the clock's extension, it no longer yields reliable results. It must be emphasized that under such circumstances the construction does not simply break down; it produces a wrong empirical outcome instead. This means that it detects second clock effects where, in fact, none are present. Accordingly, one has to make sure that the flatness condition is satisfied before employing the clock. Marzke and Wheeler recommend two tests to this effect but neither of them is fit to fulfill the intended purpose. The first one takes recourse to parallel particle trajectories and the second one even relies on a metric (Marzke and Wheeler 1964 pp. 49–50, 52–53); but both parallelism and
distance are only available by making use of the clock construction. It seems as if we need the results of a successful application of a Marzke-Wheeler clock in order to make sure that its successful application is at all possible.

Here again we encounter a situation that exhibits all the notorious marks of a circularity of testing. Do we have to conclude, accordingly, that the Marzke-Wheeler construction is bound to fail? Is it necessary to decide about flatness in a way analogous to Reichenbach’s coordinative definitions so that any physical geometry would only hold relative to a presupposed flatness convention? In fact, there is a way out of this dilemma. As in the procedure of Ehlers and Schild, outlined in section 2, one introduces an affine parameter which non-metrically describes the size of Marzke-Wheeler clocks. Subsequently, one steadily diminishes the extension of the clock and determines the prevailing duration relations. If the series of the results obtained is indeed convergent, one may identify the outcome of this procedure with the physical geometry holding at that space-time location. Accordingly, the physically realized metric is obtained from a limiting sequence of experiments.

Well, one might comment, this procedure shows that GTR suffices to establish metric intervals. But at what price? For we started with the aim of coping with distortions; and what do we end up with? What limit does our experimental series approach? Obviously, it approaches the ideal case in which distortions due to non-vanishing curvature no longer occur. So what we did, in fact, was select pure instances. But this suggests that the Marzke-Wheeler clock represents at best an alternative way of constructively axiomatizing the observation basis of GTR and fails to show that GTR is Einstein-Feigl-complete. If this judgment turned out to be warranted it would certainly induce serious doubts concerning the mere feasibility of EFC. So is EFC a chimera?

Not quite. For the essential point here is that we can, by invoking GTR principles, account perfectly well for curvature distortions and, accordingly, correct them if the metric is known. On the basis of several measurements performed by means of small Marzke-Wheeler clocks we can, for instance, predict the readings of a larger clock and we can explain why there is a difference between the two results (if there is one). From this point of view, what the Marzke-Wheeler clock virtually does is feed in initial values for the correction procedure. Relying on the results obtained by examining pure cases we can analyze the distorted ones and perform successfully all corrections of the second kind. In this sense (that

---

7It should be noted that the Marzke-Wheeler concept of completeness (contrary to my notion of EFC) does not encompass the requirement that a theory deal with distortions. This means that, judged by their own standards, the clock construction is, at any rate, successful.
is a rather indirect one and, moreover, hardly the one intended by its authors) the Marzke-Wheeler clock indeed provides a basis for the EFC of GTR.

So it comes out (as a surprise, I think) that constructive axiomatizations of GTR space-time and attempts to complete physical geometry start with the same sort of procedures; they start by concentrating on pure instances. But then the two approaches separate. Whereas the constructive procedure just stops here and is content with its achievements, the completeness method goes a step further and uses ideal case measurements to account for the distorted ones. Sometimes EFC can only be reached by following a rather winding road. This applies also to the case of differential correction circularities as sketched in section 3. I mentioned that although we are able to identify the pure cases (if they exist) we are at a loss to deal with differential perturbational influences. But what cannot be reached in one step can perhaps be reached in two. Just as in the Marzke-Wheeler procedure one could try to establish the metric structure by making use of perturbation-free processes and, subsequently, consider the distortions. Note that such a two-step back-and-forth procedure is not available to constructive axiomatizations. For the latter approach aims at immediately capturing the observation basis of a theory. The evidential basis of a theory is to be grounded upon simple experiences and should not be established via theoretical inferences. If a theory is used to clarify its observational basis the latter can no longer adequately be regarded as the foundation of the former. I conclude that (1) EFC can be achieved, that is, that the possibly ensuing circles are not fatal to testability and that (2) in the cases discussed it can only be achieved by violating the linearity requirement that is constitutive of constructive axiomatizations.

