Experimental Test of Special Relativity from a High-γ Electron \( g - 2 \) Measurement

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We report a verification of the theory of special relativity at a value of \( \gamma = 2.5 \times 10^4 \)
based upon a comparison of electron \( g - 2 \) measurements at meV and GeV kinetic energies. Specially we obtain a measure of the equivalence between the quantities \( \gamma = (1 - \beta^2)^{-1/2} \) and \( \gamma = (p/m) \frac{dE}{dt} \).

A recent publication\(^1\) has pointed out that an experimental test of special relativity is provided by comparing the values of the electron \( g \)-factor anomaly \( a \) \( [a = \frac{3}{2}(g - 2)] \) for electrons with different velocities or \( \gamma \) values. Special relativity predicts that the value of \( a \) should be independent of the electron velocity. Newman \etal\(^1\), refer to two measurements of \( a \), one done with electrons of 1 meV kinetic energy \( (\gamma - 1 = 10^3) \) and the other done with electrons of 100 keV kinetic energy \( (\gamma = 1.2) \). These two measured values agree. We point out here that another measurement of \( a \) has been done with electrons of about 12 GeV kinetic energy \( (\gamma - 2.5 \times 10^4) \), which is relevant to this test of special relativity.\(^2\)

The high-\( \gamma \) \( g - 2 \) measurement\(^3\) was obtained as a by-product of the measurement of the polarization of the high-energy longitudinally polarized electron beam at the Stanford Linear Accelerator Center (SLAC). After acceleration to high energy the longitudinally polarized beam was deflected through the beam switchyard by an angle \( \theta_e = 24.5^\circ \) into the experimental area, with the spin precessing relative to the momentum by an angle

\[ \theta_a = \gamma a \theta_e. \]  

(1)

The longitudinal component of the beam polarization is then given by

\[ P(E) = P_0 \cos(\theta_E/E_0 + \varphi_0), \]  

(2)

in which \( P_0 \) is the magnitude of the initial vector polarization, \( \overrightarrow{P}_0 \), of the electron beam before the magnetic deflection, \( \varphi_0 \) is projected angle of \( \overrightarrow{P}_0 \) with respect to the electron momentum in the plane of the bent trajectory, \( E \) is the electron energy, and \( E_0 \) is defined as

\[ E_0 = \left( \frac{180^\circ}{24.5^\circ} \right) \frac{m_e c^2}{a} \approx 3.2 \text{ GeV}, \]  

(3)

where \( m_e \) is the electron rest mass.

The longitudinal polarization of the deflected beam was measured by Möller scattering\(^4\) from a Supermendur target foil magnetized to saturation in a 90-G longitudinal magnetic field and inclined at 20° with respect to the beam direction in order to provide a large component of longitudinal polarization. Reversal of the 90-G field reversed the polarization of the target. The Möller-scattered electrons were observed by conventional particle-detection techniques with the SLAC 8-GeV/c spectrometer.\(^5\)

The results of the Möller measurement are shown in Fig. 1 together with the fitted curve \( P(E) \) given by Eq. (2) with \( P_0 \) and \( a \) as free parameters and \( \varphi_0 \) fixed at zero. The data points shown are taken from the earlier publication.\(^3\) From the fit, the value \( a = (1.1622 \pm 0.0200) \times 10^{-3} \) is obtained, where the quoted 1.7% uncertainty is the linear contribution of counting statistics (0.7%) and possible systematic effects (1.0%). The systematic contributions are the estimated 0.3% uncertainty in the absolute momentum calibration of the beam switchyard magnet system,\(^4\) and an uncertainty of 82 mrad in the value of \( \varphi_0 \), which
results in a 0.7% uncertainty in \( a \). The estimate of the uncertainty in \( \phi_0 \) is obtained by considering the calculated upper bound to \( \phi_0 \) for a single electron in the beam\(^7\) as a 3-standard-deviation effect. Had data been taken at more than one zero crossing point, \( \phi_0 \) and \( a \) could have been separately determined. However, with only one zero crossing the two parameters are highly correlated, which requires \( \phi_0 \) to be estimated independently as we have done above. In this respect the measurement of \( a \) could be significantly improved by the addition of data taken at one or more of the five remaining zero-crossing points in the present SLAC energy range. Finally, the fitted value of \( P_0 \) is 0.755 ± 0.026, which agrees with both the theoretical expectations and experimental measurements of the polarization of the injected electrons.

We note that our value of \( a \) from the high-\( \gamma \) measurement agrees with the more precise values of \( a \) determined in the lower-\( \gamma \) measurements. In Table I, we summarize the lepton \( g - 2 \) measurements for electrons and muons. For the high-\( \gamma \) electron \( g - 2 \) measurement reported in this paper, we use the average value of \( \gamma \) over the range covered by the experiment; namely, \( \gamma = 1.27 \times 10^4 \) to \( \gamma = 3.81 \times 10^4 \). We note that this average value, \( \gamma = 2.5 \times 10^4 \), is very close to the only zero-crossing point, \( \gamma = 2.22 \times 10^4 \), at which we have obtained data. Since the measured value of \( P(E) \) obtained at the crossing point most strongly influences our value of \( a \), the choice of \( \gamma = 2.5 \times 10^4 \) to characterize our measurement of \( a \) seems well justified.

