Kant’s Relational Theory of Absolute Space

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The discussion on the nature of space traditionally clusters around two different positions, namely, substantivalism and relationalism. Substantivalism maintains that space is a kind of container of or arena for physical events. The structure of space is independent of its material content, i.e., of the events possibly happening in space. The structure of space logically precedes the spatial relations among physical bodies. Space is thus absolute. Relationalism contends, by contrast, that space is nothing but the spatial order of bodies. Only spatial relations among bodies are physically significant; there is no independent spatial entity besides these relations. Events don’t happen in space; events constitute space. They spread out the canvas and supply the paint at the same time.

Relationalism requires that all motions are to be construed as relative motions, i.e., as motions with respect to other physical bodies. In particular, there must not exist absolute motions, that is, motions without respect to any material body. A primary challenge to this claim was set up by Newton’s famous “bucket experiment” which was intended to establish the existence of absolute motion and of absolute space on physical grounds. In the wake of Newton’s theory, every relationist position had to come to terms with this argument.

Relationalism thus usually goes along with a denial of absolute space. It is characteristic of Kant’s position, however, that he tried to combine both aspects. My claim is that Kant sought to establish a relational theory of absolute space. In particular, he tried to demonstrate, first, that there is true motion. This means, I contend, that there are true velocities in absolute space. This existence claim is supposed to hold for circular as well as for rectilinear motion. Second, Kant attempted to show that this true motion can be construed as relative motion. I try to defend these claims by an interpretation of the “mechanics” and “phenomenology” chapters of the Metaphysical Foundations of Natural Science published in 1786.  

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I proceed along the following lines. The first two sections are of an introductory nature. In the first one I sketch Newton’s argument for absolute space and describe an early relationist response, namely, that of Huygens. In the second one I outline Kant’s general relationist position. In the third section I give a reconstruction of Kant’s version of the law of the equality of action and reaction. I argue that what Kant aimed at here was the demonstration that Newtonian mechanics actually contains the concept of “true rectilinear velocity.” In the fourth section I turn to Kant’s treatment of circular motion and rotation. Kant placed emphasis on the relative nature of this kind of motion in order to salvage his relationist creed from Newton’s bucket. I try to show what kind of relative motion Kant considered relevant here. In the fifth section, finally, I develop the thesis that there is a close analogy between Kant’s accounts of circular and of rectilinear motion. Kant was getting at the demonstration that in both these cases there is true relative motion. Next Kant used this result in order to show, as I argue, that the center of mass of the universe is truly at rest. So what Newton himself considered as a mere presumption of “hypothesis” is in fact, according to Kant, a theorem of Newtonian mechanics. Since this rest frame is rightly to be identified with absolute space, and since its existence is suggested by taking only relative motions into account, Kant eventually arrived at a relational theory of absolute space.

I. Newton’s Argument for Absolute Space

Newton’s argument for absolute space relies on the special status that Newtonian mechanics confers to accelerated and rotational motions. According to the classical principle of relativity, rectilinear uniform motions have no tangible physical effects. Accelerated motions or rotations, however, are indicated by the presence of inertial forces. The salient point of Newton’s argument is that rotations that give rise to such forces can in no way be interpreted as relative motions but are instead to be conceived as absolute rotations.

In order to establish this conclusion, Newton presented the following experiment. Take a water-filled bucket and put it into rotation. In the beginning, there is relative motion between the bucket and the water, and the flat surface of the water shows that there are no centrifugal forces present. After a while the water has been set in motion by the rotating bucket with the result that there is relative rest between bucket and water. The concave shape of the surface of water indicates that in this case centrifugal forces indeed occur. If the bucket is then suddenly stopped there is relative motion between bucket and water together with a deformed shape of the water surface. So in this case relative rotation goes along with the existence of centrifugal forces.³

This scenario is supposed to demonstrate that the occurrence of inertial forces can’t be understood in terms of relative motions. After all, inertial forces occur or don’t occur in cases of relative motion as well as in cases of relative rest. There is no correlation between the presence of such forces and the relevant states of relative motions. Consequently, the emergence of inertial forces has to be interpreted as indicating true rotational motion or rotation against absolute space. Absolute space thus constitutes a privileged reference frame, and its existence leads to tangible physical consequences.

Moreover, Newton adduced a thought experiment that was intended to make direction and magnitude of absolute circular velocities accessible to experience. Newton imagined two globes connected to one another by a cord and rotating around their common center of mass. By inserting a spring into the cord, the magnitude of the ensuing centrifugal force can be measured. Since the mass $m$ of the globes and their distances $r$ to the center of rotation are measurable as well, the expression of the centrifugal force ($F_c = mv^2/r$) immediately gives the magnitude of the rotational velocity $v$. In the light of the bucket argument this velocity is to be understood as true velocity, i. e., as the magnitude of the motion against absolute space. If in addition a tangential force is exerted on the rotating globes, the centrifugal force between them increases or decreases depending on the fact if the direction of the tangential force is the same as or opposite to the direction of the velocity of the globes. By this procedure one obtains the direction of the globes’ motions with respect to absolute space. Newton concluded:

And thus we might find both the quantity and the determination [direction] of this circular motion, even in an immense vacuum, where there was nothing external or sensible with which the globes could be compared.\(^4\)

Consequently, all characteristics of circular motion can be specified without recourse to other physical bodies or to materially realized frames of reference. They can be specified by relying on dynamical effects, i. e., on the forces these motions give rise to. Absolute motions can thus be distinguished experimentally from relative motions, and this leaves no doubt as to the existence of absolute space.