There is an additional point that should be taken into consideration. Up to now I have not really exploited the power of EFC because I restricted employment of GTR principles to the action of space-time on matter. I was only concerned with the possible deformations of bodies or trajectories due to differential or gravitational influences. This limited application of the theory originated from my intention to compare, as directly as possible, the relation between theory and evidence in constructive axiomatizations to that relation in approaches committed to the completeness postulate. So I had to leave any action of matter on space-time out of consideration because these actions are governed by Einstein's field equations. But nobody has ever proposed a constructive axiomatization of these equations and, as things stand, trying to do this is a forlorn hope. For establishing these equations relies on much guesswork that cannot be supplanted by straightforward derivations from simple experiences. As will become apparent immediately, a large number of the problems the completeness requirement ran into stemmed from an attempt to account
for the whole observation basis of GTR without supplying the necessary resources. We wanted to gain more without correspondingly raising the stakes. So let us remove all restrictions rooted in a commitment to a close relation between theory and evidence and employ GTR right from the beginning. This means, as regards the Marzke-Wheeler clock, that we can ensure the flatness of space-time regions of interest by calculating the curvature from the field equations. That is, we do not have to rely on measurements of space-time structure to check whether curvature is vanishing; instead we can explore the possible sources of curvature (such as masses, mechanical stresses or electromagnetic radiation) and derive the prevailing curvature from them. On the basis of this extended completeness approach we are not obliged to establish flatness directly. We may use GTR for obtaining empirical information and not alone (as in the restricted completeness approach described above) for processing information gained otherwise. Note that the testability of Einstein’s equations remains unimpaired by such a procedure. Because these equations do not enter the construction of the clock no circularity of testing arises.\footnote{The subtle point is, however, that one has to know the metric in order to calculate curvature from its sources because the metric enters the energy-momentum tensor which serves as a basis of such a calculation. This problem is usually overcome by using a procedure of successive approximation analogous to the one employed by Grünbaum (as described in section 2); (compare Stephani 1980, p. 89).}

Even if the field equations do, in fact, figure in the account of a measuring procedure, test circularities need not occur. Consider, for example, the precession of the perihelion of Mercury. The value of this precession can be obtained by means of Einstein’s equations and it constitutes one of the most sensitive tests of these equations. But to observe the precession reliably one has to take into account that the light indicating Mercury’s position is deflected by the sun. Because of the small distance between Mercury and the sun this effect is of considerable importance. Accordingly, Mercury’s position, as directly observed, has to be corrected and these corrections can only be made by relying in turn on the field equations (compare Weinberg 1972, pp. 190–191, 197–198).\footnote{In practice, one might do without corrections because the perihelion advance is cumulative. Let me neglect this aspect for the sake of argument.} But this peculiarity in no way detracts from the testability of the field equations. The reason is that applying the equations twice leads to relations between quantities that can be observed without recourse to these equations. What we finally obtain is a prediction concerning the shift of spots (images of Mercury) on a photographic plate. And this prediction can be checked without an eye to GTR. I mentioned earlier on several occasions that EFC does not necessarily issue in test circularities, and we now see why. Employing the same theory for data supply and for data explanation
may lead to regularities whose verification does not depend on this theory. 10

If it is agreed that EFC is, in general, feasible without running into
vicious circles, how should we proceed? After all, constructive axiomatizations are, in general, feasible, too. Which option is the methodologically preferable one? Let me begin by briefly summarizing the characteristic features of both approaches. The constitutive principles of constructive axiomatizations are (1) the methodological requirement of
direct testability and (2) the semantical postulate of linearity. A theory is
to be founded on propositions that are immediately amenable to experi-
ence; and the concepts employed by that theory should be clarified first
(by relating them to experiences) and afterwards used to build up the
theoretical edifice. The theory must not be used to elucidate the concepts;
never turn your eyes back. EFC, on the other hand, is guided (1) by the
methodological idea of explanatory power (that is, the idea that a theory
should explain a large range of phenomena in a precise fashion by in-
voking as few independent assumptions as possible) and (2) by a se-
mantical account that allows for a reciprocal clarification of concepts and
theories. The measurement procedures associated with a theory should be
explainable by its own lights and this entails that the meaning of the
observation terms is influenced by theoretical considerations. Concerning
the drawbacks of both approaches, note that the whole observation basis
of a theory cannot be constructively axiomatized and that complete the-
ories (if only in extreme cases) may suffer from test restrictions.