Any discussion of the sensitivity of these measurements as a test of special relativity requires, of course, some theoretical model for, or at least a parametrization of, a breakdown of special relativity. As a theoretical problem applied to the \( g - 2 \) measurements, some breakdown of relativistic quantum field theory may be involved. This is a very profound problem which must involve some preferred frame of reference, perhaps determined from cosmological considerations. A systematic phenomenological viewpoint might involve an analysis of the accuracy with which the coefficients of the Lorentz transformation are tested. A recent theoretical model\(^13\) for a breakdown of special relativity predicts effects proportional to \( \gamma^2 \).

In their parametrization, Newman et al. introduce \( \gamma = (P/m_\gamma)dp/dE \) which they allow to be different from \( \gamma = (1 - \beta^2)^{-1/2} \). Hence the cyclotron, spin, and \( g - 2 \) precession frequencies for motions perpendicular to a magnetic field \( B \) are given, respectively, by

\[
\omega_c = eB/\gamma m_\gamma c, \tag{4}
\]

\[
\omega_s = geB/2m_\gamma c + (1 - \gamma)\omega_c, \tag{5}
\]

\[
\omega_a = \omega_s - \omega_c = (3g - \gamma/\gamma)eB/m_\gamma c. \tag{6}
\]

The term \((1 - \gamma)\omega_c \) in Eq. (5) is the Thomas precession frequency and is regarded by Newman et al. as of kinematic origin and hence involves the usual \( \gamma \) term, whereas the term \( \gamma \) in Eq. (4) is regarded as arising from electron dynamics and hence as possibly different. The \( g - 2 \) experiments determine the quantity \( \omega_a(eB/m_\gamma c)^{-1} \) which in the conventional theory equals \( \frac{3}{2}(g - 2) = a \). Following the parametrization of Newman et al., we
TABLE II. Summary of lepton $g-2$ relativity tests.

<table>
<thead>
<tr>
<th>Method</th>
<th>References</th>
<th>$\gamma^{(1)}$</th>
<th>$\gamma^{(2)}$</th>
<th>$C_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^+e^-\gamma$</td>
<td>10 and 11</td>
<td>12</td>
<td>29.2</td>
<td>$(1.4\pm1.8)\times10^{-8}$</td>
</tr>
<tr>
<td>$e^+\gamma$</td>
<td>8 and 9</td>
<td>1</td>
<td>1.2</td>
<td>$(-2.4\pm1.8)\times10^{-8}$</td>
</tr>
<tr>
<td></td>
<td>8 and the present work</td>
<td>1</td>
<td>$2.5\times10^{14}$</td>
<td>$(-1.0\pm8.0)\times10^{-10}$</td>
</tr>
</tbody>
</table>

*The $g$ factor was measured over the $\gamma$ interval $(1.3-3.8)\times10^4$.

As can be seen from Table II, the limit on $|C_1|$ of $<1.7\times10^{-9}$ measurement is the most sensitive upper limit obtained to date. Within the framework of the relativity-breaking model expressed by Eq. (8), we have thus demonstrated the equivalence of $\gamma$ and $\bar{\gamma}$. Of course, a linear dependence on $\gamma - 1$ is but one possible choice. Indeed, Rêdei, in a discussion of the validity of special relativity at small distances and the existence of a universal length, suggests that for the lifetime of the muon a modification with a leading term quadratic in $\gamma$ should be introduced. In the context of higher-order terms we wish to point out that the relative sensitivity of our measurement is enhanced by any higher-order dependence on $\gamma - 1$.

In conclusion, we emphasize that we have included in our discussion only those tests of special relativity which are directly comparable to ours. For reference to other tests see Newman et al. and Bailey et al.

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$\omega_{\gamma}(\gamma B/m_{\gamma}c) = \frac{1}{3}g = \gamma = a$

and regard the various $g-2$ measurements done at different electron velocities as determining $\gamma/\bar{\gamma}$.

In the accompanying Letter by Combley et al., a more general phenomenological viewpoint of a breakdown of special relativity is taken and four distinct $\gamma$ factors—$\gamma_1, \gamma_2, \gamma_3, \gamma_4$—are introduced for the transformation of time, electromagnetic fields and mass, and for determining the Thomas precession. With certain assumptions about relations among these four $\gamma$ factors, the parametrization of Combley et al. reduces to that of Newman et al.

We use the phenomenological model of Newman et al., and, in addition, as do Combley et al., and, in addition, as do Combley et al., assume a power-series expansion for $\gamma/\bar{\gamma}$ of the form

$$\gamma/\bar{\gamma} = 1 + C_1(\gamma - 1) + \ldots ,$$

which preserves the nonrelativistic equivalence of $\gamma$ and $\bar{\gamma}$ in the limit $\gamma - 1$. In order to define a figure of merit, we retain only the leading nonconstant term in Eq. (8). Then for each lepton, $g-2$ measurements at two values of $\gamma$ suffice to determine $C_1$ according to

$$C_1 = \frac{a^{(2)} - a^{(1)}}{b^{(1)} - b^{(2)}}$$

for measurements $a^{(1)}$ and $a^{(2)}$ at $\gamma^{(1)}$ and $\gamma^{(2)}$, respectively. In Table II, we present the values of $C_1$ derived from various pairs of lepton $g-2$ measurements given in Table I. Implicit in this parametrization are the assumptions that any violation of special relativity vanishes as one approaches the nonrelativistic limit, and that $g$ is a constant independent of $\gamma$. We note that although our measurement of $a$ is relatively imprecise, our value of $\gamma$ is comparatively very large. Thus our experiment provides a sensitive determination of the coefficients in a power-series expansion such as given by Eq. (8).
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