In the face of Newton’s bucket, the crucial problem for relationalism was to reconcile the apparent independence of the effects of circular motion and rotation from the surrounding bodies with the claim that only rotations — and, consequently, relative motions — should be attributed physical significance. One attempt to achieve this reconciliation was made by Huygens. Huygens argued that the parts of a rotating wheel are indeed in relative motion; opposite points on the circumference of such a wheel, for instance, move in opposite directions. It is this relative motion that generates the centrifugal forces.

Still, this phenomenon [the occurrence of centrifugal forces] showed only that the parts of the wheel, owing to the pressure acting on the circumference, are driven in relative motion

among themselves in different directions. Rotational motion is therefore only a relative motion of the parts, which are driven to different sides, but held together by a rope or other connection.

Now, is it possible to move two bodies relatively without changing their distance? This is indeed possible if an increase in their distance is prevented. An opposite relative motion exists on the circumference.\(^5\)

The gist of this argument is to construe "relative motion" not as motion with respect to some other bodies but instead as relative motion between the parts of the moving body themselves.

Huygen's argument, let's just note it in passing, is actually unsuited to achieve its aim. In a frame of reference rotating along with the wheel (i.e., having the same angular velocity and origin) the relative motions of the parts vanish but the centrifugal forces are still present. The latter can thus not be expressed in terms of the former.\(^6\) Huygens' argument was found in his Nachlass only in 1920 and hence could not have been influential on the 18th-century debate. The reason for nonetheless mentioning it here is that — as I will argue in section IV — the treatment of rotational motions Kant came up with several decades later closely resembles Huygens' approach.

II. Kant's Relationalist Position

Apart from a brief interlude, Kant always remained faithful to the relationalist creed.\(^7\) Empty space is not an object of experience, and for this epistemological reason the description of every motion requires reference to physical bodies (which are indeed objects of experience). As he concluded:

Firstly, all motion or rest can be merely relative and neither can be absolute, i.e., matter can be thought of as moved or at rest only in relation to matter and never as regards mere

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space without matter. Therefore, absolute motion, i.e., such as is thought of without any reference of one matter to another, is simply impossible (559.23–28).

This statement reproduces the central claim of relationalism.

This rejection of absolute motion does not imply for Kant, on the other hand, that absolute space is to be equally rejected wholesale. Unlike the earlier relationalists such as Leibniz and Huygens, Kant tried to find a home for absolute space in his conception. It is clear that for this purpose the Newtonian concept of absolute space had to be reinterpreted. This reinterpretation concerned the status of absolute space as well as the arguments adduced in its favor.

To begin with the latter, Kant introduced absolute space by a procedure of successive embedding of reference frames. Every description of motion requires a reference frame, and the motion of this reference frame can only be described by means of a further reference frame and so forth. The comprehensive description of motion thus necessitates a successive embedding procedure for reference frames. If one aims at an unambiguous description of motions this procedure must come to an end somewhere; on the other hand, there is no empirically accessible reference frame which can legitimately be claimed to represent this end point. Absolute space has thus to be conceived as the imaginary limit of the embedding procedure (cf. 481.22–482.6).

Kant's point here is that the multiplicity of relative motions calls for a unifying perspective. The notion of absolute space supplies this perspective in that it achieves a systematic ordering of the diverse reference frames. This notion thus exhibits a peculiar combination of properties: It displays unifying power and is at the same time purely imaginary. There is no real object corresponding to this notion. This combination of properties is characteristic of Kantian “ideas of reason”, and Kant consequently considered absolute space as such an idea (cf. 559.28–560.7).

This means, on the whole, (1) that Kant's absolute space, in contrast to the Newtonian one, is not an independently existing entity but rather a fictitious framework introduced for the sake of a coherent description of motion; and (2) that for Kant, again in contrast to Newton, the need for introducing absolute space has nothing to do with the special status of rotation. Rather, the introduction of absolute space is called for so as to make a definite description of motion possible.

This description is supposed to be accomplished by recourse to the successive embedding of reference frames. However, carrying out this procedure in an unambiguous fashion requires a clear distinction between true and apparent motions.8

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Only the former, but not the latter, are to be unified in a coherent scheme. Without such a distinction there would arise a wild variety of options for embedding coordinate systems into one another and no definite result could be expected to emerge eventually. This unwelcome consequence is to be avoided by recourse to Newtonian forces. Taking forces into account enables us to avoid arbitrariness and to establish coherence. Forces are the guide for carrying out the embedding procedure uniquely. This holds for rectilinear motions (more specifically, for the interaction of rectilinearly moved particles) as well as for circular motions or rotations.

So we have the following tasks before us: First, to derive the true motions from the apparent motions in (a) rectilinear and (b) circular motions and, second, demonstrate that these true motions can be construed as relative motions. Let me move on to a discussion of the relevant Kantian accounts.