What to do in the face of these conflicting ideals? Should we prefer a
more comprehensive account to a more directly testable one or should
we proceed the other way around? I propose to choose the first option
for the reason that on its basis alone can the problem of the convention-
ality of physical geometry be solved. Remember Reichenbach's problem
of universal forces. The difficulty that arises is that the presence or ab-
sence of such forces cannot be directly detected by empirical means and
that this peculiarity issues in an underdetermination of physical geometry
by experience. This feature, however, does not necessarily result in con-
ventionality. There may exist non-empirical criteria that uniquely deter-
mine the structure of a theory (physical geometry in this case). These
criteria are methodological in character. They refer to the way in which
a theory solves its empirical (or conceptual) problems. 11 In the light of

10 For a more systematic treatment of this option see my (1989), section II.
11 This appears to be the core of the arguments in (Glymour 1977, pp. 237–238, 244)
and (Friedman 1983, pp. 299–300). I presume that Reichenbach in his later days would
have agreed with this assessment. Compare, for example, his presentation of the conven-
tionality thesis in Reichenbach (1938). There he argues that for theoretical alternatives to
be conventional it is necessary that, besides being untestable, none of them be more prob-
criteria of this kind (such as explanatory power or Lakatos’s requirement of empirically progressive problem-shifts) two empirically equivalent theories may prove not to be equivalent. Thus it seems inappropriate to conclude that empirical equivalence necessarily engenders the need for a conventional choice.

It is essential to realize that this assessment exclusively holds with respect to a deductive approach. Methodological criteria are only applicable to theoretical principles, not to empirical facts. If one tries to construct physical geometry from scratch there is no room for methodological considerations. What matters is which simple facts allow for the construction of a theory, and if the facts do not speak for themselves the construction is in serious trouble. This does not mean that the construction fails under such circumstances but rather that the abyss between evidence and theory can only be bridged by an arbitrary decision. We simply have to decide what to regard as a free particle. Here we cannot, as in the deductive vein, apply methodological criteria to theoretical principles and choose in their light what is most reasonably to be viewed as the natural state of motion. In constructive axiomatizations untestability indeed entails conventionality.

One might assume that the conventionality problem can be solved on the basis of Reichenbach’s recommendation that universal forces be set equal to zero. And it is indeed possible to view this recommendation as a methodological criterion whose application is intended to uniquely determine physical geometry. But in this role Reichenbach’s rule is less than convincing. Why in the world should we follow this recommendation and deem rods deformed by gravitational influences to be of constant length? One might as well choose the more extravagant option Verdi’s Duca di Mantova points out when he assures us: “La costanza, tiranna del core, detestiamo qual morbo crudele” (Rigoletto, Act I). Put more seriously, the problem of Reichenbach’s rule is that it is obviously tailored to the special case of physical geometry. Where, besides physical geometry, do universal forces occur, after all? In other words, Reichenbach’s recommendation is an ad hoc methodological rule.

This consideration allows for a more precise formulation of the problem of the conventionality of physical geometry. In order for geometry to be non-conventional it is not requisite that the facts pure and simple suffice to single out the prevailing space-time structure. Here, as in other theories, the facts do not determine unambiguously their theoretical descrip-

---

able in the light of the available background knowledge (compare Reichenbach 1938 pp. 127–129). Accordingly, untestability should not entail conventionality.

12 For the status of such criteria and the ways to justify them compare (Carrier 1986, pp. 202–206).
tion. What is at issue, instead, is whether in physical geometry special decisions have to be made; that is, whether methodological criteria explicitly designed to cope with this case have to be invoked in order to arrive at an unambiguous theoretical description. If methodological criteria that are sufficient to explain other cases of theory choice in science are also sufficient to settle the question about which physical geometry holds, then geometry should not be regarded as conventional. The conventionality problem of physical geometry is whether physical geometry is more conventional than other theories. I will not start a discussion as to which criteria are possibly suited to support a non-conventionality claim in this sense. But it is apparent from the foregoing discussion that geometrizing gravitation can only be warranted within a deductive approach. So an unambiguous answer to the question about the prevailing physical geometry can (if at all) only be given if we complete and not construct our accounts of space-time structure.

REFERENCES


