

Correction to Bounds on Conditional Probabilities with Applications in Multi-User Communication

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J. Wolfowitz kindly made us aware of an error in the above paper, in the proof of Theorem 5. Along with its correction we correct here also some misprints in the same proof.

1. In the line after Formula (42), we put $a \triangleq \frac{1}{4} m_W (-\ln m_W)^{-1/2}$ instead of $\frac{1}{3} m_W (-\ln m_W)^{-1/2}$.

2. From Formulas (43) we do not need the first, and the right form of the second is $f'' = -1/f$ instead of $f'' = -\frac{1}{\sqrt{2\pi}} f$.

3. We do not need the estimates given by Formulas (44)–(45), instead we do need another estimate, easy to verify:

$$\frac{f(s)}{s} \leq \sqrt{-8 \ln s} \quad (*)$$

4. The sentence after Formula (45) takes the form: Starting the induction proof, in the case $n=1$ we have either $f(W(\mathcal{B} | \mathbf{x}))=0$ or $\partial \mathcal{B} = \mathcal{B}$. Using (*), the inequality

$$s \geq a \cdot f(s)$$

is now easily verified for $s \geq m_W$.

5. In the third line after (45) we defined $c \triangleq a/m_W = \frac{1}{3} (-\ln m_W)^{-1/2}$. Now we will have $c \triangleq a/m_W$ with the new a , i.e. $c \triangleq \frac{1}{4} (-\ln m_W)^{-1/2}$.

6. In the second line after Formula (47) the right definition is:

$$\Delta \triangleq [\min_{y \in S_{x_n}} s_y, \max_{y \in S_{x_n}} s_y], \text{ instead of } \Delta \triangleq [\min_y s_y, \max_y s_y].$$

7. The final part of the proof, beginning after Formula (48), must be replaced by the following.

By the concavity of f and since $f(0)=f(1)=0$ we clearly have

$$f(s_0)(f(s))^{-1} \geq 1 - \frac{d}{\bar{s}}.$$

Let us now apply first (46), then (*) and the fact that $\bar{s} \geq m_w^n$:

$$\begin{aligned} f(s_0)(f(s))^{-1} &\geq 1 - \frac{c \cdot f(\bar{s})}{\sqrt{n \cdot \bar{s}}} \geq 1 - c \cdot \sqrt{\frac{-8 \ln \bar{s}}{n}} \\ &\geq 1 - c \cdot \sqrt{-8 \ln m_w} = 1 - \frac{1}{4} \sqrt{8} = 1 - \frac{1}{\sqrt{2}}. \end{aligned}$$

However,

$$c^2 \leq \frac{1}{16 \ln 2} < \frac{1}{8} < 1 - \frac{1}{\sqrt{2}}$$

establishes (48) and the proof is complete.