III. Kant's Theorem of the Equality of Action and Reaction

In this section I give a reconstruction of Kant's account of true rectilinear motion. This account starts from a consideration of interactions, or, more specifically, central collisions, between particles. Kant invokes Newton's third law, i.e., the equality of action and reaction, so as to show that there is a class of privileged frames of reference, namely, center of mass frames. Motion described in such a frame can — at least as a first approximation — be considered true motion.

Kant's argument runs roughly as follows. He states: "Third mechanical law: In all communication of motion, action and reaction are always equal to one another" (544.32–33). This law implies that every action of a body A on a body B is counterbalanced by an equal and opposite reaction of B on A. Every action is thus reciprocal action. It will be seen shortly that in Kant's view this law entails that there exists a distinguished reference frame for collision processes; this is the center of mass frame (henceforth abbreviated "CM-frame") in which the two colliding bodies have equal and opposite linear momenta. Hence, taking the interactions or forces between bodies into account gives us a clue for identifying the true motions of these bodies.

The problem is how this Kantian argument is to be understood precisely. Michael Friedman, in his seminal and perceptive paper (1986), has proposed the following interpretation. Kant aimed at specifying inertial frames of reference; in such frames the apparent accelerations coincide with the true ones. Newton's third law is applied in order to single out these inertial frames. If we have a sufficiently isolated system of bodies we know from this law that its center of mass moves inertially. Such a system can thus be used to operationally define the whole class of inertial frames. So, Kant used Newton's theory as to elucidate the spatiotemporal framework inherent in it.¹⁰

¹⁰ Cf. Friedman, The Metaphysical Foundations of Science, p. 34.
Friedman's interpretation is based on the (generally accepted) premise that Newton's third law coincides with Kant's "third mechanical law." This premise leads to the following proposals for translating the relevant Kantian terms: "action" means "force," "true motion" is to be reconstructed as "true acceleration," and "absolute space" is to be understood as "inertial frame." My claim is that the premise, along with the translation rules derived from it, is actually mistaken. My suggestion is that "action" means "linear momentum," "true motion" is "true velocity," and, finally, "absolute space" is intended to refer to "rest frame." Consequently, Kant's action theorem is quite different from Newton's third law. Let me try to support this suggestion by unpacking the (somewhat obscure) "proof" Kant gave for his theorem.

Kant prepared his argument by the following consideration. If the interaction of bodies is taken into account it is no longer true that all frames of reference have equal status. If, for instance, two bodies collide there is a communication of motion. One body, by virtue of its motion, causes another body to move. Recourse to causality allows for distinguishing privileged reference frames. Every interaction must be reciprocal action, that is, action and reaction must be of the same magnitude. For collisions this means that the "motions" of the two colliding bodies must coincide, "since there is no ground for attributing more motion to one of them than to the other" (§45.26 – 27). This condition is supposed to single out the CM-frame of the involved bodies, and this privileged frame is called "absolute space" by Kant (cf. §45.5 – 37, §47.14 – 26).

This consideration leads Kant to the following account of the communication of motion in a central and perfectly inelastic collision of two uniformly moved bodies (cf. Fig. 1):

The action in the community of both bodies is constructed as follows. Let a body A be in motion toward the body B with a velocity = AB with regard to the relative space; the body B is at rest with regard to the same space. Let the velocity AB be divided into two parts, Ac and Bc, which are related to one another inversely as the masses B and A. Represent A as moved with the velocity Ac in absolute space, but B with the velocity Bc in the opposite direction together with the relative space. Thus both motions are opposite and equal to one another; and since they mutually destroy one another, both bodies put themselves relatively to one another, i.e., in absolute space, in a state of rest. But, now, B together with the relative space was in motion with the velocity Bc in the direction BA; this direction is exactly opposed to that of the body A, namely, AB. Hence if the motion of the body B is destroyed by impact, then the motion of the relative space is not therefore destroyed. Hence after the impact, the relative space with regard to both bodies A and B (which now rest in absolute space) moves in the direction BA with the velocity Bc, or, what is the same thing, both bodies after the impact move with equal velocity Bd = Bc in the direction of the impacting AB. According to the foregoing, however, the quantity of motion of B in the direction and with the velocity Bc, and hence likewise the quantity of motion of B in the direction Bd with the same velocity, is equal to the quantity of motion of the body A with the velocity and in the direction Ac. Consequently, the effect, i.e., the motion Bd, which the body B receives by impact in relative space, and hence also the action of the body A with the velocity Ac, is always equal to the reaction Bc. (§46.1 – 32; Ellington's translation slightly amended).
This quote represents the core paragraph of Kant's technical argument for his action theorem. The difficulty is, however, that it is not obvious at the outset which theorem precisely Kant tries to buttress by this argument. The key question is: What does "action" actually mean here? I will give a reformulation of this argument in modern terms in order to elucidate the content of its conclusion.

As a preliminary, a sketch of Newton's third law is called for. This law applies to forces between interacting bodies and it says that the force of a body A on a body B is equal and opposite to the force of B on A. So "action" means "force" here. This law is a consequence of momentum conservation. From the latter it follows that in collision processes the gain in momentum of one body equals the loss of momentum of the other; and since gain and loss come about during the same time interval, the relevant forces (i.e., $\Delta p/\Delta t$) are also equal in magnitude and opposite in direction. This Newtonian law holds in every inertial (i.e., non-accelerated and non-rotating) frame of reference.

![Fig. 1](Taken from Kant 1786, 546)

In order to explain his third mechanical law Kant refers to a situation in which a uniformly moved body A collides with a resting body B (cf. Fig. 1). Since the collision is supposed to be perfectly inelastic the two bodies move after the collision with a common velocity. According to Kant's preliminary consideration, there is a privileged frame for describing this process, namely, the CM-frame of the colliding
bodies. CM-frames are characterized by the feature that the total momentum of the pertinent system of bodies vanishes. For our two-body system this means that the momenta of the bodies are equal and opposite. This constitutes the first step in Kant’s argument. The velocities are distributed among the two bodies involved so that their momenta coincide (i.e., \( m_A v_{Ac} = m_B v_{Bc} \)). After the impact the two bodies are at rest with respect to the CM-frame (i.e., “absolute space”).

Next Kant introduces another frame of reference, namely, the rest frame of B, i.e., the frame in which B is relatively at rest before the collision. The velocity of this B-frame is supposed to remain unchanged by the collision; that is, after the collision there is still a relative velocity \( v_{Bc} \) between the CM-frame and the B-frame. With respect to the B-frame both bodies thus jointly move with a velocity of the magnitude of \( v_{Bc} \) in the \( x/x’ \)-direction (cf. Fig. 2). Since \( v_{Bc} \) was supposed to designate the velocity in the negative \( x \)-direction, this reversed velocity is called \( v_{Bd} \) (i.e., \( v_{Bc} = -v_{Bd} \)). This implies: The momentum of B after the collision with respect to the B-frame \( (m_B v_{Bd}) \) is equal and opposite to the momentum of B before the collision with respect to the CM-frame \( (m_B v_{Bc}) \). And these momenta are in turn equal in magnitude to the momentum of A before the collision with respect to the CM-frame \( (m_A v_{Ac}) \). This double equality constitutes Kant’s conclusion as expressed in the last sentence of the quote above. These momenta are identified with the “effect,” the “reaction,” and the “action” respectively.

So far, so good; but how is this to be understood? The first question to be asked is: What is to be made of Kant’s reference to the CM-frame? After all, Newton’s third law holds in all inertial frames. And the second question is: Which purpose serves the invocation of the B-frame? From the perspective of Newton’s third law this Kantian maneuver appears utterly mysterious; there is no use for the B-frame. I suggest the following answers to these questions. Kant wanted to prove, first, that it is the momenta (and not the changes in momentum) of the colliding bodies that coincide, and he wished to show, second, that the momentum of the body impacted upon is equal before and after the collision. The demonstration of the first thesis requires reference to the CM-frame, and the proof of the second one demands recourse to the B-frame.

Let me treat the second problem first. Kant’s terminology deserves careful inspection here. The action theorem states the equality between „Wirkung“ and „Gegenwirkung.“ In the conclusion of the argument the „Wirkung“ is identified with the momentum of the body B after the collision from the perspective of the B-frame; and the „Gegenwirkung“ is supposed to refer to the momentum of B before the collision within the CM-frame. In the English translation this terminological parallelism is veiled by the rendering of „Wirkung“ as “effect” and of „Gegenwirkung“ as “reaction.” Accordingly, Kant’s action theorem apparently means in the first place that the scalar momentum of the body acted upon remains unchanged by the collision. It is clear that this only holds true if the body’s motion is described with respect to different frames of reference before and after the collision. And this is the reason for introducing the B-frame.
There is, however, a second aspect involved in Kant's action theorem, namely, the equality of "Wirkung" and "Gegenwirkung" to the Handlung (action), i.e., to the momentum of A in the CM-frame before the collision. This second aspect expresses the fact that the two bodies have equal values of momentum before and after the collision with respect to the same frame of reference. This is what distinguishes the CM-frame. So whereas the first aspect applies to the equality of momentum values of the same body as viewed from different reference frames, the second aspect applies to the equality of momentum values of different bodies as viewed from the same reference frame. Both versions of the action theorem accordingly refer to momenta, not to changes in momentum.\textsuperscript{11}

Kant's comment on this second aspect is noteworthy. In his view it abolishes the kinematical equivalence of uniformly moved reference frames. The claim is that if causal interactions between bodies come into play, the CM-frame is necessarily distinguished (cf. 547.7–26). I take this as meaning that Kant's "phoronomical proposition," which states precisely this kinematical equivalence (cf. 490.8–13) and thus renders the classical principle of relativity, is supposed not to apply to motions generated by the causal interaction of bodies. In the case of force-induced motions a privileged reference frame exists, namely, the CM-frame of the bodies involved. As I will elaborate further in section V, this implies for Kant that in such cases we have empirical access to the "true quantity of motion" (i.e., true linear momentum) and, consequently, to the true velocities of the interacting bodies.\textsuperscript{12}

I admit, however, that there is an ambiguity in Kant's argument that precludes giving conclusive evidence in favor of this interpretation. In Kant's construction all relevant momenta numerically coincide with the corresponding momentum changes. Either a moving body comes to rest or a body at rest is set in motion. Because of this peculiar arrangement, the second aspect of Kant's construction indeed constitutes a special instance of Newton's third law. On the other hand, this construction simply makes no sense if it is regarded as an argument for or as an explanation of that law. On the latter perspective it contains superfluous elements that serve no recognizable purpose. In addition, in summarizing the upshot of his construction,

\textsuperscript{11} It was already noted by Buroker that Kant's third law was intended to apply to linear momenta; cf. Buroker, Space and Incongruence, p. 123.

\textsuperscript{12} If this is in fact Kant's conclusion it constitutes an up-dated and more sophisticated version of C. Wren's views on collisions. Around 1660, Wren introduced the concept of "proper velocity." The ratio of the proper velocities of two colliding bodies equals the reciprocal ratio of their weights (cf. M. Kalmar, Some Collision Theories of the Seventeenth Century: Mathematicism versus Mathematical Physics, Diss. Johns-Hopkins University, 1981, 172–180). This amounts indeed to distinguishing the CM-frame and to interpreting the motions occurring therein as "true velocities." On the other hand, Wren was unaware that he was actually referring to frames of reference rather than to natural properties of the colliding bodies themselves. Accordingly, Kant's use of this feature as an instrument for distinguishing frames of reference and eventually absolute space (see sec. V) is unprecedented by Wren.
Kant refers to the equality of the quantity of motion as the salient feature of the action theorem (cf. 547.30–37).

This discussion supports the following conclusion. Newton’s third law and Kant’s action theorem do not agree with one another. Newton’s law applies to the equality of the temporal changes in momentum and holds in every inertial frame. Kant’s theorem, by contrast, expresses the stronger claims that the values of the involved momenta themselves are left unaltered by the interaction (first aspect) and numerically coincide (second aspect). To these stronger claims correspond restricted classes of reference frames in which they hold true. As regards the second aspect, the class of admissible frames is narrowed down to the CM-frames, and this feature — as I will argue in section V — constitutes an important step in identifying absolute space.

IV. Rotation as Relative Motion

I turn now to Kant’s response to Newton’s physical arguments in favor of absolute space. As explained in section II, Kant adhered to a relationist interpretation of the nature of space, and for this reason he could not accept Newton’s conclusion that there is absolute motion, i.e., motion with respect to space itself and, correspondingly, without respect to any matter. Since Newton’s argument was based on the special dynamical status of rotation, Kant had to come to terms with centrifugal forces or with inertial forces in general. What Kant tried to do, in fact, was to give an account of inertial forces which made them appear as results of relative motions. In this section I reconstruct the particulars of this account.

Kant frankly admitted, first, that rotation is indeed true or actual motion. True motion is characterized by the feature that it is not equivalent to an equal and opposite motion of a reference frame, and the occurrence of this motion is testified by the presence of inertial forces. The problem is to do justice to this fact without being led astray to the Newtonian fallacy of absolute motion. Kant took the rotation of the earth as an example and developed for this case his central argument for a solution to his problem. The rotation of the earth can’t be ascertained directly, but it can be demonstrated by its dynamical effects. Kant mentions the occurrence of Coriolis accelerations (as we call these inertial accelerations today). The occurrence of such inertial effects is to be understood as follows.

But this motion [the earth’s rotation], even though it is no change of relation to empirical space [since it does not ensue in relative motions on the surface of the earth], is nevertheless no absolute motion but a continuous change of the relations of matters to one another, although it is imagined in absolute space and hence is actually only relative motion and, for just this reason alone, is true motion. The fact that this circular motion is true rests upon the

13 Cf. 556.30 – 32, 557.1 – 3, 557.14 – 17. For more on Kant’s concept of true motion see section V.
notion of the reciprocal continuous removal of each part of the earth (outside of the earth’s axis) from every other part that lies at an equal distance from the center at the opposite side of the earth’s diameter. For this [outward] motion is actual in absolute space, in that thereby the loss of the imagined distance, which gravity acting alone would produce in the body, is continuously compensated — and, in fact, without any dynamical repulsive cause (as one can see from the example chosen by Newton at Princ. Ph. N. pag. 10 Edit. 1714*). Therefore, this loss is compensated by actual motion which, however, is determined with respect to the inside of the moved matter (namely its center) and is not referred to the surrounding space.14

In the footnote to this paragraph Kant refers to Newton’s thought experiment of the two rotating globes. From this thought experiment Newton had derived the magnitude and direction of the globes’ velocities in absolute space (see sec. 1). With this experiment Newton shows, as Kant comments,

how the actuality of their [the globes’] motion together with its direction can nonetheless be discovered through experience. I have sought to show this under somewhat altered circumstances by means of the earth moving around its axis as well.15

Kant’s argument clearly refers to the counteraction of gravitational and centrifugal forces. That is, in transferring Newton’s “globes-scenario” to the case of the rotating earth the cohesive force of the cord is supplanted by the gravitational force of the earth. The problem though is to identify the relative motions the argument is intended to unveil.

I suggest the following reconstruction. The rotation of the earth is not absolute motion. That is, it is not motion in Newton’s absolute space and is consequently motion with respect to matter. On the other hand, it is “imagined in absolute space” and that means it is motion in Kant’s embedding absolute space. If one only takes those forces into account that originate from the motion (and consequently disregards every “dynamical repulsive cause,” i.e., the basic repulsive force that is constitutive of matter and that Kant treats in the “dynamics” chapter of the Metaphysical Foundations), then Newton has shown that these forces, namely, centrifugal forces, are brought about by motions. As the presence of these forces indicates, rotation is qua rotation true or actual motion. But these forces in turn generate a tendency to increase the relative distance of opposite parts of the rotating body. This increase is real because it is induced by forces; the centrifugal force thus truly separates or “removes” the parts of a rotating body. This is why rotation is relative motion.

On the other hand, in the case of the earth the repulsive effect of the centrifugal force is outweighed by the attractive effect of gravitation and for this reason the centrifugal relative motion does not actually express itself as apparent motion. This, however, makes it no less true. What matters is that all bodies would be accelerated

14 561.35 – 562.8. The first half of this quote is Ellington’s translation amended; its second half is Friedman’s translation; cf. Friedman, The Metaphysical Foundations of Science, p. 42.
more intensely to the earth's surface than they actually are if the earth's gravitational attraction were not counteracted by this centrifugal force. That is, gravitational attraction induces a "loss of the imagined distance," meaning an imagined loss of distance, among the parts of the earth. This loss is only imagined since it is partly compensated by the outward centrifugal motion. Hence gravitational attraction partly loses its effect through this centrifugal motion, and this is why the earth's rotation is actual motion.

It is helpful for a clearer understanding of Kant's line of argument to observe his distinction between "absolute motion" and "motion in absolute space." The former is motion without reference to any matter and is considered impossible (cf. 559.1 - 3, 24 - 28). The latter, by contrast, is true motion (cf. 562.17), and the whole point of Kant's argument is to show that true motion can be construed as relative motion. This is essentially done by claiming that forces always co-occur with true motion and vice versa. If there is a force present there is at least "virtual" relative motion; i.e., motion that would appear as relative motion if it were not suppressed by other forces.

In this interpretation Kant's argument for the relational nature of rotation runs roughly analogous to that of Huygens (which Kant, to be sure, couldn't be aware of) (see sec. 1).16 Both tried to avoid Newton's inference to absolute space by construing rotation as relative motion of the parts of the rotating body. In addition, both stressed the virtual nature of the relevant motion. Huygens identified the relevant relative motion with the motion of these parts along the circumference; or, more precisely, with the rectilinear motion these parts would assume if the cohesive forces of the spinning body didn't hold them back.17 Kant, by contrast, regarded the pertinent relative motion as the motion of the parts away from the axis of rotation. So Huygens' relative motion is tangential to the circumference while Kant's is perpendicular to it. In both accounts, however, it is a kind of counterfactual relative motion on which the occurrence of centrifugal forces rests.

This interpretation has to come to terms with an observation of Friedman's. Friedman points to the fact that Kant says in the above quoted argument that the rotation of the earth can be "discovered through experience." But what, he asks, would happen to a body resting on the earth's surface if the rotation were absent? And the answer is, of course: precisely nothing. All bodies would simply remain in their places. Accordingly, as Friedman argues, we should impute to Kant the notion of a little satellite orbiting the earth. In that case the centrifugal force would not be overpowered but exactly balanced by gravitation, and the absence of the former would consequently issue in observable effects.18 According to this reconstruction,
the relative motion which Kant considers relevant here is the circular motion of a
body rotating around the earth with respect to the earth's surface.

Noteworthy as it is, this interpretation is beset with two difficulties. The first is
that the textual evidence seems to militate against it. Immediately before Kant sets
out to develop his argument for the relative character of rotation (as quoted above),
he demonstrates its actuality by giving two examples as to how the earth's rotation
could be discovered empirically. These examples envisage the inertial (Coriolis)
forces acting on a body thrown perpendicularly in the air or falling through a hole
toward the center of the earth (cf. 561.21-31). In both examples the body is not
supplied initially with any horizontal motion — as it would have to if it were to
become a satellite. So it seems a bit farfetched to presume that Kant, in the argument
that immediately follows on these examples, had the notion of a satellite in mind.

The second difficulty of the "satellite-interpretation" is that it fails to make sense
of the counterfactual aspect of Kant's treatment. As explained above, Kant feels the
need to stress that circular motion is true although it sometimes does not appear
as motion; and he emphasizes that the increase in distance brought about by the
centrifugal force is actual although it is outweighed by the simultaneous action of
gravity. I take this as meaning that circular motion is actual even in those cases in
which its effects are not immediately observable — though indirectly detectable on
closer scrutiny. It is sufficient for its actuality that it would issue in directly
observable effects if some counteracting influences were absent. On the satellite-
interpretation, however, the circular motion is directly amenable to experience and
there would be no need for Kant to adduce a proviso of counterfactuality. Imputing
to Kant the view, therefore, that it is axial and not horizontal motion on which the
relative character of circular motion rests, makes a greater bulk of Kant's text
coherent.

V. Rotational and Translational Motion as True Relative Motion

In this final section I wish to tie together the threads of the story. That is, I want
to combine the results of sections III and IV so as to show that Kant's treatment of
force-induced rectilinear motion and his account of circular motion or rotation run
to a great extent along parallel lines. In both cases the issue is to identify true
relative motions, and in both cases this is supposed to be achieved by recourse to
the causes or effects of true motion, i.e., forces. This parallism is veiled by a shift
of emphasis in Kant's discussion of the respective cases. In the rectilinear case Kant
placed emphasis on the actuality of motion, whereas in the rotational case he
stressed the relative nature of motion. This shift of emphasis is due to the fact that
the respective complementary aspect (i.e., the actuality of circular motion and the
relative nature of rectilinear motion) is considered rather unproblematic. I now
elaborate the particulars of this interpretation and afterwards try to clarify its
impact on the nature of Kantian absolute space.
Let me begin by rehearsing Kant's definition of true motion. True motion is defined by the feature that it cannot be described with respect to arbitrary reference frames. The empirical indicator of true motion is that it is accompanied by a force (i.e., such motion either brings about or is brought about by a force). If these criteria are applied to *circular motion* it appears that it qualifies as true motion since centripetal forces on a body cannot be generated by simply rotating that body's reference frame (cf. 556.30–557.17). Accordingly, non-rotating frames are privileged for the description of this kind of motion. In addition, circular motion is relative motion as the argument expounded in section IV was supposed to show.

As regards *rectilinear motion*, there is a privileged reference frame as well, provided that the rectilinearly moved body interacts with some other body. As the analysis in section III was intended to make plausible, Kant regarded the CM-frame of the involved bodies as privileged. What is important to notice is that Kant conceived this kind of rectilinear motion to be actual or true in the same sense as circular motion. He explicitly attributed the predicates "actual" and "true" to it (558.15, 562.9–10) and applied his definitional criterion of true motions to them; it makes a difference here whether the motion is predicated of the body or of the frame of reference (cf. 547.14–19). The reason is again that in the cases under consideration the rectilinear motion is generated by forces so that recourse to the causes of motion is suitable for identifying a privileged frame of reference (cf. 558.12–15). In this vein, the action-theorem is supposed to single out true rectilinear motion (see sec. III). Moreover, this true motion is relative motion as well. After all, the CM-frame is constructed from the relative motions of the involved bodies (cf. 562.13–24).

So there is a conspicuous parallelism between Kant's treatment of rectilinear and circular motion. In both cases true motion can be identified. It becomes clear now why Kant did not follow Newton in relying on the dynamical special status of rotation for introducing absolute space. Kant seeks to ground absolute space on the consideration of rectilinear motion as well. Accordingly, his procedure for introducing absolute space is so broadly conceived that it applies likewise to circular and rectilinear motion (see sec. II).

The next step is to determine what "motion" is supposed to mean here. I claim that "motion" means "velocity." As I took pains to argue in section III, Kant's action theorem is intended to refer to linear momenta and, consequently, to velocities. As regards circular motion, Kant refers to Newton's rotating-globes experiment from which the latter had calculated magnitude and direction of the globes' velocities. Kant summarizes this Newtonian result by saying that Newton had derived the globes' "motion together with its direction" (see p. 410). So in this case "motion" evidently means "magnitude of velocity." Moreover, Kant explicitly clarifies his general usage of the term "motion." As he says, motion is determined or "constructed" by giving the magnitude and direction of velocity (cf. 487.5–10). Although there are thus slight uncertainties in Kant's usage in that "motion" is to refer sometimes to the scalar quantity of velocity and sometimes to its vectorial quantity,
the term is always applied to velocities and not to accelerations. So we are on the safe side by concluding that Kant uses “motion” roughly equivalent to “velocity.” In view of this rendering, Kant’s account of true motion can be summarized so that in his view the classical principle of relativity does not apply to true motion. This principle is confined to kinematics and it fails if forces are present.19

This interpretation is further borne out by the way in which Newton conceived of centrifugal forces. Newton thought that in orbital motion the centrifugal force is caused by gravity; the former is just the reaction to the latter. The invocation of the third law shows, then, that both are equal and opposite. In other cases, such as a rotating body, things are different. Here centrifugal force and gravity are considered to be of a different nature.20 This Newtonian view was taken up by Keill21 whose work Kant was familiar with.

What is noteworthy here is that this Newtonian reasoning established a link between the third law and centrifugal forces that in turn indicate true motions. And if Kant gained a similar impression through the study of Keill’s writings, it is natural to impute to him the conclusion that singling out true motions on the basis of the third law might also succeed in cases other than orbital motion. Apparently, he believed that he could extend Newton’s recipe for determining the magnitude of orbital velocities to all cases in which in the third law is applicable. And by misreading Newton’s third law Kant purportedly managed to bear out this claim.

Accordingly, Kant thought that Newtonian physics implied the existence of privileged reference frames, namely, non-rotating CM-frames. These frames were called “absolute space” by him (cf. 545.23–26, 545.34–37, 546.10–16, 561.38, 562.2–5). The problem that surfaces at this juncture is that different systems of bodies have different CM-frames. But a multiplicity of absolute spaces obviously makes no sense.

In order to solve that problem Kant invoked his embedding procedure (as sketched in sec. II). This procedure is to be applied to CM-frames, i.e., one CM-frame is embedded in a more comprehensive one and so forth. With this move we are better off than before, to be sure, but the situation is still bad enough. For there is still a whole bundle of options for embedding CM-frames. Absolute space, however, is supposed to furnish a “determinate concept of experience” (560.6); and this requires that there is an unambiguous procedure for singling out absolute space.

Fortunately enough, there is a Kantian principle at hand that allows for a solution to this difficulty, namely, Kant’s third analogy of experience. According to this principle, all simultaneously existing things interact with one another.22 Everything

19 This view, incidentally, is perfectly compatible with the law of inertia that Kant clearly understood (cf. 543.17–20, 550.27–39). Forces tend to bring about changes in the true velocity.


interacts with everything; strictly speaking, there are no isolated systems. This implies that the example of a two-body interaction which Kant used to explain his action theorem (see sec. III) is unrealistic. The consistent application of the action theorem requires instead to take all involved bodies into account, and in view of the third analogy this comprehensive system can be nothing short of the whole universe. This leads immediately to the conclusion that the sought-for overarching frame of reference is the center of mass of the universe. This frame constitutes absolute space.

So it is clear now where the embedding procedure has to end up eventually, and for this reason it can be performed unambiguously. On the other hand, the center of mass of the universe cannot actually be identified by means of any procedure accessible to human beings. It remains always beyond our empirical reach. Therefore, absolute space has the status of an idea of reason (as explained in sec. II).23

There is a terminological peculiarity in Kant that should be briefly mentioned here so as to avoid confusion. As explained above, Kant applies the term "absolute space" to the CM-frame of the bodies whose motions he analyzes. On the other hand, this CM-frame is clearly accessible to experience; it makes no sense to call it an idea of reason. So we have to distinguish between the CM-frame of the bodies involved in a particular interaction and the CM-frame of all existing bodies. The former frames constitute the individual steps of the embedding procedure and the latter frame constitutes its end-point. And what appears misleading is that Kant designates both types of frames by the term "absolute space." It is clear, however, that in light of the third analogy only the latter frame rightly deserves that expression. Application of the action theorem to the former is strictly speaking illicit and is to be understood as a provisional and preliminary consideration that is set straight by the following, more refined argumentation.

We are left with the task of clarifying which properties Kant attributes to absolute space. In fact, there is only one essential property, namely, true rest. As he puts it (following Newton), absolute space is "immovable."24 Kant's reasoning is to be reconstructed as follows. What we have now is a definite concept of true motion; it is motion with respect to the CM-frame of the universe. Motion is to be understood as velocity, and this implies that we have a definite concept of true rest as well. It is rest with respect to the CM-frame of the universe. This entails that this CM-frame is truly at rest; after all, it is at rest with respect to itself. This reasoning, I suggest, underlies Kant's claim that the action theorem entails the rest of the universe;

23 The suggestion to identify Kant's absolute space with the center of mass of the universe is due to Friedman; cf. Friedman, The Metaphysical Foundations of Science, p. 35, 41; Friedman, Kant on Space, p. 240 – 241. The foregoing discussion contains arguments to further buttress this suggestion.

24 559.2; cf. Newton, Mathematical Principles, p. 6.
the action theorem rules out that the center of mass of the universe is in rectilinear motion.25

With this result Kant thought he had arrived at a physical confirmation of an epistemological postulate. Epistemological considerations make it clear that only motion with respect to other bodies (i.e., relative motion) can become an object of experience. Rectilinear motion of the universe as a whole would not be motion with respect to other bodies and is therefore impossible for epistemological reasons (cf. 481.14–22, 562.24–563.2; see also sec. II). The point is that precisely this tenet is a nomological consequence of Newton’s theory; or at least that is what Kant thought he had demonstrated. That is, Kant believed he had unearthed a consequence of Newton’s theory that had gone unnoticed by its author. After all, Newton had only framed the “Hypothesis I: That the centre of the system of the world is immovable.”26 Kant, by contrast, thought on the basis of his misconstrual of Newton’s third law that this statement is not an independent hypothesis but a theorem of Newtonian mechanics. It can be shown that the CM-frame of the universe is as immovable as absolute space so that both can plausibly be identified with one another. Epistemology and physics thus converge to jointly support the claim that the cosmic system does not move.

So, all in all we have an inference from mechanical laws, or what Kant took to be as such, to the existence of absolute space. These mechanical laws refer to the relative motions of bodies, as Kant was at pains to argue. The combination of both items constitutes Kant’s relational theory of absolute space.

25 Cf. 562.29–563.5. On the other hand, Kant does explicitly not preclude an overall rotation of the universe (cf. 563.6–9). This appears puzzling at first sight since the embedding procedure should equally work for rotating frames as for linearly moved ones; both procedures are likewise guided by forces. The point is, however, that the absence of rotation of the endpoint of this procedure cannot be rigorously established. Rotation is relative motion of the parts of the rotating body (see sec. IV). Accordingly, in contradistinction to translational motion, rotation is possible without reference to some external body. For this reason the universe as a whole may rotate.
26 Newton, Mathematical Principles, p. 419.